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ABSTRACT

In a previous paper it was shown that in field theory there are two possible conditions under which an elementary particle lies on a Regge trajectory, i.e., the first is that the proper vertex function vanishes and that the proper vertex poles are not the poles of scattering amplitudes, and the other, due to Kaus and Zachariasen, is that the form factor and Z_3 both vanish. In the present paper it is shown that under the latter condition the polology approach, due to Bernstein et al., and the original approach, due to Goldberger and Treiman, of obtaining the Goldberger-Treiman relation both fail. Therefore, this condition may be inadequate as the condition for Reggeization.

I. INTRODUCTION

In a previous paper[1] we explored the connections between the elementary pion and the Regge pion. We found that if in field theory the proper vertex function with the elementary pion off the mass shell vanishes and that the proper vertex poles are not the poles of scattering amplitudes, then the elementary pion disappears completely but the bootstrapped pion takes its place, lying on the Regge trajectory. However, we also found a different condition for Reggeization, namely, that the form factor $\underset{\sim}{K}(s)$ with the pion off the mass shell and Z_3 (the wave-function renormalization constant of the pion) should both vanish:

$$\underset{\sim}{K}(s) \to 0, \text{ and } Z_3 \to 0. \tag{1.1}$$

(When bound-state poles exist, the vanishing of the form factor does not always mean the vanishing of the coupling constant,[2] as was shown in a previous paper.) This latter condition (1.1) is essentially the same one as that derived by Kaus and Zachariasen[3] (see Sec. II in their paper).

In this paper we apply the condition (1.1) to the π-μ decay process. It is then shown that the polology approach, due to Bernstein et al.,[4] and the original approach, due to Goldberger and Treiman,[5] of obtaining the Goldberger-Treiman relation both fail. These approaches are essentially based on the assumption that the divergence of the axial-vector current is a highly convergent operator whose matrix elements satisfy unsubtracted dispersion relations in the momentum transfer squared. A validity of this assumption, known to a partially conserved axial-vector current, has been established by experiments.[4-6] Therefore, the condition (1.1) may be inadequate as the condition for Reggeization.

II. UNSUBTRACTED DISPERSION RELATIONS IN WEAK INTERACTIONS

We assume that any matrix elements of the divergence of the axial-vector current satisfy unsubtracted dispersion relations (UDR). Let us define F, the invariant amplitude for the π-μ decay, by

$$(2q_0)^{\frac{1}{2}} \left\langle \pi^- | \partial_\mu A_\mu | 0 \right\rangle = q^2 F = \mu^2 F, \tag{2.1}$$

where A_μ denotes the axial-vector current, q is the four-momentum of the elementary pion, and μ is the pion rest mass. From our assumption the off-shell amplitude $F(q^2 = s)$ satisfies UDR,

$$F(s) = \frac{1}{\pi} \int_{9\mu^2}^{\infty} ds' \frac{\mathrm{Abs}\ F(s')}{s' - s} . \tag{2.2}$$

The absorptive part of $F(s)$ is given by

$$s\ \mathrm{Abs}\ F(s) = \pi \sum_{\alpha} \left\langle 0 | J_\pi | s, \alpha \right\rangle \left\langle s, \alpha | \partial_\mu A_\mu | 0 \right\rangle \delta^4(q_\alpha - q), \tag{2.3}$$

where J_π is the source of the pion field, and α denotes all the variables other than s. By summing up over spins and separating out kinematical factors, Eq. (2.3) can be written as

$$s \text{ Abs } F(s) = \pi \sum_{\beta} K_{\beta}^{*}(s)\, \rho_{\beta}(s)\, f_{\beta}(s)$$

$$= \underset{\sim}{K}^{\dagger}(s)\, \underset{\sim}{\rho}(s)\, \underset{\sim}{f}(s) \qquad (2.4)$$

in matrix notation. Here the two invariant amplitudes, $\underset{\sim}{K}(s)$ and $\underset{\sim}{f}(s)$, represent virtual dissociation of the pion into intermediate states and their annihilation into a lepton pair, respectively; $\underset{\sim}{\rho}(s)$ is a phase-space factor. Then Eq. (2.2) is written as

$$F(s) = \int_{9\mu^2}^{\infty} ds' \frac{\underset{\sim}{K}^{\dagger}(s')\, \underset{\sim}{\rho}(s')\, \underset{\sim}{f}(s')}{s'(s'-s)} . \qquad (2.5)$$

