Lawrence Berkeley National Laboratory
Recent Work

Title

MEASUREMENT OF THE T = 3/2 Rrt ELASTIC SCATTERING CROSS SECTION

Permalink

https://escholarship.org/uc/item/97c1j7s7

Authors

Kirschbaum, A.R.
Colton, E.
Dauber, P.M.
et al.

Publication Date

1971-02-01

A. Kirschbaum

UCRL-19426 Rev.

c. 3

MEASUREMENT OF THE
T = 3/2 Kπ ELASTIC
SCATTERING CROSS SECTION

A. R. Kirschbaum, E. Colton, P. M. Dauber,
J. H. Friedman and P. E. Schlein

February 1971

DISCLAIMER

MEASUREMENT OF THE T = 3/2 Kπ ELASTIC SCATTERING CROSS SECTION[*]

A. R. Kirschbaum, E. Colton, P. M. Dauber,[†]
and J. H. Friedman

Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

and

P. E. Schlein

Physics Department
University of California
Los Angeles, California 90024

February 1971

ABSTRACT

We present a reanalysis of our original pole-extrapolation measurements of the $K^-\pi^-$ elastic scattering cross section for two intervals in Kπ invariant mass from threshold to 0.84 GeV. These measurements were obtained from the reaction $K^-p \to K^-\pi^-\Delta^{++}$ at 2.05 GeV/c and 2.63 GeV/c. We now show how the background from the competing OPE process $K^-p \to (K^-\pi^+)(\pi^-p)$ can be approximately accounted for. In addition we summarize some recent measurements of the T = 3/2 Kπ cross section.

We present a reanalysis of our earlier work,[1] in which we employed a Chew-Low[2] extrapolation to the pion pole, using the reaction

$$K^-p \rightarrow K^-\pi^-\Delta^{++} \tag{1}$$

in order to measure the $T = 3/2$ $K\pi$ elastic scattering cross section. In our original study we noted the presence of a substantial background contribution from the competing reaction

$$K^-p \rightarrow K^{*0}(890)\pi^-p. \tag{2}$$

In this paper we calculate the background contribution explicitly, using a one-pion exchange model for Reaction (2), so that we may perform a background subtraction and thereby obtain a more reliable value for the extrapolated $K\pi$ cross section. We also present a summary of other recent measurements, as well as the predictions of the Lovelace-Veneziano model[3] and current algebra.[4]

Events of the type $K^-p \rightarrow K^-p\pi^+\pi^-$ were obtained in exposures of the LRL 72-inch hydrogen bubble chamber to a separated K^- beam[5] with incident momenta in the range 2.0 to 2.7 GeV/c. The two samples selected for the $K\pi$ cross-section measurement consist of 7045 events[6] at 2.0 to 2.1 GeV/c (mean momentum = 2.05 GeV/c) and 9148 events at 2.51 to 2.78 GeV/c (mean momentum = 2.63 GeV/c). We use cross sections for the reaction $K^-p \rightarrow K^-\pi^-\pi^+p$ obtained by interpolating between values given by Dauber et al.[6]

In order to obtain an enriched sample of Reaction (1) we require the invariant mass of the $p\pi^+$ system, M, to lie between 1.14 and 1.31 GeV. In order to enhance the contribution to Reaction (1) from the one-pion exchange (OPE) process diagrammed in Fig. 1(a), we further limit the four-momentum transfer squared from the target proton to the Δ^{++} to $|t| < 0.3$ GeV^2 (t is negative in the physical region). As the invariant mass of the $K^-\pi^-$ system, m, increases, $|t|_{min}$ also increases, thereby increasing the extrapolation distance to the pion pole and making the extrapolated values less reliable. Therefore we have performed extrapolations only for m < 0.84 GeV. After the selections described above are made, 836 events remain.

Figure 2 shows histograms of m, M, and $|t|$. Also shown in Fig. 2 are the π^-p and $K^-\pi^+$ invariant masses and the four-momentum transfer squared from the target proton to the π^-p system for the 836 events in the selected sample. The superimposed curves are the predictions by a model to be discussed below. It is evident from Fig. 2(d) that most of the background to Reaction (1) in our selected sample comes from Reaction (2).

