Title
Forecasting Inflation in Real Time

Author
Jia, Mingyuan

Publication Date
2019

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA
RIVERSIDE

Forecasting Inflation in Real Time

A Dissertation submitted in partial satisfaction
of the requirements for the degree of

Doctor of Philosophy

in

Economics

by

Mingyuan Jia

September 2019

Dissertation Committee:

Dr. Marcelle Chauvet, Chairperson
Dr. Aman Ullah
Dr. Dongwon Lee

The Dissertation of Mingyuan Jia is approved:

Committee Chairperson

University of California, Riverside

Acknowledgments

The past five years has been the most important and fruitful period in my life. It gets me ready to be an intellectually adequate economist. I would like to take this opportunity to express my sincere gratitude to many people for their contributions to my graduate study and research. First, I would like to thank my advisor, Professor Marcelle Chauvet, without whom this thesis would not have been possible. She not only taught me how good empirical macroeconomic research is done, but also guide me through practical issues in policy making. I appreciate all her contributions of time and ideas to make my Ph.D. experience productive and stimulating. I am grateful to the rest of my committee members, Professor Aman Ullah and Professor Dongwon Lee, for their great support and invaluable advice. Professor Lee provided me extensive personal and professional guidance and taught me a great deal about economic research, job market and academic life in general. I extend my thanks to the cohort of 2014 econ students. They are supportive in both my life and research. They are Yun Luo, Jianghao Chu, Shiyun Zhang, Hien Nguyen, Yi Mao, Taghi Farzad, Hanbyul Ryu, Deepshikha Batheja, Yoon Ro, and all the alumni who have helped me both in research and daily life.

ABSTRACT OF THE DISSERTATION

Forecasting Inflation in Real Time

by

Mingyuan Jia

Doctor of Philosophy, Graduate Program in Economics
University of California, Riverside, September 2019
Dr. Marcelle Chauvet, Chairperson

This dissertation is intended to model the dynamics of inflation and forecast short-run and long-run inflation using high frequency data. It first proposes a mixed-frequency unobserved component model in which the common permanent and transitory inflation components have time-varying stochastic volatilities. The key aspects of the model are its flexibility to describe the changing inflation over time, and its ability to represent distinct time series properties across price indices at mixed frequencies. More importantly, the model is applied to builds short-run and long-run coincident indicators of US inflation at the weekly frequency. The dynamics of the latent inflation factor shows that the persistence of US inflation has reduced since 1990s due to different components over time. Next, it proposes a nowcasting model for headline and core inflation of US CPI. The final selected variables include daily energy price, commodity price, dollar index, weekly gas price, money stock and monthly survey index. The model's nowcasting accuracy improves as information accumulates over the course of a month, and it easily outperforms a variety

of statistical benchmarks. Moreover, it uses a Nelson-Siegel Dynamic Factor model to fit the monthly term structure of inflation expectation and describes its dynamics over time. The extracted inflation factors correspond to the level, slope and curvature of the term structure of inflation expectation. It shows that a decomposition of the yield curve spread into its expectation and risk premia components helps disentangle the channels that connect fluctuations in Treasury rates and the future state of the economy. In particular, a change in the yield curve slope due to expected real interest path and inflation expectation path, is associated with future industrial production growth and probability of recession.

This dissertation adds to the literature by building a mixed-frequency model that can track inflation in real time and produce better nowcasting results than the existing method, by fitting the inflation expectation with a dynamic factor model that can describe the dynamics of the whole term structure and by proving the usefulness of both inflation expectation slope and real yield spread in predicting future economic activity.

Contents

List of Figures

List of Tables

Chapter 1

Real-Time Indicator of Weekly Inflation with A Mixed-Frequency Unobserved Component Model with Stochastic Volatility

Preview of Chapter 1

This chapter builds short-run and long-run coincident indicators of inflation at the weekly frequency. The author proposes a mixed-frequency unobserved component model in which the common permanent and transitory inflation components have time-varying stochastic volatility (MF-UCSV model). The key aspects of the model are its flexibility to describe the changing inflation over time, and its ability to represent distinct time series properties across price indices at mixed frequencies. The model is estimated using Bayesian Gibbs Sampler and data on weekly commodity inflation, monthly consumer inflation, expenditures inflation, and quarterly GDP deflator inflation. The empirical results show that the constructed weekly inflation indicator closely matches monthly consumer and expenditure inflation. Additionally, the paper proposes and estimates a measure of high frequency trend inflation, which are in line with survey forecast and core inflation, and provides alternative to existing trend measures. We also study the changing persistence of inflation, and find that although it has reduced since the 1990s, it was due to different components over time. Interestingly, we also find that inflation volatility increased during the Great Recession, but this did not change the mean-reversion property of inflation. Overall, the model provides a strategy for real-time multivariate tracking and nowcasting of inflation at the weekly frequency, as new data are released.

1.1 Introduction

Inflation is one of the most watched economic series by policymakers and the public in general. Monetary policymakers continuously monitor inflation releases to update their expectation about future economic conditions and to control price stability. Market practitioners also rely on inflation reports in forming expectations when negotiating long-term nominal commitments. There is a growing recent literature on rich data environment (large datasets) and mixed frequency framework that has yielded major advances in assessing real economic activity, nowcasting, and forecasting output. However, this method has not been extensively applied to study inflation dynamics. Building an inflation indicator based on a set of variables is challenging given the important time-varying properties of inflation.[1] This is especially the case across price indices at different frequencies. Aruoba and Diebold (2010) construct a real-time monthly inflation indicator with the same framework used in Aruoba, Diebold and Scotti's (ADS, 2009) business condition indicator. However, the model does not provide a characterization of changing inflation dynamics, in particular, the evolving local mean and time-varying volatility.

This chapter proposes a framework with underlying trend and cycle components representing long- and short-run inflation dynamics, which are used to construct high frequency coincident indicators of inflation. The proposed model encompasses price measures sampled at different frequencies, including weekly, monthly and quarterly price indices. The output is estimated weekly inflation indicators, which depict historical inflation trend and cycle, and that can be used to assess current inflation in real-time. The possibility of a high frequency inflation indicator providing more timely measurement than the official publication is very appealing. Official inflation measures can only be observed at the monthly frequency and with publication lags. For example, U.S. CPI is announced at the middle of the month for measures of inflation for the month prior.

The increasing availability of data at higher frequency has sparked interest in mixed

[1] see e.g. Cogley and Sargent (2005), Stock and Watson (2007), etc.

frequency models. On one hand, mixed-frequency factor model and MIDAS (mixed-data sampling) model have become important tools for nowcasting and forecasting, using daily or weekly time series. Monteforte and Moretti (2013), Modugno (2013), Breitung and Roling (2014) examine the predictability of commodity prices and asset prices with this framework. On the other hand, researchers from other fields such as computer science and statistics have advanced methods to study high frequency data. With the recent large data set collected by electronic commerce system, such as Amazon, Walmart in the U.S. or Alibaba in China, among many others, daily price information is extracted and aggregated in few minutes using web crawler technology.[2] These measures are gradually accepted by private agents to complement the official inflation publications. However, the existing methods (e.g. machine learning) are designed to predict but not to obtain inferences regarding time series dynamics. In particular, the data collecting and filtering approaches that are used by high frequency price indices (e.g. online price index and commodity price index) are distinct from those that are used by official statistical agencies, and a formal statistical treatment of inflation dynamics at high frequency is still lacking.

Our approach involves formal modeling of inflation dynamics characterizing its trend, cycle, and volatility, while allowing for mixed-frequency. In particular, this paper proposes a mixed frequency small-scale unobserved component factor model with stochastic

[2]For e.g. The Billion Prices Project (BPP) operated by MIT Sloan and Harvard Business School use big data to estimate dynamics in prices and implications for economic theory. This project uses prices collected from hundreds of online retailers on a daily basis to build inflation index and already has been applied to measure Argentina's inflation. In China, the companies Alibaba and Tsinghua University are collaborating to publish a daily internet-based consumer price index (icpi). This project not only provides the aggregate price index, but also price indices in sub-categories.

volatility - the MF-UCSV model. In the proposed model, underlying inflation process is approximated as a sum of unobserved common permanent and transitory factors. The permanent component captures long-run trend inflation, while the transitory component captures short-run deviations of inflation from its trend value. Additionally, the variances of the permanent and transitory disturbances are allowed to evolve over time according to a stochastic volatility process. Thus, the persistence of the inflation process is summarized by the relative importance of the variability of permanent and transitory components. The key aspects of the model are its flexibility to describe the changing inflation over time, and its ability to represent distinct time series properties across price indices at mixed frequencies. Price measures differ in terms of data collecting process, categories, utilization, but are highly correlated and may be driven by a set of common latent factors. The proposed flexible model allows for the potential distinct dynamics of underlying inflation, and also extracts common trend and common cyclical movements across the series. The underlying inflation indicators are extracted from a set of weekly, monthly, and quarterly price indices. The results indicate that the weekly inflation index tracks historical inflation dynamics well in the sense that it successfully identifies important inflation cycle and trend phases, including their severity and duration. Additionally, the estimated inflation indicator closely matches monthly consumer and expenditures inflation at the weekly frequency in real-time. Overall, the model provides a strategy for real-time multivariate tracking and nowcasting of inflation as new data are released. In particular, the real-time trend inflation estimates are in line with survey forecast and core inflation, which provide alternative to existing trend measures.

This chapter has several contributions to the literature. To our knowledge, this is the first one that builds high frequency short-run and long-run U.S. inflation coincident indicators that can be updated in real-time. Previous works do not use high frequency data or do not use them to build coincident indicators. Aruoba and Diebold (2010) build a U.S. inflation indicator but based on monthly and quarterly series. Similar to Aruoba and Diebold (2010), Modugno (2013) uses a dynamic factor model with three factors corresponding to weekly, monthly and quarterly series to forecast U.S. Consumer Price Index and Harmonised Index of Consumer Price for the Euro Area. Monteforte and Moretti (2013) use MIDAS regression framework with daily data to forecast inflation. However, these papers do not construct a short-run and long-run coincident indicators of inflation as proposed here.

Second, the MF-UCSV model takes into account potential nonstationarity in inflation dynamics. Many researchers suggest models that take account of slow-varying local mean for inflation perform reasonably well in forecasting inflation. Atkeson and Ohanian (2001) show that forecasts from simple random walk model cannot be statistically beaten by alternatives. Following their work, Stock and Watson (2007, 2016) proposed characterization of quarterly rate of inflation as an unobserved component model with stochastic volatility. Faust and Wright (2013) compare various inflation forecasts and find that models based on stationary specifications for inflation do consistently worse than non-stationary models. However, the related literature that focuses on extracting inflation indicator (Diebold and Aruoba, 2010) or nowcasting (Giannone, et al. 2006) with mixed frequency framework are all based on stationary specifications for inflation. This raises the question on whether the consideration of nonstationarity in these frameworks could also improve characterization of the dynamic properties of latent inflation. For example, the temporal aggregation of latent autoregressive factor is also autoregressive, while the GDP deflator inflation is better approximated as integrated moving average. In our framework, the factor loading along with changing volatilities can solve this problem by providing appropriate approximation for each series. Our mixed frequency MF-UCSV model takes into account nonstationarity and yields an estimated trend inflation at high frequency. Trend inflation is an important tool for monetary policy as it conveys information on long run inflation expectations.

Finally, filtering out the noise in multiple inflation measures has not been done in the mixed frequency literature. Generally, there are two approaches in the literature to approximate trend inflation. Clark (2011), Faust and Wright (2013), among others, use measures of long-run inflation expectations from surveys forecasts (either Survey of Professional Forecasters or Blue Chip) to capture trend inflation. Survey-based trend leads to an improvement in the accuracy of model-based forecasts (see, e.g., Ang, et al. 2007). However, surveys of inflation expectation can not be replicated as it is a result of a combination of many objective (models) and subjective information. Alternatively, a range of studies has modeled trend inflation as an unobserved component (e.g. Stock and Watson 2007, Cogley and Sbordone 2008, or Mertens 2011). In this chapter, we follow this approach, using time series smoothing methods to extract trend inflation, which is additionally obtained from a multivariate framework. The use of factor model mitigates the problem of estimating weights separately and downweighting sectors that may have large variations over time.

This chapter is organized as follows. The model is described in section 2, along with the estimation method, which uses the Gibbs Sampler used for simulating the posterior distribution of the parameters. The third section presents and interprets the empirical results. The conclusion are discussed in the fourth section.

1.2 The Model

1.2.1 The Underlying Inflation Process

Following Stock and Watson (2007), inflation is characterized by an unobserved component model with stochastic volatility. We assume that the underlying inflation process evolves daily. This assumption can be adjusted to other frequencies, like weekly or monthly.

Let π_t denote the underlying inflation at day t, which evolves following a stochastic process:

$$\pi_t = \tau_t + \eta_t \tag{1.1}$$

where τ_t represents the permanent component of underlying inflation and η_t represents the transitory component. Permanent component takes the form of a simple random walk by equation (2):

$$\tau_t = \tau_{t-1} + \sigma_{\tau,t}\varepsilon_{\tau,t} \tag{1.2}$$

and transitory component has a finite order AR(p) representation:

$$\Phi(L)\eta_t = \sigma_{\eta,t}\varepsilon_{\eta,t} \tag{1.3}$$

where function $\Phi(L)$ is a finite lag polynomial with order p, and has all the roots outside the unit cycle, $\varepsilon_{\tau,t}$ and $\varepsilon_{\eta,t}$ are mutually independent $i.i.d.$ $N(0,1)$ stochastic processes. $\sigma_{\tau,t}$ and $\sigma_{\eta,t}$ represent the variability of innovations to permanent component and transitory component. They together determine the relative importance of random walk disturbance. To model the changing volatility of inflation components, it is assumed that their log-volatility follows a random walk with no drift,

$$ln(\sigma^2_{\tau,t}) = ln(\sigma^2_{\tau,t-1}) + \nu_{\tau,t} \tag{1.4}$$

$$ln(\sigma^2_{\eta,t}) = ln(\sigma^2_{\eta,t-1}) + \nu_{\eta,t} \tag{1.5}$$

where $\nu_{\tau,t} \sim N(0,\sigma^2_{\nu\tau})$ and $\nu_{\tau,t} \sim N(0,\sigma^2_{\nu\eta})$. The magnitudes of time variation in $\sigma_{\tau,t}$ and $\sigma_{\eta,t}$ depend on the variances of $\nu_{\tau,t}$ and $\nu_{\eta,t}$. In particular, a large $\sigma^2_{\nu\tau}$ means the variability of trend components can undergo large period changes, which affect the inflation persistence indirectly.

1.2.2 Unobserved Component Model

A vector of price measures and other variables displaying comovement is modeled to depend on the latent permanent and transitory inflation factors. The daily economic variable is a linear combination of daily common permanent and transitory components. Let y_t^i denote the ith daily price measures at day t and we have below the relationship:

$$y_t^i = \beta_i \tau_t + \gamma_i \eta_t + u_t^i \tag{1.6}$$

where $u_t^i \sim N(0, \sigma_{u_i}^2)$ are contemporaneously and serially uncorrelated innovations that capture idiosyncratic shocks to the specific price measures. β_i and γ_i are the factor loadings on the common permanent and transitory components.

In the mixed frequency framework, the relationship between the observed data and daily variables need to be specified. Most of the economic variables are observed at lower frequency, for example, CPI inflation and GDP inflation are monthly and quarterly measures respectively. Inflation measures growth rate of price level, then relationship between observed inflation series and underlying daily variables depends on the temporal aggregation of price index. Here we approximate the price index observed at low frequency as the systematic sampling of the daily variables, i.e. end of period value. Thus, the inflation measures can be processed as flow variable.[3] Our approximation method is different from the commonly used method for approximating GDP growth rate in Mariano

[3] A comprehensive description of temporal aggregation of flow and stock variables can be seen in Aruoba, et al (2008).

and Murasawa (2003). Their method is doable but will complicate our model in high frequency, by making the state variable extremely large and computation unattainable. In contrast, our approximation can cast the model into a linear form. Let $\tilde{y}_t^i$ denote the ith observed flow variable in low frequency. Then $\tilde{y}_t^i$ is the intra-period sums of the corresponding daily values,

$$
\begin{aligned}
\tilde{y}_t^i &= \begin{cases} \sum_{j=0}^{D_i-1} y_{t-j}^i & if\, y_t^i\, can\, be\, observed \\ NA & otherwise \end{cases} \qquad (1.7) \\
&= \begin{cases} \beta_i \sum_{j=0}^{D_i-1} \tau_{t-j} + \gamma_i \sum_{j=0}^{D_i-1} \eta_{t-j} + u_t^{*i} & if\, y_t\, can\, be\, observed \\ NA & otherwise \end{cases}
\end{aligned}
$$

where D_i is the number of days per observational period. For example, D_i for monthly CPI of January equals 31. u_t^{*i} adds up the daily white noise disturbances and thus follows $MA(D_i - 1)$ process. Here we can appropriately treat u_t^{*i} as white noise following Aruoba et al. (2009).

Following Harvey(1990), we apply the accumulator variables to handle temporal aggregation. This could greatly reduce the state of the system. Let $C_{\tau,t}$ and $C_{\eta,t}$ denote the permanent and transitory component accumulator:

$$\begin{aligned} C_{\tau,t} &= \theta_t C_{\tau,t-1} + \tau_t \qquad (1.8) \\ &= \theta_t C_{\tau,t-1} + \tau_{t-1} + \sigma_{\tau,t}\varepsilon_{\tau,t} \end{aligned}$$

$$\begin{aligned} C_{\eta,t} &= \theta_t C_{\eta,t-1} + \eta_t \qquad (1.9) \\ &= \theta_t C_{\eta,t-1} + \Phi(L)\eta_{t-1} + \sigma_{\eta,t}\varepsilon_{\eta,t} \end{aligned}$$

where θ_t is an indicator variable which is defined as:

$$\theta_t = \begin{cases} 0 & If\, t\, is\, the\, first\, day\, of\, the\, period \\ 1 & otherwise \end{cases}$$

Then equation (7) can be written as:

$$\tilde{y}_t^i = \begin{cases} \beta_i C_{\tau,t}^i + \gamma_i C_{\eta,t}^i + u_t^{*i} & if\ y_t\ can\, be\, observed \\ NA & otherwise \end{cases}$$

1.2.3 State-Space Form

A more compact state-space representation of the MF-UCSV model is the following:

$$Y_t = C_t' \alpha_t + w_t \tag{1.10}$$

$$\alpha_{t+1} = A_t \alpha_t + R_t v_t \tag{1.11}$$

$$\Lambda_{t+1} = \Lambda_t + \zeta_t \tag{1.12}$$

$$w_t \sim N(0, H), \quad v_t \sim N(0, Q_t) \tag{1.13}$$

$$\zeta_t \sim N(0, W) \tag{1.14}$$

where Y_t is an $N \times 1$ vector of observed variables with missing values. State vector α_t includes 8 state variables, w_t and v_t are Gaussian and orthogonal measurement and transitory shocks. The time-varying variance matrix Q_t is a diagonal matrix with elements of $\sigma^2_{\tau,t}$ and $\sigma^2_{\tau,t}$. Λ_t is the vector of unobserved log-volatilities, and W is a diagonal matrix containing the variance of log-volatility disturbances.

There are two special cases nested in our model. First, the changing volatility crucially depend on the covariance matrix W. When we set $W = 0$, Λ_t is constant, then we return to the normal mixed-frequency dynamic factor model. In this case, Kalman filter and smoother can be used to extract the state variables and the corresponding state disturbances. The algorithm is classical Kalman filter in the textbook. Second, instead of shutting off the stochastic volatility, we may assume $\sigma^2_{\tau,t} = \sigma^2_{\eta,t}$ and reduce the dimension of W to unity. This is the case of common stochastic volatility. Koopman (2004) propose a method using importance sampling and Kalman filter to estimate the model. These two models can be estimated by maximizing the log likelihood function. For our model with great flexibility in setting the conditional variance, MLE is not feasible.

1.2.4 Estimation

We use Bayesian MCMC method with Metropolis-within-Gibbs sampler to estimate our model. The estimation procedure, model identification, and priors will be described briefly, and more details can be obtained in the appendix.

