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Author

Gorman, D.J.

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A COMPUTER PROGRAM FOR THE SMOOTHING AND DIFFERENTIATION OF DATA FROM MULTICHANNEL ANALYZERS*

D. J. Gorman

Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

In the interpretation of spectra taken on multichannel analyzers it is frequently helpful, especially in cases where the statistics are poor, to smooth the data in order to reduce the statistical fluctuations.

When presented with such a spectrum one would normally draw through the points a smooth curve which best fits them. This can be done numerically once we define what we mean by best fit. The most common criterion is to minimize the sum of the squares of the residuals between the actual and fitted points.

This method has been described by Savitzky and Golay[1] who fit a polynomial over a range of (2m + 1) data points and use that to evaluate the smoothed value at the mid-point. The data points taken are then shifted by one, dropping the point at the left and picking up one at the right. This process is repeated to cover the entire spectrum.

Upon doing one least squares analysis one obtains a set of coefficients, one for each point over which the fit is performed. The value at the mid-point of one interval of 2m + 1 points is found by summing the products of the counts in each channel and the particular coefficient for that channel.

$$\text{i.e.} \quad \bar{Y}_j = \sum_{i=-m}^{+m} C_i Y_{i+j} \quad ,$$

where the Y_{i+j} are the counts in channel i + j, the C_i are a set of convoluting factors and $\overline{Y}_j$ is the best calculated value for the number of counts in channel j. j is an index which runs over all of the channels in the spectrum.

The main part of the least-squares analysis of Savitzky and Golay is reproduced here along with some corrections and additions.

Convolution Using a Least Squares Procedure

A set of 2m + 1 consecutive values are to be used to determine the best least squares fit through the values of a polynomial of degree $n < 2m + 1$. The polynomial has the form:

$$f_i = \sum_{k=0}^{n} b_{nk} i^k = b_{n0} + b_{n1} i + b_{n2} i^2 + \cdots\cdots + b_{nn} i^n \quad . \qquad (I)$$

The derivatives of this polynomial are

$$\frac{df}{di} = b_{ni} + 2b_{n2} i + 3b_{n3} i^2 + \cdots\cdots + nb_{nn} i^{n-1} \qquad (IIa)$$

$$\frac{d^2 f}{di^2} = 2b_{n2} + 6b_{n3} i + \cdots\cdots + n(n-1) b_{nn} i^{n-2} \qquad (IIb)$$

$$\frac{d^n f}{di^n} = n!\, b_{nn} \quad . \qquad (IIc)$$

The value of i ranges from -m to +m, and i is equal to zero at the central point of the set of 2m + 1 values. Therefore the value of the s^{th} derivative at the point i = 0 is:

$$\left(\frac{d^s f_i}{di^s}\right)_{i=0} = s!b_{ns} = a_{ns} \quad , \tag{III}$$

where

$$f_0 = b_{n0} = a_{n0} \tag{IVa}$$

$$\frac{df_0}{di} = b_{n1} = a_{n1} \tag{IVb}$$

$$\frac{d^2 f_0}{di^2} = 2b_{n2} = a_{n2} \quad . \tag{IVc}$$

The least squares criterion requires that the sum of the squares of the differences between the observed values Y_i and the calculated ones f_i be a minimum.

$$\text{i.e.} \quad \frac{\partial}{\partial b_{nk}} \left[\sum_{i=-m}^{+m} (f_i - y_i)^2 \right] = 0 \quad . \tag{V}$$

Minimizing with respect to b_{n0} we have

$$\frac{\partial}{\partial b_{n0}} \left[\sum_{i=-m}^{+m} (b_{n0} + b_{n1}i + \cdots\cdots + b_{nn}i^n - y_i)^2 \right] \tag{VIa}$$

$$= 2 \sum_{i=-m}^{+m} (b_{n0} + b_{n1}i + \cdots\cdots + b_{nn}i^n - y_i) = 0 \quad .$$

Minimizing with respect to b_{n1} we have

$$\frac{\partial}{\partial b_{n1}}\left[\sum_{i=-m}^{+m}(b_{n0}+b_{n1}i+\cdots\cdots+b_{nn}i^{n}-y_i)^2\right] \qquad \text{(VIb)}$$

$$=2\sum_{i=-m}^{+m}(b_{n0}+b_{n1}i+\cdots\cdots+b_{nn}i^{n}-y_i)i=0 \quad .$$

With respect to the general b_{nr} we get

$$\frac{\partial}{\partial b_{nr}}\left[\sum_{i=-m}^{+m}(b_{n0}+b_{n1}i+\cdots\cdots+b_{nn}i^{n}-y_i)^2\right] \qquad \text{(VIc)}$$

$$=2\sum(b_{n0}+b_{n1}i+\cdots\cdots+b_{nr}i^{n}-y_i)i^{r}=0$$

$$2\sum_{i=-m}^{+m}\left[\left(\sum_{k=0}^{n}b_{nk}i^{k}\right)-y_i\right]i^{r}=0$$

$$\sum_{i=-m}^{+m}\sum_{k=0}^{n}b_{nk}i^{k+r}=\sum_{i=-m}^{+m}y_i i^{r} \quad , \qquad \text{(VII)}$$

r is an index representing the equation number which runs from 0 to n.

Since b_{nk} is independent of i we can write the equation as

$$\sum_{k=0}^{n}b_{nk}\sum_{i=-m}^{+m}i^{k+r}=\sum_{i=-m}^{+m}y_i i^{r}=F_r^{\dagger}$$

or

$$\sum_{k=0}^{n} b_{nk} S_{r+k} = F_r^{\dagger} \quad , \tag{VIIIa}$$

where

$$S_{r+k} = \sum_{i=-m}^{+m} i^{r+k} \quad , \tag{VIIIb}$$

and

$$F_r^{\dagger} = \sum_{i=-m}^{+m} i^r y_i \quad . \tag{VIIIc}$$

When $r+k$ is odd $S_{r+k} = 0$. Since S_{r+k} exists for even values of $r+k$ only, the set of $n+1$ equations can be separated into two sets, one for even values of k and one for odd values.

---

†Note: In the paper of Savitzky and Golay this is given as F_k, when it should be F_r.

For n even the secular equations VIIIa are

$$(IXa)\begin{cases} S_0 b_{n0} + S_2 b_{n2} + S_4 b_{n4} + \cdots + S_n b_{nn} = F_0 \\ S_2 b_{n0} + S_4 b_{n2} + S_6 b_{n4} + \cdots + S_{n+2} b_{nn} = F_2 \\ S_4 b_{n0} + S_6 b_{n2} + S_8 b_{n4} + \cdots + S_{n+4} b_{nn} = F_4 \\ \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\qquad\quad \vdots \qquad\quad \vdots \\ S_n b_{n0} + S_{n+2} b_{n2} + S_{n+4} b_{n4} + \cdots + S_{2n} b_{nn} = F_n \end{cases}$$

$$(IXb)\begin{cases} S_2 b_{n1} + S_4 b_{n3} + S_6 b_{n5} + \cdots + S_n b_{n,n-1} = F_1 \\ S_4 b_{n1} + S_6 b_{n3} + S_8 b_{n5} + \cdots + S_{n+2} b_{n,n-1} = F_3 \\ S_6 b_{n1} + S_8 b_{n3} + S_{10} b_{n5} + \cdots + S_{n+4} b_{n,n-1} = F_5 \\ \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\qquad\quad \vdots \qquad\quad \vdots \\ S_n b_{n1} + S_{n+2} b_{n3} + S_{n+4} b_{n5} + \cdots + S_{2n-2} b_{n,n-1} = F_{n-1} \end{cases}$$

For n odd they are:

$$(IXc)\begin{cases} S_0 b_{n0} + S_2 b_{n2} + S_4 b_{n4} + \cdots + S_{n-1} b_{n,n-1} = F_0 \\ S_2 b_{n0} + S_4 b_{n2} + S_6 b_{n4} + \cdots + S_{n+1} b_{n,n-1} = F_2 \\ S_4 b_{n0} + S_6 b_{n2} + S_8 b_{n4} + \cdots + S_{n+3} b_{n,n-1} = F_4 \\ \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\qquad\quad \vdots \qquad\quad \vdots \\ S_{n-1} b_{n0} + S_{n+1} b_{n2} + S_{n+3} b_{n4} + \cdots + S_{2n-2} b_{n,n-1} = F_{n-1} \end{cases}$$

$$
(\text{IXd})\quad \begin{cases}
S_2 b_{n1} + S_4 b_{n3} + S_6 b_{n5} + \cdots\cdots + S_{n+1} b_{nn} = F_1 \\
S_4 b_{n1} + S_6 b_{n3} + S_8 b_{n5} + \cdots\cdots + S_{n+3} b_{nn} = F_3 \\
S_6 b_{n1} + S_8 b_{n3} + S_{10} b_{n5} + \cdots\cdots + S_{n+5} b_{nn} = F_5 \\
\vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\qquad\qquad\quad \vdots \qquad\quad \vdots \\
S_{n+1} b_{n1} + S_{n+3} b_{n3} + S_{n+5} b_{n5} + \cdots\cdots + S_{2n} b_{nn} = F_n
\end{cases}
$$

The convoluting factors are the $a_{ns} (= s! b_{ns})$ and to find them we must solve the secular equations (IX) for the b_{ns}.

Example

For a third degree polynomial (n=3) the secular equations are:

$$S_0 b_{30} + S_2 b_{32} = F_0$$

$$S_2 b_{30} + S_4 b_{32} = F_2$$

$$S_2 b_{31} + S_4 b_{33} = F_1$$

$$S_4 b_{31} + S_6 b_{33} = F_3$$

Solving for the coefficients b we have

$$b_{30} = \frac{F_0 S_4 - F_2 S_2}{S_0 S_4 - S_2^2}$$

$$b_{32} = \frac{F_2 S_0 - F_0 S_2}{S_0 S_4 - S_2^2}$$

$$b_{31} = \frac{F_1 S_6 - F_3 S_4}{S_2 S_6 - S_4^2}$$

$$b_{33} = \frac{F_3 S_2 - F_1 S_4}{S_2 S_6 - S_4^2} \quad .$$

If $m = 3$; $2m + 1 = 7$. Then from Eqs. (VIIIb) and (VIIIc)

$$S_0 = 7, \quad S_2 = 28, \quad S_4 = 196, \quad S_6 = 1588$$

$$F_0 = Y_{-3} + Y_{-2} + Y_{-1} + Y_0 + Y_1 + Y_2 + Y_3$$

$$F_1 = -3Y_{-3} - 2Y_{-2} - Y_{-1} + Y_1 + 2Y_2 + 3Y_3$$

$$F_2 = 9Y_{-3} + 4Y_{-2} + Y_{-1} + Y_1 + 4Y_2 + 9Y_3$$

$$F_3 = 27Y_{-3} - 8Y_{-2} - Y_{-1} + Y_1 + 8Y_2 + 27Y_3$$

$$b_{30} = \frac{1}{588} [196F_0 - 28F_2]$$

$$= \frac{1}{588} [-56Y_{-3} + 84Y_{-2} + 168Y_{-1} + 196Y_0 + 168Y_1 + 84Y_2 - 56Y_3]$$

$$b_{32} = \frac{1}{588} [7F_2 - 28F_0]$$

$$= \frac{1}{588} [35Y_{-3} - 21Y_{-1} - 28Y_0 - 21Y_1 + 35Y_3]$$

$$b_{31} = \frac{1}{6048} [1588F_1 - 196F_3] = \frac{1}{1512} [397F_1 - 49F_3]$$

$$b_{31} = \frac{1}{1512}\left[132Y_{-3} - 402Y_{-2} - 348Y_{-1} + 348Y_1 + 402Y_2 - 132Y_3\right]$$

$$b_{33} = \frac{1}{6048}\left[28F_3 - 196F_1\right] = \frac{1}{1512}\left[7F_3 - 49F_1\right]$$

$$= \frac{1}{1512}\left[-42Y_{-3} + 42Y_{-2} + 42Y_{-1} - 42Y_1 - 42Y_2 + 42Y_3\right]$$

From Eq. (III) the value of the s^{th} derivative at the point i = 0 is given by a_{ns}

$$a_{30} = b_{30} = -0.09524Y_{-3} + 0.14286Y_{-2} + 0.28571Y_{-1} + 0.33333Y_0 + 0.28571Y_1 + 0.14286Y_2 - 0.09524Y_3$$

$$a_{31} = b_{31} = 0.08730Y_{-3} - 0.26587Y_{-2} - 0.23016Y_{-1} + 0.0Y_0 + 0.23016Y_1 + 0.26587Y_2 - 0.08730Y_3$$

$$a_{32} = 2b_{32} = 0.11905Y_{-3} - 0.07143Y_{-1} - 0.095238Y_0 - 0.07143Y_1 + 0.11905Y_3$$

$$a_{33} = 6b_{33} = -0.16667Y_{-3} + 0.16667Y_{-2} + 0.16667Y_{-1} - 0.16667Y_1 - 0.16667Y_2 + 0.16667Y_3 \quad .$$

The coefficients of the Y's are the convoluting factors which give the value of the s^{th} derivative at the point i = 0. They are given in tabular form in Appendix B, where N = 3, M = 7. As an example of the use of these factors take the Gaussian array $Y_i = 1000e^{-0.09(i-8)^2}$ in Table I.

The smoothed value at i = 6 is given by the derivative of zero order.

$$\bar{Y}_6 = -0.095238Y_3 + 0.142857Y_4 + 0.285714Y_5 + 0.333333Y_6 + 0.285714Y_7$$

$$+ 0.142857Y_8 - 0.095238Y_9 = 700 \quad .$$

The value of the first derivative of Y_i at i = 9 is given by

$$Y'_9 = 0.087302Y_6 - 0.265873Y_7 - 0.230159Y_8 + 0.230159Y_{10} + 0.265873Y_{11}$$

$$- 0.087302Y_{12} = -153 \quad .$$

Table I. Effect of a seven point cubic convolution on a Gaussian

i	Y_i		First Derivative		Second Derivative		Third Derivative	
	Actual	Calc[a]	Actual[b]	Calc	Actual[b]	Calc	Actual[b]	Calc
1	12							
2	39							
3	105							
4	237	253	171	174	80	48	-4	-16
5	445	462	240	237	50	14	-60	-51
6	698	700	251	240	-35	-42	-103	-70
7	914	894	165	153	-135	-97	-84	-51
8	1000	970	0	0	-180	-119	0	0
9	914	894	-165	-153	-135	-97	84	51
10	698	700	-251	-240	-35	-42	103	70
11	445	462	-240	-237	50	14	60	51
12	237	253	-171	-174	80	48	4	16
13	105							
14	39							
15	2							

[a] This is the smoothed value of Y_i.

[b] The actual values are calculated by differentiation of the function $Y = 1000e^{-0.09(i-8)^2}$.

Table I lists the value of the function Y_i, the smoothed value, and the first, second and third derivatives. They are plotted in Fig. 1. The derivatives are useful in computerized peak finding routines (1, 2, 3).

A computer program has been written to calculate these convolution factors for any degree of polynomial and taking any number of points. Appendix A contains a listing of the program and explanation as to its use. Some tables of the more commonly used convolution factors are given in Appendix B.

Repeated Convolution

We may wish to perform two or more convolutions in succession. For example, a smoothing using p points with a polynomial of degree n followed by a differentiation using q points and a polynomial of degree m.