The Chew-Low extrapolation technique consists of extrapolating the function

$$\sigma(t,m,M) = \left(\frac{d^3\sigma}{dt\,dm\,dM}\right) \bigg/ \left(\frac{d^3\sigma}{dt\,dm\,dM}\right)_{OPE} \tag{3}$$

to the unphysical point $t=m_\pi^2$. The numerator is the experimentally measured differential cross section for Reaction (1), and the denominator is the calculated differential cross section for the OPE process shown in Fig. 1(a). Explicitly,

$$\left(\frac{d^3\sigma}{dt\,dm\,dM}\right)_{OPE} = \frac{(1/\hbar c)^2}{4\pi^3 P_L^2 M_p^2}\frac{1}{(t-m_\pi^2)^2}m^2 q\sigma(m)M^2 Q\sigma(M)\,F(t,m,M). \tag{4}$$

In Eq. (4) the quantities t, m, and M are as defined above; M_p is the proton mass; m_π, the pion mass; q, the K^- momentum in the $K^-\pi^-$ rest frame; Q, the momentum of the π^+ in the π^+p rest frame; and P_L, the laboratory beam momentum; further, $\sigma(M)$ is the real π^+p elastic scattering cross section; $\sigma(m)$ is the real $K^-\pi^-$ elastic scattering cross section, which is set to 1 mb for calculating (3). The function $F(t,m,M)$ is a form factor to aid the extrapolation; it has the property that $F(m_\pi^2,m,M) = 1$. Therefore, $\sigma(m_\pi^2,m,M)$ is the real $K^-\pi^-$ elastic cross section, $\sigma_{3/2}(m)$.

In this experiment statistics are limited, and as can be seen from Fig. 2(d), background is a serious problem. In order to minimize the uncertainty in the result due to the extrapolation procedures, we have employed a form factor, $F(t,m,M)$, derived by Dürr and Pilkuhn,[7-9] which has been found in other experiments to approximately describe observed pion-exchange differential cross sections.[8,10-12] In addition, we have made background subtractions, which are described below.

If $F(t, m, M)$ correctly accounts for off-mass-shell effects and if there were no background in our sample, then measurements of $\sigma(t, m, M)$ would yield values independent of t and M and equal to $\sigma_{3/2}(m)$. In general, however, neither is true, and the equality holds only at the exchange pole $t = m_\pi^2$. If, however, $\sigma(t, m, M)$ is a continuous function of t with continuous derivatives —especially if it monotonically approaches $\sigma_{3/2}(m)$ as t approaches m_π^2—and if the background is less forward peaked than $(t-m_\pi^2)^{-2}$ near the boundary of the physical region, then it is possible to estimate $\sigma_{3/2}(m)$ by extrapolating measurements of $\sigma(t, m, M)$ from the physical region to $t = m_\pi^2$. The less accurate the measurements of $\sigma(t, m, M)$, and the larger the background in the sample, the more important it is that $\sigma(t, m, M)$ be a well-behaved slowly varying function of t.

The extrapolation is performed in two 0.1-GeV intervals in $K^-\pi^-$ invariant mass, m, starting at 0.64 GeV. Each of these intervals is subdivided into intervals in $|t|$ For each of these subintervals the quantity

$$
\sigma(\bar{t}, \bar{m}) = \frac{\int\limits_{\Delta m, \Delta M, \Delta t} (d^3\sigma/dt\,dm\,dM)\,dt\,dm\,dM}{\int\limits_{\Delta m, \Delta M, \Delta t} (d^3\sigma/dt\,dm\,dM)_{OPE}\,dt\,dm\,dM}
\tag{5}
$$

is evaluated, where the integral is over the regions defined by the intervals in t, m, and M, and $\bar{t}$ and $\bar{m}$ are the mean values of $|t|$ and m in each of these intervals.

Figure 3 plots $\sigma(\bar{t}, \bar{m})$ as a function of $\bar{t}$ for each of the four intervals in m (square data points). As can be seen, these measurements show a strong dependence on $\bar{t}$. This is not surprising, because of the large amount of background known to be in the sample.