From our assumption, the amplitude f(s) also satisfies UDR,

$$\underset{\sim}{f}(s) = \frac{\mu^2}{\mu^2 - s} F\underset{\sim}{g} + \frac{1}{\pi} \int_{9\mu^2}^{\infty} ds' \frac{\underset{\sim}{T}_0^{\dagger}(s')\, \underset{\sim}{\rho}(s')\, \underset{\sim}{f}(s')}{s' - s} , \qquad (2.6)$$

where $\underset{\sim}{g}$ is a coupling strength between the pion and the channel which can be produced by the virtual pion, and $\underset{\sim}{T}_0(s)$ is the scattering amplitude in the pseudoscalar sector. The "form factor" $\underset{\sim}{K}(s)$ satisfies the unitarity relation

$$\text{Im}\, \underset{\sim}{K} = \underset{\sim}{T}_0^{\dagger} \underset{\sim}{\rho} \underset{\sim}{K} = \underset{\sim}{K}^{\dagger} \underset{\sim}{\rho} \underset{\sim}{T}_0 \qquad (2.7)$$

above $9\mu^2$. Let us apply the usual N/D method to $\underset{\sim}{T}_0$:

$$\underset{\sim}{T}_0 = \underset{\sim}{N}_0 \underset{\sim}{D}_0^{-1}, \tag{2.8}$$

where $\underset{\sim}{N}_0(s)$ is real in the physical region, and $\underset{\sim}{D}_0(s)$ is given by

$$\underset{\sim}{D}_0(s) = \underset{\sim}{1} - \frac{s}{\pi} \int_{9\mu^2}^{\infty} ds' \frac{\underset{\sim}{\rho}(s')\, \underset{\sim}{N}_0(s')}{s'(s'-s)} . \tag{2.9}$$

Then Eq. (2.7) gives[7]

$$\underset{\sim}{K}(s) = \underset{\sim}{D}_0^{-1}(s)\, \underset{\sim}{D}_0(\mu^2)\, \underset{\sim}{g} = \underset{\sim}{D}_0^{-1}(s)\, \underset{\sim}{K}(0). \tag{2.10}$$

The solution of Eq. (2.6) is[7]

$$\underset{\sim}{f}(s) = \left[\mu^2/(\mu^2 - s)\right] F\, \underset{\sim}{K}(s) + \left[\underset{\sim}{G}(s) - F\, \underset{\sim}{K}(s)\right] , \tag{2.11}$$

$$\underset{\sim}{G}(s) = \underset{\sim}{D}_0^{-1}(s)\, \underset{\sim}{f}(0). \tag{2.12}$$

Here the first term in Eq. (2.11) is a special solution of Eq. (2.6), while the second term is a solution of the homogeneous equation of Eq. (2.6), normalized at $s = 0$. If $\underset{\sim}{G}(s) = F\, \underset{\sim}{K}(s)$ for all s, Eq. (2.11) reduces to

$$\underset{\sim}{f}(s) = \left[\mu^2/(\mu^2 - s)\right] F\, \underset{\sim}{K}(s), \tag{2.13}$$

a result known to Gell-Mann and Lévy[8], who conjectured the relation,

$\partial_\mu A_\mu = \mu^2 F\phi_\pi$, from which Eq. (2.13) immediately follows. But Eq. (2.13) contradicts Eq. (2.5), because, after inserting Eq. (2.13) into Eq. (2.5) and putting $s = \mu^2$, what we get is $F = 0$. Therefore, the second term in Eq. (2.11) is absolutely necessary.[9] If the pole term dominates in Eq. (2.11) or in Eq. (2.6) for $|s| < \mu^2$, then we get the Goldberger-Treiman relation, $\underset{\sim}{f}(0) \approx F\, \underset{\sim}{K}(0) \approx F\, \underset{\sim}{g}$. This is the polology approach, due to Bernstein et al.[4]

