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Bayesian Networks in Radiology

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A Bayesian network is a graphical model that uses probability theory to represent relationships among its variables. The model is a directed acyclic graph whose nodes represent variables, such as the presence of a disease or an imaging finding. Connections between nodes express causal influences between variables as probability values. Bayesian networks can learn their structure (nodes and connections) and/or conditional probability values from data. Bayesian networks offer several advantages: *(a)* they can efficiently perform complex inferences, *(b)* reason from cause to effect or vice versa, *(c)* assess counterfactual data, *(d)* integrate observations with canonical ("textbook") knowledge, and *(e)* explain their reasoning. Bayesian networks have been employed in a wide variety of applications in radiology, including diagnosis and treatment planning. Unlike deep learning approaches, Bayesian networks have not been applied to computer vision. However, hybrid artificial intelligence systems have combined deep learning models with Bayesian networks, where the deep learning model identifies findings in medical images and the Bayesian network formulates and explains a diagnosis from those findings. One can apply a Bayesian network's probabilistic knowledge to integrate clinical and imaging findings to support diagnosis, treatment planning, and clinical decision-making. This article reviews the fundamental principles of Bayesian networks and summarizes their applications in radiology.

Supplemental material is available for this article.

Diagnosis and decision-making in radiology require that physicians integrate an imaging study's findings with large amounts of information, including the patient's clinical symptoms, laboratory values, comorbidities, prior medical history, and the likelihood of various diseases. Artificial intelligence (AI) systems can complement physician decision-making. These systems can store large amounts of data and perform complex calculations consistently, quickly, and without biases. A Bayesian network is an AI formalism, first introduced in 1985, that can learn, store, and apply knowledge in the form of probability values to reason under conditions of uncertainty (1). Bayesian networks have been used in radiology to integrate clinical and imaging findings for differential diagnosis and clinical decision-making (2–4). This article offers a brief refresher of probability theory, introduces Bayesian networks as a graphical representation of probabilistic knowledge, reviews current applications of Bayesian networks in radiology, and discusses future applications. A glossary of Bayesian network terms (Table S1), Bayesian networks' advantages and disadvantages (Table S2), and additional resources (Table S3) are included.

Probability: Background and Definitions

Bayesian networks use probability to reason in settings of uncertainty. Probability theory offers a set of methods to conceptualize and quantify uncertainty. Probability values obey three fundamental properties: *(a)* an event's probability value must be greater than or equal to 0, *(b)* the values of all possible events must sum to 1, and *(c)* the probability of A or B is the sum of their individual probabilities if the two events are mutually exclusive.

Conditional probability expresses the likelihood of one event given another and is defined as probability that the two events, A and B, occur together (their *joint probability*) divided by the probability of the event conditioned upon. Thus, the conditional probability of A given B is defined as $P(A|B) = P(A,B)/P(B)$. Conditional probabilities are familiar concepts in diagnostic medical testing, such as sensitivity, specificity, positive predictive value, and negative predictive value. Conditional probabilities of A and B are related using Bayes theorem,

$$P\left(A|B\right) = P\left(B|A\right) \cdot \frac{P(A)}{P(B)}.$$

Bayes theorem allows probabilistic reasoning in both directions, that is, from the probability of B given A to the probability of A given B, or vice versa. In the setting of a diagnostic test, where A and B might represent a disease and a positive test result, respectively, one could compute the probability of the test result given the disease, or that of the disease given the test result.

Bayesian Networks

In medical applications, Bayesian networks typically express relationships between uncertain variables, such as the presence of a disease, and observable variables, such as demographic data, symptoms, vital signs, laboratory

Abbreviations

AI = artificial intelligence, BI-RADS = Breast Imaging Reporting and Data System

Summary

Bayesian networks are graphical models that use probability theory to integrate clinical and imaging findings for diagnosis and clinical decision-making, with the advantages of explainability and ability to train on small datasets.