As observed above, the OPE Reaction (2), depicted in Fig. 1(b), is a major contributor to the background. This reaction and other reactions which produce $K^{*0}(890)\pi^-p$ have been shown to be well described by using an OPE model with Dürr-Pilkuhn form factors, especially when the πp system forms a $\Delta(1238)$.[10,13] Since the $K^-\pi^+$ elastic-scattering cross section has been measured in other experiments,[10]

we may use this information to calculate the contribution of Reaction (2) to the background, assuming incoherence of the production amplitudes.[14] This contribution can be subtracted from the experimentally measured $d^3\sigma/dt\,dm\,dM$ to reduce the effect of the background on the extrapolation.

Figure 3 also plots $\sigma(\bar{t}, \bar{m})$ as measured for this background-subtracted sample (triangular data points). As can be seen, this background subtraction has dramatically reduced the dependence of $\sigma(\bar{t}, \bar{m})$ on $\bar{t}$.

Linear fits of the form $a-bt$ were attempted for both sets of $\sigma(\bar{t}, \bar{m})$, and in each case the points at $-t = -0.02$ GeV^2 represent the extrapolated value and the associated statistical error. Only the points with $|t| < 0.3$ GeV^2 were used in the fits. Table I summarizes the results of these fits, including fit parameters, extrapolated cross sections, and the associated statistical errors. The $\sigma_{3/2}(m)$ is obtained by averaging the background-subtracted results for the 2.05-GeV/c and 2.63-GeV/c samples for each bin in m.

In order to check that the background subtraction is reasonable, we have calculated the differential cross sections $d\sigma/dM$, $d\sigma/dt$, $d\sigma/dt_{\pi^-p}$, $d\sigma/dM_{\pi^-p}$, and $d\sigma/dm_{K\pi^+}$ for our selected sample of data, assuming that only Reactions (1) and (2) contribute incoherently to the sample. The contribution from Reaction (1) is calculated by inserting our extrapolated value of $\sigma_{3/2}(m)$ in Eq. (4). We assume for this purpose that $\sigma_{3/2}(m)$ is 0 in the range 0.64-0.74 GeV, since the extrapolated cross section is slightly negative in this region. These calculated differential cross sections are plotted as the solid lines superimposed upon the corresponding experimentally measured quantities in Figs. 2(b-f). As can be seen, this simple calculation gives a reasonably good description of these data. This is a reflection of the small slopes of $\sigma(t, m)$ as a function of $\bar{t}$ for the subtracted extrapolations. The discrepancies between the calculation and the data are responsible for the nonzero slopes that are present. We emphasize that we deduce nothing directly about the $K^-\pi^-$ elastic cross section from the extent to which the curves of Fig. 2 describe the data. These figures illustrate only that Reaction (2) is a large contributor to the background and that our procedure for calculating and subtracting this contribution is reasonable.

In Fig. 4 the results of our background-subtracted extrapolations are presented along with the results of other attempts to measure the T = 3/2 Kπ elastic scattering cross section. In interpreting our results it is important to keep in mind the uncertainty inherent in this extrapolation procedure. The errors indicated on our measurements are just the statistical ones that are propagated from the least-squares fit to the linear form for $\sigma(\bar{m}, \bar{t})$. Although it might be reasonable to assume that $\sigma(\bar{t}, m, M)$ is a well-behaved function of t, there is nothing that requires it. Also, it is reasonable to assume that the background is less forward-peaked than $(t-m_\pi^2)^{-2}$, but again, a priori, there is nothing to require it. To the extent that these assumptions are not true there will be systematic errors in interpreting the extrapolated results as the $K^-\pi^-$ elastic cross section.[15]

Cho et al.[16] obtain the $K^-\pi^-$ elastic cross section by performing a pole extrapolation, using the reaction $K^-n \to K^-\pi^-p$. For this reaction the equation for $\sigma(t, m)$ analogous to Eq. (4) has a pole at t = 0 unless $d^2\sigma/dtdm$ has a zero there. Any contribution to $d^2\sigma/dtdm$ other than simple one-meson exchange in general gives a nonzero contribution to the amplitude at t = 0. Cho et al. assume no such contribution and therefore constrain $\sigma(t, m)$ to be finite at t = 0.