Sampling of the parameters, including latent factors and volatilities, can be proceeded in several steps. First, since the model can be cast into state-space form, the unobserved state variables τ_t and η_t can be easily drawn using Kalman smoother (Koopman and Durbin, 2003). Second, conditioning on α_t and Y_t, elements in C_t' and H can be drawn row by row in equation (10). Taking the ith measurement equation $\tilde{y}_t^i = \beta_i C_{\tau,t}^i + \gamma_i C_{\eta,t}^i + u_t^{*i}$, we can draw the β_i, γ_i and variance of u_t^{*i} following the conventional method for linear model. Third, equation (11) can be broken down to equation (3), which is AR(p) model with heteroscedastic disturbance. Dividing by $\sigma_{\eta,t}$, one can obtain a standard linear regression model and draw the AR coefficients from the conjugate normal distribution. Forth, we use Jacquier, Polson, and Rossi (1994)'s algorithm and Kim, Shephard and Chib (1998)'s Metropolis rejection method to draw the stochastic volatilities, which are the unobserved components in equation (12). Fifth, conditional on the log-volatilites, $\sigma_{\nu\tau}^2$ and $\sigma_{\nu\eta}^2$ in covariance matrix W can be drawn from conjugate inverse gamma distribution.

1.3 Empirical Application

1.3.1 Data

The empirical application uses weekly GSCI commodity price index, monthly CPI-all items, monthly personal consumption expenditure deflator, and quarterly GDP deflator. The inflation measures are observations on 100 times first difference of the logarithm of each price indices. The sample ranges from 1970/02/01 through 2016/12/31. The extracted inflation indicator can be updated weekly, by including the high frequency commodity price index GSCI inflation (Goldman Sachs Commodity Index). GSCI index is a weighted future prices that almost covering all the sectors of commodities. It is published by Standard and Poor's and recognized as a leading measure of general price movements in the global economy. In this paper, the daily GSCI is averaged to build our weekly GSCI index. Similar high frequency indices include daily CRB index (Commodity Research Bureau Index) which is calculated by Commodity Research Bureau, World Market Price of Raw Materials (RMP) produced by OCED and other energy prices. The commodity price index is obtained from Global Financial Data, and all other price measures are from FRED Economic Data.

We choose the data set for the following reasons. First, a small-scale factor model is sufficient to achieve our goal and illustrate the implementation of our model. Second, we only use data up to weekly frequency since daily observations are far too noisy. Third, the indicators are all price measures that assess the change of inflation from different aspects. The choice of the variable set can also be extended beyond, for example, asset prices,

Table 1.1: Augmented Dicky-Fuller Unit Root Test

	1970 to 1983	1984 to 2016	1970 to 2016
GSCI inflation	-24.13	-42.021	-48.727
CPI inflation	-1.861	-4.823	-1.914
PCE infaltion	-1.578	-3.06	-1.722
GDP Deflator inflation	-2.57	-3.816	-2.23

Note: The ADF test includes a constant. The number of lags is chosen based on SIC criteria.

monetary base and survey data. These variables have some predicative power for future rate of inflation, thus sometimes are used in the literature. However, correlations between those variables and inflation are weak, which may disturb our signal extraction. So we exclude them in our estimation.

Examination of our data indicates our model is an appropriate approximation to different inflation measures. First, we use the Augmented Dickey–Fuller (ADF) test to examine the stationarity of the series. The test is done for three sample ranges: 1970 to 1983, 1985 to 2016 and the full sample period 1970 to 2016. The first sample period corresponds to the Great Inflation, while the second sub-sample includes Inflation Stabilization period when both the level of inflation and the volatility declined dramatically. The ADF test in Table 1.1 suggests a unit root in pre-1984 period and the full sample period for low frequency inflation measures (monthly and quarterly). However, the null hypothesis of unit root in the post-1984 period is rejected. This may suggest that innovations of transitory component tend to play a greater role in the inflation process.

Table 1.2: Autocorrelations of the First Difference of Inflation

	1970 to 1983			1984 to 2016			1970 to 2016		
lags	1	2	3	1	2	3	1	2	3
GSCI inflation	-0.513	0.008	0.032	-0.476	-0.006	-0.011	-0.523	0.013	0.042
CPI inflation	-0.461	0.12	-0.111	-0.158	-0.294	-0.080	-0.267	-0.153	-0.085
PCE inflation	-0.336	-0.048	-0.099	-0.233	-0.226	-0.059	-0.264	-0.175	-0.068
GDP inflation	-0.219	-0.103	-0.031	-0.404	-0.066	-0.009	-0.303	-0.074	-0.029

Second, equation (1) to (3) imply that the first-order autocorrelation is negative for the first difference of inflation. Table 1.2 presents estimated autocorrelation for the change in inflation over three sample periods. The first-order autocorrelation is negative for each of the measures in all sample periods. For GDP inflation, $\Delta\pi_t$ is negatively correlated, with the first autocorrelation much larger in absolute magnitude in the second period than the first.

1.3.2 Model Implementation

We assume that the transitory component follows AR(1) process. Modeling the persistence with AR(1) process would be inadequate, high-order dynamics nevertheless is not statistically better, as the transitory shock would decay too quickly when we assume the latent factors evolve daily.

The equations applied to the data are

$$\begin{bmatrix} \tilde{y}_t^{GSCI} \\ \tilde{y}_t^{CPI} \\ \tilde{y}_t^{PCE} \\ \tilde{y}_t^{GDPD} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \beta_1 & \gamma_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_2 & \gamma_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_3 & \gamma_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_4 & \gamma_4 \end{bmatrix} \begin{bmatrix} \tau_t \\ \eta_t \\ C_{\tau,t}^{W} \\ C_{\eta,t}^{W} \\ C_{\tau,t}^{M} \\ C_{\eta,t}^{M} \\ C_{\tau,t}^{Q} \\ C_{\eta,t}^{Q} \end{bmatrix} + \begin{bmatrix} u_t^{GSCI} \\ u_t^{CPI} \\ u_t^{PCE} \\ u_t^{GDPD} \end{bmatrix}, \tag{1.15}$$

$$\begin{bmatrix} \tau_t \\ \eta_t \\ C_{\tau,t}^{W} \\ C_{\eta,t}^{W} \\ C_{\tau,t}^{M} \\ C_{\eta,t}^{M} \\ C_{\tau,t}^{Q} \\ C_{\eta,t}^{Q} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & \theta_t^W & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi & 0 & \theta_t^W & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & \theta_t^M & 0 & 0 & 0 \\ 0 & \phi & 0 & 0 & 0 & \theta_t^M & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & \theta_t^Q & \\ 0 & \phi & 0 & 0 & 0 & 0 & & \theta_t^Q \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ \eta_{t-1} \\ C_{\tau,t-1}^{W} \\ C_{\eta,t-1}^{W} \\ C_{\tau,t-1}^{M} \\ C_{\eta,t-1}^{M} \\ C_{\tau,t-1}^{Q} \\ C_{\eta,t-1}^{Q} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_{\tau,t} \\ v_{\eta,t} \end{bmatrix}, \tag{1.16}$$

$$\begin{bmatrix} u_t^{GSCI} \\ u_t^{CPI} \\ u_t^{PCE} \\ u_t^{GDPD} \end{bmatrix} \sim N(0_{4\times 1}, H), \quad \begin{bmatrix} v_{\tau,t} \\ v_{\eta,t} \end{bmatrix} \sim N(0,\ Q_t),$$

$$H = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix}, \tag{1.17}$$

$$Q_t = \begin{bmatrix} \sigma_{\tau,t}^2 & 0 \\ 0 & \sigma_{\eta,t}^2 \end{bmatrix}, \tag{1.18}$$

$$\begin{bmatrix} ln(\sigma_{\tau,t}^2) & 0 \\ 0 & ln(\sigma_{\eta,t}^2) \end{bmatrix} = \begin{bmatrix} ln(\sigma_{\tau,t-1}^2) & 0 \\ 0 & ln(\sigma_{\eta,t-1}^2) \end{bmatrix} + \begin{bmatrix} v_{\tau,t} \\ v_{\eta,t} \end{bmatrix}, \tag{1.19}$$

$$\begin{bmatrix} v_{\tau,t} \\ v_{\eta,t} \end{bmatrix} \sim N(0,\, W),$$

$$W = \begin{bmatrix} \sigma_{v\tau}^2 & 0 \\ 0 & \sigma_{v\eta}^2 \end{bmatrix}. \tag{1.20}$$

We identify the model and set the prior hyperparameters in the following ways: First, we restrict the factor loadings β_4 and γ_4 to be 1 to identify the scale of factor loadings and of the unobserved components (See equation (15)). Then, we obtain the initial guess value of β_i , γ_i , ϕ and σ_i as estimates of the state-space model using MLE with time-invariant variability of state disturbances. Along with factor loadings, the initial guess of the latent state variables in α_t can also be estimated. Second, for the initial guess of time-varying volatilities $\sigma_{\tau,t}^2$ and $\sigma_{\eta,t}^2$, we estimate a GARCH(1,1) model to obtain the conditional variance. Third, the prior distributions of β_i , γ_i and ϕ are conjugate independent diffuse normal with mean fixed to initial guess value and their variance set to 10^3. Forth, we impose independent inverse gamma with degrees of freedom to 1 for σ_i in H, $\sigma^2_{\nu\tau}$ and $\sigma^2_{\nu\eta}$ in W. Finally, following Del Negro and Otrok (2008), we fix the initial condition of stochastic volatility to zero.

1.3.3 Empirical Result

Inflation Indicator and Factor Loading

We build the coincident inflation indicator as the sum of latent permanent component and transitory component. The extracted weekly inflation indicator is plotted in Figure 1.1.[4] Several observations and desirable properties are noteworthy: First, our estimated inflation indicators are available at high frequency, whereas the monthly CPI and PCE inflation are released only monthly and with weeks of lags. Therefore, our inflation indicator can be applied to nowcast CPI and PCE inflation.

Second, our inflation indicator broadly coheres with the dynamics of inflation in the past 50 years. The Great Inflation in 1970s is apparent, along with the inflation stabilization staring from 1982. The average annual inflation indicator is 6.3648 during 1970s, compared with an average value of 2.3418 after 1982 in our estimation. For the Great Inflation, we find the first peak occurred on October 6, 1974 with a weekly indicator value of 0.199632, and the second peak was on November 30, 1980, with an indicator value of 0.188673. The recent 2007 recession experienced unprecedented price decline. However,

[4] The estimated inflation factors follow daily evolution. But information of price comes on Friday of each week as assumed in our model, so we aggregate the daily inflation factors to get the weekly inflation index as plotted. By doing so, we can mimic the real time updating of inflation index.

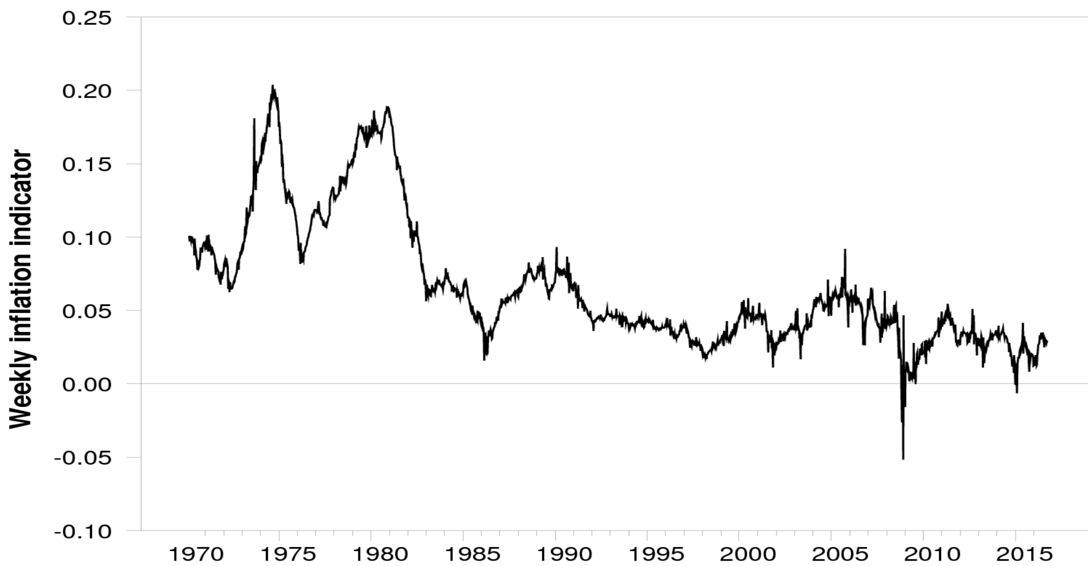

Figure 1.1: Extracted Weekly Inflation Index

Note: The weekly inflation indicator is the weekly sum of the daily inflation factors.

this deflation was quite brief and lasted two months from 10/26/2018 to 01/04/2019. In addition, the estimated inflation indicator also indicates varying volatility of inflation, which is consistent with the observations in the literature (Stock and Waston, 2007). We will examine this property in the later sections with estimated conditional volatility.

Third, our inflation indicator coheres with consumer inflation and expenditure inflation but plays no leading role in identifying the turning points. Figure 1.2 graphs the weighted inflation indicator along with monthly CPI inflation and PCE inflation. The fact that the weekly inflation indicator has no leading performance can be explained in two ways. On one hand, commodity price is made up of commodity future contracts, thus may convey limited leading information in the consumer and personal expenditure inflation. On the other hand, monthly indicators account for a large part of the extracted inflation factor as

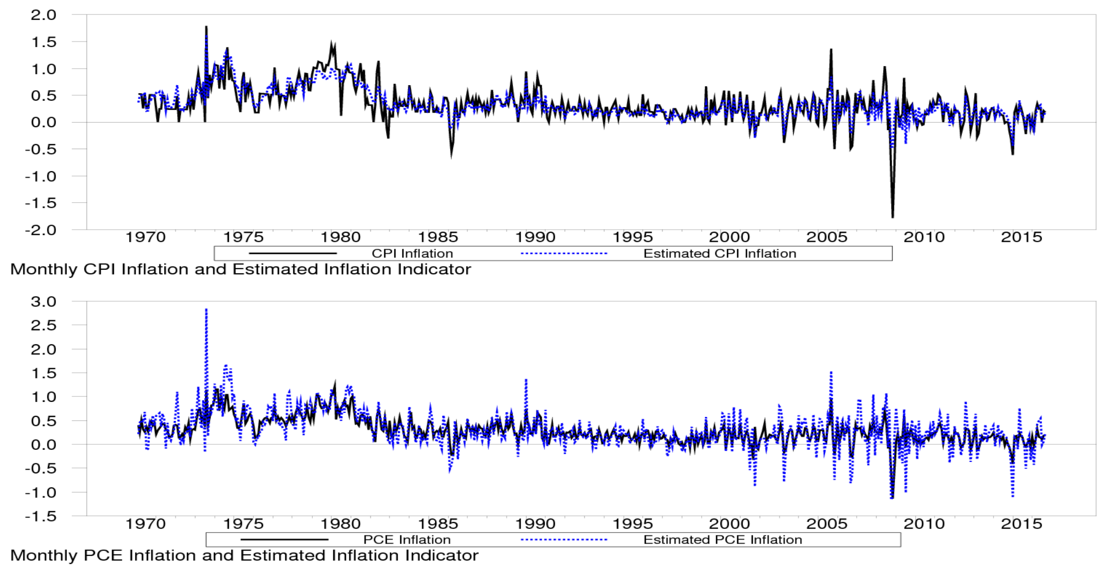

Figure 1.2: Estimated Inflation Indicator, CPI and PCE Deflator

Note: The upper graph shows estimated monthly trend inflation along with CPI inflation; the lower one shows estimated monthly trend inflation along with PCE inflation.

indicated by the values of the factor loading. It is not surprising that the weekly inflation indicator tracks monthly consumer and personal expenditure inflation well. Adding leading variables such as term premium and M2 may improve the leading performance of our indicator.

Estimated factor loadings measure the sensitivity of input variables to latent permanent and transitory components. The full sample posterior mean estimates of the factor loadings are reported in Table 1.3. The relative importance of our chosen indicators is given by the full sample posterior mean estimate of the factor loadings. For the common trend component, the monthly inflation indices have the highest posterior means, and followed by the quarterly GDP inflation. The weakest contribution comes from weekly commodity inflation (0.26). This suggests that commodity inflation is less persistent and thus few of the variation itself are from the variation of common persistent component.

Table 1.3: Factor loading Estimates

Parameters	Posterior Mean
Common Persistent component	
β_1	0.262034
β_2	1.151936
β_3	1.103199
β_4	1.0
Transitory component	
γ_1	-8.687555
γ_2	13.347247
γ_3	31.176756
γ_4	1.0

Note: β_1 and γ_1 are the factor loadings on GSCI commodity inflation; β_2 and γ_2 are the factor loadings on CPI inflation; β_3 and γ_3 are the factor loadings on PCE inflation; β_4 and γ_4 are the factor loadings on GDP deflator inflation. The estimated AR coefficient in equation (3) is -0.69.

Among inflation measures at different sampling frequencies, monthly CPI and PCE inflation capture both the persistent and transitory components well, comparing with quarterly inflation index in extracting low frequency movements and with weekly commodity inflation in modeling high frequency variations. This may suggest that transitory shocks in general price level vanish within a quarter. Thus, using quarterly average of inflation may overrate the model implied persistence of inflation either in univariate time series model or multivariate VAR model and New Keynesian model.

Trend Inflation and Volatilities

The model provides a measure of trend inflation. Figure 1.3 plots the full sample posterior means of τ_t , $\sigma^2_{\tau,t}$ and $\sigma^2_{\eta,t}$. The estimated trend inflation is quite smooth and shows substantial variation over time. Trend inflation declined continuously over the two decades. Regarding the recent 2007-2009 recession, trend inflation did not plunge deeper and go under zero line, but recovered steadily to the Fed inflation target. Compared with inflation indicator which indicates a short period deflation, trend inflation only suggests a pressure of disinflation. Therefore, the decline of price in 2008 was more likely due to a one-time large shock which decayed very quickly.

There are important similarities between $\sigma^2_{\tau,t}$ and $\sigma^2_{\eta,t}$, most notably the larger variation in 1970s coincided with high trend inflation, and persistently low volatility in 1990s followed by a remarkable increase in early 2000s. Stock and Watson (2007) suggest the recent rise of volatility as the potential reason for the decreased forecastability of inflation in recent decades. However, there are also differences between the changes in these two series. In 1990s, volatility of permanent component decreased strikingly compared to

1980s and 1970s, whereas there was only slight decrease in the volatility of transitory component. During 2000s, the volatility of both permanent and transitory components increased, but transitory component increased much more than permanent component. The persistence of inflation which depends on the relative importance of the variances of the permanent and transitory innovations is also examined. The change in inflation indicator has a negative first-order autocorrelation which summarizes the persistence of inflation process (Cecchetti, et al. 2007). The analytical expression can be calculated as:

$$\rho_{\Delta\pi} = \frac{Cov(\Delta\pi_t, \Delta\pi_{t-1})}{Var(\Delta\pi_t^2)} = \frac{-\frac{1-\phi}{1+\phi}\sigma_{\eta,t}^2}{\sigma_{\tau,t}^2 + \frac{2}{1+\phi}\sigma_{\eta,t}^2} \tag{1.21}$$

Note that $\rho_{\Delta\pi}$ has a range that depends on the AR coefficient ϕ. With the estimated value, the closer it is to -0.845, the less persistent the inflation process is. Additionally, the higher $\sigma_{\tau,t}^2$ is relative to $\sigma_{\eta,t}^2$, the closer inflation is to a pure random walk, and the closer the first-order autocorrelation of $\Delta\pi_t$ is to zero. By contrast, when $\sigma_{\eta,t}^2$ is dominant, inflation is close to a stationary AR process. From our estimation of the weekly inflation indicator, $\rho_{\Delta\pi}$ decreased by 74.12% from 1970s to the current decade. Alternatively, when inflation does change unexpectedly, how much of the surprise should we assume to be part of the new trend? We calculate the share of inflation surprise that the model currently attributes to the new trend. In our estimation, 54% of the unexpected inflation change assumed to be part of a new trend in 1970s, and this share decreased to 22% in 1990s and 19% in current decade. Overall, inflation persistence has reduced since the 1990s, but due to different components over time. In this period with a lower persistence, inflation tended to revert to a stable trend during the 2000s, whereas in the 70s and 80s the trend moved to track inflation.