Now, let us consider the limits (1.1) : $\underset{\sim}{K}(s) \to 0$, and $Z_3 \to 0$. The former limit, $\underset{\sim}{K}(s) \to 0$, gives $\underset{\sim}{D}_0(\mu^2)\underset{\sim}{g} \to 0$, and hence[10] $|\underset{\sim}{D}_0(\mu^2)| \to 0$. Dynamical bound states occur when $|\underset{\sim}{D}_0(s)| = 0$. Hence, when $|\underset{\sim}{D}_0(\mu^2)|$ is small, one can expect there to be a bound state[3] at $s = s_B$ near μ^2. Let us suppose that this pole comes out of the second Riemann sheet as the coupling strength increases; then the integral paths in Eqs. (2.5) and (2.6) should be deformed, yielding the new pole terms. Dispersion relations of $F(s)$, $\underset{\sim}{f}(s)$, and $\underset{\sim}{K}(s)$ now turn out to be of the forms

$$F(s) = \frac{\mu^2 F_B \lambda}{s_B(s_B - s)} + \int_{9\mu^2}^{\infty} ds' \frac{\underset{\sim}{K}^{\dagger}(s')\, \underset{\sim}{\rho}(s')\, \underset{\sim}{f}(s')}{s'(s' - s)}, \qquad (2.14)$$

$$\underset{\sim}{f}(s) = \frac{\mu^2}{\mu^2 - s} F\underset{\sim}{g} + \frac{\mu^2}{s_B - s} F_B \underset{\sim}{g}_B + \frac{1}{\pi} \int_{9\mu^2}^{\infty} ds' \frac{\underset{\sim}{T}_0^{\dagger}(s')\underset{\sim}{\rho}(s')\underset{\sim}{f}(s')}{s' - s}, \qquad (2.15)$$

$$\underset{\sim}{K}(s) = \underset{\sim}{g} - \frac{s - \mu^2}{s - s_B} \frac{\lambda \underset{\sim}{g}_B}{s_B - \mu^2} + \frac{s - \mu^2}{\pi} \int_{9\mu^2}^{\infty} ds' \frac{\underset{\sim}{T}_0^{\dagger}(s')\underset{\sim}{\rho}(s')\underset{\sim}{K}(s')}{(s'-\mu^2)\ (s'-s)}, \qquad (2.16)$$

where F_B is the decay constant of the bound state into the lepton pair, λ is the coupling strength between the pion and the bound state, and $\underset{\sim}{g}_B$ is the coupling strength between the bound state and the channel which can be produced by the virtual pion. That the residues of the bound-state pole are factorized in the above forms will be shown in Appendix (see also Fig. 1). In spite of the appearance of the new pole terms, the solutions of Eqs. (2.15) and (2.16) are given by (2.11) and (2.10), respectively, where $|\underset{\sim}{D}_0(s_B)| = 0$. Here it should be noted that the second term in (2.11) does not contain the new pole because this term is the solution of the homogeneous equation. Then a relation

$$F_B \underset{\sim}{g}_B = F \underset{\sim}{g}_B \lambda/(\mu^2 - s_B) \tag{2.17}$$

should hold. In quite the same way, Z_3 is now

$$Z_3^{-1} = 1 + \left(\frac{\lambda}{s_B - \mu^2}\right)^2 + \int_{9\mu^2}^{\infty} ds\, \sigma(s) , \tag{2.18}$$

where $\sigma(s)$ is the Lehmann weight function, given by

$$\sigma(s) = \underset{\sim}{K}^{+}(s)\, \underset{\sim}{\rho}(s)\, \underset{\sim}{K}(s)/(s - \mu^2)^2 . \tag{2.19}$$

In the first limit, $\underset{\sim}{K}(s) \to 0$, Eq. (2.16) tends to[11]

$$0 = \underset{\sim}{g} - \lim \underset{\sim}{g}_B \lambda/(s_B - \mu^2) , \tag{2.20}$$

and hence Eq. (2.17) becomes

$$\lim F_B \underset{\sim}{g}_B \;=\; -F\underset{\sim}{g}. \tag{2.21}$$

As already mentioned, the limit, $\underset{\sim}{K}(s) \to 0$, means $|\underset{\sim}{D}_0(\mu^2)| \to 0$. Therefore, the parameter μ^2 must take the value, $\mu^2 = s_B$, because $|\underset{\sim}{D}_0(s_B)| = 0$. Then the pole terms in Eq. (2.15) are canceled out. This can also be seen from the solution (2.11). Since the second term in (2.11) does not contain any poles, the residue $\underset{\sim}{\gamma}$ of $\underset{\sim}{G}(s)$ at $s = s_B$ must equal to the residue of $F\,\underset{\sim}{K}(s)$ at $s = s_B$, i.e., $\underset{\sim}{\gamma} = -\lambda \underset{\sim}{g}_B F$. The equation (2.20) shows $\lambda \underset{\sim}{g}_B \to 0$ as $\mu^2 \to s_B$, then $\underset{\sim}{\gamma} \to 0$. The solution of $\underset{\sim}{f}(s)$ is now given by $\underset{\sim}{f}(s) = \underset{\sim}{G}(s)$, and therefore there is no pole in $\underset{\sim}{f}(s)$. This shows that in the limit, $\underset{\sim}{K}(s) \to 0$, the polology approach, due to Bernstein et al.,[4] fails.