Jongejans et al.[17] perform a pole extrapolation, using 669 events of the type $K^-p \to K^-\pi^-\Delta^{++}$, where the momentum transfer squared to the Δ^{++} is less than 0.4 GeV^2. They use a maximum-likelihood polynomial fit to describe the t dependence of their extrapolation function. They do not account for the $K^{*0}(890)\pi^-p$ background in their sample. They point out that the values they quote for the highest two bins in Kπ mass may be unreliable, since the smallest accessible value of |t| for these bins is rather large, 0.2 GeV^2.

Bakker et al.[18] perform a pole extrapolation, using 1009 events of the type $K^-n \to K^-\pi^-p$ having a momentum transfer squared to the nucleon less than 0.32 GeV^2. They find that their results are not substantially different whether or not they use a form factor to describe the t dependence of their extrapolating function, which is well fitted by a straight line.

In addition, phase shift analyses[19,20] have been performed using the high statistics data of the International K^+ Collaboration on the reaction $K^+p \to K^0\pi^0\pi^+p$ in which both $K^+\pi^-$ elastic and charge exchange scattering were studied. In each case, there is a solution for $\delta_{3/2}$ which is compatible with our results on $\sigma_{3/2}$ and the summary in Fig. 4.

The solid curve on Fig. 4 is the T = 3/2 cross section as predicted by the Veneziano model of Lovelace.[3] This prediction overestimates our measurements as well as the others shown in the figure.

Current algebra predicts[4] s-wave Kπ scattering lengths of $a_{1/2} = 0.26$ F, $a_{3/2} = -0.13$ F, corresponding to a threshold cross section for the reaction $K^-\pi^- \to K^-\pi^-$ of 2.1 mb. This number is consistent with the results of Fig. 4.[21]

FOOTNOTES AND REFERENCES

* Supported in part by the United States Atomic Energy Commission.

† Present address: Danish Space Research Institute, Lundtoftevaj 7, 2800 Lyngby, Denmark.

1. A. R. Kirschbaum, P. M. Dauber, J. H. Friedman, E. Colton, and P. E. Schlein, (presented by Peter E. Schlein), in Proceedings of the Conference on ππ and Kπ Interactions, Argonne National Laboratory, 1969, p. 446.

2. G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

3. C. Lovelace, in Proceedings of the Conference on ππ and Kπ Interactions, Argonne National Laboratory, 1969, p. 562.

4. Steven Weinberg, Phys. Rev. Letters 17, 616 (1966); Y. Tomozawa, Nuovo Cimento 46A, 707 (1966); A. P. Balachandran, M. G. Gundzik, and F. Nicodemi, Nuovo Cimento 44A, 1257 (1966).

5. J. J. Murray, J. Button-Shafer, D. M. Siegel, J. S. Linsey, and D. Merrill, Lawrence Radiation Laboratory Report UCRL-11426, July 1964 (unpublished).

6. P. M. Dauber, P. E. Schlein, W. E. Slater, and H. K. Ticho, Phys. Rev. 153, 1403 (1967).

7. H. P. Dürr and H. Pilkuhn, Nuovo Cimento 40, 899 (1965).

8. G. Wolf, Phys. Rev. Letters 19, 925 (1967).

9. For a complete description of F(t,m,M), see e. g., E. Colton and E. Malamud, Lawrence Radiation Laboratory Report UCRL-20111 (to be published in Physical Review).

10. T. G. Trippe, C. -Y. Chien, E. Malamud, J. Mellema, P. E. Schlein, W. E Slater, D H. Stork, and H. K. Ticho, Physics Letters 28B, 203 (1968).

11. Peter E. Schlein, in Meson Spectroscopy, ed. by C. Baltay and A. H. Rosenfeld (Benjamin, New York, 1968), p. 161.

12. G. Wolf, Phys. Rev. 182, 1528 (1969).

13. E. Colton, S. Prum, P. E. Schlein, A. J. Verglas, Nucl. Phys. B17, 117 (1970).

14. For a description and the use of F(t,m,M) for the background process involving low-energy π⁻p scattering, see Eugene Colton, Peter E. Schlein, Eugene Gellert, and Gerald A. Smith, Phys. Rev. 3D, 1063 (1971).