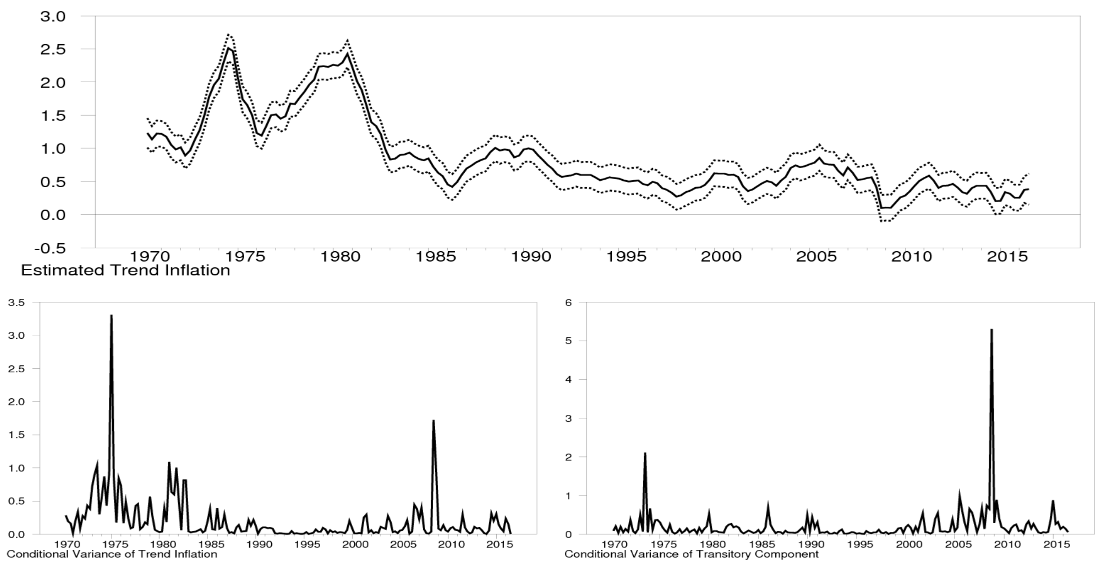

Figure 1.3: Estimated Trend Inflation and Stochastic Volatility

Note: The upper graph shows the weekly trend inflation; the lower left graph depicts the stochastic volatilies of persistent component; the lower right graph depicts the stochastic volatilities of transitory component of inflation.

Comparing with univariate model using quarterly data, our model tends to overrate the role of high frequency variations. This is shown in the higher contribution of transitory innovations to the variability of inflation process. It is not surprising that quarterly data filter out the high frequency innovations due to temporal aggregation. In contrast, weekly data highlights the volatile movements in commodity inflation, thus assign higher weights

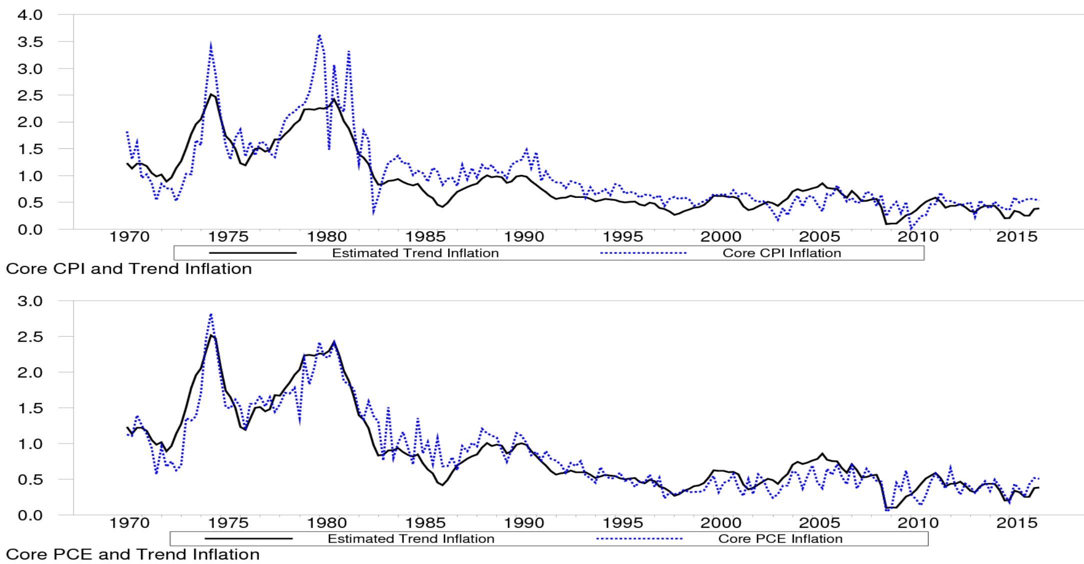

Figure 1.4: Trend Inflation and Core Inflation
Note: Trend inflation are monthly trend inflation component .

to them in signal extracting process (See Table 3 the factor loadings). Modeling transitory components as autoregressive process rather than white noise also weight more on the variability of transitory shocks.

Our model hinders the smoothness to stochastic volatility. The two spikes in 1974 and 2008 are more likely to be occasional large jumps in inflation. The 1974 spike was due to the oil crisis, and the2008 spike was due to the recent financial crisis. Hence it is possible that 2007 recession can be viewed as a temporary period with a high level of volatility in a longer period when moderate volatility is the norm.

Figure 1.4 compares model implied trend inflation with core inflation (CPI and PCE). Our trend estimates are broadly in line with the alternative measures of trend inflation. They together reflect the common low frequency variability in inflation series. However,

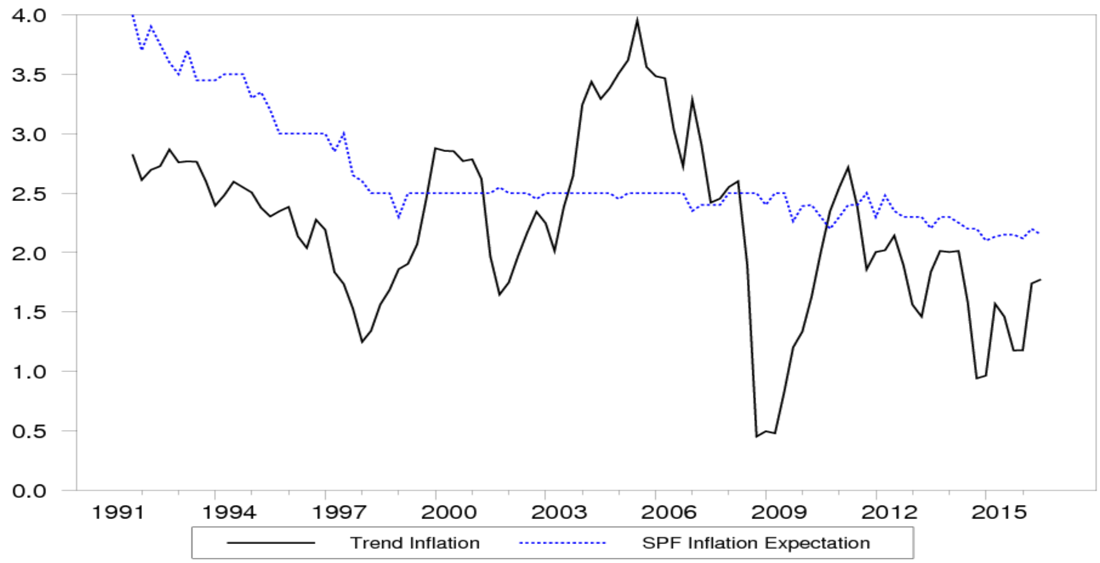

Figure 1.5: Estimated Trend Inflation and Inflation Expectation

Note: Long-term inflation expectation is measured with Survey of Professional Forecasters 10-years inflation expectation.

there are important differences between trend inflation and distinct core inflation measures. Core CPI inflation is much persistently higher than trend inflation and core PCE deflator during late 1970s and early 1980s, which indicates that sectors in CPI categories besides food and energy also contribute to large short-term variations.

Figure 1.5 plots our model implied trend inflation along with the median 10-year ahead forecast that has been reported in the Survey of Professional Forecasters since 1991. Trend inflation lines up with the survey forecasts but lies below trend inflation during 1990s and the current decade. The reason is that survey forecasts are always upward biased. Especially, long-term forecasts of PCE inflation from the SPF have often been a bit higher than long-term projections from the FOMC. After a slightly decline together in 1990s, survey forecasts kept stable henceforth while trend inflation became volatile. Although many concerns disinflation due to large output gaps and unemployment in 2004 and 2008 (Williams, 2009), the substantial increase in expectations anchoring mute these pressures and revert the trend to local mean.

A subtle feature in Figure 5 is that the model implied trend inflation leads long-run survey forecast movements. This is especially obvious for the drop around 1997 and the decline after 2012. This raises the question of how inflation expectation reacts to the changes in trend inflation. A simple linear regression between one-year ahead inflation expectation and trend inflation indicates that there exists statistically significant evidence that trend inflation help forecasting trend inflation expectation. This suggests a rise of inflation that is not accompanied by a rise of inflation expectations is less likely to persist.

1.4 Summary of Chapter 1

This article introduces a mixed-frequency unobserved component model with stochastic volatility and estimate the model using Bayesian Gibbs Sampler. MF-UCSV model provides a flexible mixed-frequency framework for extracting high frequency inflation indicator, estimating trend inflation, and describing persistence of inflation. Inflation indicator and trend inflation could be flagged in the real-time as new data are released. The framework allows ragged-edge data, publication lags and non-synchronization in real time monitoring and nowcasting.

Our paper supports the desirability of using models that account for a slowly-varying trend. The changing time series properties of inflation imparts the forecasting performance of most univariate and activity-based inflation forecast (Stock and Watson, 2007). Apart from accounting for local mean and varying volatility in this paper, one could also apply methods that take account of parameter instability, such as time-varying coefficients VAR by Cogley and Sargent (2005). It is noteworthy that researches which impose a structural break also do well in some specific models (Goren, et al., 2013). However, our initial try of a regime switching model fails to detect a structural break endogenously. One possibility is that high frequency data contains too many noises which can largely disturb the inference of Markov Switching model.

Compared with the conventional way of modeling low-frequency movement from quarterly data and high frequency variations from daily or weekly data separately, we estimate the variability of both components jointly. However, our model underrate the transitory innovation due to the quickly decay in transitory dynamics. This suggests a mixed frequency model with monthly and quarterly observations should be a future research direction to examine the relative importance of transitory components.

Chapter 2

Nowcasting Headline Inflation

Preview of Chapter 2

Nowcasting contemporaneous inflation using high frequency data is difficult. This chapter proposes a mixed frequency unobserved component model for nowcasting headline inflation of consumer price index (CPI). The model is a small-scale factor model that relies on relatively few variables. These variables are sampled at different frequencies and are selected based on the forecasting performance in real time. The final selected variables includes daily energy price, commodity price, dollar index, weekly gas price, money stock and monthly survey index. The model's nowcasting accuracy improves as information accumulates over the course of a month, and it easily outperforms a variety of statistical benchmarks. In particular, the nowcasting model outperforms the existing nowcasting models that ignore slowly moving local trend and stochastic volatility of inflation.

2.1 Introduction to Nowcasting Inflation

The annual inflation rate has been below the Fed's target for a long time in spite of the tight labor market since 2009. In view of this situation, broad discussion concerning the current 2% inflation targeting strategy has been generated among policy makers and researchers. One popular viewpoint in the 2019 Monetary Policy Forum held by Chicago University Booth School of Business is that the Fed should adopt a symmetric 2% inflation target, or define a range of inflation between 1.5% and 3%. In particular, Fed could set a medium-term goal within that range, and revisit it periodically to take account of the changing economic circumstances. Under an inflation targeting scenario, either with an explicitly announced target or a defined range, monitoring of the inflation path plays a fundamental role in assessing policy effectiveness. The goal of this chapter is to provide early nowcasts of monthly inflation that can be useful to inform monetary policy and market practitioners. The seminal work of Giannone, Reichlin and Small (2008) is the starting point of a large literature that utilizes the high-frequency data to forecast the present condition of the economy. Accordingly, nowcasts of monthly inflation are computed using incoming information on a wide range of relevant available data series within the reference month.

Following Giannone, et al. (2008) and many others, this chapter proposes a parsimonious state-space model with a small number of variables sampled at different frequencies, which are used to produce nowcasts of the monthly CPI inflation. The proposed econometric framework allows updating inflation forecasts continuously, following growing amounts of incoming information on a wide range of relevant available indicators. The relevant data group includes variables which are highly correlated with inflation and which are released earlier than the relevant inflation releases. In addition, the proposed model encompasses price measures sampled at different frequencies, including daily, weekly and monthly price indices. Within the model, long- and short-run inflation dynamics are represented by underlying trend and cycle components and modeled by different variables. The possibility of an early inflation nowcast providing more timely signals than the official publication is very appealing. Official inflation measures are observed with publication lags. For example, U.S. CPI is announced at the middle of the month for measures of inflation for the month prior.

Our approach involves formal modeling of inflation dynamics characterizing its trend, cycle and volatility, while allowing for mixed-frequency. In the proposed model, underlying inflation process is approximated as a sum of unobserved common permanent and transitory factors. The permanent component captures long-run trend inflation, while the transitory component captures short-run deviations of inflation from its trend value. Additionally, the variances of the permanent and transitory disturbances are allowed to evolve over time according to a stochastic volatility process. In the end, the daily factor model is capable to capture the slowly moving local mean and time varying volatility in the low frequency variable and volatile component in the high frequency variables. The proposed model allows for the potential distinct dynamics of underlying inflation, and also extracts common trend and common cyclical movements across the series. Then the daily forecast of common trend and cyclical factors within the reference month are produced and aggregated to make our monthly nowcast.

The nowcasting literature has yield major advances in handling mixed-frequency time series, missing observations and ragged data. These methods are extensively applied to forecast GDP growth and inflation. Modugno (2013) applies factor model and a large number of series to nowcast US CPI inflation. Breitung and Roling (2014) extend the MIDAS (mixed-data sampling) model in a non-parametric setting and examine the predictive power of high frequency financial variables and commodity prices. Additionally, Monteforte and Moretti (2013) is the only one that considers both the low- and high-frequency variability of inflation separately. Their paper uses the factor model to extract the low frequency inflation dynamics and a MIDAS model to model the high frequencies variations. In contrast, Knotek and Zaman (2017) choose a small number of data series at different frequencies to estimate nowcasts and do not use factor models.

Following this literature, we explore the valuable information contained in the high frequency energy prices and financial variables. The early signals in the high frequency data should be useful for improving the accuracy of inflation forecasting. However, our paper departures from the existing literature in several ways. First, our nowcasting model takes into account potential non-stationarity in inflation dynamics. An unobserved component model with stochastic volatility is used to model the latent dynamics of inflation process. Many researchers suggest models that consider slow-varying local mean of inflation perform reasonably well in forecasting inflation. Atkeson and Ohanian (2001) show that forecasts from simple random walk model cannot be statistically beaten by alternatives. Following their work, Stock and Watson (2007, 2016) propose a characterization of quarterly inflation as an unobserved component model with stochastic volatility. Faust and Wright (2013) compare various inflation forecasts and find that models based on stationary specifications for inflation do consistently worse than non-stationary models. However, the related literature in nowcasting (Giannone, et. al. 2006) are all based on stationary specifications for inflation.

Second, we use disaggregated indicators, including energy prices, commodity prices to capture the transitory component of inflation. Most of the variables available at daily and weekly frequency are disaggregated data that only have limited predictive content over consumer prices, and especially the core inflation. Also, disaggregated data is more volatile than the low frequency aggregate data. When we increase the observation frequency, it is increasingly difficult to perceive the trend inflation and where it is likely to be in the future. Knotek and Zaman (2017) utilize the disaggregated data judiciously only when sufficient high frequency data are available to be informative to the aggregates. Monteforte and Moretti (2013) model high frequency variation separately using daily financial variables in MIDAS framework. In contrast to their approaches, we extract the common transitory factor of inflation-the co-movement of transitory component in mixed-frequency data-through optimal inference. The high frequency variables and low frequency variables are modeled in a coherent model to produce the nowcast of monthly CPI inflation.

We examine the combinations of leading indicators that yield the best forecasting performance, and compare the predictive ability of the model with alternative univariate and multivariate specifications. We show that the model's nowcasts (zero and one month ahead) outperform a variety of statistical benchmarks. First and foremost, our model outperforms the existing nowcasting models that ignore slowly moving local trend and stochastic volatility of inflation. The results provide evidence of substantial gains in real-time nowcasting accuracy when allowing for non-stationarity and stochastic volatility. Second, the results also indicate that the mixed-frequency UCSV models with energy prices, commodity prices, dollar index increases the forecasting accuracy substantially compared with benchmark univariate models. In the out-of-sample comparisons, the model's nowcasts of headline CPI inflation outperform those from autoregressive and random walk models, with especially significant outperformance as the month goes on.

This chapter is organized as follows. The model is described in section 2, along with the estimation method. The third section describes the alternative univariate and multivariate models. Section 4 presents the empirical results. The conclusion is discussed in the last section.

2.2 Nowcasting Framework

In this section, we specify the inflation nowcasting model that allows the inclusion of data sampled at different frequencies. CPI inflation are monthly observations that track the changes in consumer price index. The nowcasting of monthly CPI inflation yields early signals of CPI inflation with the information of high frequency daily and weekly time series. Thus we first need to specify the inflation dynamics at high frequency and express low frequency data in high frequency terms. Many financial indicators are available at daily frequency, so the underlying inflation process is assumed to evolves at daily frequency. Let P_t denote the underlying daily price index. Then inflation at day t is

$$\pi_t = logP_t - logP_{t-1}. \tag{2.1}$$

Most indicators are available at lower frequency, for example, fuel prices are weekly time series, CPI index is at monthly and GDP deflator is available at quarterly frequency. Thus, we aggregate the daily inflation indicator to produce our low frequency inflation index, in order to compare the model results with observed data. The low frequency inflation indicator is expressed as

$$\begin{aligned} \Pi_t &= \sum_{j=0}^{D-1} \pi_{t-j} = \sum_{j=0}^{D-1} (logP_{t-j} - logP_{t-j-1}) \\ &= logP_t - logP_{t-D} \end{aligned} \tag{2.2}$$

where D is the number of days per observational period. For example, D for monthly CPI of January equals 31.

Following Stock and Watson (2016), underlying inflation is characterized with an unobserved component model with stochastic volatility:

$$\pi_t = \tau_t + \eta_t \tag{2.3}$$

$$\tau_t = \tau_{t-1} + \sigma_{\tau,t}\varepsilon_{\tau,t} \tag{2.4}$$

$$\Phi(L)\eta_t = \sigma_{\eta,t}\varepsilon_{\eta,t} \tag{2.5}$$

$$ln(\sigma^2_{\tau,t}) = ln(\sigma^2_{\tau,t-1}) + v_{\tau,t} \tag{2.6}$$

$$ln(\sigma^2_{\eta,t}) = ln(\sigma^2_{\eta,t-1}) + v_{\eta,t} \tag{2.7}$$

where τ_t represents the permanent component of underlying inflation and η_t represents the transitory component. Permanent component takes the form of a simple random walk by equation (2.4). The transitory component has a finite order AR(p) representation by equation (2.5), where function $\Phi(L)$ is a finite lag polynomial with order p, and has all the roots outside the unit cycle. $\varepsilon_{\tau,t}$ and $\varepsilon_{\eta,t}$ are mutually independent $i.i.d.N(0,1)$ stochastic processes. $\sigma_{\tau,t}$ and $\sigma_{\eta,t}$ represent the variability of innovations to permanent component and transitory component. They together determine the relative importance of random walk disturbance. To model the changing volatility of inflation components, it is assumed that their log-volatility follows a random walk with no drift, as described in equation (2.6) and (2.7). $v_{\tau,t}$ and $v_{\eta,t}$ are disturbance to the stochastic volatility which are assumed to be i.i.d. $v_{\tau,t} \sim N(0, \sigma^2_{v\tau})$ and $v_{\eta,t} \sim N(0, \sigma^2_{v\eta})$.