Essentials

- Bayesian networks have been used in radiology to integrate clinical and imaging findings for differential diagnosis and clinical decision-making.
- Efficient algorithms enable Bayesian networks to compute posterior probabilities (updated probabilities given additional information), analyze the value of information (to identify the most useful next diagnostic test), perform sensitivity analysis, and generate explanations of the model's reasoning.
- Bayesian networks offer advantages over various other artificial intelligence approaches, including explainability and accurate inference from small datasets and in the setting of missing data.
- Although Bayesian networks have not been applied to computer vision, hybrid artificial intelligence systems have combined deep learning models with Bayesian networks; the deep learning model identifies and extracts findings in medical images, and the Bayesian network formulates and explains a diagnosis from those findings and other clinical and laboratory data.

Keywords

Bayesian Network, Machine Learning, Abdominal Imaging, Musculoskeletal Imaging, Breast Imaging, Neurologic Imaging, Radiology Education

values, and imaging findings. Bayesian networks can integrate large amounts of probabilistic knowledge in a mathematically robust way that eliminates many of the heuristic biases of human decision-making. A Bayesian network model consists of a graph and an associated set of conditional probability tables (1,5,6). The graph incorporates a set of nodes and a set of edges, or arcs, that connect pairs of nodes. The nodes in a Bayesian network represent stochastic, or random, variables. Variables can have continuous or discrete values; we limit our discussion to variables with discrete values, as they are seen most frequently in medical applications. Continuous variables, such as age or laboratory values, are typically discretized. Each node thus has two or more states that each have an associated probability value. The states must be exhaustive (they cover all possibilities) and mutually exclusive (only one can be true). Thus, the probability values for any node's states always sum to 1. The edges of a Bayesian network form a directed acyclic graph, as the edges have specified direction and cannot form a closed-loop cycle.

Each node has a conditional probability table that specifies its states' probabilities given the values of the node's incoming edges. For nodes without incoming edges, the conditional probability table specifies the prior probability values of the nodes' states. An edge between two nodes represents probabilistic influence between the two variables, expressed as a set of conditional probabilities. By convention, the orientation of the edge conveys the direction of causal influence.

To illustrate, Figure 1 presents a simple Bayesian network with three variables: *(a) age*, a continuous variable that has been discretized into five states (eg, *40–49 years*); *(b) breast cancer*, with states *present* and *absent*, influenced by the patient's age; and *(c) mammogram*, with states *normal* and *abnormal*, influenced by the presence or absence of breast cancer. Prior probabilities have been assigned for *age*; conditional probabilities have been specified for *breast cancer* and *mammogram*.

Inference

Inference is the computational procedure that applies a Bayesian network's probabilistic knowledge. Highly efficient algorithms enable Bayesian networks to compute posterior probabilities, analyze the value of information, perform sensitivity analysis, and generate explanations of the model's reasoning.