15. A discussion of some of the problems in pole extrapolations can be found in Peter E. Schlein, Review of Some Experimental Results on the ππ and Kπ Interactions, to be published in Proceedings of the International School of Subnuclear Physics, Erice, Sicily, 1970, and G. L. Kane, in Experimental Meson Spectroscopy, eds. Charles Baltay and Arthur H. Rosenfeld, Columbia University Press, 1970, p. 1.

16. Y. Cho et al., Physics Letters 32B, 409 (1970).

17. B. Jongejans et al., Amsterdam-Nijmegen Collaboration (preprint).

18. A. M. Bakker et al., Nuc. Phys. B24, 211 (1970).

19. Brussels-CERN-UCLA Collaboration Analysis of $K^+\pi^-$ Scattering Using Data of the International K^+ Collaboration, Nucl. Phys. B (1971) (to be published); preliminary results presented to Kiev Conference by P. Herquet and T. Trippe, CERN/P. Ph. II/ Phys. 70-29 (1970).

20. John Hopkins University Analysis of $K^+\pi^-$ Scattering Using Data of the International K^+ Collaboration Nucl. Phys. B (1971) (to be published); preliminary results presented at the Washington meeting of the American Physical Society, 1970.

21. In the zero-effective-range approximation, the cross section is predicted to be nearly constant up to about 1 GeV.
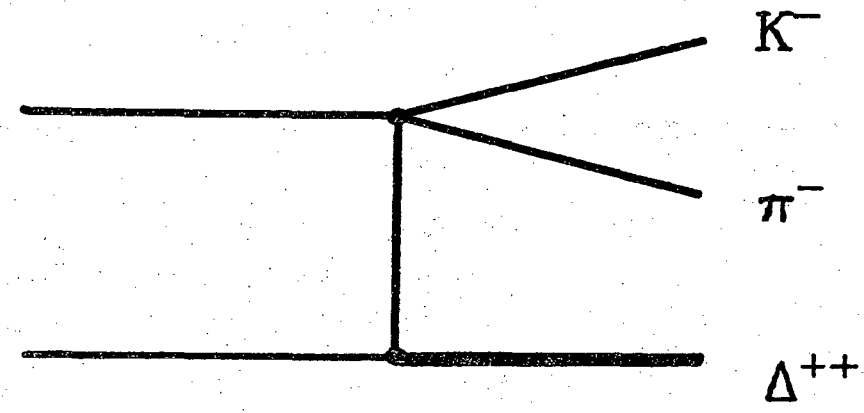
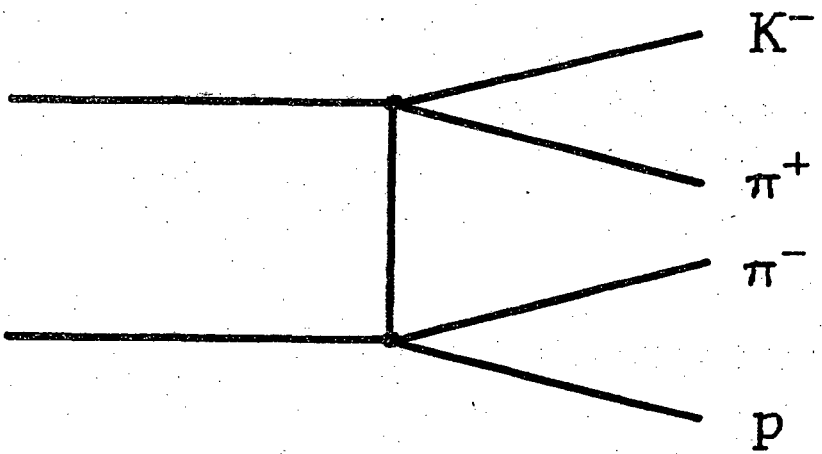Table I. Results of fits of the experimental points shown in Fig. 3 to the assumed form $\sigma(t) = a-bt$.