For the first convolution we have

$$\left(\frac{d^{s_1} f_i}{di^{s_1}}\right)_{i=0} = S_1! b_{ns_1} = a_{ns_1} = \sum_{i=-p}^{+p} C^n_{is_1} Y_i$$

to produce the new array $Y_i^{(s_1)}$.

For the second convolution

$$\left[\frac{d^{s_2}}{dj^{s_2}}\left(\frac{d^{s_1} f_i}{di^{s_1}}\right)_{i=0}\right]_{j=0} = \sum_{j=-q}^{+q} C^m_{js_2} Y_j^{(s_1)}$$

$$= \sum_{j=-q}^{+q} C^m_{js_2} \sum_{i=-p}^{+p} C^n_{is_1} Y_{i+j}$$

$$= \sum_{j=-q}^{+q} \sum_{i=-p}^{+p} C^{m}_{js_2} C^{n}_{is_1} Y_{i+j} \quad . \qquad \text{(XI)}$$

If we set h = i+j then we have

$$a_{ns_1,ms_2} = \sum_{h=-(p+q)}^{p+q} d_h Y_h \qquad \text{where ,}$$

$$d_h = \sum_{j} \sum_{i} C^{m}_{js_2} C^{n}_{is_1} \quad , \qquad \text{(XII)}$$

where the sum is over all values of i and j such that i + j = h and $-q \leq j \leq q$, $-p \leq i \leq p$. As an example take the case where we have a five point quadratic smooth, followed by a seven point convolution to obtain the quadratic first derivative. Then p = 2, and q = 3, and we have

$C_{-2} = -0.08571$
$C_{-1} = 0.34286$
$C_0 = 0.48571$
$C_1 = 0.34286$
$C_2 = -0.08571$

$C'_{-3} = -0.10714$
$C'_{-2} = -0.07143$
$C'_{-1} = -0.03571$
$C'_0 = 0$
$C'_1 = 0.03571$
$C'_2 = 0.07143$
$C'_3 = 0.10714$

$d_{-5} = C_{-2} C'_{-3} = + 0.009183$

$d_{-4} = C_{-2} C'_{-2} + C_{-1} C'_{-3} = -0.03061$

$$d_{-3} = C_{-2}\,C'_{-1} + C_{-1}\,C'_{-2} + C_0\,C'_{-3} = -0.07347$$

$$d_{-2} = C_{-2}\,C'_0 + C_{-1}\,C'_{-1} + C_0\,C'_{-2} + C_1\,C'_{-3} = -0.08367$$

$$d_{-1} = C_{-2}\,C'_1 + C_{-1}\,C'_0 + C_0\,C'_{-1} + C_1\,C'_{-2} + C_2\,C'_{-3} = -0.03571$$

$$d_0 = C_{-2}\,C'_2 + C_{-1}\,C'_1 + C_0\,C'_0 + C_1\,C'_{-1} + C_2\,C'_{-2} = 0$$

$$d_1 = C_{-2}\,C'_3 + C_{-1}\,C'_2 + C_0\,C'_1 + C_1\,C'_0 + C_2\,C'_{-1} = 0.03571$$

$$d_2 = C_{-1}\,C'_3 + C_0\,C'_2 + C_1\,C'_1 + C_2\,C'_0 = 0.08367$$

$$d_3 = C_0\,C'_3 + C_1\,C'_2 + C_2\,C'_1 = 0.07347$$

$$d_4 = C_1\,C'_3 + C_2\,C'_2 = 0.03061$$

$$d_5 = C_2\,C'_3 = -0.009183$$

Use of the Convolution Factors

1. Smoothing

Figure II shows an alpha spectrum of $^{239}Am^{4}$ and ^{241}Am taken with a 6 mm Au-Si detector and recorded in a 400 channel pulse-height analyzer. In IIa is plotted the raw data and IIb is the spectrum after performing a smoothing operation using a quadratic polynomial and smoothing over eleven points at a time.

The smoothing operation has not changed either the peak heights or the peak shapes but it has particularly accentuated the two α_0's which in the raw data are barely discernable above the statistical fluctuations of the background. It is much easier to perform a stripping operation on the smoothed spectrum than on the raw data.

Figure IIIa shows an $^{239}Am^{5}$ alpha spectrum taken on a magnetic alpha particle spectograph. The statistics are quite poor. The improvement after smoothing (over eleven points using a quadratic) shown in IIIb is remarkable. The computer program which performs the smoothing using the convolution factors is listed in Appendix C.

2. Peak Location

Figure IVa shows a gamma spectrum of ^{241}Am and ^{57}Co. In Fig. IVb is plotted the first derivative of this spectrum formed from a five point quadratic convolution. The peak maximum is the point at which the first derivative is zero.

In Fig. VIa is plotted an alpha spectrum of ^{239}Am and ^{241}Am. The first and second derivatives of this spectrum are shown in Vb and Vc. The first derivative is more useful than the second derivative for determining peak locations.

Appendix A

The computer program for calculating convolution factors

A complete listing of the program is given below.

For each set of factors one data card is required giving the values of POWER and NPOINT in a 2I10 format. POWER is the degree of the polynomial (i.e. 2 for a quadratic) and NPOINT is the total number of points (2m + 1) over which the convolution is to be performed. Reading is terminated by a blank card.

In accordance with the dimensions of the matrices

$$\text{POWER} \leqslant 20 \qquad \text{and} \qquad \text{NPOINT} \leqslant 30 \ .$$

The program runs in a field length of 51000.

The subroutine MATIV was written by Burton S. Garbow of the Argonne National Laboratory.

```
                      PROGRAM CONVOL (INPUT,OUTPUT,TAPE2=INPUT,TAPE3=OUTPUT)
                C
                C     CONVOL CALCULATES SETS OF CONVOLUTING FACTORS FOR SMOOTHING AND
                C     CALCULATING DERIVATIVES BY A LEAST SQUARES PROCEDURE
                C
000043                DIMENSION S(30),FO(30),F(20,30),A(20,30),B(20,30),C(20,30),D(20,30
                     1)
CC0C43                DIMENSION OARR(10,30)
000043                INTEGER POWER,POW,SUMS,SO,FC,S,F,C,D
                C
                C     POWER IS THE DEGREE OF THE FITTING POLYNOMIAL
                C     NPOINT IS THE NUMBER OF POINTS USED TO FIT THE POLYNOMIAL
                C     NPCINT MUST BE ODD
                C
000043              5 READ(2,1000) POWER,NPCINT
C00052                DO 6 I = 1,NPCINT
000061              6 B(2,I) = 0.
000063                IF(POWER.LE.0) CALL EXIT
000065                MPT = (NPCINT - 1)/2
0C0070                POW = 2 * POWER
0C0071                IPWR = POWER + 1
C00072                SUMS = 0
000073                SO = NPOINT
000074                NM1 = NPCINT - 2
                C
                C     SET S(I) = 0 FOR ODD I
                C
000075                DO 10 I = 1,NM1,2
C00102             1C S(I) = 0
                C
                C     CALCULATE S(I) FOR EVEN I
                C
000104                DO 30 I = 2,POW,2
CC0105                DO 20 J = 1,MPT
000111             20 SUMS = SUMS + 2*(J**I)
000115                S(I) = SUMS
C00117             3C SUMS = 0
                C
                C     SET UP F MATRIX
                C
000122                DO 40 I = 1,NPCINT
C00126             4C FO(I) = 1
000130                DO 50 J = 1,POWER
C00131                DO 50 I = 1,NPCINT
000140             50 F(J,I) = (-MPT + I - 1)**J
                C
                C     TEST TO SEE IF POWER EVEN OR ODD
                C
CC0150                PWD = POWER/2
C00151                PW2 = PWD * 2
000153                IF(PW2.NE.POWER) GO TO 160
                C
                C     IF POWER IS EVEN DO THIS SECTION
                C     SET UP MATRIX FOR EVEN DERIVATIVES
                C
000155                MXDIM = (POWER + 2)/2
CC0157                C(1,1) = SO
000161                DO 60 J = 2,MXDIM
000167             60 C(1,J) = S(2*J) - 2)
000173                DO 70 I = 2,MXDIM
000174                DO 70 J = 1,MXDIM
CC0204             7C C(I,J) = S((2*I) + (2*J) - 4)
000212                DO 80 J = 1,NPOINT
C00222             80 D(1,J) = FO(J)
000224                DO 90 I = 2,MXDIM
000226                DO 90 J = 1,NPCINT
C00241             90 D(I,J) = F((2*I - 2),J)
000246                DO 100 I = 1,MXDIM
000247                DO 100 J = 1,MXDIM
C00256            100 A(I,J) = FLOAT(C(I,J))
000263                DO 110 I = 1,MXDIM
000264                DO 110 J = 1,NPOINT
000273            11C B(I,J) = FLOAT(D(I,J))
C00300                CALL MATINV(A,MXCIM,B,NPOINT,DETERM)
C00303                WRITE(3,3000) POWER,NPCINT,POWER,MPT
000316                WRITE(3,3001)
000321                WRITE(3,3002)
000324                WRITE(3,3CC3)
                C
                C     MULTIPLY THE B MATRIX BY THE FACTORIAL OF THE DERIVATIVE NUMBER
                C
000327                DO 115 I = 2,MXDIM
000331                DO 115 J = 1,NPOINT
000332                KX = 2*I - 2
CC0334            115 B(I,J) = B(I,J) * FACTCR(KX)
000346                DO 118 I = 1,MXDIM
000347                IMZ = 2*I - 1
000351                DO 118 J = 1,NPOINT
000363            118 OARR(IMZ,J) = B(I,J)
                C
                C     SET UP MATRIX FOR ODD DERIVATIVES
                C
000370                MXDIM = POWER/2
000372                DO 120 I = 1,MXDIM
000373                DC 120 J = 1,MXDIM
C00403            12C C(I,J) = S((2*I) + (2*J) - 2)
000411                DO 130 I = 1,MXDIM
C00412                DO 130 J = 1,NPCINT
000425            130 D(I,J) = F((2*I - 1),J)
000432                DC 140 I = 1,MXDIM
000433                DO 140 J = 1,MXDIM
000442            14C A(I,J) = FLOAT(C(I,J))
C00447                DO 150 I = 1,MXDIM
000450                DO 150 J = 1,NPOINT
000457            150 B(I,J) = FLOAT(D(I,J))
000464                CALL MATINV(A,MXCIM,B,NPOINT,DETERM)
                C
                C     MULTIPLY THE B MATRIX BY THE FACTORIAL OF THE DERIVATIVE NUMBER
                C
000467                DO 155 I = 2,MXDIM
000471                DO 155 J = 1,NPOINT
000472                KX = 2*I - 1
C00474            155 B(I,J) = B(I,J) * FACTCR(KX)
000506                DO 158 I = 1,MXDIM
C00507                IMZ = 2*I
CC051C                DO 158 J = 1,NPOINT
000522            158 OARR(IMZ,J) = B(I,J)
000527                JPWR = IPWR
000531                IF(JPWR.GT.7) JPWR = 7
000534                DO 159 N = 1,NPOINT
CC0536                MIN = (NPCINT - 1)/2
000540                NIT = -MIN - 1 + N
000542            159 WRITE(3,3004) NIT,(OARR(J,N),J=1,JPWR)
```

```
000563          IF(IPWR.LE.7) GO TO 5
000565          WRITE(3,3000) POWER,NPCINT,POWER,MPT
000600          WRITE(3,3001)
000603          WRITE(3,3005)
000606          WRITE(3,3003)
000611          DO 300 N = 1,NPOINT
000613          MIN = (NPOINT-1)/2
000615          NIT = -MIN - 1 + N
000617      300 WRITE(3,3004) NIT,(OARR(J,N),J=8,IPWR)
000640          GO TO 5
                C
                C       IF POWER IS ODD DO THIS SECTION
                C       SET UP MATRIX FOR EVEN DERIVATIVES
                C
000640      160 MXDIM = (POWER + 1)/2
000643          C(1,1) = S0
000644          DO 170 J = 2,MXDIM
000655      170 C(1,J) = S(2*J - 2)
000657          DO 180 I = 2,MXDIM
000661          DO 180 J = 1,MXDIM
000672      180 C(I,J) = S(2*I + 2*J - 4)
000677          DO 190 J = 1,NPOINT
000707      190 D(1,J) = F0(J)
000711          DO 200 I = 2,MXDIM
000713          DO 200 J = 1,NPOINT
000726      200 D(I,J) = F((2*I - 2),J)
000733          DO 210 I = 1,MXDIM
000734          DO 210 J = 1,MXDIM
000743      210 A(I,J) = FLOAT(C(I,J))
000750          DO 220 I = 1,MXDIM
000751          DO 220 J = 1,NPCINT
000760      220 B(I,J) = FLOAT(D(I,J))
000765          CALL MATINV(A,MXDIM,B,NPOINT,DETERM)
000770          WRITE(3,3000) POWER,NPCINT,POWER,MPT
001003          WRITE(3,3001)
001006          WRITE(3,3002)
001011          WRITE(3,3003)
                C
                C       MULTIPLY THE B MATRIX BY THE FACTORIAL OF THE DERIVATIVE NUMBER
                C
001014          DO 225 I = 2,MXDIM
001016          DO 225 J = 1,NPOINT
001017          KX = 2*I - 2
001021      225 B(I,J) = B(I,J) * FACTOR(KX)
001033          DO 228 I = 1,MXDIM
001034          IMZ = 2*I - 1
001036          DO 228 J = 1,NPOINT
001050      228 OARR(IMZ,J) = B(I,J)
                C
                C       SET UP MATRIX FOR ODD DERIVATIVES
                C
001055          MXDIM = (POWER + 1)/2
001057          DO 230 I = 1,MXDIM
001061          DO 230 J = 1,MXDIM
001072      230 C(I,J) = S(2*I + 2*J - 2)
001077          DO 240 I = 1,MXDIM
001100          DO 240 J = 1,NPOINT
001113      240 D(I,J) = F((2*I - 1),J)
001120          DO 250 I = 1,MXDIM
001121          DO 250 J = 1,MXDIM
001130      250 A(I,J) = FLOAT(C(I,J))
001135          DO 260 I = 1,MXDIM
001136          DO 260 J = 1,NPOINT
001145      260 B(I,J) = FLOAT(D(I,J))
001152          CALL MATINV(A,MXDIM,B,NPOINT,DETERM)
                C
                C       MULTIPLY THE B MATRIX BY THE FACTORIAL OF THE DERIVATIVE NUMBER
                C
001155          DO 265 I = 2,MXDIM
001157          DO 265 J = 1,NPOINT
001160          KX = 2*I - 1
001162      265 B(I,J) = B(I,J) * FACTOR(KX)
001174          DO 268 I = 1,MXDIM
001175          IMZ = 2*I
001176          DO 268 J = 1,NPOINT
001210      268 OARR(IMZ,J) = B(I,J)
001215          JPWR = IPWR
001217          IF(JPWR.GT.7) JPWR = 7
001222          DO 269 N = 1,NPOINT
001224          MIN = (NPOINT - 1)/2
001226          NIT = -MIN - 1 + N
001230      269 WRITE(3,3004) NIT,(OARR(J,N),J=1,JPWR)
001251          IF(IPWR.LE.7) GO TO 5
001253          WRITE(3,3000) POWER,NPCINT,POWER,MPT
001266          WRITE(3,3001)
001271          WRITE(3,3005)
001274          WRITE(3,3003)
001277          DO 400 N = 1,NPOINT
001301          MIN = (NPOINT-1)/2
001303          NIT = -MIN - 1 + N
001305      400 WRITE(3,3004) NIT,(OARR(J,N),J=8,IPWR)
001326          GO TO 5
                C
001326       1000 FORMAT(2I10)
001326       3000 FORMAT(1H1,29X,*DEGREE*,I3,* POLYNOMIAL*,2X,I3,* POINTS*,5X,*N=*,I
                    13,5X,*M=*,I3)
001326       3001 FORMAT(1H0,5X,6HPOINTS,45X,10HDERIVATIVE)
001326       3002 FORMAT(1H ,20X,1H0,15X,1H1,15X,1H2,15X,1H3,15X,1H4,15X,1H5,15X,1H6
                    1)
001326       3003 FORMAT(1H )
001326       3004 FORMAT(1H ,I9,4X,7(F12.8,4X))
001326       3005 FORMAT(1H ,20X,1H7,15X,1H8,15X,1H9,14X,2H10,14X,2H11,14X,2H12,14X,
                    12H13)
001326          END
```

```
      FUNCTION FACTOR(I)
C
C     THIS FUNCTION COMPUTES THE FACTORIAL OF I
C
010111      IF(I.EQ.0 .OR. I.EQ.1) GO TO 30
010117      IPROD = I
010120   10 I = I - 1
010122      IF(I.EQ.1) GO TO 20
010123      IPROD = IPROD * I
010125      GO TO 10
010125   20 FACTOR = IPROD
010127      RETURN
010127   30 FACTOR = 1.
010131      RETURN
010131      END
```

```
      FORTRAN IV SUBROUTINE MATINV(A,N,B,M,DETERM)                          F4020003
C                                                                           F4C20002
C     MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS       ANF40201
C                                                                           F4020004
      DIMENSION IPIVOT(20),A(20,30),B(20,1),INDEX(20,2),PIVOT(20)
      COMMON /LSP/ PIVOT,IPIVOT,INDEX
      EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM), (AMAX, T, SWAP)             F4C20007
C                                                                           F4020008
C     INITIALIZATION                                                        F4020009
C                                                                           F402001C
   10 DETERM=1.0                                                            F4020011
   15 DO 20 J=1,N                                                           F4020012
   20 IPIVOT(J)=0                                                           F4020013
   30 DO 550 I=1,N                                                          F4020014
C                                                                           F4020015
C     SEARCH FOR PIVOT ELEMENT                                              F4020016
C                                                                           F4020017
   40 AMAX=0.0                                                              F4020018
   45 DO 105 J=1,N                                                          F4020019
   50 IF (IPIVOT(J)-1) 60, 105, 60                                          F402C020
   60 DO 100 K=1,N                                                          F4020021
   70 IF (IPIVOT(K)-1) 80, 100, 740                                         F4020022
   80 IF (ABS(AMAX)-ABS(A(J,K))) 85, 100, 100
   85 IROW=J                                                                F4020024
   90 ICOLUM=K                                                              F4020025
   95 AMAX=A(J,K)                                                           F4020026
  100 CONTINUE                                                              F4020027
  105 CONTINUE                                                              F4020028
      IF(AMAX) 110,800,110                                                  F402REV.
  110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1                                       F4C20029
C                                                                           F4020030
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL                     F4020031
C                                                                           F4020032
  130 IF (IROW-ICOLUM) 140, 260, 140                                        F4020033
  140 DETERM=-DETERM                                                        F4C20C34
  150 DO 200 L=1,N                                                          F4020035
  160 SWAP=A(IROW,L)                                                        F4020036
  170 A(IROW,L)=A(ICOLUM,L)                                                 F4020037
  200 A(ICOLUM,L)=SWAP                                                      F4020038
  205 IF(M) 260, 260, 210                                                   F4020039
  210 DO 250 L=1, M                                                         F4020040
  220 SWAP=B(IROW,L)                                                        F4020041
  230 B(IROW,L)=B(ICOLUM,L)                                                 F4020042
  250 B(ICOLUM,L)=SWAP                                                      F4020043
  260 INDEX(I,1)=IROW                                                       F4020044
  270 INDEX(I,2)=ICOLUM                                                     F4020045
  310 PIVOT(I)=A(ICOLUM,ICOLUM)                                             F4020046
  320 DETERM=DETERM*PIVOT(I)                                                F4C20C47
C                                                                           F4020048
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT                                     F4020049
C                                                                           F4C20050
  330 A(ICOLUM,ICOLUM)=1.0                                                  F4020051
  340 DO 350 L=1,N                                                          F4020052
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)                                      F4020053
  355 IF(M) 380, 380, 360                                                   F4020054
  360 DO 370 L=1,M                                                          F4C20055
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT(I)                                      F4020056
C                                                                           F4020057
C     REDUCE NON-PIVOT ROWS                                                 F4020058
C                                                                           F4020059
  380 DO 550 L1=1,N                                                         F4C20060
  390 IF(L1-ICOLUM) 400, 550, 400                                           F4020061
  400 T=A(L1,ICOLUM)                                                        F4020062
  420 A(L1,ICOLUM)=0.0                                                      F4C20063
  430 DO 450 L=1,N                                                          F4020064
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T                                         F4020065
  455 IF(M) 550, 550, 460                                                   F4020066
  460 DO 500 L=1,M                                                          F402C067
  500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T                                         F4020068
  550 CONTINUE                                                              F4020069
C                                                                           F4C2007C
C     INTERCHANGE COLUMNS                                                   F4020071
C                                                                           F4020072
  600 DO 710 I=1,N                                                          F4020073
  610 L=N+1-I                                                               F4020074
  620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630                              F402C075
  630 JROW=INDEX(L,1)                                                       F4020076
  640 JCOLUM=INDEX(L,2)                                                     F4020077
  650 DO 705 K=1,N                                                          F4C20078
  660 SWAP=A(K,JROW)                                                        F4020079
  670 A(K,JROW)=A(K,JCOLUM)                                                 F4C20080
  700 A(K,JCOLUM)=SWAP                                                      F4020081
  705 CONTINUE                                                              F4020082
  710 CONTINUE                                                              F4C20083
  740 RETURN                                                                F4020084
  800 DETERM = 0.                                                           F402REV.
      RETURN                                                                F402REV.
      END
```