Next let us turn our attention to $F(s)$. When $\underset{\sim}{K}(s) \neq 0$, inserting Eq. (2.11) into Eq. (2.14) and putting $s = \mu^2$, we have

$$F\left[1 + \frac{\mu^2}{s_B}\left(\frac{\lambda}{\mu^2 - s_B}\right)^2 + \int_{9\mu^2}^{\infty} ds\,\sigma(s)\right] = \int_{9\mu^2}^{\infty} ds\,\frac{\underset{\sim}{K}^{\dagger}\,\underset{\sim}{\rho}\,\underset{\sim}{G}}{s(s-\mu^2)}. \tag{2.22}$$

In the limits, $\underset{\sim}{K}(s) \to 0$, and $Z_3 \to 0$, the above equation becomes

$$F\left[1 + \lim \lambda^2/(s_B - \mu^2)^2\right] = 0, \tag{2.23}$$

where the bracket term tends to infinity, as is seen from Eq. (2.18).

Then what we get is $F = 0$. This shows that in the limits (1.1) the original approach, due to Goldberger and Treiman,[5] fails. This failure can also be seen in the following way: In Eq. (2.14) let us put $s = \mu^2$. Then we have

$$F = \frac{\mu^2}{s_B} \frac{\lambda F_B}{s_B - \mu^2} + \int_{9\mu^2}^{\infty} ds \frac{\underset{\sim}{K}^{\dagger}(s)\, \underset{\sim}{\rho}(s)\, \underset{\sim}{f}(s)}{s(s - \mu^2)} \cdot \qquad (2.24)$$

In the limits (1.1) the above equation tends to

$$F = \lim \lambda F_B/(s_B - \mu^2) , \qquad (2.25)$$

while Eq. (2.14) becomes

$$F(s) = \lim (\lambda F_B)/(s_B - s) . \qquad (2.26)$$

Both equations (2.25) and (2.26) are consistent with each other as long as $F = 0$. Note that this difficulty is independent of that of $\underset{\sim}{f}(s)$.

III. ONCE-SUBTRACTED DISPERSION RELATIONS IN WEAK INTERACTIONS

If $F(s)$ satisfies the once-subtracted dispersion relation (ODR)

$$F(s) = F - \frac{s - \mu^2}{s - s_B} \frac{\lambda F_B}{s_B - \mu^2} + \frac{s - \mu^2}{\pi} \int_{9\mu^2}^{\infty} ds' \frac{\underset{\sim}{K}^{\dagger}(s')\underset{\sim}{\rho}(s')\underset{\sim}{f}(s')}{(s'-\mu^2)\,(s'-s)} \cdot \qquad (3.1)$$

and if $\underset{\sim}{f}(s)$ satisfies UDR, then in the limits (1.1) the above equation tends to

$$F(s) = F\left[1 + \lim \lambda^2/(s_B - \mu^2)^2\right], \tag{3.2}$$

owing to Eq.(2.17). The bracket term again tends to infinity, so that ODR for $F(s)$ fails, too. Note that this difficulty is closely related with that of $\underset{\sim}{f}(s)$.