Posterior Probabilities

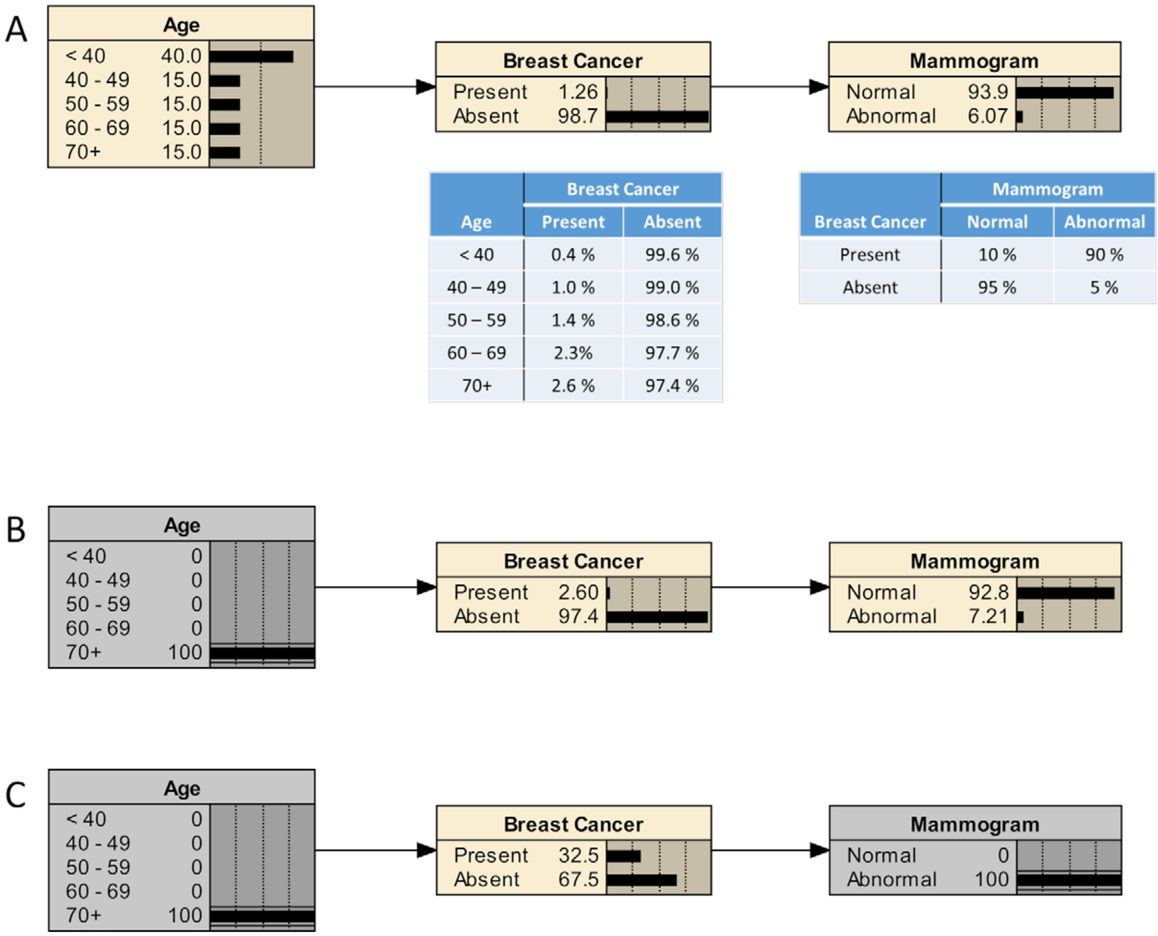
The most fundamental Bayesian network inference task is to compute posterior probabilities, which are the probabilities of the model's states after evidence is presented. Given a set of evidence *E,* specified by fixing the states of any subset of nodes in the network, one can calculate the marginal posterior probabilities, $P(\square|E)$, of all other nodes. Because evidence in Bayesian networks can be transmitted in either direction across a link, Bayesian networks can be used to perform predictive inference (from causes to effects), diagnostic inference (from effects to causes), or any combination thereof. Efficient algorithms can propagate beliefs throughout the network as additional evidence is acquired (6,7). Additionally, one can analyze a counterfactual query, such as "If *A* were true (when it is known to be false), what is the probability of *C* given the other known information?" (8).

Value of Information

In settings in which evidence can be obtained incrementally (eg, through additional laboratory tests or imaging examinations) a user may wish to know which piece of additional information would have the greatest influence on the probability of the hypothesis node or nodes. In other words, which test result will add the greatest certainty to the diagnosis? A Bayesian network can calculate the value of information by computing the mutual information (or cross-entropy) of the hypothesis node H given a particular observation node O, written $I(H;O)$. The most informative evidence for that hypothesis is the one with the highest value of $I(H;O)$ (9).