Appendix B

Convolution Factors

In this appendix are given the convolution factors calculated using fitting polynomials from first to sixth degree and smoothing up to 23 points.

DEGREE 1 POLYNOMIAL 3 POINTS N= 1 M= 1

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-1	.33333333	-.50000000					
0	.33333333	0.					
1	.33333333	.50000000					

DEGREE 1 POLYNOMIAL 5 POINTS N= 1 M= 2

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-2	.20000000	-.20000000					
-1	.20000000	-.10000000					
0	.20000000	0.					
1	.20000000	.10000000					
2	.20000000	.20000000					

DEGREE 1 POLYNOMIAL 7 POINTS N= 1 M= 3

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-3	.14285714	-.10714286					
-2	.14285714	-.07142857					
-1	.14285714	-.03571429					
0	.14285714	0.					
1	.14285714	.03571429					
2	.14285714	.07142857					
3	.14285714	.10714286					

DEGREE 1 POLYNOMIAL 9 POINTS N= 1 M= 4

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-4	.11111111	-.06666667					
-3	.11111111	-.05000000					
-2	.11111111	-.03333333					
-1	.11111111	-.01666667					
0	.11111111	0.					
1	.11111111	.01666667					
2	.11111111	.03333333					
3	.11111111	.05000000					
4	.11111111	.06666667					

DEGREE 1 POLYNOMIAL 11 POINTS N= 1 M= 5

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-5	.09090909	-.04545455					
-4	.09090909	-.03636364					
-3	.09090909	-.02727273					
-2	.09090909	-.01818182					
-1	.09090909	-.00909091					
0	.09090909	0.					
1	.09090909	.00909091					
2	.09090909	.01818182					
3	.09090909	.02727273					
4	.09090909	.03636364					
5	.09090909	.04545455					

DEGREE 1 POLYNOMIAL 13 POINTS N= 1 M= 6

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-6	.07692308	-.03296703					
-5	.07692308	-.02747253					
-4	.07692308	-.02197802					
-3	.07692308	-.01648352					
-2	.07692308	-.01098901					
-1	.07692308	-.00549451					
0	.07692308	0.					
1	.07692308	.00549451					
2	.07692308	.01098901					
3	.07692308	.01648352					
4	.07692308	.02197802					
5	.07692308	.02747253					
6	.07692308	.03296703					

DEGREE 1 POLYNOMIAL 15 POINTS N= 1 M= 7

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-7	.06666667	-.02500000					
-6	.06666667	-.02142857					
-5	.06666667	-.01785714					
-4	.06666667	-.01428571					
-3	.06666667	-.01071429					
-2	.06666667	-.00714286					
-1	.06666667	-.00357143					
0	.06666667	0.					
1	.06666667	.00357143					
2	.06666667	.00714286					
3	.06666667	.01071429					
4	.06666667	.01428571					
5	.06666667	.01785714					
6	.06666667	.02142857					
7	.06666667	.02500000					

DEGREE 1 POLYNOMIAL 17 POINTS N= 1 M= 8

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-8	.05882353	-.01960784					
-7	.05882353	-.01715686					
-6	.05882353	-.01470588					
-5	.05882353	-.01225490					
-4	.05882353	-.00980392					
-3	.05882353	-.00735294					
-2	.05882353	-.00490196					
-1	.05882353	-.00245098					
0	.05882353	0.					
1	.05882353	.00245098					
2	.05882353	.00490196					
3	.05882353	.00735294					
4	.05882353	.00980392					
5	.05882353	.01225490					
6	.05882353	.01470588					
7	.05882353	.01715686					
8	.05882353	.01960784					

DEGREE 1 POLYNOMIAL 19 POINTS N= 1 M= 9

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-9	.05263158	-.01578947					
-8	.05263158	-.01403509					
-7	.05263158	-.01228070					
-6	.05263158	-.01052632					
-5	.05263158	-.00877193					
-4	.05263158	-.00701754					
-3	.05263158	-.00526316					
-2	.05263158	-.00350877					
-1	.05263158	-.00175439					
0	.05263158	0.					
1	.05263158	.00175439					
2	.05263158	.00350877					
3	.05263158	.00526316					
4	.05263158	.00701754					
5	.05263158	.00877193					
6	.05263158	.01052632					
7	.05263158	.01228070					
8	.05263158	.01403509					
9	.05263158	.01578947					

DEGREE 1 POLYNOMIAL 21 POINTS N= 1 M= 10

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-10	.04761905	-.01298701					
-9	.04761905	-.01168831					
-8	.04761905	-.01038961					
-7	.04761905	-.00909091					
-6	.04761905	-.00779221					
-5	.04761905	-.00649351					
-4	.04761905	-.00519481					
-3	.04761905	-.00389610					
-2	.04761905	-.00259740					
-1	.04761905	-.00129870					
0	.04761905	0.					
1	.04761905	.00129870					
2	.04761905	.00259740					
3	.04761905	.00389610					
4	.04761905	.00519481					
5	.04761905	.00649351					
6	.04761905	.00779221					
7	.04761905	.00909091					
8	.04761905	.01038961					
9	.04761905	.01168831					
10	.04761905	.01298701					

DEGREE 1 POLYNOMIAL 23 POINTS N= 1 M= 11

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-11	.04347826	-.01086957					
-10	.04347826	-.00988142					
-9	.04347826	-.00889328					
-8	.04347826	-.00790514					
-7	.04347826	-.00691700					
-6	.04347826	-.00592885					
-5	.04347826	-.00494071					
-4	.04347826	-.00395257					
-3	.04347826	-.00296443					
-2	.04347826	-.00197628					
-1	.04347826	-.00098814					
0	.04347826	0.					
1	.04347826	.00098814					
2	.04347826	.00197628					
3	.04347826	.00296443					
4	.04347826	.00395257					
5	.04347826	.00494071					
6	.04347826	.00592885					
7	.04347826	.00691700					
8	.04347826	.00790514					
9	.04347826	.00889328					
10	.04347826	.00988142					
11	.04347826	.01086957					

DEGREE 2 POLYNOMIAL 3 POINTS N= 2 M= 1

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-1	.00000000	-.50000000	1.00000000				
0	1.00000000	0.	-2.00000000				
1	.00000000	.50000000	1.00000000				

DEGREE 2 POLYNOMIAL 5 POINTS N= 2 M= 2

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-2	-.08571429	-.20000000	.28571429				
-1	.34285714	-.10000000	-.14285714				
0	.48571429	0.	-.28571429				
1	.34285714	.10000000	-.14285714				
2	-.08571429	.20000000	.28571429				

DEGREE 2 POLYNOMIAL 7 POINTS N= 2 M= 3

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-3	-.09523810	-.10714286	.11904762				
-2	.14285714	-.07142857	.00000000				
-1	.28571429	-.03571429	-.07142857				
0	.33333333	0.	-.09523810				
1	.28571429	.03571429	-.07142857				
2	.14285714	.07142857	.00000000				
3	-.09523810	.10714286	.11904762				

DEGREE 2 POLYNOMIAL 9 POINTS N= 2 M= 4

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-4	-.09090909	-.06666667	.06060606				
-3	.06060606	-.05000000	.01515152				
-2	.16883117	-.03333333	-.01731602				
-1	.23376623	-.01666667	-.03679654				
0	.25541126	0.	-.04329004				
1	.23376623	.01666667	-.03679654				
2	.16883117	.03333333	-.01731602				
3	.06060606	.05000000	.01515152				
4	-.09090909	.06666667	.06060606				

DEGREE 2 POLYNOMIAL 11 POINTS N= 2 M= 5

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-5	-.08391608	-.04545455	.03496503				
-4	.02097902	-.03636364	.01398601				
-3	.10256410	-.02727273	-.00233100				
-2	.16083916	-.01818182	-.01398601				
-1	.19580420	-.00909091	-.02097902				
0	.20745921	0.	-.02331002				
1	.19580420	.00909091	-.02097902				
2	.16083916	.01818182	-.01398601				
3	.10256410	.02727273	-.00233100				
4	.02097902	.03636364	.01398601				
5	-.08391608	.04545455	.03496503				

DEGREE 2 POLYNOMIAL 13 POINTS N= 2 M= 6

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-6	-.07692308	-.03296703	.02197802				
-5	0.	-.02747253	.01098901				
-4	.06293706	-.02197802	.00199800				
-3	.11188811	-.01648352	-.00499500				
-2	.14685315	-.01098901	-.00999001				
-1	.16783217	-.00549451	-.01298701				
0	.17482517	0.	-.01398601				
1	.16783217	.00549451	-.01298701				
2	.14685315	.01098901	-.00999001				
3	.11188811	.01648352	-.00499500				
4	.06293706	.02197802	.00199800				
5	0.	.02747253	.01098901				
6	-.07692308	.03296703	.02197802				

