

# Lawrence Berkeley National Laboratory

## LBL Publications

### Title

PION ABSORPTION IN HIGHLY EXCITED NUCLEAR MATTER

### Permalink

<https://escholarship.org/uc/item/9661j4kc>

### Author

Ko, C-M.

### Publication Date

1979

PION ABSORPTION IN HIGHLY EXCITED NUCLEAR MATTER

Che-Ming Ko

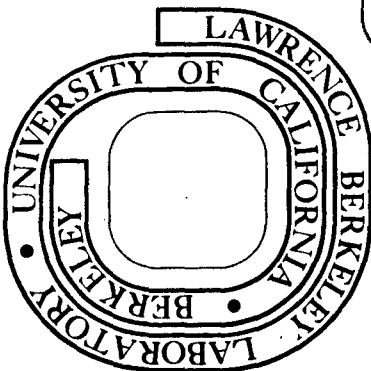
January 1979

RECEIVED  
JAN 17 1979  
PHYSICS DEPARTMENT  
UNIVERSITY OF CALIFORNIA  
BERKELEY, CALIF. 94720

Prepared for the U. S. Department of Energy  
under Contract W-7405-ENG-48

TWO-WEEK LOAN COPY

*This is a Library Circulating Copy  
which may be borrowed for two weeks.  
For a personal retention copy, call  
Tech. Info. Division, Ext. 6782*



LBL-8593 C.2

## **DISCLAIMER**

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

## PION ABSORPTION IN HIGHLY EXCITED NUCLEAR MATTER

Che-Ming Ko

Nuclear Science Division  
Lawrence Berkeley Laboratory  
University of California  
Berkeley, California 94720

## ABSTRACT

Motivated by pion productions in high-energy heavy-ion collisions, we study the p-wave pion absorption in highly excited nuclear matter. The basic absorption mechanism is the two-nucleon model with both pion and rho-meson rescattering. The excited nuclear matter is modeled by a finite temperature Fermi gas. We deduce that the pion absorption rate depends weakly on the temperature of the nuclear matter.

[ Intermediate Energy: High-energy heavy-ion collisions, pion production and absorption, highly excited nuclear matter . ]

## I. INTRODUCTION

The production of pions in high-energy heavy-ion collisions is a subject being extensively studied both experimentally<sup>1-3</sup> and theoretically.<sup>4-9</sup> The energy spectrum of the produced pions in the inclusive cross-sections implies that these pions are produced from a source with a very high temperature. In the collision of Ne and NaF at energy 0.8 GeV/A, Nagamiya et al<sup>1</sup> has deduced that the temperature of the pion source is 62 MeV. The space-time structure of this source presumably can be determined from the correlation measurement of two pions.<sup>10,11</sup>

In the scattering of pions from ordinary nuclei, it has been known that pions are strongly absorbed in nuclear medium.<sup>12</sup> For example, the total absorption cross-section for 130 MeV  $\pi^+$  in  $^{12}\text{C}$  in a bubble chamber experiment is  $\sim 120$  mb and is about one-third of the total cross section.<sup>13</sup> Recently, Ginocchio<sup>14</sup> has done detailed cascade calculations for deep inelastic pion-induced nuclear reactions. He found, using the isobar model, that the mean free path for pion absorption in the nucleus is of the order of the nuclear surface thickness, i.e., 1 fm. Are pions still strongly absorbed in a highly excited nuclear matter produced in high-energy heavy-ion collisions?

From previous studies on pion absorptions in normal nuclei, it is well established that the most important absorption mechanism involves a pair of nucleons.<sup>12,15-17</sup> After the absorption of a pion, each nucleon therefore receives on the average more than 70 MeV in kinetic energy, implying that the final two nucleons are well above the Fermi energy. Hence we expect that the Pauli principle does not play an important role in pion absorptions and that pion absorptions in highly excited nuclear

matter are similar to that in normal unexcited nuclei. We shall show in this paper quantitatively that this is indeed so and suggest that the pions observed in high-energy heavy-ion collisions are from the surface of the source, as those produced inside are absorbed.

We shall take into account only the p-wave pion absorption as it is the dominant part above threshold. The study of p-wave pion absorptions in a zero temperature Fermi gas model has been reported in Ref. 16. Here we generalize it to finite temperatures.

In Section II we outline the p-wave pion absorption in nuclei using the Fermi gas model. In Section III numerical results are presented. Conclusions are drawn in Section IV, and detailed formulations are given in the Appendix for completeness.

