

UC San Diego

UC San Diego Previously Published Works

Title

The Absence of the Fractional Quantum Hall Effect at High Landau Levels

Permalink

<https://escholarship.org/uc/item/97j7j88x>

Authors

Fogler, MM

Koulakov, AA

Publication Date

1996-02-25

Peer reviewed

The Absence of the Fractional Quantum Hall Effect at High Landau Levels

M. M. Fogler and A. A. Koulakov

*Theoretical Physics Institute, University of Minnesota, 116 Church St. Southeast, Minneapolis,
Minnesota 55455*

Abstract

We compare the energies of the Laughlin liquid and a charge density wave in a weak magnetic field for the upper Landau level filling factors $\nu_N = 1/3$ and $1/5$. The charge density wave period has been optimized and was found to be $\simeq 3R_c$, where R_c is the cyclotron radius. We conclude that the optimal charge density wave is more energetically preferable than the Laughlin liquid for the Landau level numbers $N \geq 2$ at $\nu_N = 1/3$ and for $N \geq 3$ at $\nu_N = 1/5$. This implies that the $1/3$ fractional quantum Hall effect cannot be observed for $N \geq 2$, in agreement with the experiment.

PACS numbers: 73.40.Hm

arXiv:cond-mat/9602128v1 25 Feb 1996

The fractional quantum Hall effect (FQHE) was first discovered at the lowest Landau level (LL) [1]. This remarkable phenomenon occurring at certain unique values of the filling factor $\nu = 1/3, 1/5, \dots$ has been associated with the formation of a uniform incompressible quantum state, or the Laughlin liquid [2]. The traditional alternative to the Laughlin liquid is a charge density wave (CDW), which does not exhibit the FQHE. The FQHE occurs because the Laughlin liquid is lower in energy than the optimal CDW, which at the lowest LL has the same spacial periodicity as the triangular Wigner crystal [3].

Later, the FQHE was found at the first excited LL also. The theoretical work, driven by this discovery [4], addressed the question of what exactly ground state, liquid or crystalline, is formed at high LL's? The liquid state at the N -th LL was defined as follows:

$$|\Psi_L^N\rangle = \prod_i \frac{(a_i^\dagger)^N}{\sqrt{N!}} |\Psi_L^0\rangle \quad (1)$$

Here a_i^\dagger is the *inter*-LL ladder operator, raising the i -th electron to the next LL, and $|\Psi_L^0\rangle$ is the Laughlin state at the lowest LL. As a crystalline state Ref. [4] continues using the Wigner crystal as well as at the lowest LL. However, a recent more elaborate investigation of the CDW state [5] shows that the optimum one should have a period of the order of R_c , the cyclotron radius. The formation of the CDW with this period, which is a characteristic spread of electron wave functions, enables the system to reach a lower value of the interaction energy. Below we compare the energies of this optimal CDW and the Laughlin liquid. We show that the crystalline ground state rules out the FQH states at high LL's.

Our calculation is based on the following model [6]. We explicitly consider only the electrons at the upper LL, which is assumed to be spin-polarized. All the other LL's are completely filled. The role of these lower LL's is reduced to the screening of the Coulomb interaction among the electrons at the upper LL. The screening is accounted for by means of the dielectric function [6]:

$$\epsilon(q) = 1 + v(q) \Pi(q),$$

$$\Pi(q) = \frac{2}{\pi l^2} \sum_{\substack{m < N \\ N \leq n}} \frac{(-1)^{n-m}}{\hbar \omega_c (n-m)} F_{nm}(q) F_{mn}(q), \quad (2)$$

$$F_{nm} = L_m^{n-m} \left(\frac{q^2 l^2}{2} \right) e^{-\frac{q^2 l^2}{4}},$$

where l is the magnetic length, κ is the bare dielectric constant, $v(q) = 2\pi e^2/\kappa q$ is the Coulomb potential, $L_n^m(x)$ is the Laguerre polynomial, and ω_c is the cyclotron frequency. This dielectric function tends to unity in the limits $q \rightarrow 0$ and $q \rightarrow \infty$ and reaches its largest value of $1 + \sqrt{2} N r_s$ at $q \sim R_c^{-1}$. Here $r_s = \sqrt{2}/k_F a_B$, with k_F being the Fermi wave vector, and a_B being the effective Bohr radius. This model correctly renders the low energy physics of the system in the limit $r_s \ll 1$ and $N r_s \gg 1$ [Ref. [6]]. Moreover, the results obtained within the framework of this model remain correct to the leading order in r_s even for $N r_s \ll 1$. In the latter limit $\epsilon(q) \simeq 1$, which is consistent with the fact that the LL mixing can be ignored completely.