Sensitivity Analysis

When using a Bayesian network in practice, a natural question is how much confidence one should have in the network's output. In other words, how sensitive are the results to small changes in the network's parameters? A model's sensitivity can be determined by calculating the derivatives of the posterior probability distributions of the set of target nodes (eg, diagnostic hypotheses) over each of the conditional probability table entries in the Bayesian network (10–12). A large derivative means that small changes in the parameter may lead



	Breast Cancer	
Age	Present	Absent
< 40	0.4 %	99.6 %
40 – 49	1.0 %	99.0 %
50 – 59	1.4 %	98.6 %
60 – 69	2.3%	97.7 %
70+	2.6 %	97.4 %

	Mammogram	
Breast Cancer	Normal	Abnormal
Present	10 %	90 %
Absent	95 %	5 %

Figure 1: An example Bayesian network model with three nodes. Tan-colored nodes are inferred from known information; nodes in gray have been set (instantiated) to a specific value, typically in response to an observation. An arrow indicates the direction of probabilistic influence; if an arrow points from node *A* to node *B*, then the probabilities of *B*'s states are conditioned on *A*. For example, the probability of abnormal mammographic findings is conditioned on the presence or absence of breast cancer. **(A)** The conditional probability tables for *breast cancer* and *mammogram* are shown beneath those nodes; here the sensitivity of mammography for breast cancer, *P* (*mammogram* = *abnormal* | *breast cancer* = *present*), is 90%. Prior probabilities have been assigned to the age ranges. Before any evidence is presented, there is a 1.26% probability that breast cancer is present. **(B)** If we know that the patient is of age 70 or greater, the model infers a 2.60% probability that breast cancer is present. **(C)** If that patient has an abnormal mammogram, the probability of breast cancer rises to 32.5%. Note that although the arrow points from *breast cancer* to *mammogram*, information can flow in each direction. As one would expect, the result of the mammogram influences the patient's likelihood of breast cancer.

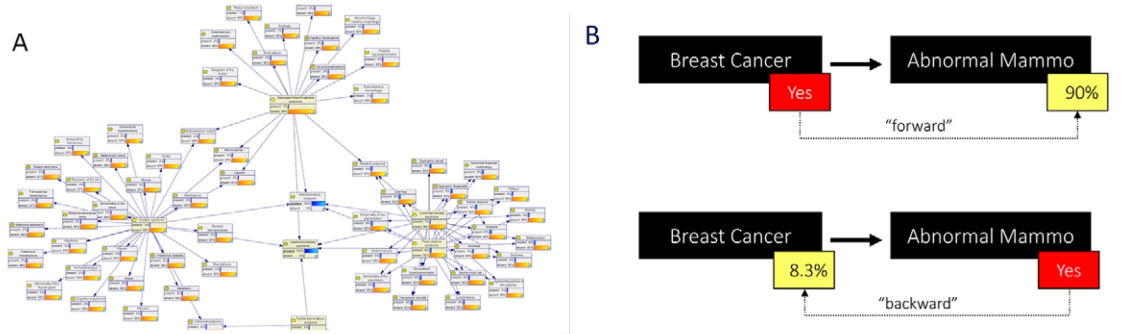
to large changes in the target nodes' posterior probabilities. In most cases, one can identify a small subset of parameters, called the sensitivity set, that influences the probability of the target nodes.

Explanation

Although deep neural networks have been applied successfully to numerous image- and text-based applications in radiology, their "black-box" property has rekindled interest in explainable AI (13). Bayesian networks offer strong advantages over neural networks; one can exploit a Bayesian network's structure to describe direct and indirect influences between variables. A Bayesian network model explicitly specifies probabilistic relationships between variables and can quantify those relationships. Two types of explanations, microlevel and macrolevel, have been described (14). Microlevel explanations identify the pieces of evidence that had the greatest influence on the hypothesis of interest. Explanation is generated using a computation similar to that for value of information; one can quantify the impact of omitting that evidence (15). Macrolevel explanations identify the network paths through which evidence flows to influence the hypothesis. The technique identifies all paths through the network from evidence nodes to the hypothesis and measures the strength of each, defined as the amount by which the evidence influences the probability of each node along the path. The idea is that the path is as strong as its weakest link. The paths can then be displayed and ranked in terms of strength (eg, Fig 2D) (15,16).