If $\underset{\sim}{f}(s)$ satisfies ODR

$$\underset{\sim}{f}(s) = \underset{\sim}{f}(0) + \frac{s}{\mu^2 - s} F\underset{\sim}{g} + \frac{s}{s_B - s}\,\frac{\mu^2}{s_B} F_B \underset{\sim}{g}_B + \frac{s}{\pi}\int_{9\mu^2}^{\infty} ds' \frac{\underset{\sim}{T}_0^{\dagger}(s')\underset{\sim}{\rho}(s')\underset{\sim}{f}(s')}{s'(s'-s)}, \tag{3.3}$$

then the solution of this equation is

$$\underset{\sim}{f}(s) = \underset{\sim}{G}(s) - \left[s/(s-\mu^2)\right] F\,\underset{\sim}{K}(s) + s\,\underset{\sim}{\chi}(s), \tag{3.4}$$

where $s\,\underset{\sim}{\chi}(s)$ is the solution of the homogeneous equation, while $\underset{\sim}{G}(s) - \left[s/(s-\mu^2)\right] F\,\underset{\sim}{K}(s)$ is the special solution. In this case the relation (2.17) is no longer valid, but the different relation

$$F_B \underset{\sim}{g}_B = F \underset{\sim}{g}_B\, \lambda s_B/\mu^2(\mu^2 - s_B) - \underset{\sim}{\gamma}/\mu^2 \tag{3.5}$$

holds. In the limit, $\underset{\sim}{K}(s) \to 0$, this relation tends to

$$\lim F_B \underset{\sim}{g}_B = -F\underset{\sim}{g} - \underset{\sim}{\gamma}/s_B, \tag{3.6}$$

owing to Eq(2.20), and hence no cancellation of the pole terms in $\underset{\sim}{f}(s)$ occurs. Therefore, there is no contradiction in ODR for $\underset{\sim}{f}(s)$, but one cannot get the Goldberger-Treiman relation.

If $F(s)$ and $\underset{\sim}{f}(s)$ both satisfy ODR, then in the limits (1.1) Eq. (3.1) becomes

$$F(s) = F - \lim F_B \left[\lambda/(s_B - \mu^2) \right] , \qquad (3.7)$$

because in this case Eq. (2.17) is no longer valid. Since $F(s = \mu^2) = F$, the above limit should tend to zero, i.e.,

$$\lim F_B \left[\lambda/(s_B - \mu^2) \right] = 0. \qquad (3.8)$$

Therefore, it follows that $\lim F_B = 0$, because the bracket term goes to infinity. Since $\underset{\sim}{g}_B \to 0$ as $Z_3 \to 0$, we get from Eq. (3.6)

$$\gamma = -s_B \, Fg \, . \qquad (3.9)$$

After all, there are no contradictions in ODR for $F(s)$ and $\underset{\sim}{f}(s)$, but one cannot again calculate the π-μ decay rate.

The unsubtracted dispersion relation for $F(s)$ fails even when $\underset{\sim}{f}(s)$ satisfies ODR, because, as was shown in Sec. II, this failure is independent of subtraction of $\underset{\sim}{f}(s)$.

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APPENDIX

It is shown that the residues of the bound-state pole in $F(s)$, $\underset{\sim}{f}(s)$, and $\underset{\sim}{K}(s)$ are factorized in the forms (2.14), (2.15), and (2.16).

The form factor $\underset{\sim}{K}(s)$ can be continued into the second Riemann sheet through the interval between $9\mu^2$ and the next threshold s_1, by the formula

$$K_\alpha^{II} = K_\alpha - 2i(T_0^{II})_{\alpha 1}\, \rho_1\, K_1 , \tag{A.1}$$

where $\underset{\sim}{T}_0^{II}$ is the scattering amplitude continued into the second sheet, given by

$$\begin{aligned}(T_0^{II})_{\alpha 1} &= (T_0)_{\alpha 1} - 2i(T_0^{II})_{\alpha 1}\, \rho_1\, (T_0)_{11} \\ &= (T_0)_{\alpha 1}\left[1 + 2i\, \rho_1 (T_0)_{11}\right]^{-1} .\end{aligned} \tag{A.2}$$

Since the pole s_B is not the pole of K_α^{II}, we have

$$R_1/g_{B1} = R_\alpha/g_{B\alpha} \equiv -\lambda , \tag{A.3}$$

from Eq. (A.1), where $\underset{\sim}{R}$ is the residue of $\underset{\sim}{K}(s)$ at $s = s_B$. The equation (A.3) shows that $\underset{\sim}{R}$ is factorized in the form $R = -\lambda \underset{\sim}{g}_B$.