Let us now describe the CDW state at the upper LL. In the limit of a weak magnetic field ($N \gg 1$) a simple quasiclassical picture can be given. In this case electrons can be viewed as classical particles rotating in cyclotron orbits. The only constraint imposed by the Landau quantization is that the concentration of the centers of the cyclotron circles at any point does not exceed $1/2\pi l^2$. One can fill in a disk with these centers at their maximum concentration (see Fig. 1). We call this disk a bubble and the triangular crystal built out of such disks a bubble phase. It is shown in Ref. [5] that the optimum number of electrons in a bubble is

$$\tilde{M} \simeq 3\nu_N N, \quad (3)$$

which corresponds to the separation $\simeq 3R_c$ between nearest bubbles. The nonuniform distribution of guiding centers with such a period, chosen in accordance with the form

factor of the electron wave function, does not create significant variations of the charge density. Hence, the electrostatic (Hartree) energy of the system does not increase too much. However, the exchange interaction in this ferromagnetic state favors an increase of overlap among the wave functions and thus the most compact arrangement of the guiding centers.

For the comparison of different trial states we use the cohesive energy, which is the energy per electron relative to the uniform state formed at high temperature. The cohesive energy of the CDW state has been shown to be $E_{\text{coh}}^{\text{CDW}} = -r_s \hbar \omega_c \propto -B$ [Ref. [5]], which is of the same order of magnitude as the exchange-enhanced spin splitting [6].

As for the cohesive energy of the Laughlin liquid, arguments can be given [6] that it is very close to the cohesive energy of the Wigner crystal. The lower bound for the latter was estimated in Ref. [5] to be $E_{\text{coh}}^{\text{L}} \sim -\hbar \omega_c / \sqrt{N} \propto -B^{3/2}$. In the limit of small magnetic field the CDW should obviously be more energetically preferable, ruling out the FQH states.

It is interesting to know at what LL the transition from liquid to the crystalline ground state occurs? To answer this question one cannot use the quasiclassical approach and the CDW should be defined more accurately. We do so in several steps. First, we introduce the wave function of one bubble consisting of M electrons at the *lowest* LL:

$$\Psi_0 \{ \mathbf{r}_k \} = \left\{ \prod_{1 \leq i < j \leq M} (z_i - z_j) \right\} \exp \left(- \sum_{i=1}^M \frac{|z_i|^2}{4l^2} \right). \quad (4)$$

Here $z_j = x_j + iy_j$ is the complex coordinate of the j -th electron. Second, we define the wave function of a bubble at the N -th LL centered at point R by raising every electron onto this LL and shifting its position by the magnetic translation operator [7]

$$\Psi_{\mathbf{R}} \{ \mathbf{r}_k \} = \prod_{i=1}^M \frac{(a_i^\dagger)^N}{\sqrt{N!}} \exp \left(\frac{b_i^\dagger \bar{R} - b_i R}{l\sqrt{2}} \right) \Psi_0 \{ \mathbf{r}_k \}, \quad (5)$$

where b_i is an *intra*-LL ladder operator. To finally obtain the wave function of the CDW we build an antisymmetric combination of the bubbles centered at the triangular lattice sites \mathbf{R}_i

$$\Psi_{\text{CDW}} = \sum_P \text{sign}(P) \prod_{\mathbf{R}_i} \Psi_{\mathbf{R}_i} \{ P(\mathbf{r}_k) \}. \quad (6)$$

Here P are the permutations of electrons between bubbles. For the case $M = 1$ this trial state coincides with the Maki-Zotos ansatz wave function for the Wigner crystal [8]. It can be easily seen that Ψ_{CDW} is of the Fock type, and that the overlap between the wave functions of different bubbles is negligible.