A vector of price measures and other variables displaying co-movements is modeled to depend on the latent permanent and transitory inflation factors. In the mixed frequency framework, the relationship between the observed data and daily inflation factors need to be specified. Following the approach developed in chapter 1, we let $\tilde{y}_t^i$ denote the ith observed flow variable in low frequency. Then $\tilde{y}_t^i$ is the weighted sums of the corresponding daily inflation factors,

$$\tilde{y}_t^i = \begin{cases} \sum_{j=0}^{D_i-1} \beta_i \tau_{t-j} + \sum_{j=0}^{D_i-1} \gamma_i \eta_{t-j} + u_t^i & if\ y_t\ can\ be\ observed \\ NA & otherwise \end{cases} \tag{2.8}$$

Following Harvey (1978) and Modugno (2013), we define the corresponding low frequency factor accumulator. Let $C_{\tau,t}$ and $C_{\eta,t}$ denote the permanent and transitory component accumulator. Then weekly inflation accumulator is define as

$$\begin{aligned} C_{\tau,t}^W &= \theta_t^W C_{\tau,t-1} + \tau_t \\ &= \theta_t^W C_{\tau,t-1} + \tau_{t-1} + \sigma_{\tau,t}\varepsilon_{\tau,t} \end{aligned} \tag{2.9}$$

and

$$\begin{aligned} C_{\eta,t}^W &= \theta_t^W C_{\eta,t-1} + \eta_t \\ &= \theta_t^W C_{\eta,t-1} + \Phi(L)\eta_{t-1} + \sigma_{\eta,t}\varepsilon_{\eta,t} \end{aligned} \tag{2.10}$$

where θ_t^W is an indicator variable which is defined as:

$$\theta_t = \begin{cases} 0 & If\, t\, is Monday \\ 1 & otherwise \end{cases}$$

The monthly inflation accumulator is define as

$$\begin{aligned} C^M_{\tau,t} &= \theta^M_t C_{\tau,t-1} + \tau_t \\ &= \theta^M_t C_{\tau,t-1} + \tau_{t-1} + \sigma_{\tau,t}\varepsilon_{\tau,t} \end{aligned} \tag{2.11}$$

and

$$\begin{aligned} C^M_{\eta,t} &= \theta^M_t C_{\eta,t-1} + \eta_t \\ &= \theta^M_t C_{\eta,t-1} + \Phi(L)\eta_{t-1} + \sigma_{\eta,t}\varepsilon_{\eta,t} \end{aligned} \tag{2.12}$$

where θ^M_t is defined as:

$$\theta_t = \begin{cases} 0 & If\, t\, is the\ first\ day\ of\ the\ month \\ 1 & otherwise \end{cases}$$

Then equation (2.8) can be written as:

$$\tilde{y}^i_t = \begin{cases} \beta_i C^i_{\tau,t} + \gamma_i C^i_{\eta,t} + u^{*i}_t & if\ y_t\ can be observed \\ NA & otherwise \end{cases} \tag{2.13}$$

The multivariate unobserved component model extracts the co-movement among the target variable CPI inflation, denoted by $y_{1,t}$, and other candidate daily indicators $y^D_{m,t}$, $m = 1, \cdots, M$, weekly indicators $y^W_{n,t}$, $n = 1, \cdots, N$, monthly indicators $y^M_{k,t}$, $k = 1, \cdots, K$. The model separates out the common trend and cyclical movements underlying these variables in the unobserved factor τ_t , η_t, and the idiosyncratic movements not representing there inter-correlations captured by the associated idiosyncratic terms u_t. The MF-UCSV is expressed as follows:

$$\begin{bmatrix} y_{1,t} \\ y^D_{1,t} \\ \vdots \\ y^D_{M,t} \\ y^W_{1,t} \\ \vdots \\ y^W_{M,t} \\ y^M_{1,t} \\ \vdots \\ y^M_{K,t} \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{M_t-1} \beta_1 \tau_{t-j} + \sum_{j=0}^{M_t-1} \gamma_1 \eta_{t-j} \\ \beta^D_1 \tau_t + \gamma^D_1 \eta_t \\ \vdots \\ \beta^D_M \tau_t + \gamma^D_M \eta_t \\ \sum_{j=0}^{6} \beta^W_1 \tau_{t-j} + \sum_{j=0}^{6} \gamma^W_1 \eta_{t-j} \\ \vdots \\ \sum_{j=0}^{6} \beta^W_N \tau_{t-j} + \sum_{j=0}^{6} \gamma^W_N \eta_{t-j} \\ \sum_{j=0}^{M-1} \beta^M_1 \tau_{t-j} + \sum_{j=0}^{M-1} \gamma^M_1 \eta_{t-j} \\ \vdots \\ \sum_{j=0}^{M_t-1} \beta^M_K \tau_{t-j} + \sum_{j=0}^{M_t-1} \gamma^M_K \eta_{t-j} \end{bmatrix} + \begin{bmatrix} u^1_t \\ u^D_{1,t} \\ \vdots \\ u^D_{M,t} \\ u^W_{1,t} \\ \vdots \\ u^W_{N,t} \\ u^M_{1,t} \\ \vdots \\ u^M_{K,t} \end{bmatrix}$$

where β and γ are the factor loadings on the common permanent and transitory components, which measure the sensitivity of the common factor to the observable variables. The dynamics of the unobserved permanent and transitory component is described by equation

(2.4)-(2.5) and equation (2.9)-(2.12).

The MF-UCSV model can be cast into a state-space representation as follows:

$$Y_t = C_t^{'}\alpha_t + w_t \tag{2.14}$$

$$\alpha_{t+1} = A_t\alpha_t + R_t v_t \tag{2.15}$$

$$\Lambda_{t+1} = \Lambda_t + \zeta_t \tag{2.16}$$

$$w_t \sim N(0,H), \;\; v_t \sim N(0,Q_t)$$

$$\zeta_t \sim N(0,W)$$

where Y_t is an $N \times 1$ vector of observed variables with missing values. State vector α_t includes common trend and cyclical inflation factors, w_t and v_t are Gaussian and orthogonal measurement and transitory shocks. The time-varying variance matrix Q_t is a diagonal matrix with elements of $\sigma^2_{\tau,t}$ and $\sigma^2_{\eta,t}$. Λ_t is the vector of unobserved log-volatilities, and W is a diagonal matrix containing the variance of log-volatility disturbances.

In the state-space system, equation (2.15) corresponds to the measurement equation that relates observed variables with unobserved common trend and cyclical component, and idiosyncratic terms. Equation (2.16) is the state equation,which specifies the dynamics of the trend and cyclical component. Equation (2.17) describes the dynamics of the stochastic volatility, which governs the relative importance of trend and cyclical component. Through the state-space model, we can use Kalman filter to obtain the optimal inferences on the state variable. The estimation follows the procedure in chapter 1.

2.3 Model Comparison

2.3.1 Univariate Model

We compare the nowcasts obtained from the MF-UCSV model with those obtained from benchmark univariate models. We consider the following inflation forecasting models which are widely used in the literature:

- Univariate autoregressive AR(p) model: $\pi_T^{CPI} = \beta_0 + \sum_{j=1}^{p} \pi_{T-j}^{CPI} + \varepsilon_T$
- Pure Random Walk (RW): $\pi_T^{CPI} = \pi_{T-1}^{CPI} + \varepsilon_T$
- Random Walk on annual inflation(RW-AO): $\pi_T^{CPI} = \frac{1}{12}\sum_{j=1}^{12} \pi_{T-j}^{CPI} + \varepsilon_T$

The number of lags in the AR(p) model is selected with Bayesian Information Criteria. Besides the autoregressive model, we also consider two variants of random walk model. The first is the pure random walk, and the second is the 4-quarter random walk model considered by Atkeson and Ohanian (2001). Although the AO random walk model is used to forecast quarterly inflation, it yields well-established predictive performance in the literature. Thus we modify it to forecast the monthly CPI inflation.

2.3.2 Mixed-Frequency Factor model

Mixed-frequency factor model has been widely used in the literature to nowcast output and inflation. The starting paper of Giannone, Reichlin and Small (2008) forecast quarterly inflation, and Modugno (2013) first uses a relatively large dataset, including daily and weekly data, in a factor model to nowcast monthly inflation. The mixed-frequency factor model outperform the alternative competing univariate model. In the dynamic factor model, underlying daily inflation factor is extracted from a set of stationary indicators. The model is expressed as:

$$\begin{bmatrix} y_{1,t} \\ y^D_{1,t} \\ \vdots \\ y^D_{M,t} \\ y^W_{1,t} \\ \vdots \\ y^W_{M,t} \\ y^M_{1,t} \\ \vdots \\ y^M_{K,t} \end{bmatrix} = \begin{bmatrix} \beta_1 \sum_{j=0}^{M_t-1} f_{t-j} \\ \beta^D_1 f_t \\ \vdots \\ \beta^D_M f_t \\ \sum_{j=0}^{6} \beta^W_1 f_{t-j} \\ \vdots \\ \sum_{j=0}^{6} \beta^W_N f_{t-j} \\ \sum_{j=0}^{M-1} \beta^M_1 f_{t-j} \\ \vdots \\ \sum_{j=0}^{M_t-1} \beta^M_K f_{t-j} \end{bmatrix} + \begin{bmatrix} u^1_t \\ u^D_{1,t} \\ \vdots \\ u^D_{M,t} \\ u^W_{1,t} \\ \vdots \\ u^W_{N,t} \\ u^M_{1,t} \\ \vdots \\ u^M_{K,t} \end{bmatrix} \tag{2.17}$$

The dynamics of the inflation factor and error terms are modeled as autoregressive processes:

$$f_t = \phi_1 f_{t-1} + \cdots + \phi_p f_{t-p} + e_t, \;\; e_t \sim i.i.d.N(0,1) \tag{2.18}$$

$$u_t^i = \varphi_1^i u_{t-1}^i + \cdots + \varphi_q^i u_{t-q}^i + \varepsilon_t^i, \;\; \varepsilon_t \sim i.i.d.N(0, \sigma_{\varepsilon^i}^2) \tag{2.19}$$

where the unobserved factor f_t and error term u_t are assumed to be mutually independent at all leads and lags . The model can also be cast into state-space model and estimated via Kalman filter.

2.4 Nowcast Monthly Headline Inflation

2.4.1 Data and Timing of Forecast

This chapter applies a small-scale MF-UCSV model to the problem of forecasting U.S. monthly CPI inflation at short horizons. The monthly CPI inflation is released by Bureau of Labor Statistics around the middle of the month following the reference period. Compared with major NIPA variables, CPI inflation is not subject to annual seasonal adjustment. The BLS generally does not revise the data on consumer prices for reasons other than recalculating the seasonal factors. Implementing our model and nowcasting the target variable requires a set of indicators that arrive before the official release and available at high frequency. In this chapter, we apply the "confirmatory" factor analysis of Chauvet, at el. (2016). We judiciously choose candidate indicators and model specifications based on prior knowledge of economic dynamics and relationships. On one hand, the candidate indicators should be informative to consumer prices. For example, energy price volatility is highly correlated with the volatility of consumer price index. Many found that high-frequency information on energy price is an useful indicator to have both in long-horizon and short-horizon forecasting (Stock and Watson, 2003; Modugno, 2013; Monteforte and Moretti, 2013). On the other hand, the candidate variables should be available before the release of the official publication. Daily and weekly energy prices and financial series provide early signals of inflation continuously in real time. Survey data arrives at low frequency, however, always around the end of the reference month when the official information is yet published. The variables included in the models are selected based on their marginal predictive contribution to nowcast CPI inflation, and on model specification tests. In the end, our information set consist 11 candidate inflation indicators which are sampled at three frequencies.

The first set of variables are daily crude oil prices, commodity prices, term spread and exchange rate. The Texas Intermediate (WTI) crude oil price and Trade Weighted U.S. Dollar Index are from the Federal Reserve Bank of St. Louis' Archival Federal Reserve Economic Data (ALFRED). GSCI (Goldman Sachs Commodity Index) is obtained from Global Financial Data. GSCI index is a weighted future prices that almost covering all the sectors of commodities. It is published by Standard and Poor's and recognized as a leading measure of general price movements in the global economy. The interest term spread is calculated as the difference between 3-month and 10-year treasury yield in Fed

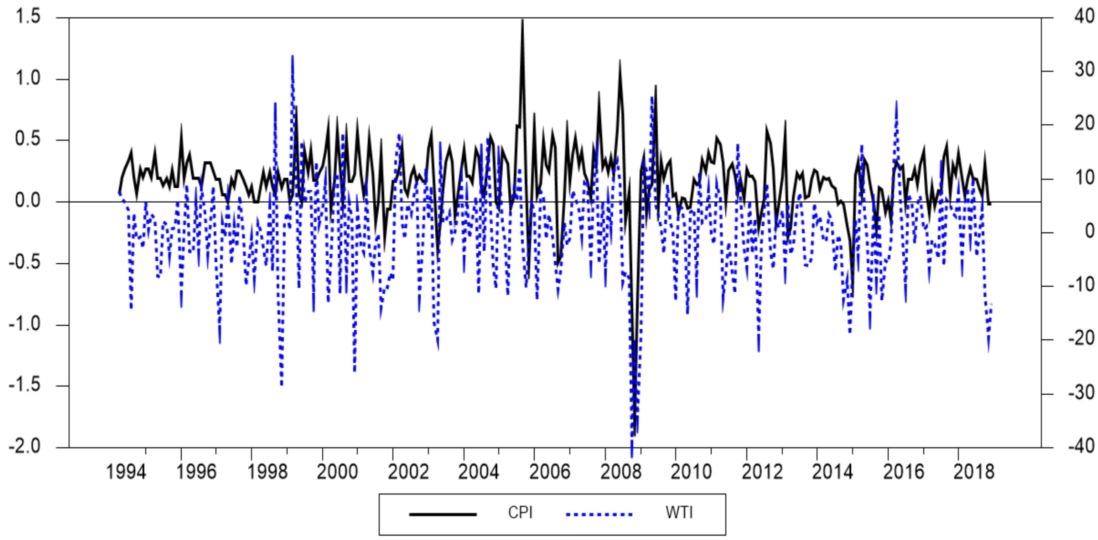

Figure 2.1: CPI Inflation and Crude Oil Price

H.15 release. The term spread was thought to be a useful indicator of future economic condition and inflation dynamics (Minskin, 1990). However, this predictive power was unstable in the past thirty years, as we will show in chapter 3. We include this variable in our model to examine its marginal predictive usefulness in our framework.

The second set of indicators are weekly gasoline prices, diesel prices and monetary aggregates. The weekly retail gasoline and diesel prices are released by U.S. Energy Information Administration (EIA) by every Monday. They are weighted average based on sampling of approximately 900 retail outlets. The M1 and M2 monetary aggregates are obtained from Fed H.6 money stock measures.

The monthly information set consists our target variable consumer price index and survey data ISM price index. Manufacturing ISM Report On Business is available on

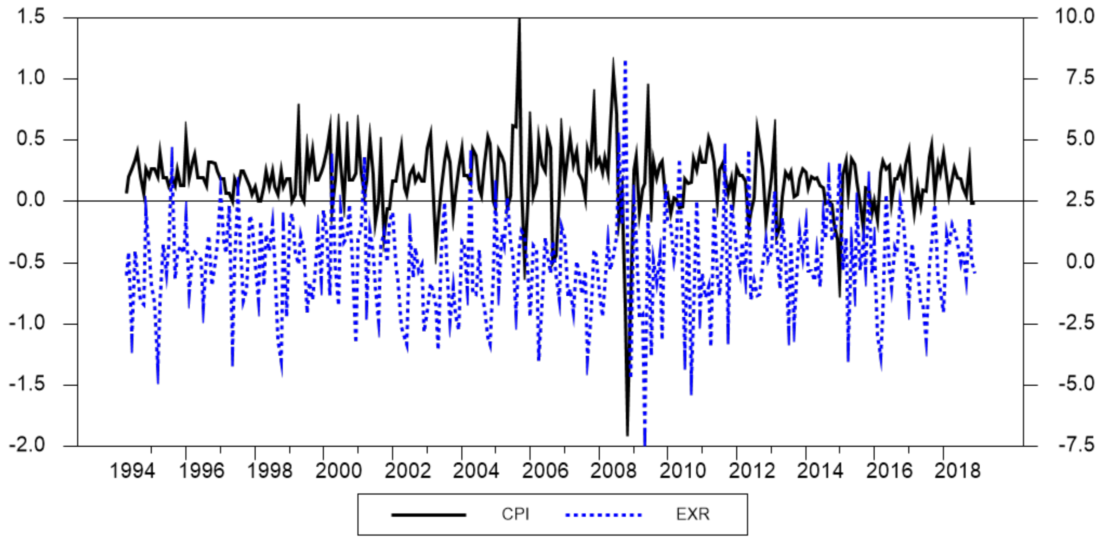

Figure 2.2: CPI Inflation and Dollar Index

the first Monday following the reference month. The price index follow the way how ISM Purchasing Managers' Index is built. A value of ISM price index of more than 50 indicates increase of the price level in comparison with the previous month.

Our sample ranges from 1993/03/28 through 2018/12/31 based on the availability of high frequency data. All variables are transformed to growth rate by log-differencing, with the exception of term spread. We plot the selected inflation indicator along with CPI inflation in Figure 2.1 to Figure 2.5. We perform out-of-sample nowcasting starting in January 2005 for CPI inflation. We focus primarily on root mean squared errors (RMSEs) as our measure of nowcasting accuracy, which give a sense of the absolute errors involved in nowcasting inflation. We use Diebold and Mariano (DM, 1995) tests for equal forecast