In quite the same way, the residue, $\underset{\sim}{\beta}$, of $\underset{\sim}{f}(s)$ at $s = s_B$ is factorized as $\underset{\sim}{\beta} = -\mu^2 F_B \underset{\sim}{g}_B$, by making use of the formula

$$f_\alpha^{\ II} = f_\alpha - 2i(T_0^{\ II})_{\alpha 1}\, \rho_1\, f_1 \, . \tag{A.4}$$

The residue, δ, of $F(s)$ at $s = s_B$ is also factorized as $\delta = -\mu^2 \lambda F_B$. This is easily shown by making use of the formula

$$F^{II} = F - 2i\, K_1^{\ II}\, \rho_1\, f_1 \, , \tag{A.5}$$

together with Eq. (A.1) and $\underset{\sim}{\beta} = -\mu^2 \bar{F}_B \underset{\sim}{g}_B$.

FOOTNOTES AND REFERENCES

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† Formerly Takesi Ogimoto. On leave of absence from Department of Physics, Osaka University, Osaka, Japan.

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10. We assume $\underset{\sim}{g} \not\equiv 0$.
11. The limit, $\underset{\sim}{K}(s) \to 0$, does not always mean $\underset{\sim}{g} \to 0$, when the bound-state pole s_B exists. When $\mu^2 = s_B$, the solution $\underset{\sim}{K}(s) = \underset{\sim}{D}_0^{-1}(s)\,\underset{\sim}{D}_0(\mu^2)\,\underset{\sim}{g}$ is indefinite at $s = \mu^2 = s_B$. Therefore, the behavior of $\underset{\sim}{K}(s)$ near $s = \mu^2 = s_B$ needs a closer study. Let us suppose $\underset{\sim}{K}(s) \equiv 0$ except at $s = \mu^2$. Then we have Eq. (2.20).

Therefore, the first term $\underset{\sim}{g}$ is cancelled by the second term, so that $\underset{\sim}{K}(s)$ is equal to zero even at $s = \mu^2$. The value of $\underset{\sim}{K}(s)$ at $s = \mu^2$ should be defined by $\lim_{s \to \mu^2} \underset{\sim}{K}(s) = \underset{\sim}{K}(\mu^2)$.

FIGURE CAPTION

Fig. 1. Factorization of the residues of the pole in $F(s)$, $\mathbf{f}(s)$, and $\mathbf{K}(s)$ at $s = s_B$.

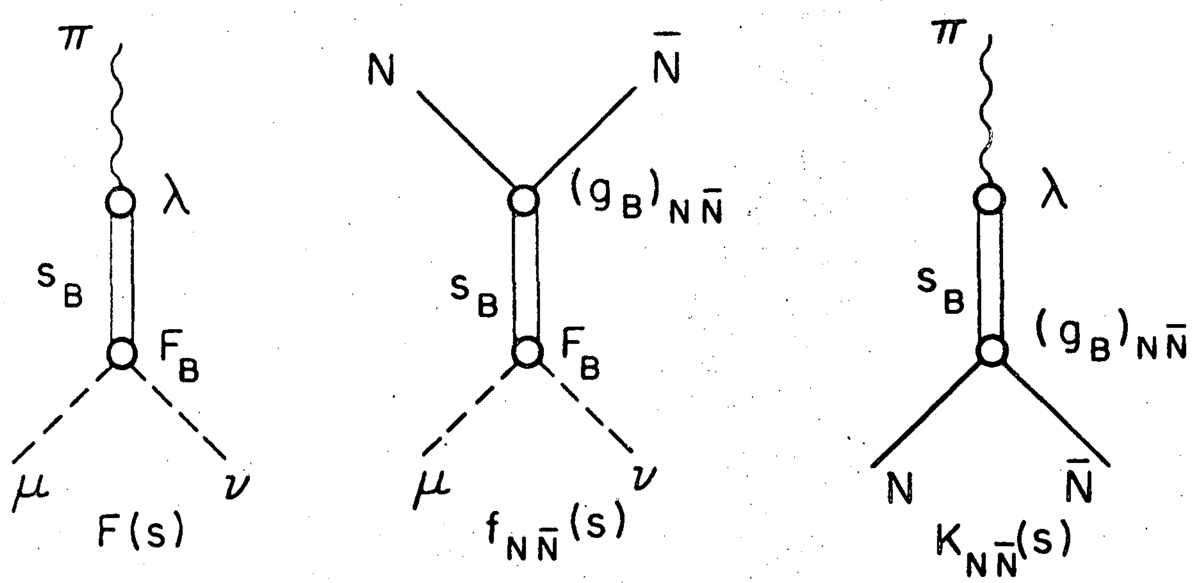

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Fig. 1