To show that Ψ_{CDW} matches our earlier quasiclassical picture we introduce the guiding center density operator:

$$\hat{\nu}(\mathbf{r}) = 2\pi l^2 \sum_i \delta(\mathbf{r} - \hat{\mathbf{R}}_i). \quad (7)$$

The summation here is carried over the electrons at the considered LL and $\hat{\mathbf{R}}_i = \mathbf{r}_i + \frac{l^2}{\hbar} \mathbf{z} \times \hat{\mathbf{P}}_i$ is the guiding center operator, with $\hat{\mathbf{P}}_i$ being the canonical momentum of the i -th particle. The average of this operator has a physical meaning of the guiding center density [9]. It can be shown that for the state defined by Eq. (6)

$$\langle \hat{\nu}(q) \rangle = \frac{\nu_N A}{M} F_{M,M-1}(q) \approx 2\nu_N A \frac{J_1(ql\sqrt{2M})}{ql\sqrt{2M}}, \quad (8)$$

$$q \ll \frac{\sqrt{M}}{l},$$

which is just the Fourier transform of a uniform disk with the radius $l\sqrt{2M}$. The last equation in (8) follows from the asymptotic formula for the Laguerre polynomials [10].

The cohesive energy of the CDW can be calculated in the same way as it has been done for the Wigner crystal [3]:

$$E_{\text{coh}}^{\text{CDW}} = \frac{1}{2\nu_N} \sum_{q \neq 0} u_{\text{HF}}(q) \left| \frac{\langle \hat{\nu}(q) \rangle}{A} \right|^2 \quad (9)$$

The summation in Eq. (9) is carried over the reciprocal vectors of the triangular lattice. The Hartree-Fock interaction potential $u_{\text{HF}}(q)$ is defined in the same way as in Ref. [5]:

$$u_{\text{HF}}(q) = u_{\text{H}}(q) - u_{\text{ex}}(q),$$

$$u_{\text{H}}(q) = \frac{v(q)}{2\pi l^2 \epsilon(q)} F_{NN}(q), \quad (10)$$

$$u_{\text{ex}}(q) = 2\pi l^2 \int \frac{d^2 q'}{(2\pi)^2} e^{i\mathbf{q}\mathbf{q}'l^2} u_{\text{H}}(q').$$

Using Eqs. (8,9), the cohesive energy for any given ν_N can be calculated numerically. The result is, of course, different for different M (see Fig. 2). Therefore, one has to find \tilde{M} corresponding to the lowest energy. The energies of the CDW optimized in this way are summarized in Tables I and II. In Table I we present the results for the case $Nr_s \ll 1$, when the LL mixing can be ignored completely, i.e., $\epsilon(q) \equiv 1$. Table II shows the results for the practically important case $r_s = \sqrt{2}$. One can see that the optimum number of electrons per bubble is the same both with and without the screening and is in perfect agreement with Eq. (3).

The above results have been tested by the self-consistent Hartree-Fock procedure, similar to that described in Ref. [3]. Starting from the initial approximation given by wave function (6), this procedure finds the optimal set of $\langle \hat{\nu}(q) \rangle$ for a given periodicity of the CDW. The obtained corrections are of the order of $10^{-5} r_s \hbar \omega_c$ and thus do not affect the significant digits displayed in Tables I and II. We associate the corrections with a slight nonorthogonality of the wave functions of different bubbles.

Let us now discuss the Laughlin liquid at high LL's. The interaction energy per electron can be calculated using the density-density correlation function

$$h_N(\mathbf{r}) \equiv \frac{\langle \hat{\rho}_N(\mathbf{r}) \hat{\rho}_N(0) \rangle - \langle \hat{\rho}_N \rangle^2}{\langle \hat{\rho}_N \rangle} \quad (11)$$

where $\hat{\rho}_N(\mathbf{r})$ is the projection of the density operator onto the N -th LL. This can be most effectively done in the Fourier space because $h_N(q)$ is very simply related to $h_0(q)$ (the correlation function for $N = 0$), obtained earlier by Monte-Carlo simulations [4]:

$$\begin{aligned}
h_N(q) &= h_0(q) \left[L_N \left(\frac{q^2 l^2}{2} \right) \right]^2, \\
E_{\text{cor}}^{\text{L}} &= \frac{1}{2} \int \frac{d^2 q}{(2\pi)^2} \frac{v(q)}{\epsilon(q)} h_N(q).
\end{aligned}
\tag{12}$$

The calculations are greatly simplified by virtue of the approximation formula for h_0 given in Ref. [11].