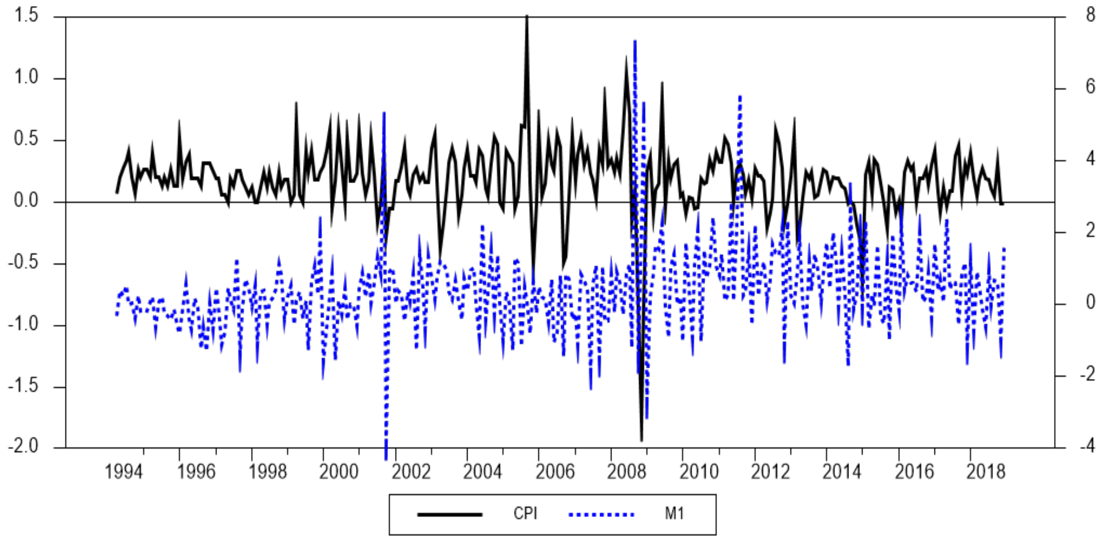

Figure 2.3: CPI Inflation and M1

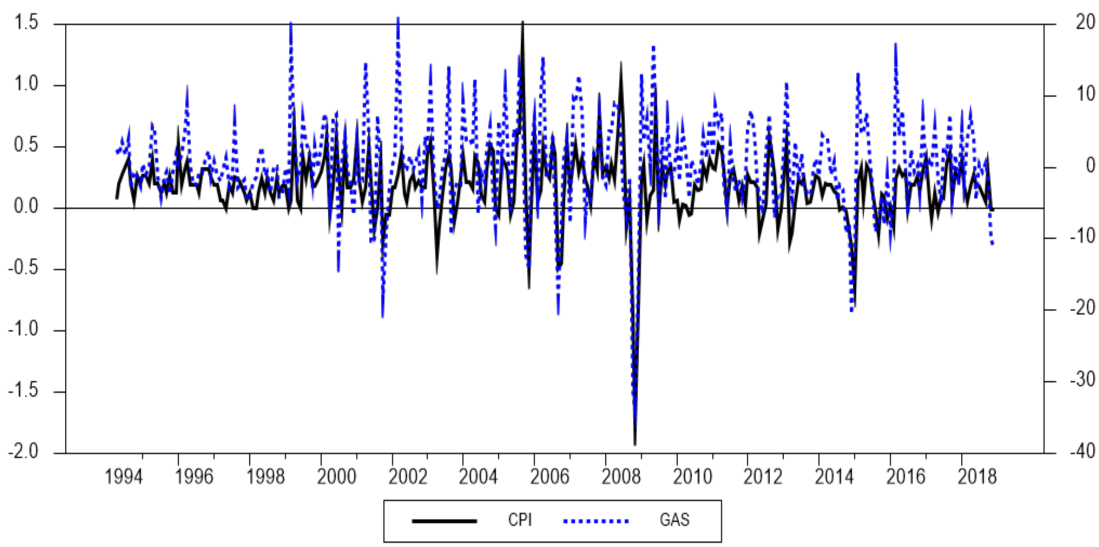

Figure 2.4: CPI Inflation and Gasoline Price

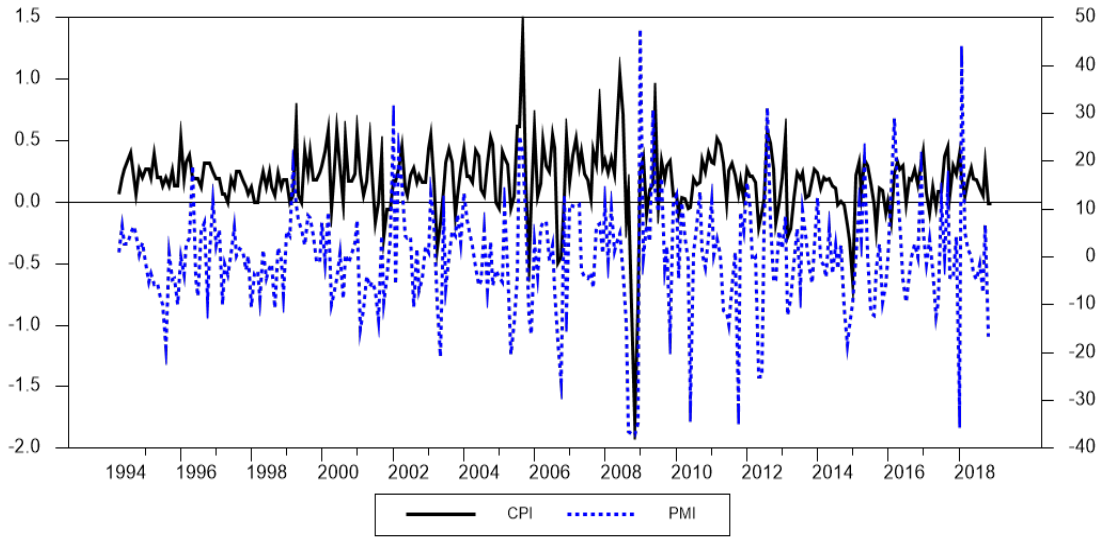

Figure 2.5: CPI Inflation and ISM Price Index

accuracy between our model's nowcasts and those from other sources.

Monthly CPI inflation readings are first released by the BLS around the middle of month, with around two weeks publication lags. Over the course of a given month, the arrival of the previous month's inflation estimate contains relevant information and influences the current month's nowcast. Oil prices and commodity prices arrive at the daily frequency, and retail gasoline prices arrive at the weekly frequency. While precise release dates of these series vary from one month to the next, we illustrate the model's monthly nowcasting performance for CPI inflation at four representative dates. The first prediction is produced on the first Monday day of the target month when the PMI of the preceding month is released; the second prediction is produced on the third Monday of the target month when we have three weekly readings; the third prediction is produced on the final day of the month when we have all the available daily and weekly information; the fourth is the first Monday following the target month when the PMI is released for the target month.

2.4.2 Model Implementation

In the empirical work, we focus on the nowcasting of monthly headline CPI. The model yields optimal inference on the underlying trend and cyclical component of inflation. We impose two assumptions on our model. First, we use the the high frequency variables to only model the transitory component of inflation. The trend component captures the slowly moving local mean of inflation process, and the daily and weekly variables did not behave a downward trend. Second, we assume that the transitory component follows AR(1) process. First order dynamics may not be adequate to describe the dynamics of inflation at high frequency. However, this assumption reduces the number of parameters to be estimated and the forecasting performance appears quite encouraging.

2.4.3 Confirmatory Factor Analysis

Many find that a small-scale factor model with pre-selected variables lead to more accurate forecast (Chauvet and Potter, 2001, 2016; Bai and Ng, 2008). To better illustrate the model implementation and find the combination of variables that can yield the best forecasting performance, we start with the construction of a 5-variable model that captures different sources of shocks. The shocks should be related to movements in inflation. The basic model construction includes two daily variables that monitors the price changes in crude oil and commodity, one weekly variable that tracks the gasoline price movement,

Table 2.1: Nowcasting Model with 5 and 6 Candidate Variables

Model	Model A	Model B	Model C	Model D	Model E	Model F
2005/01-2018/12	0.2272	0.2834	0.2268	0.2264	0.2270	0.2405
2005/01-2007/11	0.2809	0.3671	0.2803	0.2810	0.2811	0.2920
2007/12-2009/06	0.4533	0.5477	0.4541	0.4526	0.4529	0.4869
2009/07-2018/12	0.1314	0.1627	0.1304	0.1310	0.1315	0.1386

Note: model A: CPI, WTI, GSCI, GAS, ISM
model B: CPI, WTI, GSCI, GAS, ISM, SPREAD
model C: CPI, WTI, GSCI, GAS, ISM, EXR
model D: CPI, WTI, GSCI, GAS, ISM, M1
model E: CPI, WTI, GSCI, GAS, ISM, M2
model F: CPI, WTI, GSCI, GAS, ISM, DIESEL

one monthly ISM price index, and the CPI inflation itself (model A). We include ISM price index among our indicators because they are valuable survey information from the perspective of industries, and one not explicitly contained in standard price indices. They provide us a good opportunity to forecast the CPI inflation using more informative monthly observations before the release of the official data.

The next step is to assess the marginal predictive ability of additional indicators, which could improve the fit between our inflation index and CPI inflation. We consider the 6-variable mixed-frequency unobserved component model with stochastic volatility. The alternative series to be added are daily term spread (model B), dollar index (Model C), M1(Model D), M2 (Model E), weekly diesel price (Model F).

Table 2.1 shows the the RMSE of the basic 5-variable model and 6-variable models. The best 6-variable combinations correspond to model C{WTI oil price, GSCI commodity price, Dollar Index, Gasoline price, ISM price index, CPI inflation}, model D{WTI oil price, GSCI commodity price, M1, Gasoline price, ISM price index, CPI inflation}, and model E{WTI oil price, GSCI commodity price, Gasoline price, ISM price index, CPI inflation, M2}. Some interesting findings are noteworthy. First, most of the 6 variable models display an inferior performance compared with the best five-variable benchmark (Model A). The exception is model C which includes daily dollar index. However, the improvement adding additional exchange rate is not significant. Second, the RMSE increases substantially when we add term spread. Term spread was thought to be an useful leading indicator of inflation at long horizons. However, recent research find that the the predictive power of term spread for both output growth and inflation deteriorated in recent decades. Our results are in accordance with the existing evidence and confirm that interest rate is not a good predictor of short term inflation. Third, adding another fuel price-diesel price-yield decreases in the accuracy of forecasting. Large scale models are not necessarily better than the small scale model. Since large models that include all available variables in the same category can lead to large cross-correlation in the idiosyncratic errors of the series. This corroborates the results of Chauvet (2001, 2016) and Alvarez, Camacho, and Perez-Quiros (2013).

We continue to enlarge our model to incorporate more candidate variables. The results of 7 variable models are reported in Table 2.2. Once again, we find that including more than one series from the same category (monetary aggregates) does not increase the model's predictive performance. The best performed model is Model G{CPI, WTI, GSCI, Exchange rate, M1,Gasoline price, ISM index}. The RMSE of the larger model are not

Table 2.2: Nowcasting Model with 7 Variables

Model	Model G	Model H
2005/01-2018/12	0.2269	0.2260
2005/01-2007/11	0.2813	0.2805
2007/12-2009/06	0.4514	0.4532
2009/07-2018/12	0.1315	0.1298

Note: model G: CPI, WTI, GSCI, M1, M2, GAS, ISM
model H: CPI, WTI, GSCI, EXR, M1, GAS, ISM, SPREAD

substantially different from the benchmark Model A based on the Diebold-Mariano test. Our model selection process shows that the best performed model is the combination of inflation indicators in energy price, commodity price, exchange rate, monetary aggregates and survey price index.

2.4.4 Nowcasting Performance

In this section, we presents the results of the mixed-frequency unobserved component model with stochastic volatility in out-of-sample forecasting. The models are evaluated over 2005M1 to 2018M12, as described in section 4.3. We also consider three subperiods: the period before Great Recession (2005M1 to 2007M11), the Great Recession (2007M12 to 2009M06), and the period after Great Recession (2009M07 to 2018M12). The basic construction of 5-variable model (Model A) and the 7-variable model (Model I) are chosen to assess their ability to predict the current month inflation rate. We use the exact amount of data available at the time of prediction. For comparison, we also estimate the univariate

Table 2.3: Nowcast Comparison

Model	MF-UCSV(Model A)	MF-UCSV(Model I)	AR	RW	RW-AO	MF-DF
2005/01-2018/12	0.2272	0.2260	0.2883	0.3309	0.3357	0.2727
2005/01-2007/11	0.2809	0.2805	0.3341	0.4291	0.3640	0.3678
2007/12-2009/06	0.4533	0.4532	0.5562	0.5662	0.6918	0.4950
2009/07-2018/12	0.1314	0.1298	0.1916	0.2268	0.2137	0.1959

model and mixed-frequency factor model in the pseud-real-time scheme.

The results of RMSE are reported in Table 2.3. The MF-UCSV models with our top ranked model specifications yield similar forecast performance over the full sample period and sub-periods. However, we find substantial improvements of the performance over univariate model and mixed-frequency dynamic factor model (MF-DF). The RMSE of MF-UCSV model is 22% and 20% lower than the best performing univariate autoregressive model and MF-DF model, respectively. The difference is statistically significant at 5% level in DM test. Based on the performance of mixed frequency model in our analysis, it is evident that the high frequency information do contain useful information about the current movement of CPI inflation and improve the forecasting performance.

We plot the nowcasts of MF-UCSV model, AR model and MF-DF model in Figure 2.6 to Figure 2.8. NBER recessions are represented as shaded area. As can be seen, the autoregressive model tends to produce overestimated CPI inflation during most of the period. It is known that the U.S. economy experienced low inflation in our sample period. Most of the inflation models produce higher inflation forecast in the recent decade. Part

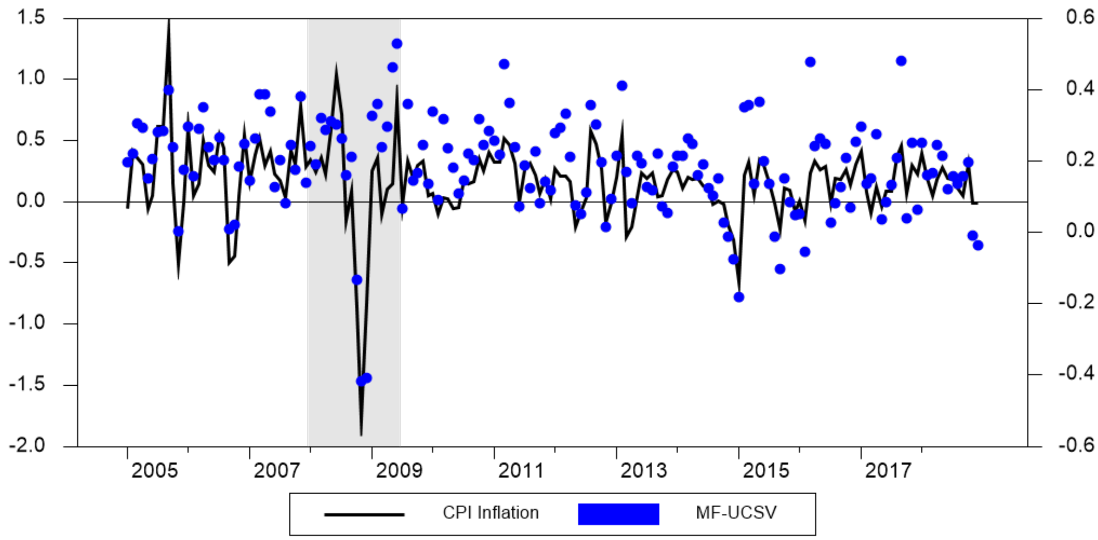

Figure 2.6: Nowcast of MF-UCSV Model

Note: the shaded area is the recession with NBER dating

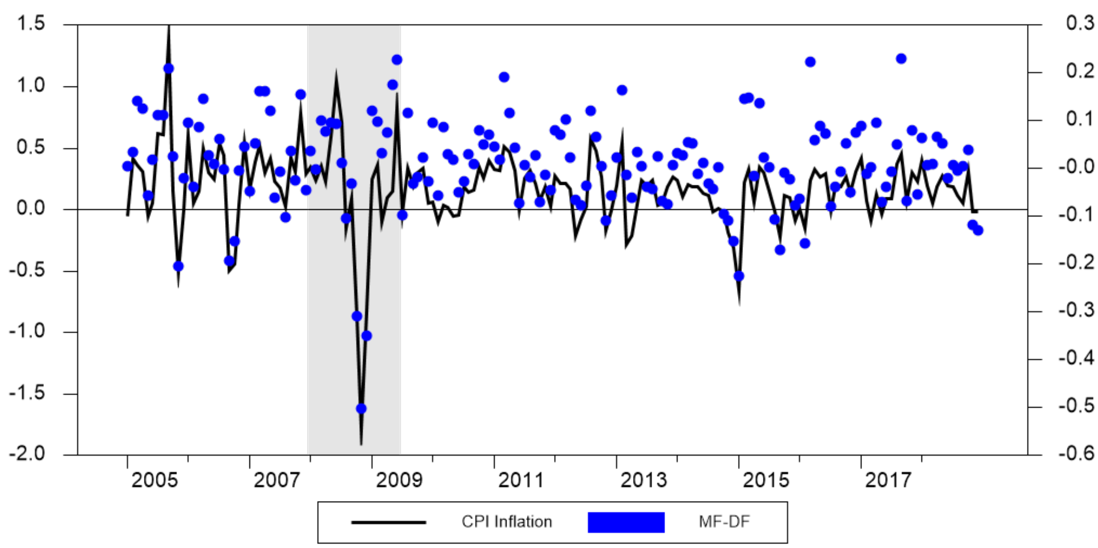

Figure 2.7: Nowcast of MF-DF

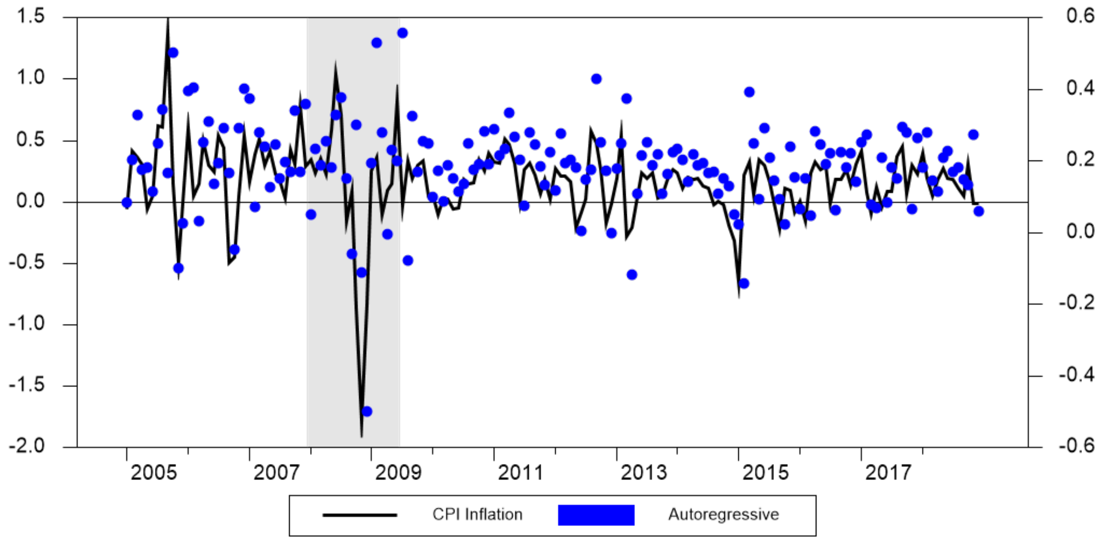

Figure 2.8: Nowcast of AR Model

of the reason is the ignorance of the slowly decreasing mean of inflation. Similar results also appear in the MF-DF model estimation. Our proposed model adds two features to the dynamic factor model in which we allow stochastic volatility and nonstationarity of inflation process. The forecasting result clearly shows that these two features are important in improving the accuracy of nowcasting inflation at zero and one month horizon.

In addition, the forecasting performance of all models seems to change over sub-periods. Inflation is more difficult to forecast during the recessions. As can be seen, the RMSE increased substantially during the Great Recession. Our result is consistent with the literature that both GDP growth and inflation are more difficult to forecast in recessions than that in expansions (Chauvet and Potter, 2013). One noteworthy phenomenon is that inflation become increasingly easy to forecast after the financial crisis. The RMSE of the post recession period drops almost 50% compared with the pre-recession period.

2.5 Summary of Chapter 2

This chapter applies the mixed-frequency unobserved component model with stochastic volatility to nowcast monthly CPI inflation in US. Nowcasts could be flagged in the real-time as new data are released. The framework allows ragged-edge data, publication lags and non-synchronization in real time nowcasting. Differently from existing mixed-frequency inflation forecasting model, our setup allows for slowly-moving local mean in inflation and the random shifts in the volatility.

We evaluate the performance of univariate and multivariate econometric models that can be useful for earlier assessments of inflation. Consistently with findings in the literature, we find that forecast accuracy improves significantly when adding high frequency inflation indicators in the model. These indicators include energy price, commodity price, exchange rate and early survey price index.

Our paper supports the desirability of using models that account for a slowly-varying inflation trend. The changing time series properties of inflation imparts the forecasting performance of most univariate and activity-based inflation forecast (Stock and Watson, 2007). When allowing for local mean and stochastic volatility, the performance of the mixed frequency model substantially increases. Given the similar time series properties shared in the global inflation, the nowcasting model developed in this paper has the potential to reduce forecast errors of inflation in euro countries. An open question for further investigation is that whether a similar model could also be useful to forecast quarterly inflation or inflation at longer horizons.

Chapter 3

The Term Structure of Inflation Expectation

Preview of Chapter 3

In this chapter, we use a Nelson-Siegel Dynamic Factor model to fit term structure of inflation expectation and describe its dynamics over time. The extracted inflation factors can be viewed as the level, slope and curvature of the term structure of inflation expectation. We also show that a decomposition of the yield curve slope into its expectation and risk premium components helps disentangle the channels that connect fluctuations in Treasury rates and the future state of the economy. In particular, a change in the yield curve slope due to expected real interest path and inflation expectation path, is associated with future industrial production growth and probability of recession.

3.1 Introduction to the Term Structure of Inflation Expectation

There have been major advances in extracting inflation expectations from nominal interest rates, inflation-indexed real bonds and swaps in recent years. However, few attention has been paid to the dynamics of term structure of inflation expectation. The term structure of inflation expectation is the relationship between inflation expectation and different forecast horizons. The graphed term structure of inflation expectation should be a continuous curve of inflation expectations from the near term to the long end, analogous to a yield curve. The term structure of inflation expectation is important for two reasons. First, inflation expectations held by household and private investors are of central importance to both policymakers and market practitioners. Long-term inflation expectations are key determinants of future inflation and output growth. Thus central banks over the world keenly monitor the dynamics of inflation expectation.