DEGREE 2 POLYNOMIAL 15 POINTS N= 2 M= 7

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-7	-.07058824	-.02500000	.01470588				
-6	-.01176471	-.02142857	.00840336				
-5	.03800905	-.01785714	.00307046				
-4	.07873303	-.01428571	-.00129282				
-3	.11040724	-.01071429	-.00468649				
-2	.13303167	-.00714286	-.00711054				
-1	.14660633	-.00357143	-.00856496				
0	.15113122	0.	-.00904977				
1	.14660633	.00357143	-.00856496				
2	.13303167	.00714286	-.00711054				
3	.11040724	.01071429	-.00468649				
4	.07873303	.01428571	-.00129282				
5	.03800905	.01785714	.00307046				
6	-.01176471	.02142857	.00840336				
7	-.07058824	.02500000	.01470588				

DEGREE 2 POLYNOMIAL 17 POINTS N= 2 M= 8

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-8	-.06501548	-.01960784	.01031992				
-7	-.01857585	-.01715686	.00644995				
-6	.02167183	-.01470588	.00309598				
-5	.05572755	-.01225490	.00025800				
-4	.08359133	-.00980392	-.00206398				
-3	.10526316	-.00735294	-.00386997				
-2	.12074303	-.00490196	-.00515996				
-1	.13003096	-.00245098	-.00593395				
0	.13312693	0.	-.00619195				
1	.13003096	.00245098	-.00593395				
2	.12074303	.00490196	-.00515996				
3	.10526316	.00735294	-.00386997				
4	.08359133	.00980392	-.00206398				
5	.05572755	.01225490	.00025800				
6	.02167183	.01470588	.00309598				
7	-.01857585	.01715686	.00644995				
8	-.06501548	.01960784	.01031992				

DEGREE 2 POLYNOMIAL 19 POINTS N= 2 M= 9

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-9	-.06015038	-.01578947	.00751880				
-8	-.02255639	-.01403509	.00501253				
-7	.01061477	-.01228070	.00280112				
-6	.03936311	-.01052632	.00088456				
-5	.06368863	-.00877193	-.00073714				
-4	.08359133	-.00701754	-.00206398				
-3	.09907121	-.00526316	-.00309598				
-2	.11012826	-.00350877	-.00383311				
-1	.11676249	-.00175439	-.00427539				
0	.11897391	0.	-.00442282				
1	.11676249	.00175439	-.00427539				
2	.11012826	.00350877	-.00383311				
3	.09907121	.00526316	-.00309598				
4	.08359133	.00701754	-.00206398				
5	.06368863	.00877193	-.00073714				
6	.03936311	.01052632	.00088456				
7	.01061477	.01228070	.00280112				
8	-.02255639	.01403509	.00501253				
9	-.06015038	.01578947	.00751880				

DEGREE 2 POLYNOMIAL 21 POINTS N= 2 M= 10

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-10	-.05590062	-.01298701	.00564653				
-9	-.02484472	-.01168831	.00395257				
-8	.00294214	-.01038961	.00243692				
-7	.02745995	-.00909091	.00109959				
-6	.04870873	-.00779221	-.00005944				
-5	.06668846	-.00649351	-.00104015				
-4	.08139915	-.00519481	-.00184255				
-3	.09284080	-.00389610	-.00246664				
-2	.10101340	-.00259740	-.00291242				
-1	.10591697	-.00129870	-.00317989				
0	.10755149	0.	-.00326904				
1	.10591697	.00129870	-.00317989				
2	.10101340	.00259740	-.00291242				
3	.09284080	.00389610	-.00246664				
4	.08139915	.00519481	-.00184255				
5	.06668846	.00649351	-.00104015				
6	.04870873	.00779221	-.00005944				
7	.02745995	.00909091	.00109959				
8	.00294214	.01038961	.00243692				
9	-.02484472	.01168831	.00395257				
10	-.05590062	.01298701	.00564653				

DEGREE 2 POLYNOMIAL 23 POINTS N= 2 M= 11

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-11	-.05217391	-.01086957	.00434783				
-10	-.02608696	-.00988142	.00316206				
-9	-.00248447	-.00889328	.00208922				
-8	.01863354	-.00790514	.00112931				
-7	.03726708	-.00691700	.00028233				
-6	.05341615	-.00592885	-.00045172				
-5	.06708075	-.00494071	-.00107284				
-4	.07826087	-.00395257	-.00158103				
-3	.08695652	-.00296443	-.00197628				
-2	.09316770	-.00197628	-.00225861				
-1	.09689441	-.00098814	-.00242801				
0	.09813665	0.	-.00248447				
1	.09689441	.00098814	-.00242801				
2	.09316770	.00197628	-.00225861				
3	.08695652	.00296443	-.00197628				
4	.07826087	.00395257	-.00158103				
5	.06708075	.00494071	-.00107284				
6	.05341615	.00592885	-.00045172				
7	.03726708	.00691700	.00028233				
8	.01863354	.00790514	.00112931				
9	-.00248447	.00889328	.00208922				
10	-.02608696	.00988142	.00316206				
11	-.05217391	.01086957	.00434783				

DEGREE 3 POLYNOMIAL 3 POINTS N= 3 M= 1

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-1	.00000000	-.50000000	1.00000000	0.			
0	1.00000000	0.	-2.00000000	0.			
1	.00000000	.50000000	1.00000000	0.			

DEGREE 3 POLYNOMIAL 5 POINTS N= 3 M= 2

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-2	-.08571429	.08333333	.28571429	-.50000000			
-1	.34285714	-.66666667	-.14285714	1.00000000			
0	.48571429	0.	-.28571429	0.			
1	.34285714	.66666667	-.14285714	-1.00000000			
2	-.08571429	-.08333333	.28571429	.50000000			

DEGREE 3 POLYNOMIAL 7 POINTS N= 3 M= 3

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-3	-.09523810	.08730159	.11904762	-.16666667			
-2	.14285714	-.26587302	.00000000	.16666667			
-1	.28571429	-.23015873	-.07142857	.16666667			
0	.33333333	0.	-.09523810	0.			
1	.28571429	.23015873	-.07142857	-.16666667			
2	.14285714	.26587302	.00000000	-.16666667			
3	-.09523810	-.08730159	.11904762	.16666667			

DEGREE 3 POLYNOMIAL 9 POINTS N= 3 M= 4

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-4	-.09090909	.07239057	.06060606	-.07070707			
-3	.06060606	-.11952862	.01515152	.03535354			
-2	.16883117	-.16245791	-.01731602	.06565657			
-1	.23376623	-.10606061	-.03679654	.04545455			
0	.25541126	0.	-.04329004	0.			
1	.23376623	.10606061	-.03679654	-.04545455			
2	.16883117	.16245791	-.01731602	-.06565657			
3	.06060606	.11952862	.01515152	-.03535354			
4	-.09090909	-.07239057	.06060606	.07070707			

DEGREE 3 POLYNOMIAL 11 POINTS N= 3 M= 5

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-5	-.08391608	.05827506	.03496503	-.03496503			
-4	.02097902	-.05710956	.01398601	.00699301			
-3	.10256410	-.10334110	-.00233100	.02564103			
-2	.16083916	-.09770785	-.01398601	.02680653			
-1	.19580420	-.05749806	-.02097902	.01631702			
0	.20745921	0.	-.02331002	0.			
1	.19580420	.05749806	-.02097902	-.01631702			
2	.16083916	.09770785	-.01398601	-.02680653			
3	.10256410	.10334110	-.00233100	-.02564103			
4	.02097902	.05710956	.01398601	-.00699301			
5	-.08391608	-.05827506	.03496503	.03496503			

DEGREE 3 POLYNOMIAL 13 POINTS N= 3 M= 6

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-6	-.07692308	.04716117	.02197802	-.01923077			
-5	0.	-.02747253	.01098901	-.00000000			
-4	.06293706	-.06568432	.00199800	.01048951			
-3	.11188811	-.07475857	-.00499500	.01398601			
-2	.14685315	-.06197969	-.00999001	.01223776			
-1	.16783217	-.03463203	-.01298701	.00699301			
0	.17482517	0.	-.01398601	0.			
1	.16783217	.03463203	-.01298701	-.00699301			
2	.14685315	.06197969	-.00999001	-.01223776			
3	.11188811	.07475857	-.00499500	-.01398601			
4	.06293706	.06568432	.00199800	-.01048951			
5	0.	.02747253	.01098901	.00000000			
6	-.07692308	-.04716117	.02197802	.01923077			

DEGREE 3 POLYNOMIAL 15 POINTS N= 3 M= 7

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-7	-.07058824	.03867102	.01470588	-.01143791			
-6	-.01176471	-.01233271	.00840336	-.00163399			
-5	.03800905	-.04234600	.00307046	.00439920			
-4	.07873303	-.05486725	-.00129282	.00729010			
-3	.11040724	-.05339486	-.00468649	.00766717			
-2	.13303167	-.04142725	-.00711054	.00615887			
-1	.14660633	-.02246283	-.00856496	.00339367			
0	.15113122	0.	-.00904977	0.			
1	.14660633	.02246283	-.00856496	-.00339367			
2	.13303167	.04142725	-.00711054	-.00615887			
3	.11040724	.05339486	-.00468649	-.00766717			
4	.07873303	.05486725	-.00129282	-.00729010			
5	.03800905	.04234600	.00307046	-.00439920			
6	-.01176471	.01233271	.00840336	.00163399			
7	-.07058824	-.03867102	.01470588	.01143791			

DEGREE 3 POLYNOMIAL 17 POINTS N= 3 M= 8

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-8	-.06501548	.03216374	.01031992	-.00722394			
-7	-.01857585	-.00421397	.00644995	-.00180599			
-6	.02167183	-.02764878	.00309598	.00180599			
-5	.05572755	-.03998968	.00025800	.00386997			
-4	.08359133	-.04308566	-.00206398	.00464396			
-3	.10526316	-.03878569	-.00386997	.00438596			
-2	.12074303	-.02893877	-.00515996	.00335397			
-1	.13003096	-.01539388	-.00593395	.00180599			
0	.13312693	0.	-.00619195	0.			
1	.13003096	.01539388	-.00593395	-.00180599			
2	.12074303	.02893877	-.00515996	-.00335397			
3	.10526316	.03878569	-.00386997	-.00438596			
4	.08359133	.04308566	-.00206398	-.00464396			
5	.05572755	.03998968	.00025800	-.00386997			
6	.02167183	.02764878	.00309598	-.00180599			
7	-.01857585	.00421397	.00644995	.00180599			
8	-.06501548	-.03216374	.01031992	.00722394			

DEGREE 3 POLYNOMIAL 19 POINTS N= 3 M= 9

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-9	-.06015038	.02711324	.00751880	-.00478469			
-8	-.02255639	.00026582	.00501253	-.00159490			
-7	.01061477	-.01816931	.00280112	.00065672			
-6	.03936311	-.02924368	.00088456	.00208744			
-5	.06368863	-.03400882	-.00073714	.00281452			
-4	.08359133	-.03351628	-.00206398	.00295525			
-3	.09907121	-.02881759	-.00309598	.00262689			
-2	.11012826	-.02096429	-.00383311	.00194671			
-1	.11676249	-.01100791	-.00427539	.00103199			
0	.11897391	0.	-.00442282	0.			
1	.11676249	.01100791	-.00427539	-.00103199			
2	.11012826	.02096429	-.00383311	-.00194671			
3	.09907121	.02881759	-.00309598	-.00262689			
4	.08359133	.03351628	-.00206398	-.00295525			
5	.06368863	.03400882	-.00073714	-.00281452			
6	.03936311	.02924368	.00088456	-.00208744			
7	.01061477	.01816931	.00280112	-.00065672			
8	-.02255639	-.00026582	.00501253	.00159490			
9	-.06015038	-.02711324	.00751880	.00478469			

DEGREE 3 POLYNOMIAL 21 POINTS N= 3 M= 10

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-10	-.05590062	.02313508	.00564653	-.00329381			
-9	-.02484472	.00276052	.00395257	-.00131752			
-8	.00294214	-.01191054	.00243692	.00013869			
-7	.02745995	-.02151184	.00109959	.00113261			
-6	.04870873	-.02667709	-.00005944	.00172203			
-5	.06668846	-.02804002	-.00104015	.00196473			
-4	.08139915	-.02623434	-.00184255	.00191850			
-3	.09284080	-.02189378	-.00246664	.00164113			
-2	.10101340	-.01565205	-.00291242	.00119039			
-1	.10591697	-.00814289	-.00317989	.00062409			
0	.10755149	0.	-.00326904	0.			
1	.10591697	.00814289	-.00317989	-.00062409			
2	.10101340	.01565205	-.00291242	-.00119039			
3	.09284080	.02189378	-.00246664	-.00164113			
4	.08139915	.02623434	-.00184255	-.00191850			
5	.06668846	.02804002	-.00104015	-.00196473			
6	.04870873	.02667709	-.00005944	-.00172203			
7	.02745995	.02151184	.00109959	-.00113261			
8	.00294214	.01191054	.00243692	-.00013869			
9	-.02484472	-.00276052	.00395257	.00131752			
10	-.05590062	-.02313508	.00564653	.00329381			

DEGREE 3 POLYNOMIAL 23 POINTS N= 3 M= 11

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-11	-.05217391	.01995541	.00434783	-.00234114			
-10	-.02608696	.00412993	.00316206	-.00106415			
-9	-.00248447	-.00769231	.00208922	-.00009121			
-8	.01863354	-.01591162	.00112931	.00060809			
-7	.03726708	-.02092835	.00028233	.00106415			
-6	.05341615	-.02314280	-.00045172	.00130739			
-5	.06708075	-.02295531	-.00107284	.00136820			
-4	.07826087	-.02076619	-.00158103	.00127698			
-3	.08695652	-.01697578	-.00197628	.00106415			
-2	.09316770	-.01198439	-.00225861	.00076011			
-1	.09689441	-.00619236	-.00242801	.00039526			
0	.09813665	0.	-.00248447	0.			
1	.09689441	.00619236	-.00242801	-.00039526			
2	.09316770	.01198439	-.00225861	-.00076011			
3	.08695652	.01697578	-.00197628	-.00106415			
4	.07826087	.02076619	-.00158103	-.00127698			
5	.06708075	.02295531	-.00107284	-.00136820			
6	.05341615	.02314280	-.00045172	-.00130739			
7	.03726708	.02092835	.00028233	-.00106415			
8	.01863354	.01591162	.00112931	-.00060809			
9	-.00248447	.00769231	.00208922	.00009121			
10	-.02608696	-.00412993	.00316206	.00106415			
11	-.05217391	-.01995541	.00434783	.00234114			

DEGREE 4 POLYNOMIAL 5 POINTS N= 4 M= 2

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-2	-.00000000	.08333333	-.08333333	-.50000000	1.00000000		
-1	.00000000	-.66666667	1.33333333	1.00000000	-4.00000000		
0	1.00000000	0.	-2.50000000	0.	6.00000000		
1	.00000000	.66666667	1.33333333	-1.00000000	-4.00000000		
2	-.00000000	-.08333333	-.08333333	.50000000	1.00000000		

DEGREE 4 POLYNOMIAL 7 POINTS N= 4 M= 3

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-3	.02164502	.08730159	-.09848485	-.16666667	.27272727		
-2	-.12987013	-.26587302	.50757576	.16666667	-.63636364		
-1	.32467532	-.23015873	-.14393939	.16666667	.09090909		
0	.54709957	0.	-.53030303	0.	.54545455		
1	.32467532	.23015873	-.14393939	-.16666667	.09090909		
2	-.12987013	.26587302	.50757576	-.16666667	-.63636364		
3	.02164502	-.08730159	-.09848485	.16666667	.27272727		