The cohesive energy per electron is then, according to the definition

$$E_{\text{coh}} = E_{\text{cor}} - E^{\text{UEL}} = E_{\text{cor}} + \frac{\nu_N}{2} u_{\text{ex}}(q=0), \tag{13}$$

where E^{UEL} is the interaction energy per particle in the uniform uncorrelated electron liquid formed at high temperature. The results of the numerical evaluation of these energies are also listed in Tables I and II.

Compare now the energies of the Laughlin liquid and the CDW. As one can see, at $N = 0$ and $N = 1$ the Laughlin liquid is more energetically preferable. At large N however the CDW wins. The transition to the bubble state both with and without screening occurs at $N = 2$ for $\nu_N = 1/3$ and at $N = 3$ for $\nu_N = 1/5$. The difference in the energies of these two states at $N = 2$ and $\nu_N = 1/5$ is very small. It should be noted in this connection that the trial state (6), being compressible in nature, can be further improved by introducing the magnetophonon correlations [12]. Another possibility arises when the system is subjected to an external impurity potential. The CDW can lower its energy by accomodating to this potential, while the incompressible $\nu_N = 1/5$ liquid state cannot [13]. Hence, the question of what exactly phase dominates at $\nu = 4\frac{1}{5}$ remains open.

One can suggest the following interpretation of our results considering the zero-point vibrations of the lattice. Electrons at any N form the Wigner crystal when the upper LL filling is so small that the average distance between electrons is larger than the characteristic spread of electron wave functions R_c . The amplitude of the zero-point vibrations of the crystal is mostly determined by the magnetic barrier since the interaction energy per electron is smaller than the cyclotron gap for the cases of interest. Hence this amplitude is $\lesssim l$. For the crystalline state to be stable this amplitude should be small compared to the lattice

constant (the Lindemann criterion). Now an important difference between the low and high LL's arises. At low LL's an increase of the electron density reduces the lattice constant. At some value of ν_N it becomes of the order of l and the crystal melts into the Laughlin liquid. At high LL's, however, the lattice constant does not change as one increases the LL filling but remains of the order of $R_c \gg l$. Hence CDW cannot be melted by quantum fluctuations at high LL's.

In conclusion, we have compared the energies of the Laughlin liquid and the CDW with the optimized period ($\sim R_c$) at the upper LL filling factors $\nu_N = 1/3$ and $1/5$. We found that the $1/3$ liquid state is unstable for $N \geq 2$, while the $1/5$ state loses to the CDW at $N \geq 3$. Our result implies that the $1/3$ fractional quantum Hall effect cannot be observed at filling factors $\nu > 4$. This conclusion is in agreement with the existing experimental data. The difference between the energies of the CDW and the Laughlin liquid at $N = 2$ and $\nu_N = 1/5$ is so small that more work is needed to distinguish them unambiguously. The authors are grateful to B. I. Shklovskii for the inspiration for this work and for numerous helpful suggestions. This work is supported by NSF under Grant DMR-9321417.

REFERENCES

- [1] *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1990).
- [2] R. B. Laughlin, Phys. Rev. Lett. **18**, 1395 (1983).
- [3] D. Yoshioka and H. Fukuyama, Journ. Phys. Soc. of Japan **47**, 394 (1979); D. Yoshioka and P.A. Lee, Phys. Rev. B **27**, 4986 (1983).
- [4] A. H. MacDonald and S. M. Girvin, Phys. Rev. B **33**, 4009 (1986); A. H. MacDonald, Phys. Rev. B **30**, 3550 (1984).
- [5] A. A. Koulakov, M. M. Fogler, and B. I. Shklovskii, Phys. Rev. Lett. **76**, 499 (1996); M. M. Fogler, A. A. Koulakov, and B. I. Shklovskii, preprint cond-mat /9601110.
- [6] I. L. Aleiner and L. I. Glazman, Phys. Rev. B **52**, 11296 (1995).
- [7] S. Kivelson, C. Kallin, D. P. Arovas, and J. R. Schrieffer, Phys. Rev. B **36**, 1620 (1987).
- [8] K. Maki and X. Zotos, Phys. Rev. B **28**, 4349 (1983).
- [9] The average density of guiding centers is in fact related to the order parameter $\Delta(q)$ introduced by H. Fukuyama, P. M. Platzman, and P. W. Anderson [Phys. Rev. B **19**, 5211 (1979)] through $\langle \hat{\nu}(q) \rangle = A\Delta(q)$.
- [10] H. Bateman and A. Erdelyi, *Higher Transcendental Functions* (McGraw-Hill, New York, 1953).
- [11] S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys. Rev. B **33**, 2481 (1986).
- [12] P. K. Lam and S. M. Girvin, Phys. Rev. B **30**, 473 (1984).
- [13] Rodney Price, P. M. Platzman, and Song He, Phys. Rev. Lett. **70**, 339 (1993).