| Data set | Fit quantities | $K^-\pi^-$ mass range (GeV) | | | |
| | | 0.64-0.74 | | 0.74-0.84 | |
		2.05 GeV/c	2.63 GeV/c	2.05 GeV/c	2.63 GeV/c
Background in	Confidence level (%)	35	66	26	17
	a(mb)	-0.1 ± 1.1	≲1.7 ± 1.0	5.5 ± 1.9	1.7 ± 1.4
	b(mb/GeV²)	43.0 ± 8.7	27.8 ± 8.8	14.5 ± 9.8	38.6 ± 9.2
	σ extrap (mb)	-1.0 ± 1.2	1.1 ± 1.1	5.2 ± 2.0	0.9 ± 1.4
Background out	Confidence level (%)	48	65	33	46
	a(mb)	-1.5 ± 1.1	1.2 ± 1.1	4.0 ± 1.9	1.3 ± 1.6
	b(mb/GeV²)	22.6 ± 8.7	-2.5 ± 10.1	5.4 ± 9.9	12.9 ± 11.1
	σ extrap (mb)	-1.9 ± 1.2	1.2 ± 1.2	3.9 ± 2.0	1.1 ± 1.7
	Avg. σ extrap (mb)	-0.3 ± 0.9		2.3 ± 1.3	

FIGURE CAPTIONS

Fig. 1. (a) The OPE process used to determine the $K^-\pi^-$ elastic cross section.

(b) The OPE process which contributes most of the background in our data.

Fig. 2. (a) $d\sigma/dM_{K^-\pi^-}$ for those events satisfying 1.14 GeV $\leqslant M_{\pi^+p}$ < 1.31 GeV and $|t|<0.3$ GeV².

(b) $d\sigma/dM_{\pi^+p}$ for those events satisfying $M_{K^-\pi^-}$ < 0.84 GeV and $|t|<0.3$ GeV².

(c) $d\sigma/dt$ for those events satisfying $M_{K^-\pi^-}$ < 0.84 GeV and 1.14 GeV < M_{π^+p} < M_{π^+p} < 1.31 GeV.

(d), (e), (f) $d\sigma/dM_{K^-\pi^+}$, $d\sigma/dM_{\pi^-p}$, $d\sigma/dt_{\pi^-p}$ for those events satisfying $M_{K^-\pi^-}$ < 0.84 GeV, 1.14 GeV < M_{π^+p} < 1.31 GeV/c, and $|t|<0.3$ GeV².

Fig. 3. Plots of $\sigma(\bar{t}, \bar{m})$ vs $\bar{t}$. Square data points are for no background subtraction, and triangular data points are with background subtracted as described in the text.