Herein, we also highlight the information contained in the term structure of inflation expectation. Inflation expectation is an important component of nominal interest rate, it helps disentangle the channels that connect fluctuations in Treasury rates and the future state of the economy. A vast literature has shown that the term structure of interest rates is useful for forecasting future economic activity, including output growth, inflation and future recessions. Examples include Harvey (1988, 1989), Estrella and Hardouvelis (1991), Mishkin (1990), Estrella and Mishkin (1998), Hamilton and Kim (2002), and among others. However, there is evidence that the predictive power of the spread is not stable over time (Chauvet and Potter, 2002, 2005; Stock and Watson, 2003, Bordo and Haubrich, 2008).

Although the literature has broad agreement that yield curve contains information about current and future economic conditions, it has been less successful at establishing why such an empirical association holds and why the relationship may have shifted. The term structure of inflation expectation and its dynamics help break down the nominal yield into inflation expectation, real interest and risk premium. These components of expectation and risk premium should contain different information about future economic scenarios that may help explore the distinct effects of these channels.

In this chapter, we investigate what the term structure of asset-price based inflation expectation can tell us about future economic condition. First, we model the dynamics of inflation expectation over time and across horizons. A Nelson-Siegel dynamic factor model is used to fit the term structure of inflation expectation and summarize the term structure of inflation with three factors. The factors can be viewed as level, slope and curvature. The end result is a smooth, continuous curve with inflation expectations from 1 year to 10 years ahead. The dynamic factor model fits the term structure of inflation expectation quite well with reasonably small measurement errors. The level factor summarizes the long term inflation expectation, the slope factor approximates the difference between short end and long end inflation forecast. This model has the substantial flexibility required to match the changing shape of the term structure of inflation expectation. More importantly, it fits into the literature of modeling yield curve.

In addition, this chapter investigates the separate contributions of expected changes in real interest rate, inflation expectation and term premium in the yield curve. We begin with a review of the predictive relationship between nominal yield spread and future real output growth, inflation and probability of recession in various favors. We find that the predictive power of the yield spread for future industrial production growth has declined at all forecasting horizons since 1990s. Moreover, the yield slope tends to be only statistically significant to forecast industrial production growth 6 quarters ahead. We decompose the nominal spread into the changes in real interest rate, inflation expectation and the term premium. The in-sample estimation results show that the term premium component appears to have lost the predictive power significantly while the predictive power of the inflation expectation slope and real interest spread has remained.

This paper is related to the literature in the following ways. First, this paper extends the existing work of term structure of inflation expectation. Aruoba (2019) uses inflation expectations from the Survey of Professional Forecasters (SPF), the Blue Chip Economic Indicators and Blue Chip Financial Forecasts to estimate a term structure of inflation expectation ranging from 3 months to 10 years. The purpose of his paper is to fill in the missing forecast horizons in the survey data and to calculate the corresponding real interest rates. Without explicitly assuming a Nelson-Siegel framework, we model the dynamics of inflation expectation from a curve fitting perspective. There are two general sources for data on inflation expectation. The widely used inflation expectations are from survey data, such as consumer, business or professional forecasters. Despite the outstanding forecast performance, survey inflation expectation suffers from the limited maturities and infrequency. For example, Survey of Professional Forecasters only contains inflation forecast one year and ten years ahead. Moreover, The survey is conducted quarterly. Therefore, one can only use limited information to investigate the dynamics of inflation expectation. In this paper, we use a panel of inflation expectations over the full range of forecasting horizons to estimate the term structure of inflation, which can avoid the sparsity of survey data.

Second, there has been resurgent interest in the literature to examine the usefulness of term spread (yield curve) as a leading indicator of economic activity. Part of the reason is the continuous evidence on the instability of the predictive relationship. Hamilton and Kim (2002) decompose the nominal spread into expectation component and term premium. They find that both components have significant predictive power in forecasting real output growth. Kim and Park (2018) use the same method and extend the sample to the recent decade to investigate the stability of such association relation. Their results show that the term premium has lost its predictive power since 1980s. In addition, the expectation also has multiple components, which represent the change in inflation expectation and real interest rate respectively. It is difficult to differentiate the separate contributions of these two components in their framework. To the best of our knowledge, this chapter is the first to explore the possible effect of inflation expectation on the predictive power of yield curve. In particular, our result sheds light on the shifted relationship between nominal yield spread and real activities.

The chapter is organized as follows. In section 2, I motivate the Nelson-Siegel Dynamic Factor model to fit the term structure of inflation expectation and describe the model. Then the model is used to extract latent factors in the term structure. In section 3, I review the predictive power of yield spread in forecasting future economic activities. Moreover, the yield spread is decomposed into various components to further examine their distinct effects. Section 4 concludes this chapter.

3.2 The Term Structure of Inflation Expectation

3.2.1 Motivation

It is widely known that the slope of nominal yield curve is a robust and powerful predictor of future macroeconomic dynamics. This suggests that the shape of yield curve contains market expectations of future fundamentals. In particular, when entering recessions, short-term nominal yield exceeds longer-maturity bond yield, which makes an inverted yield curve. The most used indicators are the difference between 10-year T-bond yield and 3-month T-bill rate and the difference between 10-year T-bond yield and 2-year T-bond yield. However, nominal yield curve contains multiple information, including expected change in real interest rate and inflation expectation. Mishkin (1990a, 1990b) show that for maturities of six months or less, the yield curve provide almost no information about future inflation but does provide information about real interest rate. However, yield curve contains information about inflation change over long run. Then it raises the question of how much of the predictive power should we assume to be part of inflation expectation? We start from two observations. First, we plot the difference between 2 year and 10-years break-even inflation rate along with current trend inflation in Figure 3.1. Here the break-even inflation rate is the difference between interest rates on a nominal Treasury bond (that is, one not indexed to inflation) and Treasury Inflation Protected Securities (TIPS)[1]. There are two things noteworthy. First, the difference of inflation expectation and current trend inflation are highly correlated[2]. This indicates that the spread between short-term inflation and long-run inflation expectation may contain information about the current economic condition. This phenomenon is very similar to the forecasting exercise that uses yield spread as a predictor of short rates and future real output growth. In addition, this is consistent with Frankel and Lown(1994)'s finding that the slope of yield curve indicate the inflation change one year ahead.

Second, the two series are negatively correlated(we use 2-year inflation expectation minus 10-year inflation expectation).In particular, when the difference of inflation expectation declines, trend inflation increases. When the monetary policy is tight, long term bond yield increases less than the short term bond yield, which makes an inverted yield curve. Additionally, long term inflation expectation also decreases less than short term inflation forecast because long term inflation expectation are well anchored. Thus, the negative correlation is not surprising since it reflects the inflation's reaction to monetary policy.

[1]Our nominal yield data are obtained from Gurkaynak, Sack, and Wright (2007). These yields are constructed by fitting a zero-coupon yield curve of the Svensson (1994) type to a large pool of underlying off-the-run Treasury bonds on a daily basis. The authors demonstrate that the model fits the underlying bonds extremely well and, by implication, provides a very good approximation to the Treasury zero-coupon yield curve. The TIPS yield data are obtained from the Federal Reserve Board of Governors; see Gurkaynak, Sack, and Wright (2010).

[2]We use the the difference between 10-year break-even inflation and 2-year break-even inflation. The trend inflation is estimated using method in Chapter 1.

Figure 3.1: Break-even Inflation and Trend Inflation

Thus it is reasonable to guess that the term structure of inflation expectation may have the same predictive power as nominal yield curve.

Third, the term structure of inflation expectation and yield curve have some points of similarity in shapes, such as inverted slope and hump shape. Figure 3.2 shows the shape of term structure of inflation expectation in four different periods. It can be seen that the term structure of inflation expectation has a variety of shapes like upward sloping, downward sloping, hump and inverted hump. This similarity suggests that we may adopt the yield curve modeling method to fit the term structure of inflation expectation.

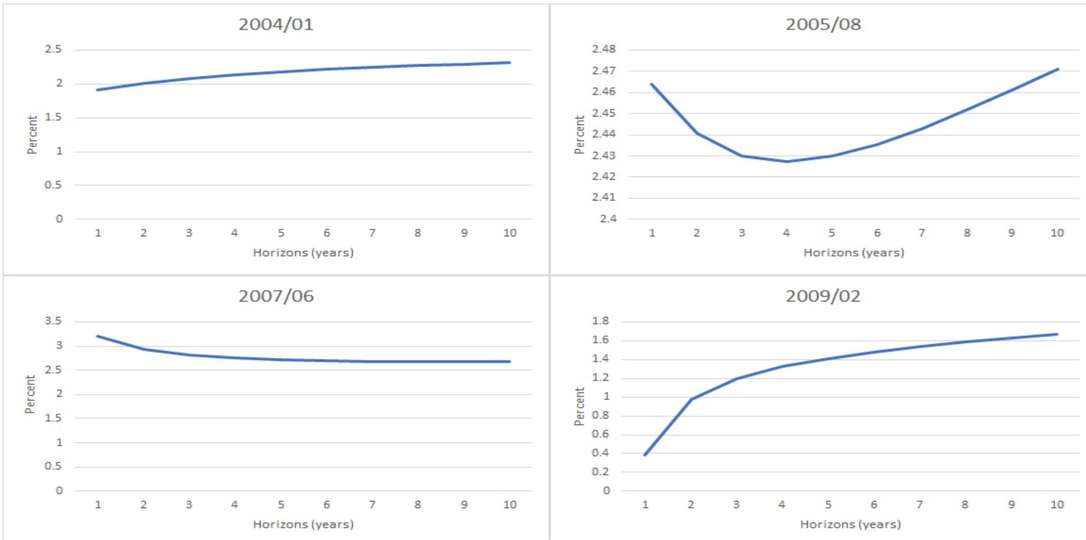

Figure 3.2: Shape of the Term Structure of Inflation Expectation

3.2.2 Modeling Inflation Expectation

In this section, we use the Nelson-Siegel term structure representation with dynamic factor form to fit a parametric curve of the term structure of inflation expectation. This model is based on the workhorse yield curve model introduced by Nelson and Siegel (1987) which is extended to a factor structure by Diebold and Li (2006). This class of model has been widely employed to model the yield curve (Diebold et. al., 2006; Christensen, et. al., 2011). It is flexible to fit the cross-section inflation expectations over forecasting horizons, and also describe the dynamics of inflation expectation over time. The Nelson-Siegel yield curve model (the NS model) links the yield of a bond with τ months to maturity, $y_t(\tau)$, to three latent factors as:

$$y_t(\tau) = L_t + (\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau})S_t + (\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau})C_t + \varepsilon_t. \tag{3.1}$$

The Nelson-Siegel yield curve can be viewed as a constant plus a polynomial times an exponential decay term, in which the exponential decay rate is governed by parameter λ_t. L_t, S_t and C_t are three latent factors that summarize the dynamics of the yield. L_t is viewed as a long-term factor, which is called level factor. S_t is called slope and can approximate the term spread. The loading on C_t is called curvature which summarizes the medium-term dynamics. Many studies show that the Nelson-Siegel dynamic factor model is a very good representation of the yield curve both in cross-section and over time.

Therefore it is a natural application of Nelson-Siegel dynamic factor model to represent inflation expectation over the entire forecasting horizons, at least from a curve-fitting perspective. Let $\pi_t(\tau)$ denote the τ-month inflation expectation from the end of month t to the end of month $t+\tau$. Inflation expectation is approximated as:

$$\pi_t(\tau) = L_t + S_t(\frac{1-e^{-\lambda\tau}}{\lambda\tau}) + C_t(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}) + \varepsilon_t. \tag{3.2}$$

And the three factors evolve according to a persistent independent autoregressive process:

$$L_t = \mu_L + \rho_1(L_t - \mu) + \eta_{1t} \tag{3.3}$$

$$S_t = \mu_S + \rho_2(S_t - \mu) + \eta_{2t} \tag{3.4}$$

$$C_t = \mu_C + \rho_3(C_t - \mu) + \eta_{3t} \tag{3.5}$$

where $\phi(L)$ is the log polynomial and has all roots lie outside of the unit cycle, and η_{it} is white noise.

The model can be cast into the state-space form:

$$\pi_t = C_t^{'} F_t + w_t$$

$$F_t = \mu + A_t F_{t-1} + v_t$$

$$w_t \sim N(0, H), \quad v_t \sim N(0, Q)$$

where π_t is an $N \times 1$ vector of observed inflation expectations with different forecasting horizons. State vector F_t includes the latent factors, w_t and v_t are Gaussian and orthogonal measurement and transitory shocks. Then we can use Kalman filter and smoother to estimate the model. The optimal filtered and smoothed estimates of the latent factors can be obtained in a conventional way.

3.2.3 Inflation Expectation Data

One of the key aspects of our paper is the use of asset-price based inflation expectation. The pioneer works use survey-based inflation expectation because professional survey data have outstanding forecast performance. However, survey data is only available for limited time horizons. In contrast to surveys, asset prices provide high frequency observations of expected inflation over a wide range of horizons. For example, the principal and coupon payments of U.S. Treasury inflation-protected securities (TIPS) vary according to changes in the Consumer Price Index (CPI). Thus, the break-even inflation (BEI) rates is closely monitored by central banks as high-frequency indicators of inflation expectations. Our data is obtained from The Federal Reserve Bank of Cleveland (hereafter Fed inflation expectation). The method is based on Haubrich et al. (2011) who use nominal yields, inflation swaps, and survey data to extract the expected inflation rate (CPI) over 30 years. This model-based inflation expectation get around the risk premium and liquidity problem in TIPS real yield. The Fed inflation expectation is sampled at monthly frequency. Our sample ranges from January 1998 to December 2018, including inflation expectations covering horizons 1 year to 10 years.

3.2.4 Model Implementation

We assume the latent factors follow independent AR(1) process for transparency and parsimony. Suppose there are N observed cross-section inflation expectations. Then the measurement equation is written as:

$$\begin{bmatrix} \pi_t(\tau_1) \\ \pi_t(\tau_2) \\ \vdots \\ \pi_t(\tau_N) \end{bmatrix} = + \begin{bmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & 1-e^{-\lambda\tau_2} & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} - e^{-\lambda\tau_2} \\ \vdots & \vdots & \vdots \\ 1 & 1-e^{-\lambda\tau_N} & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} - e^{-\lambda\tau_N} \end{bmatrix} \begin{bmatrix} L_t \\ S_t \\ C_t \end{bmatrix} + \begin{bmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_N) \end{bmatrix}$$

Table 3.1: The Term Structure of Inflation Expectation Parameter Estimates

	L_{t-1}	S_{t-1}	C_{t-1}	μ
L_t	0.996			0.044
S_t		0.944		-0.001
C_t			0.950	0.007

And the transition equation takes the form:

$$\begin{bmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{bmatrix} = \begin{bmatrix} \rho_1 & 0 & 0 \\ 0 & \rho_2 & 0 \\ 0 & 0 & \rho_3 \end{bmatrix} \begin{bmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{bmatrix} + \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \\ \eta_{3t} \end{bmatrix}$$

The model is estimated using maximum likelihood via the prediction-error decomposition and Kalman filter. Estimates of the latent factors are obtained using the Kalman smoother because the paper focuses on the historical analysis. The results are reported in Table 3.1. The estimates of the transition matrix indicates that the dynamics of L_t, S_t and C_t are highly persistent, with the own-lag coefficients of 0.996, 0.944 and 0.950, respectively. The mean level is negligible and not statistically significant. Finally, the estimated λ is 0.0218, which imply the loading on the curvature factor is maximized at maturity of less than 6.8 years.

Table 3.2 shows the mean and standard deviation of the measurement error. The three-factor model fits the term structure of inflation expectation quite well. The mean error is statistically negligible at all horizons and the model perform even better for the medium-term inflation expectation.

Table 3.2: Summary Statistics for Measurement Errors

Horizons	Mean	Standard deviation
1	-0.02***	0.14
2	-0.004***	0.026
3	-0.0008***	0.004
6	-0.0001***	0.0005
10	0.0008***	0.0037

Note: *** means the value is statistically significant at 1% level.

We interpret the three latent factors as level, slope and curvature. The three factors are comparative assessment of long-term, short-term and medium-term dynamics of inflation expectation. First, we run a linear regression and obtain

$$\pi_t(120) = 1.03L_t + \varepsilon_t. \tag{3.6}$$

The coefficient tends to unity as we increase horizon τ. Thus L_t can be interpreted as level factor which describes the long term inflation dynamics and approximates $\pi_t(\tau = \infty)$. Alternatively, note that an increase in level factor increases all inflation expectation equally, as the loading is identical at all horizons. Second, an approximation of the slope is the difference between short-term and long-term inflation expectations. We use the difference between 10-year and 1-year inflation expectation and regress on the slope factor:

$$\pi_t(\tau = 120) - \pi_t(\tau = 12) = 0.77S_t + \varepsilon_t. \tag{3.7}$$

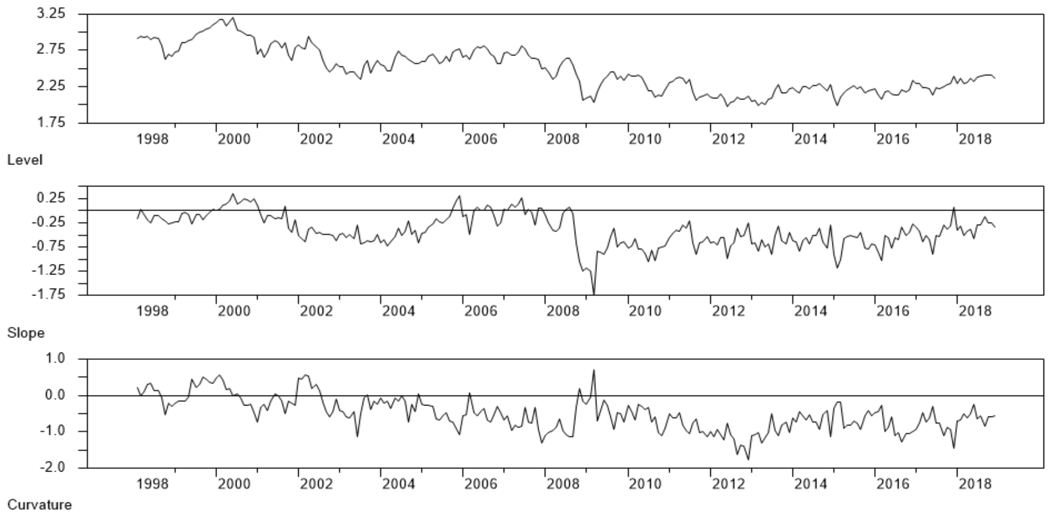

Figure 3.3: Estimates of level, slope and curvature

Finally, the third factor which summarizes the medium-run dynamics is closely related with curvature of the curve depicted in Figure 3.2. We use the twice five-year inflation expectation minus the sum of the ten-year and one year inflation expectation to approximate the curvature. We obtain the following estimation result:

$$2 * \pi_t(60) - \pi_t(\tau = 12) - \pi_t(\tau = 120) = 0.14C_t + \varepsilon_t. \tag{3.8}$$

The estimates of the three factors are presented in Figure 3.3. The level factor has a slight downward trend, which is possibly the continuation of the downward inflation trend that starts during 1980s. The slope factor is negative during most of the sample period, raises above zero briefly just before the 2001 and 2008 recession.

3.3 The Information in Term Structure of Inflation Expectation

In this section, we explore the predictive power of yield curve and its relationship to the term structure of inflation expectation. The predictions using yield curve come in two general favors. The first one uses the term spread or factors extracted from yield curve to predict the growth rate of real output and inflation rate at some point in the future, usually at horizons over 2 quarters to 8 quarters. The second one uses the same variables to forecast the probability of future recessions. To investigate the separate contributions of different yield curve components, we first develop the decomposition of yield curve.