DEGREE 4 POLYNOMIAL 9 POINTS N= 4 M= 4

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-4	.03496503	.07239057	-.07342657	-.07070707	.09790210		
-3	-.12320513	-.11952862	.21620047	.03535354	-.14685315		
-2	.06993007	-.16245791	.08799534	.06565657	-.07692308		
-1	.31468531	-.10606061	-.12296037	.04545455	.06293706		
0	.41724942	0.	-.21561772	0.	.12587413		
1	.31468531	.10606061	-.12296037	-.04545455	.06293706		
2	.06993007	.16245791	.08799534	-.06565657	-.07692308		
3	-.12820513	.11952862	.21620047	-.03535354	-.14685315		
4	.03496503	-.07239057	-.07342657	.07070707	.09790210		

DEGREE 4 POLYNOMIAL 11 POINTS N= 4 M= 5

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-5	.04195804	.05827506	-.05244755	-.03496503	.04195804		
-4	-.10489510	-.05710956	.10139860	.00699301	-.04195804		
-3	-.02331002	-.10334110	.08508159	.02564103	-.04195804		
-2	.13986014	-.09770785	.00058275	.02680653	-.00699301		
-1	.27972028	-.05749806	-.07925408	.01631702	.02797203		
0	.33333333	0.	-.11072261	0.	.04195804		
1	.27972028	.05749806	-.07925408	-.01631702	.02797203		
2	.13986014	.09770785	.00058275	-.02680653	-.00699301		
3	-.02331002	.10334110	.08508159	-.02564103	-.04195804		
4	-.10489510	.05710956	.10139860	-.00699301	-.04195804		
5	.04195804	-.05827506	-.05244755	.03496503	.04195804		

DEGREE 4 POLYNOMIAL 13 POINTS N= 4 M= 6

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-6	.04524887	.04716117	-.03789593	-.01923077	.02036199		
-5	-.08144796	-.02747253	.05090498	-.00000000	-.01357466		
-4	-.05553270	-.06568432	.06005759	.01048951	-.01974496		
-3	.04524887	-.07475857	.02766351	.01398601	-.01110654		
-2	.16042781	-.06197969	-.01664267	.01223776	.00226244		
-1	.24681201	-.03463203	-.05169340	.00699301	.01316331		
0	.27848622	0.	-.06478815	0.	.01727684		
1	.24681201	.03463203	-.05169340	-.00699301	.01316331		
2	.16042781	.06197969	-.01664267	-.01223776	.00226244		
3	.04524887	.07475857	.02766351	-.01398601	-.01110654		
4	-.05553270	.06568432	.06005759	-.01048951	-.01974496		
5	-.08144796	.02747253	.05090498	.00000000	-.01357466		
6	.04524887	-.04716117	-.03789593	.01923077	.02036199		

DEGREE 4 POLYNOMIAL 15 POINTS N= 4 M= 7

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-7	.04643963	.03867102	-.02799278	-.01143791	.01083591		
-6	-.06191950	-.01233271	.02670279	-.00163399	-.00464396		
-5	-.06358657	-.04234600	.04013853	.00439920	-.00940700		
-4	-.00357228	-.05486725	.02873700	.00729010	-.00762086		
-3	.08129641	-.05339486	.00593485	.00766717	-.00269545		
-2	.16237632	-.04142725	-.01781719	.00615887	.00271710		
-1	.21920804	-.02246283	-.03505434	.00339367	.00672238		
0	.23951590	0.	-.04129771	0.	.00818377		
1	.21920804	.02246283	-.03505434	-.00339367	.00672238		
2	.16237632	.04142725	-.01781719	-.00615887	.00271710		
3	.08129641	.05339486	.00593485	-.00766717	-.00269545		
4	-.00357228	.05486725	.02873700	-.00729010	-.00762086		
5	-.06358657	.04234600	.04013853	-.00439920	-.00940700		
6	-.06191950	.01233271	.02670279	.00163399	-.00464396		
7	.04643963	-.03867102	-.02799278	.01143791	.01083591		

DEGREE 4 POLYNOMIAL 17 POINTS N= 4 M= 8

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-8	.04643963	.03216374	-.02115583	-.00722394	.00619195		
-7	-.04643963	-.00421397	.01431889	-.00180599	-.00154799		
-6	-.06191950	-.02764878	.02670279	.00180599	-.00464396		
-5	-.02786378	-.03998968	.02386481	.00386997	-.00464396		
-4	.03215051	-.04308566	.01246328	.00464396	-.00285782		
-3	.09883306	-.03878569	-.00205406	.00438596	-.00035723		
-2	.15718028	-.02893877	-.01545011	.00335397	.00202429		
-1	.19647535	-.01539388	-.02469834	.00180599	.00369136		
0	.21028816	0.	-.02798285	0.	.00428673		
1	.19647535	.01539388	-.02469834	-.00180599	.00369136		
2	.15718028	.02893877	-.01545011	-.00335397	.00202429		
3	.09883306	.03878569	-.00205406	-.00438596	-.00035723		
4	.03215051	.04308566	.01246328	-.00464396	-.00285782		
5	-.02786378	.03998968	.02386481	-.00386997	-.00464396		
6	-.06191950	.02764878	.02670279	-.00180599	-.00464396		
7	-.04643963	.00421397	.01431889	.00180599	-.00154799		
8	.04643963	-.03216374	-.02115583	.00722394	.00619195		

DEGREE 4 POLYNOMIAL 19 POINTS N= 4 M= 9

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-9	.04576659	.02711324	-.01633035	-.00478469	.00374454		
-8	-.03432494	.00026582	.00766244	-.00159490	-.00041606		
-7	-.05453520	-.01816931	.01792117	.00065672	-.00237399		
-6	-.03903621	-.02924368	.01853761	.00208744	-.00277169		
-5	.00242294	-.03400882	.01305796	.00281452	-.00216596		
-4	.05451609	-.03351628	.00448284	.00295525	-.00102791		
-3	.10634002	-.02881759	-.00473268	.00262689	.00025698		
-2	.14941446	-.02096429	-.01267912	.00194671	.00138891		
-1	.17768206	-.01100791	-.01799255	.00103199	.00215372		
0	.18750841	0.	-.01985462	0.	.00242294		
1	.17768206	.01100791	-.01799255	-.00103199	.00215372		
2	.14941446	.02096429	-.01267912	-.00194671	.00138891		
3	.10634002	.02881759	-.00473268	-.00262689	.00025698		
4	.05451609	.03351628	.00448284	-.00295525	-.00102791		
5	.00242294	.03400882	.01305796	-.00281452	-.00216596		
6	-.03903621	.02924368	.01853761	-.00208744	-.00277169		
7	-.05653520	.01816931	.01792117	-.00065672	-.00237399		
8	-.03432494	-.00026582	.00766244	.00159490	-.00041606		
9	.04576659	-.02711324	-.01633035	.00478469	.00374454		

DEGREE 4 POLYNOMIAL 21 POINTS N= 4 M= 10

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-10	.04472050	.02313508	-.01284585	-.00329381	.00237154		
-9	-.02484472	.00276052	.00395257	-.00131752	.00000000		
-8	-.05001635	-.01191054	.01216975	.00013869	-.00124818		
-7	-.04315136	-.02151184	.01407669	.00113261	-.00166424		
-6	-.01515297	-.02667709	.01167721	.00172203	-.00150516		
-5	.02452935	-.02804002	.00670795	.00196473	-.00099365		
-4	.06789993	-.02623434	.00063837	.00191850	-.00031816		
-3	.10841682	-.02189378	-.00532924	.00164113	.00036711		
-2	.14099187	-.01565205	-.01025975	.00119039	.00094225		
-1	.16199065	-.00814289	-.01348524	.00062409	.00132160		
0	.16923254	0.	-.01460493	0.	.00145376		
1	.16199065	.00814289	-.01348524	-.00062409	.00132160		
2	.14099187	.01565205	-.01025975	-.00119039	.00094225		
3	.10841682	.02189378	-.00532924	-.00164113	.00036711		
4	.06789993	.02623434	.00063837	-.00191850	-.00031816		
5	.02452935	.02804002	.00670795	-.00196473	-.00099365		
6	-.01515297	.02667709	.01167721	-.00172203	-.00150516		
7	-.04315136	.02151184	.01407669	-.00113261	-.00166424		
8	-.05001635	.01191054	.01216975	-.00013869	-.00124818		
9	-.02484472	-.00276052	.00395257	.00131752	.00000000		
10	.04472050	-.02313508	-.01284585	.00329381	.00237154		

DEGREE 4 POLYNOMIAL 23 POINTS N= 4 M= 11

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-11	.04347826	.01995541	-.01027499	-.00234114	.00156076		
-10	-.01739130	.00412993	.00183271	-.00106415	.00014189		
-9	-.04347826	-.00769231	.00835614	-.00009121	-.00066890		
-8	-.04347826	-.01591162	.01062464	.00060809	-.00101348		
-7	-.02517162	-.02092835	.00982764	.00106415	-.00101881		
-6	.00457666	-.02314280	.00701461	.00130739	-.00079691		
-5	.03981693	-.02295531	.00309511	.00136820	-.00044486		
-4	.07551487	-.02076619	-.00116123	.00127698	-.00004481		
-3	.10755149	-.01697578	-.00512474	.00106415	.00033605		
-2	.13272311	-.01198439	-.00830564	.00076011	.00064543		
-1	.14874142	-.00619236	-.01035411	.00039526	.00084599		
0	.15423341	0.	-.01106026	0.	.00091533		
1	.14874142	.00619236	-.01035411	-.00039526	.00084599		
2	.13272311	.01198439	-.00830564	-.00076011	.00064543		
3	.10755149	.01697578	-.00512474	-.00106415	.00033605		
4	.07551487	.02076619	-.00116123	-.00127698	-.00004481		
5	.03981693	.02295531	.00309511	-.00136820	-.00044486		
6	.00457666	.02314280	.00701461	-.00130739	-.00079691		
7	-.02517162	.02092835	.00982764	-.00106415	-.00101881		
8	-.04347826	.01591162	.01062464	-.00060809	-.00101348		
9	-.04347826	.00769231	.00835614	.00009121	-.00066890		
10	-.01739130	-.00412993	.00183271	.00106415	.00014189		
11	.04347826	-.01995541	-.01027499	.00234114	.00156076		

DEGREE 5 POLYNOMIAL 5 POINTS N= 5 M= 2

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-2	-.00000000	.07108333	-.08333333	-.40812500	1.00000000	-.36750000	
-1	.00000000	-.63200000	1.33333333	.74000000	-4.00000000	1.04000000	
0	1.00000000	0.	-2.50000000	0.	6.00000000	0.	
1	.00000000	.63200000	1.33333333	-.74000000	-4.00000000	-1.04000000	
2	-.00000000	-.07108333	-.08333333	.40812500	1.00000000	.36750000	

DEGREE 5 POLYNOMIAL 7 POINTS N= 5 M= 3

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-3	.02164502	-.01666667	-.09848485	.12500000	.27272727	-.50000000	
-2	-.12987013	.15000000	.50757576	-1.00000000	-.63636364	2.00000000	
-1	.32467532	-.75000000	-.14393939	1.62500000	.09090909	-2.50000000	
0	.56709957	0.	-.53030303	0.	.54545455	0.	
1	.32467532	.75000000	-.14393939	-1.62500000	.09090909	2.50000000	
2	-.12987013	-.15000000	.50757576	1.00000000	-.63636364	-2.00000000	
3	.02164502	.01666667	-.09848485	-.12500000	.27272727	.50000000	

DEGREE 5 POLYNOMIAL 9 POINTS N= 5 M= 4

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-4	.03496503	-.02960373	-.07342657	.08741259	.09790210	-.15384615	
-3	-.12820513	.16095571	.21620047	-.39947552	-.14685315	.42307692	
-2	.06993007	-.26445221	.08799534	.22377622	-.07692308	-.15384615	
-1	.31468531	-.33554779	-.12296037	.40122378	.06293706	-.34615385	
0	.41724942	0.	-.21561772	0.	.12587413	0.	
1	.31468531	.33554779	-.12296037	-.40122378	.06293706	.34615385	
2	.06993007	.26445221	.08799534	-.22377622	-.07692308	.15384615	
3	-.12820513	-.16095571	.21620047	.39947552	-.14685315	-.42307692	
4	.03496503	.02960373	-.07342657	-.08741259	.09790210	.15384615	

DEGREE 5 POLYNOMIAL 11 POINTS N= 5 M= 5

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-5	.04195804	-.03339161	-.05244755	.05638112	.04195804	-.05769231	
-4	-.10489510	.12622378	.10139860	-.17569930	-.04195804	.11538462	
-3	-.02331002	-.07278555	.08508159	-.00480769	-.04195804	.01923077	
-2	.13986014	-.21993007	.00058275	.14860140	-.00699301	-.07692308	
-1	.27972028	-.17972028	-.07925408	.13811189	.02797203	-.07692308	
0	.33333333	0.	-.11072261	0.	.04195804	0.	
1	.27972028	.17972028	-.07925408	-.13811189	.02797203	.07692308	
2	.13986014	.21993007	.00058275	-.14860140	-.00699301	.07692308	
3	-.02331002	.07278555	.08508159	.00480769	-.04195804	-.01923077	
4	-.10489510	-.12622378	.10139860	.17569930	-.04195804	-.11538462	
5	.04195804	.03339161	-.05244755	-.05638112	.04195804	.05769231	

DEGREE 5 POLYNOMIAL 13 POINTS N= 5 M= 6

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-6	.04524887	-.03306938	-.03789593	.03676471	.02036199	-.02488688	
-5	-.08144796	.09287330	.05090498	-.08399321	-.01357466	.03733032	
-4	-.05553270	-.00004114	.06005759	-.03532497	-.01974496	.02036199	
-3	.04524887	-.11497385	.02766351	.04198375	-.01110654	-.01244344	
-2	.16042781	-.15679761	-.01664267	.07841423	.00226244	-.02941176	
-1	.24681201	-.10756890	-.05169340	.05789798	.01316331	-.02262443	
0	.27848622	0.	-.06478815	0.	.01727684	0.	
1	.24681201	.10756890	-.05169340	-.05789798	.01316331	.02262443	
2	.16042781	.15679761	-.01664267	-.07841423	.00226244	.02941176	
3	.04524887	.11497385	.02766351	-.04198375	-.01110654	.01244344	
4	-.05553270	.00004114	.06005759	.03532497	-.01974496	-.02036199	
5	-.08144796	-.09287330	.05090498	.08399321	-.01357466	-.03733032	
6	.04524887	.03306938	-.03789593	-.03676471	.02036199	.02488688	

DEGREE 5 POLYNOMIAL 15 POINTS N= 5 M= 7

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-7	.04643963	-.03109907	-.02799278	.02465170	.01083591	-.01191950	
-6	-.06191950	.06740454	.02670279	-.04287926	-.00464396	.01362229	
-5	-.06358657	.02589069	.04013853	-.03089724	-.00940700	.01165754	
-4	-.00357228	-.05180043	.02873700	.00570374	-.00762086	.00052393	
-3	.08129641	-.10573986	.00593485	.03474339	-.00269545	-.00894261	
-2	.16237632	-.11112765	-.01781719	.04221243	.00271710	-.01190760	
-1	.21920804	-.06951060	-.03505434	.02772982	.00672238	-.00803763	
0	.23951590	0.	-.04129771	0.	.00818377	0.	
1	.21920804	.06951060	-.03505434	-.02772982	.00672238	.00803763	
2	.16237632	.11112765	-.01781719	-.04221243	.00271710	.01190760	
3	.08129641	.10573986	.00593485	-.03474339	-.00269545	.00894261	
4	-.00357228	.05180043	.02873700	-.00570374	-.00762086	-.00052393	
5	-.06358657	-.02589069	.04013853	.03089724	-.00940700	-.01165754	
6	-.06191950	-.06740454	.02670279	.04287926	-.00464396	-.01362229	
7	.04643963	.03109907	-.02799278	-.02465170	.01083591	.01191950	