Fig. 4. Summary of measurements of the T = 3/2 $K\pi$ elastic cross section. Curve is the prediction of a Veneziano model of Lovelace.
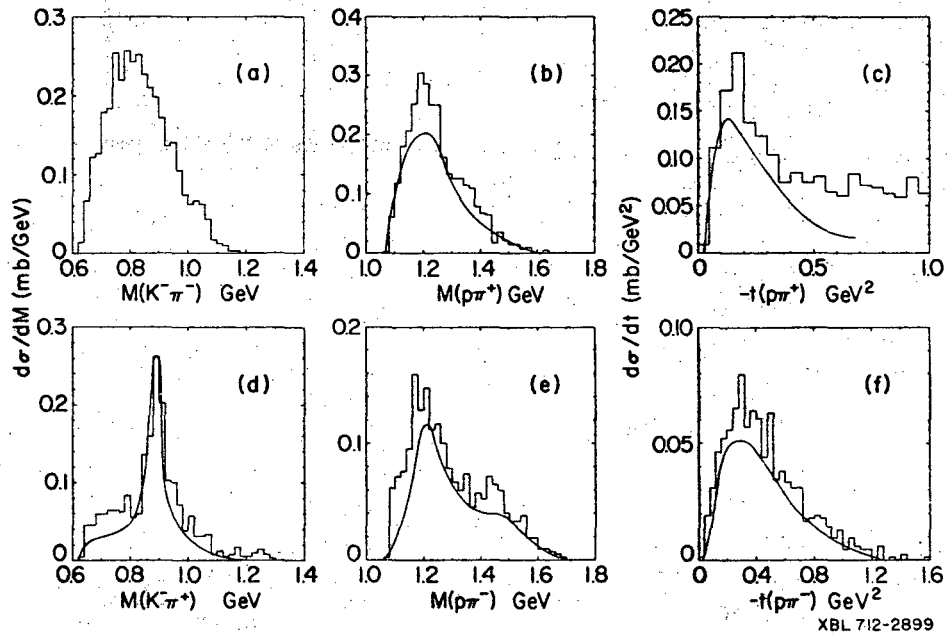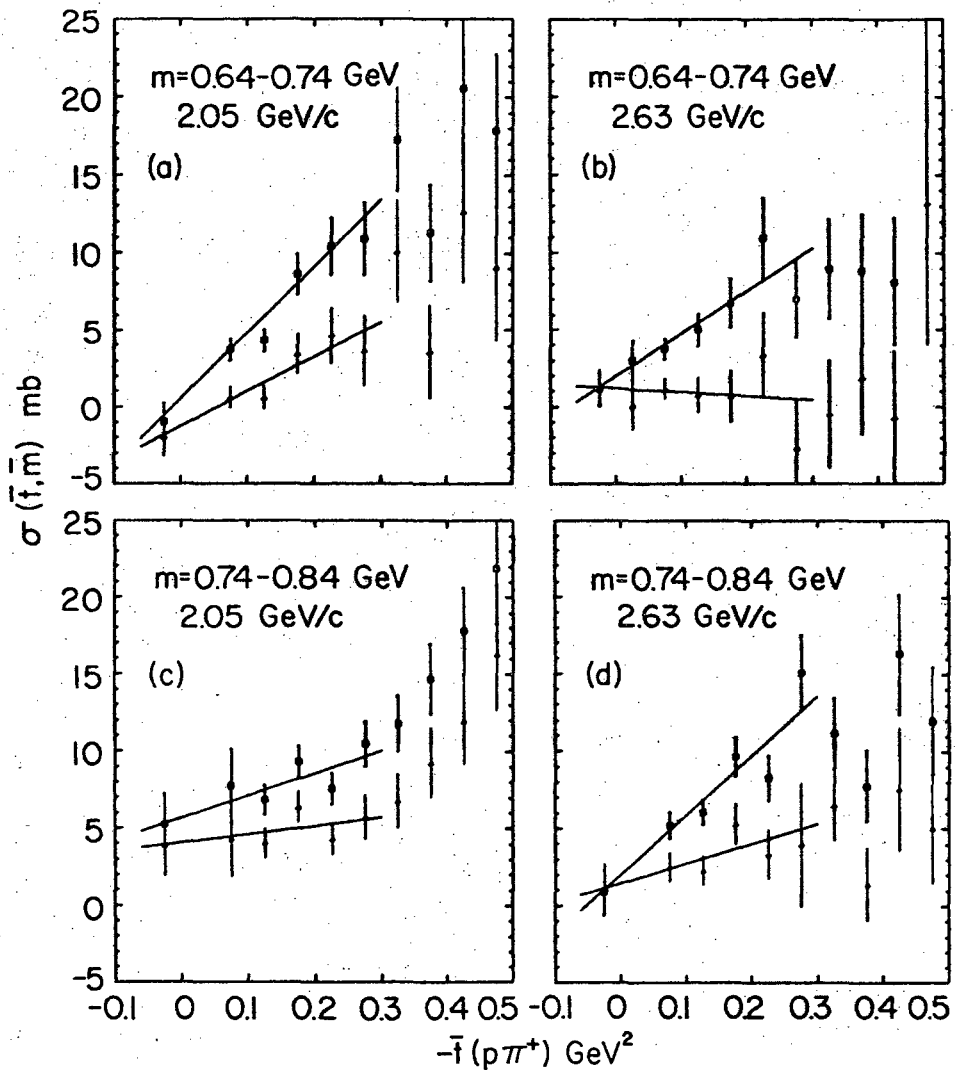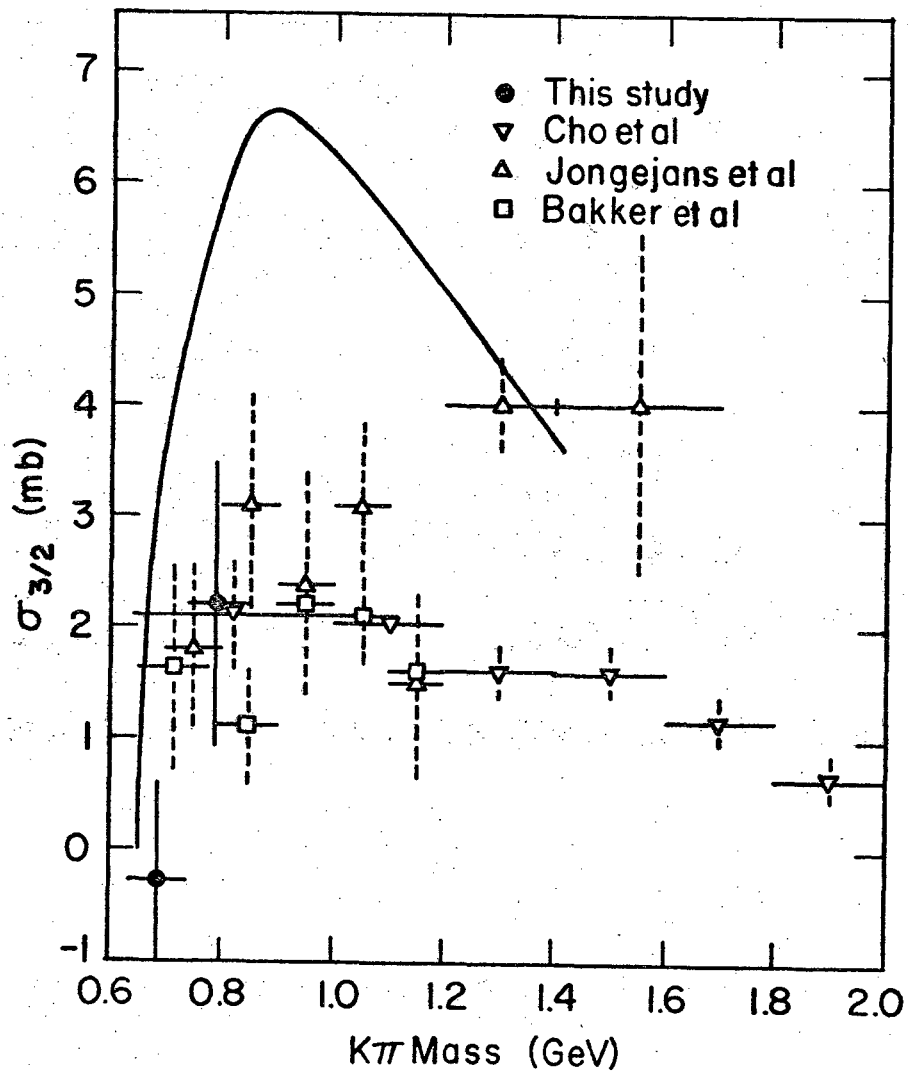
K^-

π^-

Δ^{++}

(a)

K^-

π^+

π^-

p

(b)

XBL 7010-6711

Fig. 1

(a)

(b)

(c)

(d)

(e)

(f)

$d\sigma/dM$ (mb/GeV)

$d\sigma/dt$ (mb/GeV^2)

$M(K^-\pi^-)$ GeV

$M(p\pi^+)$ GeV

$-t(p\pi^+)$ GeV^2

$M(K^-\pi^+)$ GeV

$M(p\pi^-)$ GeV

$-t(p\pi^-)$ GeV^2

XBL 712-2899

Fig. 2

m=0.64-0.74 GeV
2.05 GeV/c

(a)

m=0.64-0.74 GeV
2.63 GeV/c

(b)

m=0.74-0.84 GeV
2.05 GeV/c

(c)

m=0.74-0.84 GeV
2.63 GeV/c

(d)

$\sigma(\bar{t},\bar{m})$ mb

$-\bar{t}\,(p\pi^+)$ GeV^2

XBL 712-2900

Fig. F3

● This study
▽ Cho et al
△ Jongejans et al
□ Bakker et al

$\sigma_{3/2}$ (mb)

Kπ Mass (GeV)

XBL 712-2901

Fig. 4