Following Hamilton and Kim (2002), consider the following decomposition of nominal yield at n maturity.

$$i_t^n = \frac{1}{n}\sum_{j=0}^{n-1} E_t i_{t+j}^1 + TP_t^n \tag{3.9}$$

where $E_t i_{t+j}^1$ denotes the market's expectation of i_{t+j}^1 at time t. The expected value of i_{t+j}^1 is the sum of expected value of inflation and real interest rate. We further decompose the nominal yield into three components:

$$i_t^n = \frac{1}{n}\sum_{j=0}^{n-1} E_t(\pi_{t+j}^1 + r_{t+j}^1) + TP_t^n \tag{3.10}$$

where $E_t \pi_{t+j}^1$ and $E_t r_{t+j}^1$ denote the market's expectation of inflation and real interest at time t. Term spread is the difference between long term and short term nominal yield, which is also called the slope of the yield curve. Following equation (3.11), the slope of yield curve (*slope*) is decomposed into the slope of its expectation and risk premium components:

$$slope = \pi^{EP} + r^{EP} + TP^{EP} \tag{3.11}$$

where π^{EP} denote the average expected path (EP) of inflation over the bond's life and r^{EP} denotes the average expected path of the real rate over the same horizon. Fluctuations in each of these terms could be associated with different growth prospects.

3.3.1 Industrial Production and Expectation Components

Following the literature, we use industrial production and the following regressions to examine the forecasting ability of the yield curve:

$$IP_t^h = \beta_0 + \beta_1 Slope_t + \varepsilon_t \tag{3.12}$$

where IP_t^h is the annualized industrial production over the next h months. We also estimate the following equation that control for supplemental variables X_t:

$$IP_t^h = \beta_0 + \beta_1 Slope_t + \gamma X_t + \varepsilon_t. \tag{3.13}$$

Because current and lagged rate of industrial production growth may be useful for forecasting future industrial production, they are included in the estimated equation. In addition, many think that yield curve can predict future real economic activities because it reflects the expectation of monetary policy. To investigate whether the slope factor has

Table 3.3: Industrial Production and Yield Spread

$$IP_t^h = \beta_0 + \beta_1 Slope_t + \varepsilon_t$$

k(months ahead)	β_0	β_1	R^2
1	1.429(1.098)	-0.152(0.731)	0.0005
6	0.837(1.127)	0.148(0.586)	0.001
12	0.047(1.260)	0.526(0.447)	0.023
18	-0.592(1.405)	0.811*(0.445)	0.080
24	-1.138(1.337)	1.039**(0.449)	0.189

Note: ***, ** and * denote statistically significant at the 1%, 5%, and 10% levels respectively.

additional information beyond that contained in monetary policy, we include the Federal Fund Rates(FFR) as the contemporaneous measure of monetary policy.

Table 3.4: Industrial Production, Yield Spread and IP Lags

$$IP_t^h = \beta_0 + \beta_1 Slope_t + \sum \gamma_i IP_{t-i}^1 + \varepsilon_t$$

k(k months ahead)	β_0	β_1	R^2
1	0.209(0.648)	0.127(0.338)	0.238
6	0.349(0.791)	0.146(0.354)	0.372
12	-0.187(1.097)	0.499(0.364)	0.212
18	-0.950(1.382)	0.877**(0.439)	0.195
24	-1.610(1.366)	1.168**(0.464)	0.310

Note: ***, ** and * denote statistically significant at the 1%, 5%, and 10% levels respectively.

Table 3.3 to Table 3.5 show the results of the estimation with different control variables. The OLS estimated coefficients on the yield spread are statistically significant over 18-24 months forecasting horizons. This result is in line with Kim and Park(2018),in which they use data after 1984. Compared with previous literature, the yield spread has lost its predictive power at the short end (less than 4 quarters) after 1990s. The estimation results with lagged industrial production and federal fund rates are very similar and confirm the weakened predictive relation between yield spread and future real activity.

Table 3.5: Industrial Production, Yield Spread and FFR

$$IP_t^h = \beta_0 + \beta_1 Slope_t + \gamma FFR_t + \varepsilon_t$$

k(months ahead)	β_0	β_1	R^2
1	1.280(0.936)	0.016(0.623)	0.163
6	0.660(0.942)	0.323(0.475)	0.183
12	-0.06(1.113)	0.657*(0.390)	0.188
18	-0.651(1.307)	0.900**(0.436)	0.196
24	-1.193(1.282)	1.103**(0.447)	0.264

Note: ***, ** and * denote statistically significant at the 1%, 5%, and 10% levels respectively.

Following Equation (3.10), we decompose the yield slope into an expectation term and risk premium term:

$$IP_t^h = \alpha_0 + \alpha_1 EP_t + \alpha_2 TP_t + \varepsilon_t \tag{3.14}$$

where EP_t is the effect of the expected future changes in short rate and TP_t is the effect of the term premium. Unlike Hamilton and Kim (2002), who use instrumental variables to the unobserved two components, we use the expectation term and term premium explicitly estimated from financial data.

Table 3.6: Industrial Production, Expectation and Term Premium

$$IP_t^h = \alpha_0 + \alpha_1 EP_t + \alpha_2 TP_t + \varepsilon_t$$

k(months ahead)	α_0	α_1	α_2	R^2
1	1.342(0.903)	-0.453(0.728)	0.321(1.013)	0.002
6	0.917(0.990)	-0.111(0.732)	0.455(0.727)	0.003
12	0.332(1.148)	0.325(0.662)	0.612(0.534)	0.014
18	-0.141(1.271)	0.802(0.601)	0.514(0.501)	0.054
24	-0.620(1.242)	1.121**(0.569)	0.617(0.498)	0.146

Note: ***, ** and * denote statistically significant at the 1%, 5%, and 10% levels respectively.

Table 3.6 reports the OLS estimation of equation (3.15). The results show that the two components have different effect. In particular, the term premium has lost its predictive power in our sample for all the forecasting horizons. Only the coefficients on expectation component (EP) are statistically significant over 18-24 months horizon. Previous research shows that the term premium helps forecast future output growth. This implies that the decrease in the predictive power of yield spread mainly results from the significant reduction in the forecasting power of the term premium.

The expectation component also contains inflation expectation and real rate expectation. The distinct effects of these two components are very important. First, some policy makers argue that the deceasing variability of inflation expectation is the main reason of weakened predictive power of yield spread. The stable long term inflation expectation increases the credibility of central bank maintaining stable price level, thus weaken the association between real activities and inflation expectation. Thus we generalize the decomposition in equation (3.12):

$$IP_t^h = \gamma_0 + \gamma_1 \pi_t^{EP} + \gamma_2 r_t^{EP} + \gamma_3 TP_t^{EP} + \varepsilon_t. \tag{3.15}$$

where $\pi_t^{EP}$is measured by the difference between 10 year ahead inflation expectation and 1 year ahead inflation expectation. r_t^{EP} is the change in real interest rate. The estimation results in Table 3.7 are noteworthy. First, in the case of the inflation expectation component, the coefficients are statistically significant at forecast horizons one year to two years ahead. This is consistent with the results obtained from Table 3.6. A fall in the inflation expectation slope increases the growth rate of industrial production in the future. This also indicates that most of the variation in inflation expectation result from demand shock or monetary policy shock, in that a high future inflation expectation may be a signal of easy monetary policy. Second, real interest rate slope contains no predictive information over 1-18 month horizon. Our results about the contribution of inflation expectation slope is inconsistent with Benzoni, et al.(2018). In their paper, the measure of inflation expectation is the spread between the model's forecast for inflation six-quarters ahead and its projection of average inflation over the next three months. It turns out this measure of inflation expectation slope

Table 3.7: Industrial Production, Inflation Expectation Slope, Real Yield Slope and Term Premium

$$IP_t^h = \gamma_0 + \gamma_1 \pi_t^{EP} + \gamma_2 r_t^{EP} + \gamma_3 TP_t^{EP} + \varepsilon_t$$

k(months ahead)	γ_0	γ_1	γ_2	γ_3	R^2
1	0.395(1.078)	0.683(2.401)	0.980(0.673)	-0.592(1.294)	0.031
6	0.217(1.130)	3.197(2.430)	0.770(0.553)	-0.534(1.031)	0.044
12	-0.075(1.203)	4.692*(2.657)	0.743(0.465)	-0.448(0.760)	0.102
18	-0.256(1.163)	5.326**(2.498)	0.861*(0.452)	-0.611(0.634)	0.185
24	-0.517(0.984)	5.202***(1.960)	1.033***(0.382)	-0.578(0.490)	0.292

Note: ***, ** and * denote statistically significant at the 1%, 5%, and 10% levels respectively.

does not improve the model fit and is excluded in the regression. Using a inflation slope at long end, inflation expectation component do help predict the future path of industrial production.

Since different measures of inflation slope give distinct results about the contribution of inflation expectation term. We use the whole information in the term structure of inflation to examine whether the ex ante inflation plays a part in the estimation. We run the following regression:

$$IP_t^h = \gamma_0 + \gamma_1 S_t + \gamma_2 C_t + \gamma_3 r_t^{EP} + \gamma_4 TP_t^{EP} + \varepsilon_t. \tag{3.16}$$

where S_t is the estimated slope factor in the term structure of inflation expectation, C_t is the estimated curvature factor. We report the estimation results in Table 3.8. Most of the results are similar to Table 3.6. The only difference is that the curvature factor is not useful

Table 3.8: Industrial Production, Inflation Factor,Real Term and Term Premium

$$IP_t^h = \gamma_0 + \gamma_1 S_t + \gamma_2 C_t + \gamma_3 r_t^{EP} + \gamma_4 TP_t^{EP} + \varepsilon_t.$$

k(months ahead)	γ_0	γ_1	γ_2	γ_3	γ_4
1	3.67(1.762)	0.58(2.017)	4.38*(2.439)	1.52**(0.786)	-2.53(2.055)
6	2.56*(1.527)	-1.81(1.814)	3.84(2.391)	0.93*(0.520)	-1.82(1.660)
12	1.34(1.384)	-3.37*(1.897)	3.03(2.335)	0.60(0.385)	-1.11(1.283)
18	0.53(1.293)	-4.14**(1.812)	2.42(2.047)	0.55(0.378)	-0.88(1.075)
24	0.13(1.145)	-4.11***(1.960)	2.23(1.455)	0.72**(0.293)	-0.78(0.783)

Note: ***, ** and * denote statistically significant at the 1%, 5%, and 10% levels respectively.

to predict industrial production.

3.3.2 Probability of Recession and Yield Spread

Yield spread is more successful in predicting future recessions. The forecasting horizons are normally 4-8 quarters. We use the same independent variables in Section 3.1 but apply a Probit model to forecast the probability of recession h months ahead. Y_t^h is a binary variable of recessions that indicates the presence ($Y_t = 1$) or absence ($Y_t = 0$) of a recession in the future h months with the NBER recession dating. We estimate a model that relates the indicator variables to the information of yield curve:

$$Pr(Y_t^h = 1|X_t) = F(\beta X_t)$$

where Pr denotes probability, F is the cumulative normal distribution, and X_t represents the various slope factors in the last section. The results are reported in Table 3.9-3.12. We summarize the findings as follows. First, the nominal yield slope, either represented with yield spread or estimated slope factor, helps predict the recession one year ahead. Second, the effects of term premium is in line with the estimation of industrial production. The variation of term premium has lost the ability to forecast future recessions. Third, we find that the coefficients of inflation expectation slope and real yield slope are statistically significant. This means that it is the expectation of future monetary policy, measured by changes in the slope of the expected real rate path that contains the recession signal.

Table 3.9: Recession and Yield Spread

$$Pr(Y_{t+h} = 1|Slope) = F(\beta_0 + \beta_1 Slope_t)$$

k(months ahead)	β_0	β_1
12	-0.271(0.160)	-0.321***(0.077)
18	0.186(0.163)	-0.473***(0.079)
24	0.593(0.172)	-0.592***(0.082)

Note: ***, ** and * denote statistically significant at the 1%, 5%, and 10% levels respectively.

Table 3.10: Recession, Expectation term and Term premium

$$Pr(Y_{t+h} = 1 | EP_t, TP_t) = F(\beta_0 + \beta_1 EP_t + \beta_2 TP_t)$$

k(months ahead)	β_0	β_1	β_2
12	-0.540***(0.162)	-0.430***(0.107)	0.085(0.175)
18	-0.148(0.163)	-0.675***(0.108)	0.071(0.177)
24	0.227(0.170)	-0.909***(0.115)	0.086(0.178)

Note: ***, ** and * denote statistically significant at the 1%, 5%, and 10% levels respectively.

Table 3.11: Recession, Inflation Expectation, Real Spread and Term premium

$$Pr(Y_{t+h} = 1 | \pi_t^{EP}, r_t^{EP}, TP_t) = F(\beta_0 + \beta_1 \pi_t^{EP} + \beta_2 r_t^{EP} + \beta_3 TP_t)$$

k(months ahead)	β_0	β_1	β_2	β_3
12	-0.706***(0.188)	-2.417***(0.402)	-0.654***(0.093)	0.915***(0.268)
18	-0.335*(0.191)	-2.865***(0.418)	-0.810***(0.101)	0.969***(0.265)
24	0.055(0.206)	-3.470***(0.468)	-1.045***(0.120)	1.141***(0.278)

Note: ***, ** and * denote statistically significant at the 1%, 5%, and 10% levels respectively.

Table 3.12: Recession, Inflation Factor, Real Spread and Term premium

$$Pr(Y_{t+h} = 1|S_t, r_t^{EP}, TP_t) = F(\beta_0 + \beta_1 S_t + \beta_2 r_t^{EP} + \beta_3 TP_t)$$

k(months ahead)	β_0	β_1	β_2	β_3
12	-0.348**(0.182)	1.577***(0.301)	-0.344***(0.063)	0.428***(0.220)
18	0.067(0.191)	2.110***(0.316)	-0.445***(0.068)	0.474***(0.228)
24	0.511(0.211)	2.667***(0.353)	-0.594***(0.079)	0.595***(0.245)

Note: ***, ** and * denote statistically significant at the 1%, 5%, and 10% levels respectively.

3.3.3 Why the Yield Curve Help Forecast Future Economic Activities?

In this section, we discuss the potential reasons why the yield curve can help forecast future economic activities and what is the change in the predictive power. In general, the yield on a bond with long maturity reflects market's expectation of the path for short term interest rate over the life of the bond. Thus the view about monetary policy and business condition would affect the expected path of bond yield. First, current monetary policy has a significant influence on the yield spread and hence on real activity over the next several quarters. Suppose that the Fed adopts a tight monetary policy which would raise the short term interest. If the current short rate is higher than the short rate in the future, then the expected long term yield will be less than the short term yield according to equation (3.10). This lead to the flattened yield curve.

Second, market's expectation about future business condition may also be reflected in the yield spread. If private investors expect a recession in the near future, they likely either anticipate the Fed will cut the future policy rates during the downturn, or the future rate of return to investment to fall. In this scenario, the future short rate is expected to be low compared with current short rates. This in turn will reduce long-term yield and result in an flattened yield curve.

Third, the yield spread also reflects market's sentiment about risk. The term premium represents the compensation for the risk of changes in short term rates and future inflation variation. If the inflation risk increases in the economic boom, the long term rates will rise more than the short rate. Our estimation with the sample after 1990s show that the expectation effect account for the predictive power of the yield curve. However, the term premium has lost its forecasting ability in our estimation. Part of the reason is the decreasing inflation variability and well anchored inflation expectation. This can be seen in our analysis in Chapter 1. Inflation variability and persistence of inflation has dropped quite dramatically since 1990s. This fall in variability most likely results in a reduced inflation risk premium for long bonds, which lowers long-term interest rates and weaken the predictive ability of term premium.

Fourth, based on our estimation results, the expectation component is an excellent indicator of future economic activity. This relationship, however, is only one part of the explanation for the yield curve's usefulness. The expected interest rate path also have multiple components. Expected rate depends on market participants' views on the future evolution of both inflation and real interest rates. Alan Greenspan argued that "the key component from which the yield curve slope derives much of its predictive power for future GDP growth" is the real rate spread. That is, when the real federal funds rate is higher than its long-run level, the chance of a recession increases. Our result confirms that a change in the expected path of real interest rate is associated with an change in future industrial production and probability of recession. However, real interest is not the only determinant, the path of inflation expectation is also significant in explaining the relationship between yield spread and economic activity. In particular, the inflation expectation slope account for a larger portion of the variation in the estimation than real yield spread. This is not surprising because inflation tends to be positively related to real activity. Notice that the inflation expectation has been stable since 1990s, which means a credible regime of the monetary policy against inflation. In this case, an shock that affect inflation will increase short rates, but not long rates, because long-term expectations of inflation don't change. Thus this relationship will twist the yield curve, distorting the message of the underlying real curve, which weaken the reliability of the predictive relationship.

3.4 Summary of Chapter 3

In this chapter, we use a Nelson-Siegel Dynamic Factor model to fit term structure of inflation expectation and describe its dynamics over time. The extracted inflation slope factors help decompose the yield spread into inflation expectation slope and real yield spread. The decomposition is used to investigate the distinct contributions of those components to the predictive relationship between yield spread and real economic activity. We have confirmed earlier results on the weakened usefulness of yield spread for forecasting the growth of industrial production and probability of recession. The main reason is the significant reduction in the predictive power of the term premium component. On the other hand, both the inflation expectation slope and real interest rate spread are statistically significant.

There are two points need to be noticed. First, our results hinge on the specific model to extract inflation expectation. Christensen et al.(2012) use an econometric model with no-arbitrage condition to extract inflation expectation from nominal yields and TIPS yields. Abrahams et al.(2018) apply no-arbitrage asset pricing model to break down the nominal yields with a liquidity factor. The research on the predictive power of yield curve is controversial because different measures are used in the research. Second, the predictive power of term premium should be further examined. The term premium also contains inflation risk premium and real rate risk premium, which reflect the compensation for the uncertainty associated with the future evolution of inflation and real interest rates, respectively. In this paper, we don't differentiate the two types of risk. The reason is the lack of reliable measure of these components. Thus more research is needed to estimate the unobserved components of risk premium.

Bibliography

[1] Michael Abrahams, Tobias Adrian, Richard K Crump, Emanuel Moench, and Rui Yu. Decomposing real and nominal yield curves. *Journal of Monetary Economics*, 84:182–200, 2016.

[2] Andrew Ang, Geert Bekaert, and Min Wei. Do macro variables, asset markets, or surveys forecast inflation better? *Journal of monetary Economics*, 54(4):1163–1212, 2007.

[3] S Borağan Aruoba and Francis X Diebold. Real-time macroeconomic monitoring: Real activity, inflation, and interactions. *American Economic Review*, 100(2):20–24, 2010.

[4] S Borağan Aruoba, Francis X Diebold, and Chiara Scotti. Real-time measurement of business conditions. *Journal of Business & Economic Statistics*, 27(4):417–427, 2009.

[5] Andrew Atkeson, Lee E Ohanian, et al. Are phillips curves useful for forecasting inflation? *Federal Reserve bank of Minneapolis quarterly review*, 25(1):2–11, 2001.

[6] Jushan Bai and Serena Ng. Forecasting economic time series using targeted predictors. *Journal of Econometrics*, 146(2):304–317, 2008.

[7] William A Barnett, Marcelle Chauvet, and Danilo Leiva-Leon. Real-time nowcasting of nominal gdp with structural breaks. *Journal of Econometrics*, 191(2):312–324, 2016.

[8] Michael D Bordo and Joseph G Haubrich. The yield curve as a predictor of growth: long-run evidence, 1875–1997. *The Review of Economics and Statistics*, 90(1):182–185, 2008.

[9] JÖrg Breitung and Christoph Roling. Forecasting inflation rates using daily data: A nonparametric midas approach. *Journal of Forecasting*, 34(7):588–603, 2015.

[10] Maximo Camacho and Gabriel Perez-Quiros. Introducing the euro-sting: Short-term indicator of euro area growth. *Journal of Applied Econometrics*, 25(4):663–694, 2010.

[11] Marcelle Chauvet and Simon Potter. Recent changes in the us business cycle. *The Manchester School*, 69(5):481–508, 2001.

[12] Marcelle Chauvet and Simon Potter. Predicting a recession: evidence from the yield curve in the presence of structural breaks. *Economics Letters*, 77(2):245–253, 2002.