FIGURES

FIG. 1. The quasiclassical image of the bubble phase. (a) Top view. The bubbles (dark circles) are the places where the accumulation of the guiding centers occurs. (b) The enlarged view of one bubble. The dark region shows the guiding center density $\nu(x, y)/2\pi l^2$, while the toroidal figure illustrates the charge density distribution $\rho_N(x, y)$ around the bubble. Half of the charge density is removed. This charge density is created by electrons moving in the cyclotron orbits centered inside the bubble. One of such cyclotron orbits is shown by the arc with the arrow.

FIG. 2. The cohesive energy of the CDW as a function of ν_N for different numbers of electrons in a bubble M . The calculations are made for $N = 5$ and $r_s = \sqrt{2}$. The crosses show the Laughlin liquid energies.

TABLES

$\nu_N = 1/3$					
N	\tilde{M}	E^{UEL}	$E_{\text{coh}}^{\text{L}}$	$E_{\text{coh}}^{\text{CDW}}$	$\delta E/E_{\text{coh}}^{\text{CDW}}$
0	1	-0.0853	-0.0819	-0.0733	-11.7%
1	1	-0.1692	-0.1824	-0.1726	-5.6%
2	2	-0.1970	-0.1939	-0.2163	10.3%
3	3	-0.2135	-0.2056	-0.2433	15.5%
4	4	-0.2251	-0.2228	-0.2480	15.1%
5	5	-0.2341	-0.2367	-0.2767	14.5%
$\nu_N = 1/5$					
N	\tilde{M}	E^{UEL}	$E_{\text{coh}}^{\text{L}}$	$E_{\text{coh}}^{\text{CDW}}$	$\delta E/E_{\text{coh}}^{\text{CDW}}$
0	1	-0.0396	-0.0639	-0.0622	-2.7%
1	1	-0.0986	-0.2109	-0.2043	-3.2%
2	1	-0.1164	-0.2465	-0.2454	-0.5%
3	2	-0.1267	-0.2474	-0.2811	12.0%
4	2	-0.1340	-0.2538	-0.2990	15.1%
5	3	-0.1395	-0.2562	-0.3187	19.6%

TABLE I. The cohesive energies of the Laughlin liquid $E_{\text{coh}}^{\text{L}}$ and the CDW $E_{\text{coh}}^{\text{CDW}}$ in the limit $Nr_s \rightarrow 0$ for different N . \tilde{M} is the optimum number of electrons per bubble. All the energies are given in the units of $r_s \hbar \omega_c$. The right column shows their relative difference: $(E_{\text{coh}}^{\text{L}} - E_{\text{coh}}^{\text{CDW}})/E_{\text{coh}}^{\text{CDW}}$. The energy per electron in the uniform uncorrelated state E^{UEL} is provided for reference.

$\nu_N = 1/3$					
N	\tilde{M}	E^{UEL}	$E_{\text{coh}}^{\text{L}}$	$E_{\text{coh}}^{\text{CDW}}$	$\delta E/E_{\text{coh}}^{\text{CDW}}$
0	1	-0.1206	-0.1158	-0.1037	-11.7%
1	1	-0.1297	-0.1520	-0.1424	-6.7%
2	2	-0.1136	-0.1141	-0.1188	4.0%
3	3	-0.1034	-0.0948	-0.1018	6.9%
4	4	-0.0965	-0.0822	-0.0896	8.2%
5	5	-0.0914	-0.0743	-0.0805	7.7%
$\nu_N = 1/5$					
N	\tilde{M}	E^{UEL}	$E_{\text{coh}}^{\text{L}}$	$E_{\text{coh}}^{\text{CDW}}$	$\delta E/E_{\text{coh}}^{\text{CDW}}$
0	1	-0.0560	-0.0903	-0.0880	-2.7%
1	1	-0.0765	-0.1731	-0.1692	-2.3%
2	1	-0.0677	-0.1423	-0.1396	-1.9%
3	2	-0.0618	-0.1143	-0.1202	4.9%
4	2	-0.0577	-0.0979	-0.1050	6.8%
5	3	-0.0547	-0.0856	-0.0946	9.7%