Constructing a Model

There is considerable flexibility in the construction of Bayesian networks; one can apply manual and/or automated approaches for *structure learning* and *parameter learning*. Structure learning determines the structure of the Bayesian network's graphical model, including the dependence and independence of variables and the placement of the edges of the graph. Structure learning methods include scoring-based search algorithms (using optimization techniques), constraint-based algorithms



C

Approach	Network structure	Probability values
(1) "Hand-crafted"	Manual	Manual
(2) Fit to Model	Manual	Auto
(3) Fully Automatic	Auto	Auto

D

GUARDING is caused by CHOLECYSTITIS, which is caused by GALLSTONES.
AGE influences GALLSTONES.
The AGE of 41 increases the probability of GALLSTONES to 17%.

Figure 2: As illustrated schematically in independent examples, Bayesian networks can **(A)** perform exact and efficient computation over models with hundreds or thousands of variables, **(B)** reason "forwards" from causes to effects as well as "backwards" from effects to causes, **(C)** learn their structure and probability values from manual input and/or directly from data, and **(D)** explain their reasoning.

(to test for conditional dependence in the data), and Bayesian methods (to generate an ensemble of possible structures). These methods are beyond the scope of this review and are discussed in detail elsewhere (17,18). Parameter learning applies a given dataset to determine the conditional probability values at each node. For example, Burnside et al (19) trained a Bayesian network from a large database of patient demographics and mammographic findings to accurately predict the presence of breast cancer. Several approaches have been developed to automatically deduce both the structure and the parameters of a Bayesian network from data (20,21).

One can combine manual and automated approaches to parameter learning by blending probability values provided by experts and those learned from data. Laplace smoothing is a common approach that adds a smoothing parameter, $\lambda > 0$, to each state's count to assure a nonzero value (22). For example, for a variable with three states, Laplace smoothing would assign an initial count of λ to each state to give each state a probability of one-third. As actual data are accrued, each state's count is incremented accordingly. Laplace smoothing provides a simple way to avoid overfitting by pulling the probability estimates away from zero and toward the uniform distribution, and the effect of smoothing wanes as more data are accrued.

Applications in Clinical Radiology

Abdominal Imaging

Several diagnostic models have been developed in abdominal imaging. Decision Aid for Diagnosing Liver Lesions (DAFODILL) aided in diagnosis of 14 types of liver lesions using eight MRI-based features and four clinical features (23). A model for diagnosis of gallbladder disease incorporated probability data from the peer-reviewed literature to evaluate hepatobiliary diseases based on a patient's age and sex, imaging findings, physical examination findings, and laboratory values (24). This model served as a test bed for a system to explain a Bayesian network model's reasoning. A model for diagnosis of acute appendicitis in children applied four US imaging features (appendiceal diameter, wall hyperemia, appearance of periappendiceal mesentery, presence of appendicolith) and nine clinical and laboratory variables (including duration of symptoms, presence of fever, white blood cell count, and C-reactive protein) (25). This model was derived from existing clinical data, published literature, and expert opinions without requiring a large dataset to train on. As an example, in a pediatric patient with an appendix of 11 mm, appendicolith present, wall hyperemia, fever, and 48 hours of symptoms, the Bayesian network determined a 97% probability of acute appendicitis. In a pediatric patient with nonvisualized appendix, normal-appearing mesentery, normal white blood cell count, no fever but with rebound tenderness and nausea, the Bayesian network calculated a 34% probability of acute appendicitis. The model was able to assign a probability even if certain imaging or clinical data were missing or not available.

Musculoskeletal Imaging

Primary bone tumors are rare and can present a diagnostic challenge. A Bayesian network model was able to correctly discriminate among five benign and five malignant neoplasms of the appendicular skeleton using age, sex, and 17 radiographic characteristics (26). A naive Bayesian approach used two clinical and 16 qualitative radiographic features to diagnose 29 bone tumor types based on data of 1664 cases. This system correctly identified the diagnosis in 44% of cases and listed the correct condition among the three most likely conditions in 60% of cases (27). These examples show how Bayesian networks can be applied in settings with relatively few cases from which to learn. Unlike neural network approaches, which generally require large datasets for training, Bayesian networks can apply knowledge from the medical literature.