DEGREE 5 POLYNOMIAL 17 POINTS N= 5 M= 8

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-8	.04643963	-.02858617	-.02115583	.01702786	.00619195	-.00619195	
-7	-.04643963	.04894221	.01431889	-.02302632	-.00154799	.00541796	
-6	-.06191950	.03310114	.02670279	-.02244582	-.00464396	.00619195	
-5	-.02786378	-.01720846	.02386481	-.00522446	-.00464396	.00232198	
-4	.03215051	-.06411447	.01246328	.01303882	-.00285782	-.00214337	
-3	.09483306	-.08726879	-.00205406	.02374077	-.00035723	-.00494165	
-2	.15718028	-.08034254	-.01545011	.02387473	.00202429	-.00523934	
-1	.19647535	-.04752124	-.02469834	.01463146	.00369136	-.00327459	
0	.21028816	0.	-.02798285	0.	.00428673	0.	
1	.19647535	.04752124	-.02469834	-.01463146	.00369136	.00327459	
2	.15718028	.08034254	-.01545011	-.02387473	.00202429	.00523934	
3	.09883306	.08726879	-.00205406	-.02374077	-.00035723	.00494165	
4	.03215051	.06411447	.01246328	-.01303882	-.00285782	.00214337	
5	-.02786378	.01720846	.02386481	.00522446	-.00464396	-.00232198	
6	-.06191950	-.03310114	.02670279	.02244582	-.00464396	-.00619195	
7	-.04643963	-.04894221	.01431889	.02302632	-.00154799	-.00541796	
8	.04643963	.02858617	-.02115583	-.01702786	.00619195	.00619195	

DEGREE 5 POLYNOMIAL 19 POINTS N= 5 M= 9

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-9	.04576659	-.02601415	-.01633035	.01209174	.00374454	-.00343249	
-8	-.03432494	.03568407	.00766244	-.01284585	-.00041606	.00228833	
-7	-.05653520	.03287465	.01792117	-.01555789	-.00237399	.00329789	
-6	-.03903621	.00096601	.01853761	-.00750896	-.00277169	.00195181	
-5	.00242294	-.03557139	.01305796	.00331089	-.00216596	-.00010096	
-4	.05451609	-.06164254	.00448284	.01188983	-.00102791	-.00181720	
-3	.10834002	-.06996527	-.00473268	.01569785	.00025698	-.00265850	
-2	.14941446	-.05950768	-.01267912	.01419040	.00138891	-.00249024	
-1	.17768206	-.03392561	-.01799255	.00831202	.00215372	-.00148068	
0	.18750841	0.	-.01985462	0.	.00242294	0.	
1	.17768206	.03392561	-.01799255	-.00831202	.00215372	.00148068	
2	.14941446	.05950768	-.01267912	-.01419040	.00138891	.00249024	
3	.10834002	.06996527	-.00473268	-.01569785	.00025698	.00265850	
4	.05451609	.06164254	.00448284	-.01188983	-.00102791	.00181720	
5	.00242294	.03557139	.01305796	-.00331089	-.00216596	.00010096	
6	-.03903621	-.00096601	.01853761	.00750896	-.00277169	-.00195181	
7	-.05653520	-.03287465	.01792117	.01555789	-.00237399	-.00329789	
8	-.03432494	-.03568407	.00766244	.01284585	-.00041606	-.00228833	
9	.04576659	.02601415	-.01633035	-.01209174	.00374454	.00343249	

DEGREE 5 POLYNOMIAL 21 POINTS N= 5 M= 10

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-10	.04472050	-.02358468	-.01284585	.00880207	.00237154	-.00200669	
-9	-.02484472	.02612040	.00395257	-.00736546	.00000000	.00100334	
-8	-.05001635	.02989134	.01216975	-.01068394	-.00124818	.00179546	
-7	-.04315136	.01004449	.01407669	-.00703741	-.00166424	.00135540	
-6	-.01515297	-.01717885	.01167721	-.00073709	-.00150516	.00040796	
-5	.02452935	-.04085299	.00670795	.00528204	-.00099365	-.00055034	
-4	.06789993	-.05460851	.00063837	.00926465	-.00031816	-.00121872	
-3	.10841682	-.05587288	-.00532924	.01043841	.00036711	-.00145946	
-2	.14099187	-.04511105	-.01025975	.00881741	.00094225	-.00126531	
-1	.16199065	-.02506614	-.01348524	.00500557	.00132160	-.00072688	
0	.16923254	0.	-.01460493	0.	.00145376	0.	
1	.16199065	.02506614	-.01348524	-.00500557	.00132160	.00072688	
2	.14099187	.04511105	-.01025975	-.00881741	.00094225	.00126531	
3	.10841682	.05587288	-.00532924	-.01043841	.00036711	.00145946	
4	.06789993	.05460851	.00063837	-.00926465	-.00031816	.00121872	
5	.02452935	.04085299	.00670795	-.00528204	-.00099365	.00055034	
6	-.01515297	.01717885	.01167721	.00073709	-.00150516	-.00040796	
7	-.04315136	-.01004449	.01407669	.00703741	-.00166424	-.00135540	
8	-.05001635	-.02989134	.01216975	.01068394	-.00124818	-.00179546	
9	-.02484472	-.02612040	.00395257	.00736546	.00000000	-.00100334	
10	.04472050	.02358468	-.01284585	-.00880207	.00237154	.00200669	

DEGREE 5 POLYNOMIAL 23 POINTS N= 5 M= 11

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-11	.04347826	-.02137124	-.01027499	.00654961	.00156076	-.00122631	
-10	-.01739130	.01915780	.00183271	-.00429715	.00014189	.00044593	
-9	-.04347826	.02612040	.00835614	-.00736546	-.00066890	.00100334	
-8	-.04347826	.01414412	.01062464	-.00585791	-.00101348	.00089186	
-7	-.02517162	-.00570274	.00982764	-.00221139	-.00101881	.00045180	
-6	.00457666	-.02551562	.00701461	.00181786	-.00079691	-.00007041	
-5	.03981693	-.04015826	.00309511	.00506913	-.00044486	-.00051047	
-4	.07551487	-.04686723	-.00116123	.00689219	-.00004481	-.00077451	
-3	.10755149	-.04485643	-.00512474	.00706222	.00033605	-.00082732	
-2	.13272311	-.03492167	-.00830564	.00569469	.00064543	-.00068063	
-1	.14874142	-.01904514	-.01035411	.00316032	.00084599	-.00038139	
0	.15423341	0.	-.01106026	0.	.00091533	0.	
1	.14874142	.01904514	-.01035411	-.00316032	.00084599	.00038139	
2	.13272311	.03492167	-.00830564	-.00569469	.00064543	.00068063	
3	.10755149	.04485643	-.00512474	-.00706222	.00033605	.00082732	
4	.07551487	.04686723	-.00116123	-.00689219	-.00004481	.00077451	
5	.03981693	.04015826	.00309511	-.00506913	-.00044486	.00051047	
6	.00457666	.02551562	.00701461	-.00181786	-.00079691	.00007041	
7	-.02517162	.00570274	.00982764	.00221139	-.00101881	-.00045180	
8	-.04347826	-.01414412	.01062464	.00585791	-.00101348	-.00089186	
9	-.04347826	-.02612040	.00835614	.00736546	-.00066890	-.00100334	
10	-.01739130	-.01915780	.00183271	.00429715	.00014189	-.00044593	
11	.04347826	.02137124	-.01027499	-.00654961	.00156076	.00122631	

DEGREE 6 POLYNOMIAL 7 POINTS N= 6 M= 3

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-3	.00000000	-.01666667	.01111111	.12500000	-.16666667	-.50000000	1.00000000
-2	-.00000000	.15000000	-.15000000	-1.00000000	2.00000000	2.00000000	-6.00000000
-1	.00000000	-.75000000	1.50000000	1.62500000	-6.50000000	-2.50000000	15.00000000
0	1.00000000	0.	-2.72222222	0.	9.33333333	0.	-20.00000000
1	.00000000	.75000000	1.50000000	-1.62500000	-6.50000000	2.50000000	15.00000000
2	-.00000000	-.15000000	-.15000000	1.00000000	2.00000000	-2.00000000	-6.00000000
3	.00000000	.01666667	.01111111	-.12500000	-.16666667	.50000000	1.00000000

DEGREE 6 POLYNOMIAL 9 POINTS N= 6 M= 4

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-4	-.00543901	-.02960373	.02852629	.08741259	-.11623932	-.15384615	.26666667
-3	.04351204	.16095571	-.21709920	-.39947552	.76324786	.42307692	-1.13333333
-2	-.15229215	-.26445221	.64873608	.22377622	-1.25470085	-.15384615	1.46666667
-1	.30458430	-.33554779	-.09747216	.40122378	.00940171	-.34615385	.06666667
0	.61926962	0.	-.72538203	0.	1.19658120	0.	-1.33333333
1	.30458430	.33554779	-.09747216	-.40122378	.00940171	.34615385	.06666667
2	-.15229215	.26445221	.64873608	-.22377622	-1.25470085	.15384615	1.46666667
3	.04351204	-.16095571	-.21709920	.39947552	.76324786	-.42307692	-1.13333333
4	-.00543901	.02960373	.02852629	-.08741259	-.11623932	.15384615	.26666667

DEGREE 6 POLYNOMIAL 11 POINTS N= 6 M= 5

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-5	-.01151789	-.03339161	.03142054	.05638112	-.06900452	-.05769231	.08823529
-4	.06622789	.12622378	-.16697930	-.17569930	.31312217	.11538462	-.28235294
-3	-.12669683	-.07278555	.24722656	-.00480769	-.25648567	.01923077	.17058824
-2	.01151789	-.21993007	.20186617	.14860140	-.27330317	-.07692308	.21176471
-1	.32250103	-.17972028	-.14634855	.13811189	.11674208	-.07692308	-.07058824
0	.47593583	0.	-.33437086	0.	.33785822	0.	-.23529412
1	.32250103	.17972028	-.14634855	-.13811189	.11674208	.07692308	-.07058824
2	.01151789	.21993007	.20186617	-.14860140	-.27330317	.07692308	.21176471
3	-.12669683	.07278555	.24722656	.00480769	-.25648567	-.01923077	.17058824
4	.06622789	-.12622378	-.16697930	.17569930	.31312217	-.11538462	-.28235294
5	-.01151789	.03339161	.03142054	-.05638112	-.06900452	.05769231	.08823529

DEGREE 6 POLYNOMIAL 13 POINTS N= 6 M= 6

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-6	-.01667064	-.03306938	.02904594	.03676471	-.04104152	-.02488688	.03405573
-5	.07335080	.09287330	-.11644968	-.08399321	.13993411	.03733032	-.08513932
-4	-.07804889	-.00004114	.08440009	.03532497	-.04207351	.02036199	.01238390
-3	-.07577562	-.11487385	.15850443	.04198375	-.13112249	-.01244344	.06656347
-2	.09350830	.15679761	.05029919	.07841423	-.05914107	-.02941176	.03405573
-1	.30310247	-.10756890	-.11254965	.05789798	.06898468	-.02262443	-.03095975
0	.39106714	0.	-.18650063	0.	.12891958	0.	-.06191950
1	.30310247	.10756890	-.11254965	-.05789798	.06898468	.02262443	-.03095975
2	.09850830	.15679761	.05029919	-.07841423	-.05914107	.02941176	.03405573
3	-.07577562	.11487385	.15850443	-.04198375	-.13112249	.01244344	.06656347
4	-.07804889	.00004114	.08440009	.03532497	-.04207351	-.02036199	.01238390
5	.07335080	-.09287330	-.11644968	.08399321	.13993411	-.03733032	-.08513932
6	-.01667064	.03306938	.02904594	-.03676471	-.04104152	.02488688	.03405573

DEGREE 6 POLYNOMIAL 15 POINTS N= 6 M= 7

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-7	-.02063983	-.03109907	.02527577	.02465170	-.02516340	-.01191950	.01475748
-6	.07223942	.06740454	-.07983431	-.04287926	.06735466	.01362229	-.02951496
-5	-.03778677	.02589069	.01965062	-.03089724	.00443889	.01165754	-.00567595
-4	-.08613162	-.05180043	.09429829	.00570374	-.05192771	.00052393	.01816305
-3	-.01111376	-.10573986	.07931880	.03474339	-.05228890	-.00894261	.02033024
-2	.13892197	-.11112765	.00080817	.04221243	-.00987007	-.01190760	.00515996
-1	.27784393	-.06951060	-.08161776	.02772982	.03819031	-.00803763	-.01289990
0	.33333333	0.	-.11579918	0.	.05853245	0.	-.02063983
1	.27784393	.06951060	-.08161776	-.02772982	.03819031	.00803763	-.01289990
2	.13892197	.11112765	.00080817	-.04221243	-.00987007	.01190760	.00515996
3	-.01111376	.10573986	.07931880	-.03474339	-.05228890	.00894261	.02033024
4	-.08613162	.05180043	.09429829	-.00570374	-.05192771	-.00052393	.01816305
5	-.03778677	-.02589069	.01965062	.03089724	.00443889	-.01165754	-.00567595
6	.07223942	-.06740454	-.07983431	.04287926	.06735466	-.01362229	-.02951496
7	-.02063983	.03109907	.02527577	-.02465170	-.02516340	.01191950	.01475748

DEGREE 6 POLYNOMIAL 17 POINTS N= 6 M= 8

POINTS	DERIVATIVE 0	1	2	3	4	5	6
-8	-.02355633	-.02858617	.02150282	.01702786	-.01597344	-.00619195	.00699960
-7	.06730381	.04894221	-.05500142	-.02302632	.03447077	.00541796	-.01137434
-6	-.00942253	.03310114	-.00529120	-.02244582	.01198008	.00619195	-.00524970
-5	-.07161125	-.01720846	.05052647	-.00522446	-.01849733	.00232198	.00437475
-4	-.05399836	-.06411447	.06496624	.01303882	-.03013830	-.00214337	.00861449
-3	.03624051	-.08726879	.03609262	.02374077	-.02017820	-.00494165	.00625925
-2	.15583420	-.08034254	-.01462975	.02387473	.00159803	-.00523934	.00013461
-1	.25368359	-.04752124	-.05956358	.01463146	.02180730	-.00327459	-.00572082
0	.29105274	0.	-.07720437	0.	.02986218	0.	-.00807646
1	.25368359	.04752124	-.05956358	-.01463146	.02180730	.00327459	-.00572082
2	.15583420	.08034254	-.01462975	-.02387473	.00159803	.00523934	.00013461
3	.03624051	.08726879	.03609262	-.02374077	-.02017820	.00494165	.00625925
4	-.05399836	.06411447	.06496624	-.01303882	-.03013830	.00214337	.00861489
5	-.07161125	.01720846	.05052647	.00522446	-.01849733	-.00232198	.00437475
6	-.00942253	-.03310114	-.00529120	.02244582	.01198008	-.00619195	-.00524970
7	.06730381	-.04894221	-.05500142	.02302632	.03447077	-.00541796	-.01137434
8	-.02355633	.02858617	.02150282	-.01702786	-.01597344	.00619195	.00699960

