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Jonathan D. Young

July 1968

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ABSTRACT

Formulation for a damped cubic spline and its application as a fitting function for a set of points:

$$(t_i, x_i);\ i = 1, I \text{ with } \geq 3 \text{ and}$$

$$t_{i+1} > t_i;\ i = 1, I - 1$$

is presented.

Introduction

Let x be a function of t for which only the following is known:

$$x_i \equiv x(t_i) \qquad i = 1, I$$

and either

$$x'_1 \equiv x'(t_1) \text{ and } x'_I \equiv x'(t_I)$$

or

$$x''_1 \equiv x''(t_1) \text{ and } x''_I \equiv x''(t_I).$$

Reference (1) describes the cubic spline function, s(t), which

can be used to fit the given data and outlines its properties. References (2) and (3) point out that extreme local curvature may be implicit to the data and that in this case the cubic spline curve will contain inflection points or oscillations which are undesirable. These references propose a parametric hyperbolic spline fit, $f_p(t)$, which for sufficiently large value of a positive parameter, p, will have no such extraneous inflection points.

We propose a parametrically damped cubic spline fit, $s_p(t)$, which likewise for sufficiently large p will have no extraneous inflection points. The s_p fit is computationally more convenient than f_p. However either the hyperbolic fit or the damped cubic fit achieve their end (elimination of extraneous inflection) at some sacrifice of the curvature minimization property possessed by the cubic spline. With some data it may be desirable to try both f_p and s_p with the view to accepting whichever has the more desirable curvature.

The Damped Cubic Spline

The damped cubic spline function, $s_p(t)$ has the following properties:

(1) over any subinterval, $[t_i, t_{i+1}]$; $i = 1, I-1$, $s_p(t)$ coincides with the segment function -

$$h_i(\tau) = x_i + a_i\tau + b_i e^{-p\omega}(\tau^3 - \tau) + c_i e^{-p\tau}(\omega^3 - \omega)$$

where $\tau = (t-t_i)/(t_{i+1}-t_i)$, $\omega = 1 - \tau$,

and $a_i = (x_{i+1}-x_i)/(t_{i+1}-t_i)$

so that

(2) $s_p(t_i) = x_i$

The coefficients b_i and c_i are determined so that

(3) $s_p(t)$ has continuous first and second derivatives over the whole interval, $[t_1, t_I]$.

(4) for $p = 0$, $s(t)$ is in fact the cubic spline fit but for sufficiently large p, $s_p(t)$ has no extraneous inflection points

Rather than defining s_p in terms of the coefficients a_i, b_i and c_i it is computationally convenient to calculate

$$s'_{p_i} \equiv s'_p(t_i) \qquad i = 1, I$$

and define s_p in terms of

$$(t_i, x_i, s'_{p_i}); \qquad i = 1, I.$$

since for any i; $i = 1, I - 1$, the segment function, h_i is completely determined by (t_i, x_i, s'_{p_i}) and $(t_{i+1}, x_{i+1}, s'_{p_{i+1}})$.

First Derivative at Table Points, s'_{p_i}

The values of s'_{p_i}; $i = 1, I$ can be readily computed for any value of p.

For $s_p \in C_2$ we require;-

$$\left.\begin{aligned} s_{p_i} &= h_i(0) = h_{i-1}(1) = x_i \\ s'_{p_i} &= h'_i(0)/(t_{i+1}-t_i) = h'_{i-1}(1)/(t_i - t_{i-1}) \\ s''_{p_i} &= h''_i(0)/(t_{i+1} - t_i)^2 = h''_{i-1}(1)/(t_i-t_{i-1})^2 \end{aligned}\right\} i = 2, I-1$$

References

1. Young, J., "Numerical Applications of Cubic Spline Functions, "The Logistics Review 14 (1967) pp. 9-14

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