Breast Imaging

Breast imaging reports contain highly structured data that are particularly suitable for diagnostic decision support. The MammoNet system incorporated seven clinical features and 15 mammographic findings to predict the presence of breast cancer. It achieved an area under the receiver operating characteristic curve of 0.88 on 67 test cases (28,29). Burnside et al (19) analyzed a database of more than 48 000 screening and diagnostic mammograms, matched with outcomes data from a breast cancer registry, to create a tree-augmented naive Bayes model of breast cancer risk that achieved an area under the receiver operating characteristic curve of 0.96. A natural language processing tool that extracted Breast Imaging Reporting and Data System (BI-RADS) descriptors from textual mammography reports applied this Bayesian network model to predict BI-RADS assessment category with 98% accuracy, and the model was shown to reduce false-positive interpretations by 29% (30,31). Another Bayesian network model of 16 epidemiologic and clinical characteristics, morphologic MRI features, and quantitative MRI parameters was developed to predict the risk of triple-negative breast cancer (32).

Neurologic Imaging

Bayesian networks have been applied in a variety of neuroscience and neuroimaging studies, including resting-state functional MRI studies to predict states of belief and disbelief and early diagnosis of Alzheimer disease from structural MRI (33–35). Bayesian networks have been used in an array of multimodal MRI analyses for limited but complex diagnostic tasks in neuroradiology, including differentiating among atypical meningiomas, glioblastomas, and metastases (36); predicting glioma grade (37); and assisting in segmentation of multiple sclerosis lesions (38).

Advances in deep learning, such as convolutional neural networks, have enabled automated image analysis methods that make possible fully automated diagnostic systems to go from raw images to differential diagnoses. Recent studies explored Bayesian networks for diagnosis of 19 diseases involving the cerebral hemispheres (39) and 36 diseases involving deep gray matter (40). These efforts generated differential diagnoses with high accuracy by combining deep learning to detect and characterize abnormalities on multimodal brain MR images with Bayesian networks to integrate these imaging abnormalities with clinical features. These systems achieved performance similar to that of subspecialists and better performance than that of general radiologists and neuroradiology fellows. The approach accurately models the steps taken by radiologists when generating an image-based differential diagnosis, namely recognizing imaging abnormalities and then integrating imaging findings with clinical context.

Radiology Education

Because Bayesian networks can invoke the causal connections and probability values to explain their reasoning, researchers have explored Bayesian networks in radiology education. The Bayesian Network Tutoring and Explanation (BANTER) system tutored users in diagnosis and selection of optimal diagnostic procedures. It computed posterior probabilities based on given evidence, determined the best diagnostic procedure to affirm or exclude a potential diagnosis, quizzed users on their selection of optimal diagnostic procedures, and generated explanations of its reasoning (16,41,42). The Adaptive Radiology Interpretation and Education System, or ARIES, allowed users to interact with Bayesian network models to compute probabilities of disease, identify the features most strongly associated with a given diagnosis, and highlight the most discriminative features through sensitivity analyses (43,44).

Discussion

Advantages of Bayesian Networks

Bayesian networks offer several advantages and can be used instead of or in concert with other radiology AI approaches, such as deep neural networks, decision trees, support vector machines, and clustering algorithms. Bayesian networks can apply a variety of highly efficient algorithms to learn a model's structure and/or conditional probability parameters directly from data. It can also apply a network's probabilistic knowledge to determine the most likely diagnosis, identify the most useful diagnostic test, and determine the sensitivity of the model's conclusions to its variables. Bayesian networks can make accurate inferences from relatively small datasets and can perform well in the setting of missing data. Although convolutional neural networks and other deep learning algorithms have been applied to numerous imaging tasks, such as detection and diagnosis of abnormalities, a frequent concern is their inability to explain their reasoning (13,45). In addition, other AI learning methods often require large datasets—on the order of 1000 or 10 000 cases—to achieve useful results (46). Recent work combining deep learning to detect findings and Bayesian networks to relate those findings to diagnoses can tap the advantages of both technologies (39,40). Some key features of Bayesian networks are summarized schematically in Figure 2.