DEGREE 6 POLYNOMIAL 19 POINTS N= 6 M= 9

POINTS	DERIVATIVE						
	0	1	2	3	4	5	6
-9	-.02562929	-.02601415	.01816323	.01209174	-.01048055	-.00343249	.00356979
-8	.06086957	.03568407	-.03832901	-.01284585	.01855072	.00228833	-.00475973
-7	.00926100	.03287465	-.01386704	-.01555789	.01073541	.00329789	-.00328981
-6	-.05163548	.00096601	.02462471	-.00750896	-.00528200	.00195181	.00062996
-5	-.06407323	-.03557139	.04518434	.00331089	-.01541482	-.00010096	.00332481
-4	-.01827971	-.06164254	.03965277	.01198983	-.01553192	-.00181720	.00363979
-3	.06708844	-.06996527	.01423099	.01569785	-.00756360	-.00265850	.00196258
-2	.16093687	-.05950768	-.01824596	.01419040	.00368466	-.00249024	-.00057612
-1	.23217122	-.03392561	-.04431797	.00831202	.01301028	-.00148068	-.00272446
0	.25858124	0.	-.05419213	0.	.01658366	0.	-.00355364
1	.23217122	.03392561	-.04431797	-.00831202	.01301028	.00148068	-.00272446
2	.16093687	.05950768	-.01824596	-.01419040	.00368466	.00249024	-.00057612
3	.06708844	.06996527	.01423099	-.01569785	-.00756360	.00265850	.00196258
4	-.01827971	.06164254	.03965277	-.01198983	-.01553192	.00181720	.00363979
5	-.06407323	.03557139	.04518434	-.00331089	-.01541482	.00010096	.00332481
6	-.05163548	-.00096601	.02462471	.00750896	-.00528200	-.00195181	.00062996
7	.00926100	-.03287465	-.01386704	.01555789	.01073541	-.00329789	-.00328981
8	.06086957	-.03568407	-.03832901	.01284585	.01855072	-.00228833	-.00475973
9	-.02562929	.02601415	.01816323	-.01209174	-.01048055	.00343249	.00356979

DEGREE 6 POLYNOMIAL 21 POINTS N= 6 M= 10

POINTS	DERIVATIVE						
	0	1	2	3	4	5	6
-10	-.02705314	-.02358468	.01534134	.00880207	-.00708535	-.00200669	.00193237
-9	.05410628	.02612040	-.02705334	-.00736546	.01040258	.00100334	-.00212560
-8	.02100178	.02989134	-.01572073	-.01068394	.00810916	.00179546	-.00191203
-7	-.03295195	.01004449	.01007115	-.00703741	-.00032037	.00135540	-.00027460
-6	-.05955041	-.01717885	.02911312	-.00073709	-.00735496	.00040796	.00119532
-5	-.04296675	-.04085299	.03321525	.00528204	-.00988693	-.00055034	.00181720
-4	.01136986	-.05460851	.02283905	.00926465	-.00776656	-.00121872	.00152196
-3	.08618477	-.05587288	.00340181	.01043841	-.00256218	-.00145946	.00059856
-2	.16005743	-.04511105	-.01774725	.00881741	.00345433	-.00126531	-.00051330
-1	.21340991	-.02506614	-.03367878	.00500557	.00809660	-.00072688	-.00138436
0	.23278443	0.	-.03956324	0.	.00982735	0.	-.00171101
1	.21340991	.02506614	-.03367878	-.00500557	.00809660	.00072688	-.00138436
2	.16005743	.04511105	-.01774725	-.00881741	.00345433	.00126531	-.00051330
3	.08618477	.05587288	.00340181	-.01043841	-.00256218	.00145946	.00059856
4	.01136986	.05460851	.02283905	-.00926465	-.00776656	.00121872	.00152196
5	-.04296675	.04085299	.03321525	-.00528204	-.00988693	.00055034	.00181720
6	-.05955041	.01717885	.02911312	.00073709	-.00735496	-.00040796	.00119532
7	-.03295195	-.01004449	.01007115	.00703741	-.00032037	-.00135540	-.00027460
8	.02100178	-.02989134	-.01572073	.01068394	.00810916	-.00179546	-.00191203
9	.05410628	-.02612040	-.02705334	.00736546	.01040258	-.00100334	-.00212560
10	-.02705314	.02358468	.01534134	-.00880207	-.00708535	.00200669	.00193237

DEGREE 6 POLYNOMIAL 23 POINTS N= 6 M= 11

POINTS	DERIVATIVE						
	0	1	2	3	4	5	6
-11	-.02798601	-.02137124	.01300059	.00654961	-.00491934	-.00122631	.00109945
-10	.04757621	.01915780	-.01932691	-.00429715	.00603288	.00044593	-.00099950
-9	.02798601	.02612040	-.01491945	-.00736546	.00581120	.00100334	-.00109945
-8	-.01749125	.01414412	.00216079	-.00585791	.00134292	.00089186	-.00039980
-7	-.04944896	-.00570274	.01773465	-.00221139	-.00322018	.00045180	.00037350
-6	-.05081670	-.02551562	.02505597	.00181786	-.00581976	-.00007041	.00085221
-5	-.02022271	-.04015826	.02264975	.00506913	-.00588902	-.00051047	.00092369
-4	.03375866	-.04686723	.01243857	.00689219	-.00383109	-.00077451	.00064240
-3	.09747448	-.04485643	-.00184270	.00706222	-.00057769	-.00082732	.00015503
-2	.15617578	-.03492167	-.01594407	.00569469	.00277202	-.00068063	-.00036081
-1	.19711506	-.01904514	-.02610919	.00316032	.00523232	-.00038139	-.00074421
0	.21175883	0.	-.02979602	0.	.00613151	0.	-.00088501
1	.19711506	.01904514	-.02610919	-.00316032	.00523232	.00038139	-.00074421
2	.15617578	.03492167	-.01594407	-.00569469	.00277202	.00068063	-.00036081
3	.09747448	.04485643	-.00184270	-.00706222	-.00057769	.00082732	.00015503
4	.03375866	.04686723	.01243857	-.00689219	-.00383109	.00077451	.00064240
5	-.02022271	.04015826	.02264975	-.00506913	-.00588902	.00051047	.00092369
6	-.05081670	.02551562	.02505597	-.00181786	-.00581976	.00007041	.00085221
7	-.04944896	.00570274	.01773465	.00221139	-.00322018	-.00045180	.00037350
8	-.01749125	-.01414412	.00216079	.00585791	.00134292	-.00089186	-.00039980
9	.02798601	-.02612040	-.01491945	.00736546	.00581120	-.00100334	-.00109945
10	.04757621	-.01915780	-.01932691	.00429715	.00603288	-.00044593	-.00099950
11	-.02798601	.02137124	.01300059	-.00654961	-.00491934	.00122631	.00109945

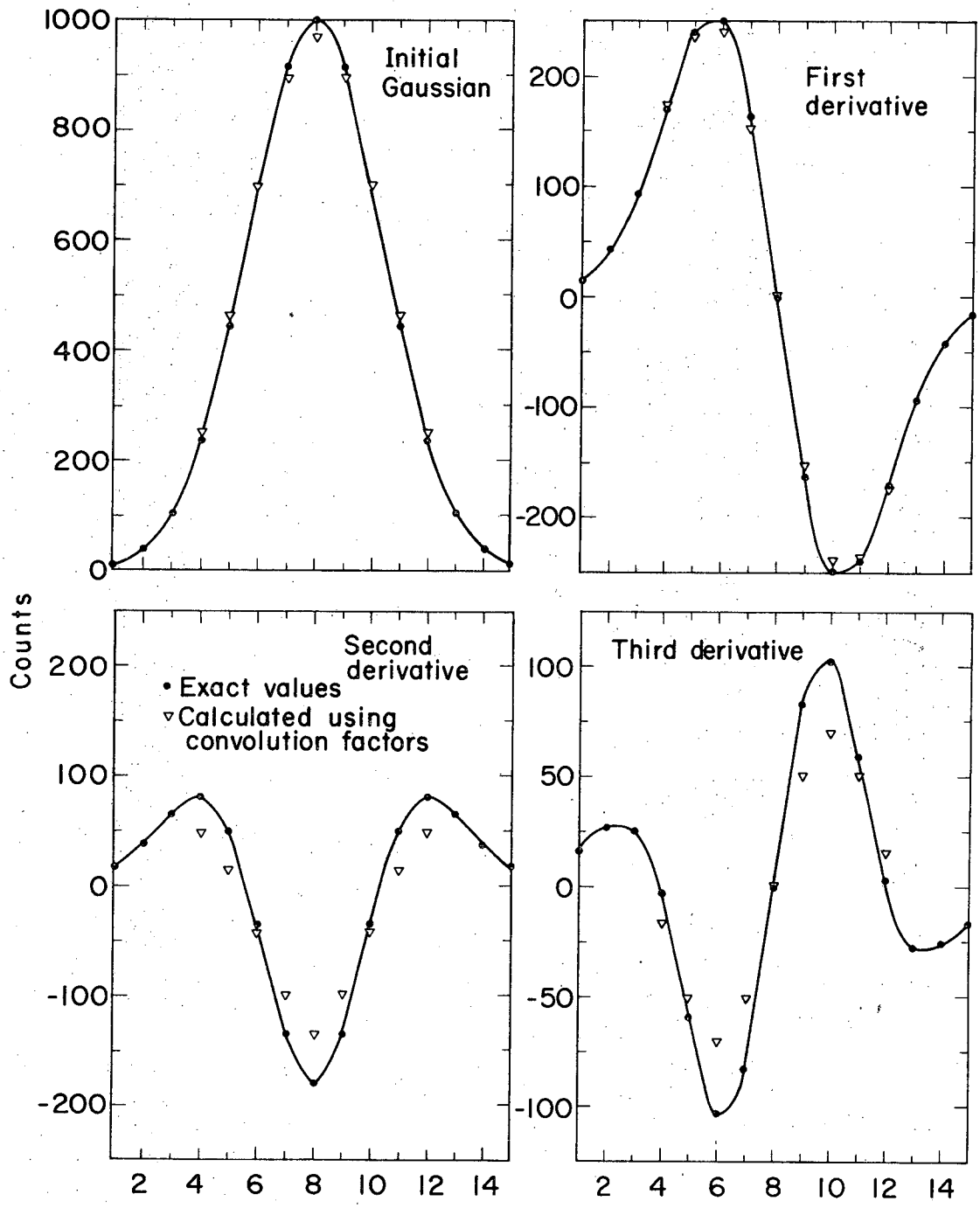
Appendix C

Computer program for smoothing and differentiation of data

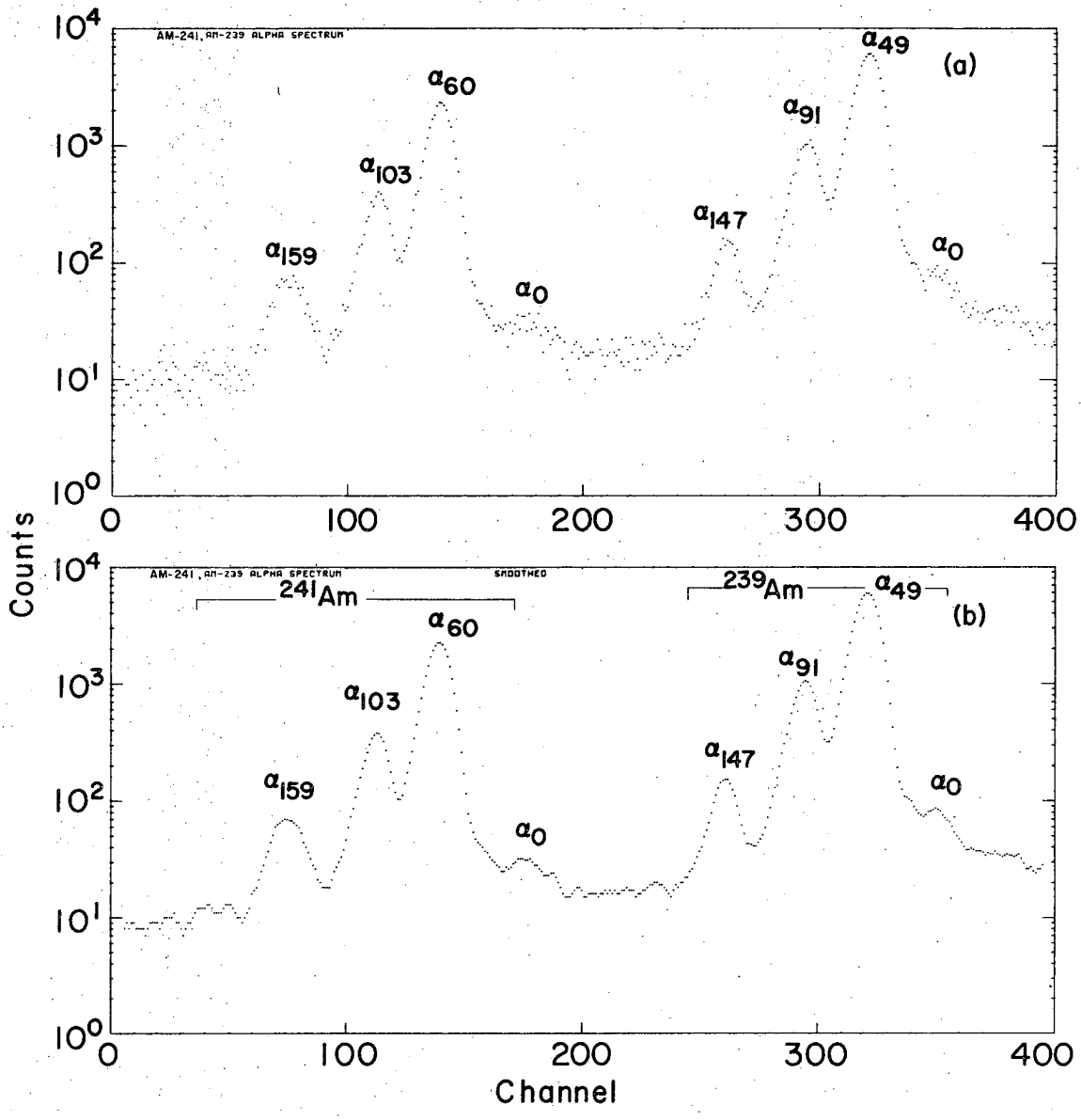
The smoothing program, in the form of a subroutine, is reproduced below. It is a more generalized form of the program given by Savitzky and Golay.[1] NDATA is the input array of N points which is to be smoothed or differentiated. CONF is the array of NP convolution factors. These four variables are set by the main program for input to the subroutine. OUTDAT is the array of M smoothed (or differentiated) data points as calculated by the subroutine.