[13] Marcelle Chauvet and Simon Potter. Forecasting recessions using the yield curve. *Journal of Forecasting*, 24(2):77–103, 2005.

[14] Marcelle Chauvet and Simon Potter. Forecasting output. In *Handbook of Economic Forecasting*, volume 2, pages 141–194. Elsevier, 2013.

[15] Jens HE Christensen, Jose A Lopez, and Glenn D Rudebusch. Inflation expectations and risk premiums in an arbitrage-free model of nominal and real bond yields. *Journal of Money, Credit and Banking*, 42:143–178, 2010.

[16] Timothy Cogley and Thomas J Sargent. Drifts and volatilities: monetary policies and outcomes in the post wwii us. *Review of Economic dynamics*, 8(2):262–302, 2005.

[17] Timothy Cogley and Argia M Sbordone. Trend inflation, indexation, and inflation persistence in the new keynesian phillips curve. *American Economic Review*, 98(5):2101–26, 2008.

[18] Francis X Diebold and Canlin Li. Forecasting the term structure of government bond yields. *Journal of econometrics*, 130(2):337–364, 2006.

[19] Francis X Diebold, Glenn D Rudebusch, and S Boragan Aruoba. The macroeconomy and the yield curve: a dynamic latent factor approach. *Journal of econometrics*, 131(1-2):309–338, 2006.

[20] Arturo Estrella and Gikas A Hardouvelis. The term structure as a predictor of real economic activity. *The journal of Finance*, 46(2):555–576, 1991.

[21] Arturo Estrella and Frederic S Mishkin. The yield curve as a predictor of us recessions. *Current issues in economics and finance*, 2(7), 1996.

[22] Jon Faust and Jonathan H Wright. Forecasting inflation. In *Handbook of economic forecasting*, volume 2, pages 2–56. Elsevier, 2013.

[23] Jeffrey A Frankel and Cara S Lown. An indicator of future inflation extracted from the steepness of the interest rate yield curve along its entire length. *The Quarterly Journal of Economics*, 109(2):517–530, 1994.

[24] Domenico Giannone, Lucrezia Reichlin, and David H Small. Nowcasting gdp and inflation: the real-time informational content of macroeconomic data releases. 2006.

[25] Jan JJ Groen, Richard Paap, and Francesco Ravazzolo. Real-time inflation forecasting in a changing world. *Journal of Business & Economic Statistics*, 31(1):29–44, 2013.

[26] James D Hamilton and Dong H Kim. A re-examination of the predictability of the yield spread for real economic activity. *Journal of Money, Credit, and Banking*, 34(2):340–360, 2002.

[27] Andrew C Harvey. *Forecasting, structural time series models and the Kalman filter*. Cambridge university press, 1990.

[28] Campbell R Harvey. The real term structure and consumption growth. *Journal of Financial Economics*, 22(2):305–333, 1988.

[29] Campbell R Harvey. Forecasts of economic growth from the bond and stock markets. *Financial Analysts Journal*, 45(5):38–45, 1989.

[30] Eric Jacquier, Nicholas G Polson, and Peter E Rossi. Bayesian analysis of stochastic volatility models. *Journal of Business & Economic Statistics*, 20(1):69–87, 2002.

[31] Sangjoon Kim, Neil Shephard, and Siddhartha Chib. Stochastic volatility: likelihood inference and comparison with arch models. *The review of economic studies*, 65(3):361–393, 1998.

[32] Edward S Knotek and Saeed Zaman. Nowcasting us headline and core inflation. *Journal of Money, Credit and Banking*, 49(5):931–968, 2017.

[33] Siem Jan Koopman and Charles S Bos. State space models with a common stochastic variance. *Journal of Business & Economic Statistics*, 22(3):346–357, 2004.

[34] Roberto S Mariano and Yasutomo Murasawa. A new coincident index of business cycles based on monthly and quarterly series. *Journal of applied Econometrics*, 18(4):427–443, 2003.

[35] Elmar Mertens. Measuring the level and uncertainty of trend inflation. *Review of Economics and Statistics*, 98(5):950–967, 2016.

[36] Frederic S Mishkin. What does the term structure tell us about future inflation? *Journal of monetary economics*, 25(1):77–95, 1990.

[37] Michele Modugno. Now-casting inflation using high frequency data. *International Journal of Forecasting*, 29(4):664–675, 2013.

[38] Libero Monteforte and Gianluca Moretti. Real-time forecasts of inflation: The role of financial variables. *Journal of Forecasting*, 32(1):51–61, 2013.

[39] Charles R Nelson and Charles R Plosser. Trends and random walks in macroeconmic time series: some evidence and implications. *Journal of monetary economics*, 10(2):139–162, 1982.

[40] Charles R Nelson and Andrew F Siegel. Parsimonious modeling of yield curves. *Journal of business*, pages 473–489, 1987.

[41] James H Stock and Mark W Watson. Forecasting output and inflation: The role of asset prices. *Journal of Economic Literature*, 41(3):788–829, 2003.

[42] James H Stock and Mark W Watson. Why has us inflation become harder to forecast? *Journal of Money, Credit and banking*, 39:3–33, 2007.

[43] James H Stock and Mark W Watson. Core inflation and trend inflation. *Review of Economics and Statistics*, 98(4):770–784, 2016.

[44] John C Williams. Heeding daedalus: Optimal inflation and the zero lower bound. *Brookings Papers on Economic Activity*, 2009(2):1–37, 2009.

A. Details of the Gibbs Sampler

We describe in more detail the sampling steps and related posterior distributions that compose our Gibbs sampler procedure. The model in state-space form is

$$y_t = C_t^{'} \alpha_t + w_t \tag{.17}$$

$$\alpha_{t+1} = A_t \alpha_t + R_t v_t \tag{.18}$$

$$\Lambda_{t+1} = \Lambda_t + \zeta_t \tag{.19}$$

$$w_t \sim N(0, H), \quad v_t \sim N(0, Q_t) \tag{.20}$$

$$\zeta_t \sim N(0, W) \tag{.21}$$

The parameters to be estimated are

$[\{\tau_t, \eta_t\},\ \{\beta_i, \gamma_i\},\ \{\phi\},\ \{\sigma_{\tau,t}, \sigma_{\eta,t}\},\ \{\sigma_{\nu\tau}, \sigma_{\nu\eta}\},\ \{\sigma_{u_i}\}]$. We partition them into 5 blocks:

$$\theta_1 = \{\tau_t, \eta_t\}$$

$$\theta_2 = \{\beta_i, \gamma_i, \sigma_{u_i}\}$$

$$\theta_3 = \{\phi_i\}$$

$$\theta_4 = \{\sigma_{\tau,t}, \sigma_{\eta,t}\}$$

$$\theta_5 = \{\sigma_{\nu\tau}, \sigma_{\nu\eta}\}$$

and let $y_t = [\tilde{y}_t^1, \tilde{y}_t^2, \cdots, \tilde{y}_t^n]$ denote the observed variables.

A.1 Step 1: drawing the unobserved state variables $\theta_1 = \{\tau_t, \eta_t\}$ from $f(\theta_1|Y_t, \theta_{\neq 1})$

In the first step of the Gibbs sampler, we draw the state variables in α_t which contains the unobserved permanent component τ_t and transitory component η_t. Since the model is in state-space form, the posterior distribution of the state vector can be obtained via the Kalman smoother proposed by Koopman and Durbin (2003). The posterior distribution of the state vector in the linear Gaussian state-space model is also Gaussian with conditional mean $\hat{\alpha}_t = E(\alpha_t|Y_T)$ and conditional covariance $V_t = Cov(\alpha_t|Y_T)$. The derivation of the conditional mean and covariance matrix follows classical forward recursion of Kalman filter and backward recursion of Kalman smoother. The classical Kalman filter recursion is

$$E(\alpha_{t+1}|Y_t) = a_{t+1} = A_t a_t + K_t \upsilon_t$$

$$Cov(\alpha_{t+1}|Y_t) = P_{t+1} = A_t P_t L_t^{'} + R_t Q_t R_t^{'}$$

where

$$K_t = A_t P_t C_t F_t^{-1}$$

$$\upsilon_t = y_t - C_t^{'} a_t$$

$$L_t = A_t - K_t C_t^{'}$$

$$F_t = C_t^{'} P_t C_t + H.$$

The smoothing backward recursion is

$$r_{t-1} = C_t F_t^{-1} \upsilon_t + L_t^{'} r_t$$

$$\hat{\alpha}_t = a_t + P_t r_{t-1}$$

$$N_{t-1} = C_t F_t^{-1} C_t^{'} + L_t^{'} N_t L_t$$

$$V_t = P_t - P_t N_{t-1} P_t$$

for $t = T, T-1, \dots 1$, with $r_n = 0$, and $N_n = 0$.

Since the state variables are not all stationary, the unconditional mean and variance is not appropriate to initialize the Kalman filter. We adopt the exact recursions for calculating the mean and mean square error matrix of the state vector in the case where the initial state vector is diffuse. The initial state vector is specified as

$$\alpha_1 = a + T\delta + R_0 \varepsilon_0$$

where $\varepsilon_0 \sim N(0, Q_0)$, δ is a $q \times 1$ vector of unknown quantities. The $m \times q$ matrix T and the $m \times (m-q)$ matrix R_0 are selection matrices, and satisfy$T^{'}R_0 = 0$ and $\delta = T^{'}\alpha_1$. The vector δ is random and we assume that

$$\delta \sim N(0, \kappa I_q)$$

where $\kappa \to \infty$. Therefore the initial conditions for the state vector become

$$E(\alpha_1) = a,\ and\ Var(\alpha_1) = P$$

where $P = \kappa P_\infty + P_*$, $P_\infty = TT^{'}$, $P_* = R_0 Q_0 R_0^{'}$.

The mean squared error P_t in the classical filtering is decomposed into

$$P_t = \kappa P_{\infty,t} + P_{*,t} + O(\kappa^{-1})$$

where the term $P_{\infty,t}$ will disappear after a limited number of updates d in the exact Kalman filter. Therefore, the state filtering equations apply without change for $t > d$.

For the initial d time periods, the exact filtering equations are

$$a_{t+1} = A_t a_t + K_{\infty,t} \upsilon_t$$

$$P_{\infty,t+1} = A_t P_{\infty,t} L_{\infty,t}^{'}$$

$$P_{*,t+1} = A_t P_{*,t} L_{\infty,t}^{'} - K_{\infty,t} F_{\infty,t} K_{*,t}^{'} + R_t Q_t R_t^{'}$$

where

$$K_{\infty,t} = A_t P_{\infty,t} C_t F_{\infty,t}^{-1}$$

$$\upsilon_t = y_t - C_t^{'} a_t$$

$$L_{\infty,t} = A_t - K_{\infty,t} C_t^{'}$$

$$F_{\infty,t} = C_t^{'} P_{\infty,t} C_t$$

$$K_{*,t} = (A_t P_{*,t} C_t + K_{\infty,t} F_{*,t}) F_{\infty,t}^{-1}$$

$$F_{*,t} = C_t^{'} P_{*,t} C_t + H$$

with the initialization $a_1 = a$, $P_{*,t} = P_*$ and $P_{\infty,t} = P_\infty$.

The the initial d time periods state smoothing recursion is given by

$$\hat{\alpha}_t = a_t + P_{*,t} r_{t-1}^{(0)} + P_{\infty,t} r_{t-1}^{(1)}$$

$$\begin{aligned} V_t &= P_{*,t} - P_{*,t} N_{t-1}^{(0)} P_{*,t} - P_{\infty,t} N_{t-1}^{(1)} P_{\infty,t} \\ &\quad - (P_{\infty,t} N_{t-1}^{(1)} P_{\infty,t})^{'} - P_{\infty,t} N_{t-1}^{(2)} P_{\infty,t} \end{aligned}$$

where

$$r_{t-1}^{(0)} = L_{\infty,t}^{'} r_{t-1}^{(0)}$$

$$r_{t-1}^{(1)} = C_t(F_{\infty,t}^{-1}\upsilon_t - K_{*,t}^{'}r_t^{(0)}) + L_{\infty,t}^{'}r_t^{(1)}$$

$$N_{t-1}^{(0)} = L_{\infty,t}^{'}N_t^{(0)}L_{\infty,t}$$

$$N_{t-1}^{(1)} = C_tF_{\infty,t}^{-1}C_t^{'} + L_{\infty,t}^{'}N_t^{(1)}L_{\infty,t} - L_{\infty,t}N_t^{(0)}K_{*,t}C_t^{'} - (L_{\infty,t}N_t^{(0)}K_{*,t}C_t^{'})^{'}$$

$$N_{t-1}^{(2)} = C_t(K_{*,t}^{'}N_t^{(0)}K_{*,t} - F_{\infty,t}^{-1}K_{*,t}F_{\infty,t}^{-1})C_t^{'} + L_{\infty,t}^{'}N_t^{(2)}L_{\infty,t} - L_{\infty,t}N_t^{(2)}K_{*,t}C_t^{'} - (L_{\infty,t}N_t^{(1)}K_{*,t}C_t^{'})^{'}$$

for $t = d, d-1, \ldots 1$, with $r_d^{(0)} = r_d$, $r_d^{(1)} = 0$ and $N_d^{(0)} = N_d$, $N_d^{(1)} = N_d^{(2)} = 0$. With the above results, the conditional mean and covariance matrix are obtained and used to sample the state vector from $N(\hat{\alpha}_t, V_t)$.

A.2 Step 2: drawing the factor loadings $\theta_2 = \{\beta_i, \gamma_i, \sigma_{u_i}\}$

Conditioning on α_t and Y_t, factor loading in C_t' and variances in H can be drawn row by row in equation (22). Taking the ith measurement equation:

$$\tilde{y}_t^i = \beta_i C_{\tau,t}^i + \gamma_i C_{\eta,t}^i + u_t^{*i},$$

where $C_{\tau,t}^i$ and $C_{\tau,t}^i$ are obtained in the first step. Conditioning on all the variables, β_i, γ_i and variance of u_t^{*i} can be drawn following the conventional method for linear model. We state the priors in terms of their precision ($\bar{H}$). Then the prior of β_i, for example, is $\beta_i \sim N(\bar{\beta}_i, \bar{H}^{\dagger})$, where $\bar{H}^{\dagger}$ is the inverse of $\bar{H}$. The data evidence is summarized as

$$\beta_i \sim N(\hat{\beta}_i, h^{-1}(X^{'}X)^{-1})$$

where $h^{-1} = \sigma^2_{u^{*i}}$. Then we can draw β_i from the posterior distribution:

$$\beta_i \sim N((\bar{H} + \hat{H})^{-1}(\hat{H}\hat{\beta}_i + \bar{H}\bar{\beta}_i), (\bar{H} + \hat{H})^{-1}).$$

where $\bar{\beta}$ is the prior mean. Finally, γ_i can be drawn is a similar way.

The natural prior for the reciprocal of the variance of u_t^{*i} is

$$\iota s^2 h \sim \chi^2_\iota.$$

The data evidence is summarized as

$$((\tilde{y}^i - \beta_i C^i_\tau + \gamma_i C^i_\eta)'(\tilde{y}^i - \beta_i C^i_\tau + \gamma_i C^i_\eta))h \sim \chi^2_T.$$

We sample σ_{u_i} from the posterior distribution:

$$((\tilde{y}^i - \beta_i C^i_\tau + \gamma_i C^i_\eta)'(\tilde{y}^i - \beta_i C^i_\tau + \gamma_i C^i_\eta) + \iota s^2)h \sim \chi^2_{T+\iota}.$$

A.3 Step 3: drawing $\theta_3 = \{\phi_i\}$ from $f(\theta_3|\eta_t, \sigma^2_{\eta,t})$

Equation (23) can be broken down to $\phi(L)\eta_t = \sigma_{\eta,t}\varepsilon_{\eta,t}$, which is AR(p) model with heteroscedastic disturbance. Dividing by $\sigma_{\eta,t}$, one can obtain a standard linear regression model

$$\phi(L)\eta^*_t = \varepsilon_{\eta,t}$$

Then the autoregressive coefficients can be draw in a similar way as drawing β in step 2.

A.4 Step 4: drawing $\theta_4 = \{\sigma_{\tau,t}, \sigma_{\eta,t}\}$ from $f(\theta_4|Y_T, \theta_{\neq 4})$

We use Jacquier, Polson, and Rossi (1994)'s algorithm and Kim, Shephard and Chib(1998)'s Metropolis rejection method to draw the stochastic volatility, that is the unobserved components in equation (24). To sample the stochastic volatilities, notice that conditional on all the parameters and on the states vectors, the orthogonal innovations $x_{\tau,t} = \sigma_{\tau,t}\varepsilon_{\tau,t}$ and $x_{\eta,t} = \sigma_{\eta,t}\varepsilon_{\eta,t}$ are observable. We can proceed on a univariate basis because the stochastic volatilities are mutually independent. Jacquier, et. al. adopted a date-by-date blocking scheme and developed the conditional kernel. We take the $\sigma_{\tau,t}$ for example and $\sigma_{\eta,t}$ can be obtained in the same way.

Let $h_t = log(\sigma^2_{\tau,t})$, the conditional distribution of h_t is

$$p(h_t|h_{-t}, x_{\tau,t}, \sigma_{\nu\tau}) = p(h_t|h_{t-1}, h_{t+1}, \varepsilon_\tau, \sigma_{\nu\tau})$$

from Markov properties of stochastic volatility. By Bayes's theorem, the conditional kernel can be expressed as

$$\begin{aligned} p(h_t|h_{t-1}, h_{t+1}, x_{\tau,t}, \sigma_{\nu\tau}) &\propto p(x_{\tau,t}|h_t)p(h_t|h_{t-1})p(h_t|h_{t+1}) \\ &\propto h_t^{-1.5} exp(-\frac{x^2_{\tau,t}}{2h_t}) exp(-\frac{(lnh_t - \frac{1}{2}(lnh_{t-1} + lnh_{t+1}))^2}{\sigma^2_{\nu\tau}}). \end{aligned}$$

Since the normalization constant in the kernel is costly to compute, we use a Metropolis step to generate a sequence of random draws from the approximate distribution. The Metropolis sampler involves calculating a ratio to decide accepting or rejecting the draw from the approximation distribution. Here we use the approximation distribution

$$q(h_t) \propto N(\mu_t, \sigma_{\nu\tau}^2/2)$$

where

$$\mu_t = \frac{h_{t-1}+h_{t+1}}{2} + \frac{\sigma_{\nu\tau}^2}{4}(x_{\tau,t}^2 exp(-\frac{h_{t-1}+h_{t+1}}{2}) - 1).$$

The acceptance probability is specified as

$$r_t = \frac{f^*}{g^*}$$

where

$$log f^* = -\frac{1}{2}h_t - \frac{x_{\tau,t}^2}{2} exp(-h_t),$$

and

$$log(g^*) = -\frac{1}{2}h_t - \frac{x_{\tau,t}^2}{2}\{exp(-h_t^*)(1+h_t^*) - h_t exp(-h_t^*)\},$$

$h_t^* = \frac{h_{t-1}+h_{t+1}}{2}$. Then the accept-reject procedure to sample h_t is first to propose a value of h_t from $q(h_t)$ and second to accept this value with probability r_t. If the value is rejected, we set $h_t^m = h_t^{m-1}$, where m denote the mth iteration.

A.5 Step 5: drawing $\theta_5 = \{\sigma_{\nu\tau}, \sigma_{\nu\eta}\}$ from $f(\theta_5|\theta_4)$

Conditioning on the log-volatilites, $\sigma_{\nu\tau}^2$ and $\sigma_{\nu\eta}^2$ in covariance matrix W can be drawn from conjugate inverse gamma distribution as in step 2. For example, the dynamics of log-volatilites is random walk with only $\sigma_{\nu\tau}$ unknown. Assume the prior for $\sigma_{\nu\tau}$ is

$$p(\sigma_{\nu\tau}) = IG(\frac{\upsilon_0}{2}, \frac{\delta_0}{2})$$

Then the posterior inverse gamma is

$$p(\sigma_{\nu\tau}|h_T) = IG(\frac{\upsilon_1}{2}, \frac{\delta_1}{2})$$

where $\upsilon_1 = \upsilon_0 + T$, and $\delta_1 = \delta_0 + \sum_{t=1}^{T}(\Delta lnh_t^2)$.