Bayesian network models can be updated continuously to learn from experience and play a role in learning health systems. The ability to directly examine a Bayesian network's probability tables can help assure that the model is performing as expected (47). Researchers have extended the capabilities of Bayesian networks into models that evolve over time and integrated them into logic-based models for automated reasoning. A dynamic Bayesian network, a model that repeats the static interactions of a conventional Bayesian network over time, has been developed to predict lung cancer risk from clinical and CT findings (17,48,49).

Several efforts have sought to integrate the probabilistic reasoning of Bayesian networks with other AI frameworks, such as logic-based knowledge representation and neural network learning mechanisms. A generalized logic-programming approach constructs a Bayesian network to answer queries based on a context-sensitive probabilistic knowledge base (50). Ontologies apply a logic-based framework to enable automated reasoning through the relationships defined between concepts (51). Several groups have explored approaches to augment ontologies with Bayesian networks' probabilistic knowledge and ability to reason under uncertainty (52–54). Bayesian neural networks consist of a stochastic artificial neural network trained using Bayesian inference (55).

Limitations of Bayesian Networks

Bayesian networks have several limitations. Unlike deep neural networks, Bayesian networks cannot be applied as directly to tasks in computer vision and natural language processing. An important limitation is the need to discretize continuous variables; automated approaches for discretization pose a technical challenge, and the only continuous distribution that Bayesian networks can work with is a conditional Gaussian distribution (56). Furthermore, the scope of Bayesian networks has been limited, especially in biomedical applications. A recent review found that almost 60% of Bayesian-network health care applications were focused in only four areas: heart conditions, cancer, psychologic disorders, and lung disease (57). As outlined by Kyrimi et al (58), despite hundreds of articles published on Bayesian networks, there is still a large implementation gap, as Bayesian networks have rarely been applied in routine clinical practice.

Future Applications

One key opportunity for Bayesian networks lies in their integration with deep learning models, as has been shown for diagnosis in neuroradiology (39,40). Such hybrid approaches use the relative strengths of each technology. Deep neural networks can effectively recognize patterns and extract features, such as increased fluid-attenuated inversion recovery signal intensity in a certain spatial distribution, and Bayesian networks can interpret and integrate the extracted image features, in combination with relevant clinical features such as age, sex, and laboratory values, to provide an explainable diagnosis. For rare diseases, a particular advantage of Bayesian networks is that one can specify a model's probability tables with values gleaned from the medical literature rather than having to present thousands of examples to train a deep learning system.

Bayesian networks are poised to play an important role in learning health systems. Because a model's conditional probability tables can be updated directly based on its experience, accrual of real-life data will allow a Bayesian network to model its population more precisely and to automatically adjust its reasoning over time to match a changing environment. For example, once a specific diagnosis is identified, the patient's clinical data can then be inputted to refine the model. One can imagine collecting data on a larger scale from a health system's laboratory, pathology, and genomics databases to continually update probabilistic models for imaging diagnosis. Standardized data dictionaries, such as the Observational Medical Outcomes Partnership Common Data Model (59) and the radiology community's Common Data Elements (60), will support such efforts.

Conclusion

Bayesian networks continue to form an important AI approach in radiology. They offer powerful machine learning and decision support capabilities across a large number of applications. Their ability to learn from small datasets and provide explainable decisions make them especially useful in a medical field that integrates multimodal data for diagnosis and medical decision-making. One can combine Bayesian networks with other AI approaches. For example, one can use deep learning techniques to identify findings in images and use Bayesian networks to integrate those findings with other data and to explain the system's reasoning. There are opportunities for greater adoption of Bayesian networks in clinical practice.

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