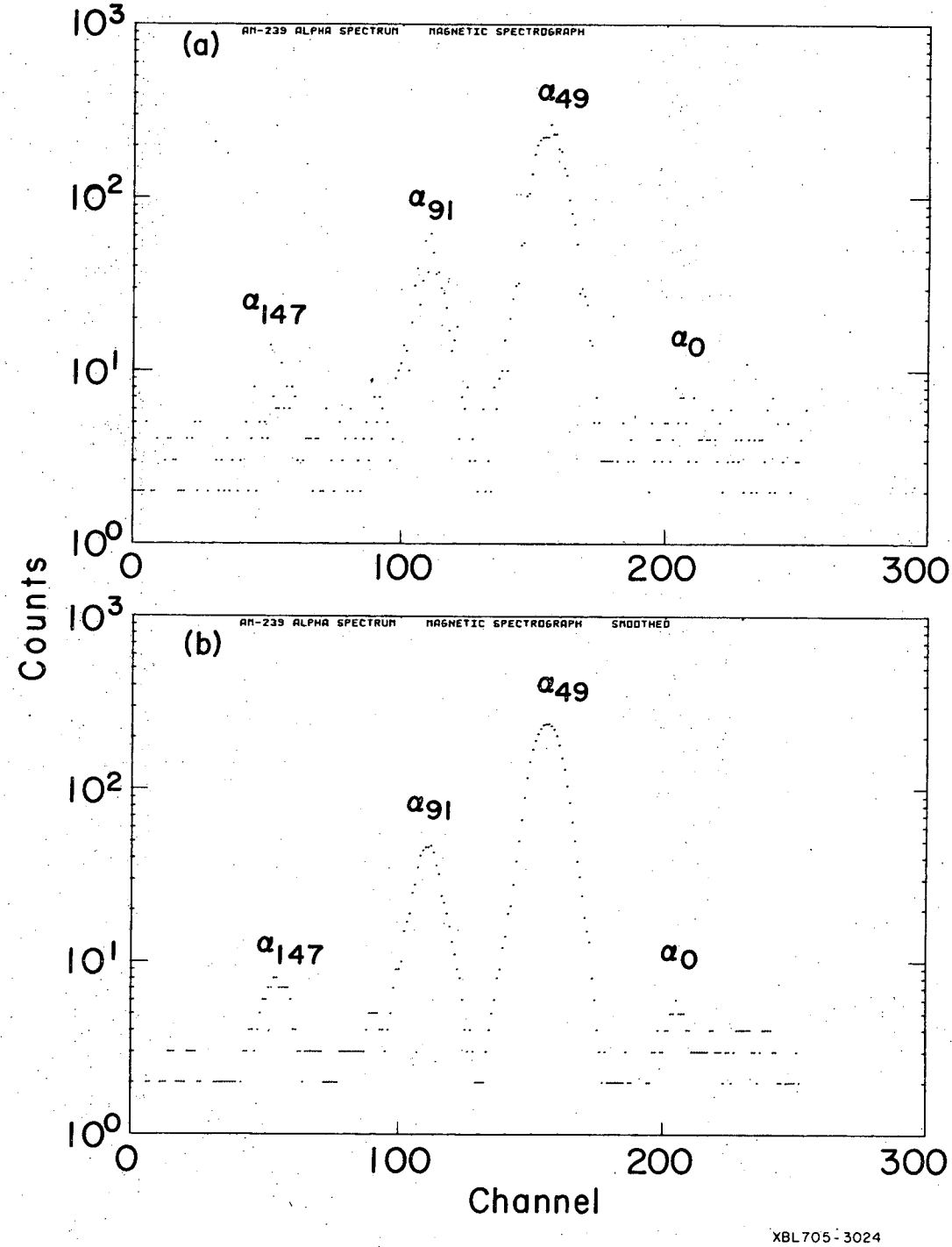
Because the dimension of the array IPOINT is set at 50, NP must be less than 50. If smoothing is desired over more than 50 points. The dimension of IPOINT can be increased.

```
                      SUBROUTINE SMOOTH(N,INDATA,M,OUTDAT,NP,CONF)
                C
                C     THIS SUBROUTINE SMOOTHS DATA OVER NP POINTS
                C
                C     N=NUMBER OF RAW DATA INPUT POINTS
                C     INDATA=ARRAY OF N INPUT DATA POINTS
                C
                C     M=NUMBER OF SMOOTHED DATA POINTS=N-NP+1
                C     OUTDAT=ARRAY OF M SMOOTHED POINTS
                C
                C     NP=NUMBER OF POINTS OVER WHICH CONVOLUTION IS PERFORMED
                C     CONF IS THE ARRAY OF CONVOLUTING FACTORS
                C
002715                DIMENSION INDATA(N),OUTDAT(N),CONF(NP),IPOINT(50)
002715                INTEGER OUTDAT
002715                INIT = (NP-1)/2
002717                SUM = 0.
002720                M = N + 1 - NP
002721                DO 10 I = 2,NP
002727                J = I - 1
002730             10 IPOINT(I) = INDATA(J)
002733                DO 100 I = 1,M
002740                NPM1 = NP - 1
002741                J = I + NPM1
002743                DO 20 K = 1,NPM1
002751                KA = K + 1
002752             20 IPOINT(K) = IPOINT(KA)
002755                IPOINT(NP) = INDATA(J)
002763                DO 30 L = 1,NP
002771             30 SUM = SUM + CONF(L) * IPOINT(L)
003000                OUTDAT(I+INIT) = SUM
003003                SUM = 0.
003004            100 CONTINUE
003006                DO 120 I = 1,INIT
003012            120 OUTDAT(I) = 0
003014                NEXT = M + INIT + 1
003022                DO 140 I = NEXT,N
003026            140 OUTDAT(I) = 0
003030                RETURN
003030                END
```

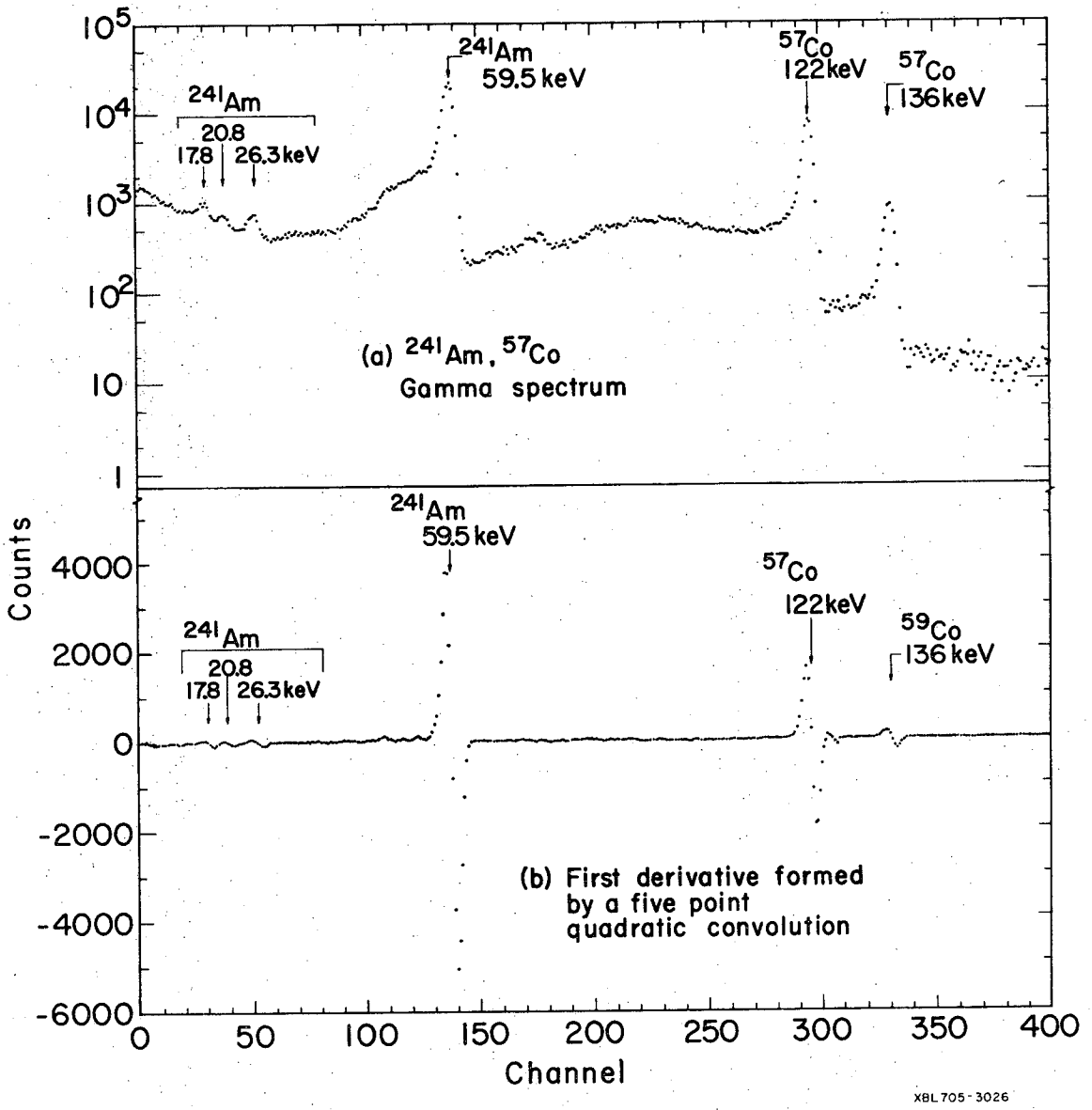

Initial Gaussian
First derivative
Second derivative
Third derivative
• Exact values
▽ Calculated using convolution factors
Counts
1000
800
600
400
200
0
200
100
0
-100
-200
100
50
-50
2
4
6
8
10
12
14

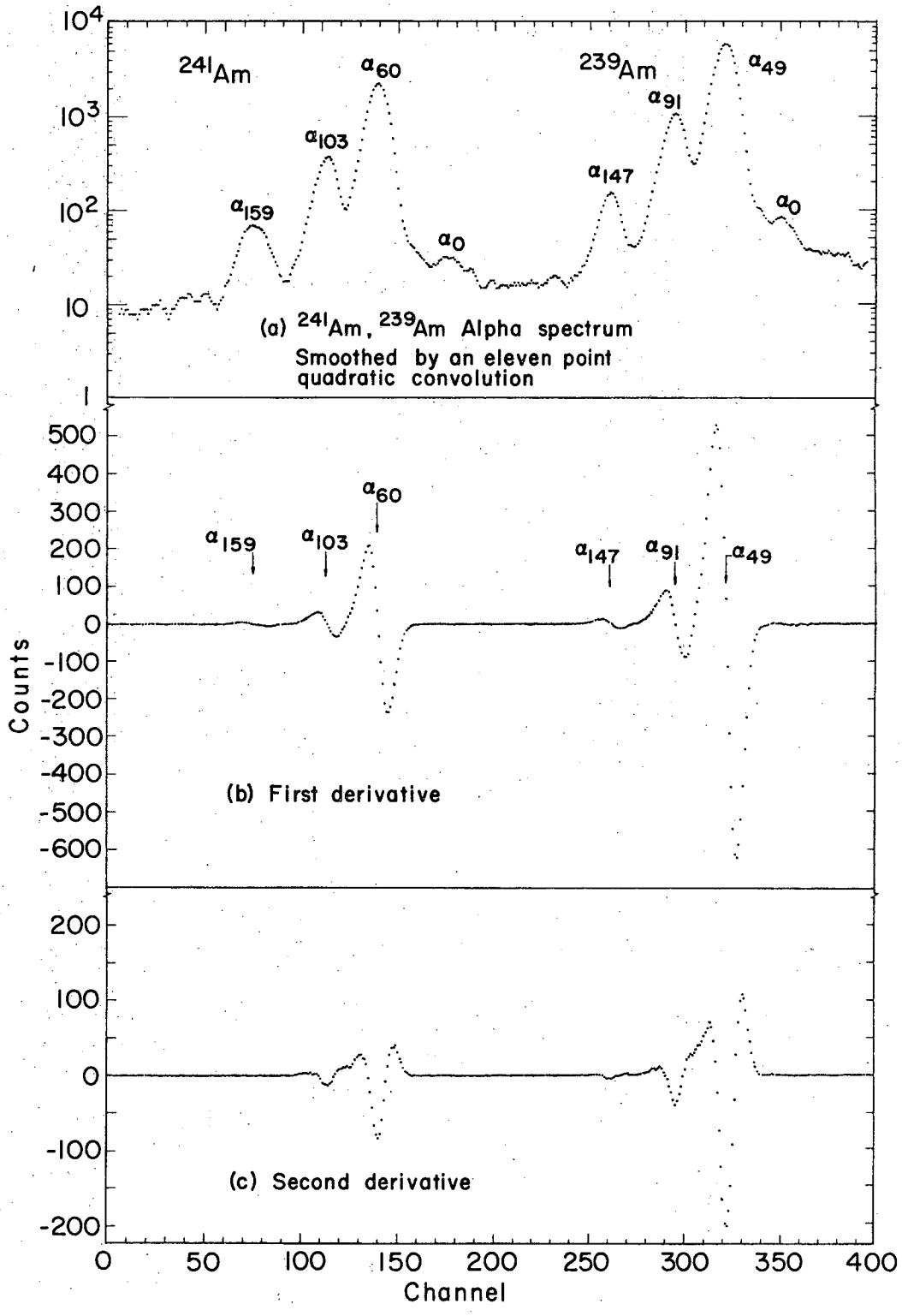
XBL705-3023


AM-241, AM-239 ALPHA SPECTRUM
(a)
α_{49}
α_{91}
α_{60}
α_{103}
α_{147}
α_{159}
α_0
α_0
AM-241, AM-239 ALPHA SPECTRUM SMOOTHED
^{241}Am
^{239}Am
(b)
Counts
Channel
XBL705-3025


Am-239 ALPHA SPECTRUM MAGNETIC SPECTROGRAPH
(a)
α_{49}
α_{91}
α_{147}
α_0
Am-239 ALPHA SPECTRUM MAGNETIC SPECTROGRAPH SMOOTHED
(b)
α_{49}
α_{91}
α_{147}
α_0
Counts
Channel
10^0
10^1
10^2
10^3
0
100
200
300

XBL705-3024


10^5
10^4
10^3
10^2
10
1
^{241}Am
59.5 keV
^{57}Co
122keV
^{57}Co
136keV
^{241}Am
20.8
17.8
26.3keV
(a) ^{241}Am, ^{57}Co
Gamma spectrum
Counts
4000
2000
0
-2000
-4000
-6000
^{241}Am
59.5 keV
^{57}Co
122keV
^{59}Co
136 keV
^{241}Am
20.8
17.8
26.3keV
(b) First derivative formed
by a five point
quadratic convolution
0
50
100
150
200
250
300
350
400
Channel
XBL705-3026


10^4
10^3
10^2
10
1
^{241}Am
α_{60}
^{239}Am
α_{49}
α_{91}
α_{103}
α_{147}
α_{159}
α_0
α_0
(a) ^{241}Am, ^{239}Am Alpha spectrum
Smoothed by an eleven point
quadratic convolution
500
400
300
200
100
0
-100
-200
-300
-400
-500
-600
Counts
α_{60}
α_{159}
α_{103}
α_{147}
α_{91}
α_{49}
(b) First derivative
200
100
0
-100
-200
(c) Second derivative
0
50
100
150
200
250
300
350
400
Channel
XBL705-3027

FOOTNOTES AND REFERENCES

*Work performed under the auspices of the U. S. Atomic Energy Commission.

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TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720