TABLE II. Same as Table I but $r_s = \sqrt{2}$, which corresponds to the electron density of $1.6 \cdot 10^{11} \text{cm}^{-2}$ in GaAs-GaAlAs heterostructures. The energies are now given in the units of $\hbar\omega_c$.

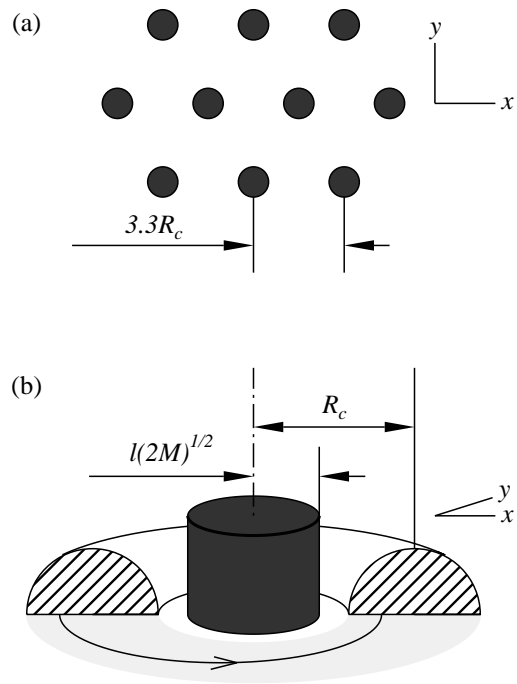


FIG.1. Fogler and Koulakov, PRL, "The absence..."

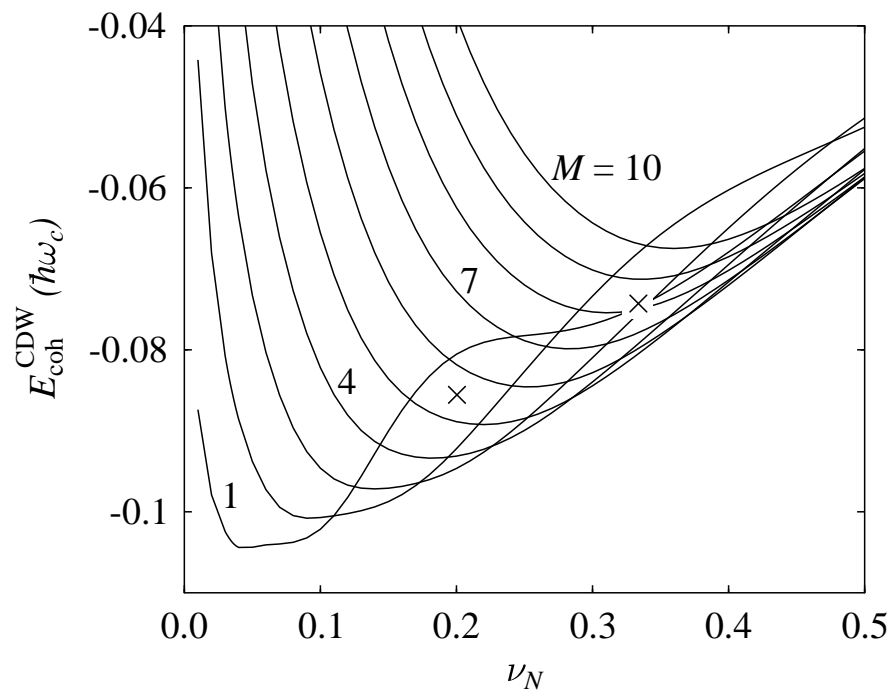


FIG.2. Fogler and Koulakov, PRL, "The absence..."