## II. FORMULATION

The pion absorption rate in a nucleus is given by

$$\Gamma = 2\pi \sum_f \delta(E_f - E_i - \omega) |T_{fi}|^2 \quad (1)$$

where  $\omega$  is the energy of the absorbed pion. The matrix element  $T_{fi}$  describes the transition of the nucleus from the initial state to the final state, their energies being  $E_i$  and  $E_f$  respectively. For pion absorption by a pair of nucleons in the Fermi gas model, Eq. (1) is written as

$$\begin{aligned} \Gamma = & 2\pi \sum_{\substack{k_1, k_2, k_3, k_4 \\ \sim \sim \sim \sim}} f(k_1) f(k_2) (1 - f(k_3))(1 - f(k_4)) \\ & * \delta\left(\frac{\hbar^2}{2m} (k_3^2 + k_4^2 - k_1^2 - k_2^2) - \omega\right) \\ & * \left| \int d(1) d(2) \psi_{\substack{k_3 k_4 \\ \sim \sim}}^*(12) T_q(12) \psi_{\substack{k_1 k_2 \\ \sim \sim}}(12) \right|^2 \end{aligned} \quad (2)$$

where

$$f(k) = \frac{1}{1 + e^{(\epsilon - \mu)/T}} \quad (3)$$

with  $\epsilon = \hbar^2 k^2 / 2m$  and  $T$  the temperature of the nuclear matter. The chemical potential  $\mu$  is determined from the total nucleon number, i.e.,  $\sum_{\underline{k}} f(k) = N/4$ . In the above,  $\underline{k}_1$  and  $\underline{k}_2$  are the initial momenta of the two nucleons, while  $\underline{k}_3$  and  $\underline{k}_4$  are their final momenta;  $\underline{q}$  is the momentum of the incident pion. The normalized antisymmetric pair wave function is denoted by  $\psi_{\underline{k}_1 \underline{k}_2}^{(12)}$ , where (1) and (2) represent all the coordinates of nucleons 1 and 2. The mass of the nucleon is  $m$ .

From conservation of momentum, the matrix element of the pion absorption operator,  $T_{\underline{q}}^{(12)}$ , must have the form

$$\begin{aligned} & \int d(1) d(2) \psi_{\underline{k}_3 \underline{k}_4}^* (12) T_{\underline{q}}^{(12)} \psi_{\underline{k}_1 \underline{k}_2} (12) \\ &= (2\pi)^3 \delta^{(3)}(\underline{k}' - \underline{k} - \underline{q}) \int d\mathbf{r} d(1') d(2') \psi_{\underline{k}'}^*(\mathbf{r}; 1' 2') \\ & \times T_{\underline{q}}(\mathbf{r}, 1' 2') \psi_{\underline{k}}(\mathbf{r}; 1' 2') \end{aligned} \quad (4)$$

where we have introduced the center-of-mass and relative momenta

$$\underline{K} = \underline{k}_1 + \underline{k}_2, \quad \underline{k} = \frac{1}{2}(\underline{k}_1 - \underline{k}_2) \quad (5a)$$

$$\underline{K}' = \underline{k}_3 + \underline{k}_4, \quad \underline{k}' = \frac{1}{2}(\underline{k}_3 - \underline{k}_4) \quad (5b)$$

and

$$\underline{R} = \frac{1}{2}(\underline{r}_1 + \underline{r}_2), \quad \underline{r} = \underline{r}_1 - \underline{r}_2. \quad (5c)$$

The notations (1') and (2') represent the spin and isospin coordinates of nucleons 1 and 2. The matrix element on the RHS of Eq. (4) has been discussed in detail in Ref. 16 for p-wave pion absorption by a pair of

nucleons with pion and rho-meson rescattering. Diagrammatically it describes the processes shown in Fig. 1. The explicit form of this matrix element is given in the Appendix.

The kinematic factors in Eq. (2) can be greatly simplified if we make the following two approximations: 1) We approximate the factors  $(1 - f(k_3))$  and  $(1 - f(k_4))$  by unity. This is justified for not too high temperatures as  $k_3$  and  $k_4$  are, on the average, well above the Fermi momentum. Certainly we should be cautious in the case of very high temperature. In this case, there are finite probabilities for nucleons in such high momentum states and the above kinematic factors are smaller than unity. 2) We approximate the angular part in the  $\delta$ -function for the energy conservation by its average value, i.e.

$$\begin{aligned} & \frac{\hbar^2}{2m} (k_3^2 + k_4^2 - k_1^2 - k_2^2) - \omega \\ &= \frac{\hbar^2}{4m} (\tilde{k} \cdot \tilde{K} - \tilde{q}^2) + \frac{\hbar^2}{m} (k'^2 - k^2) - \omega \\ &\approx \frac{\hbar^2}{m} \left( k'^2 - k^2 - \frac{q^2}{4} \right) - \omega \end{aligned} \quad (6)$$

Both approximations were used in Ref. 16.

The pion absorption cross section per nucleon is then given by Eq. (A.11) in the Appendix. The only place where the effect of temperature appears is in the function  $P(x)$  defined in Eq. (A.16). This function essentially gives the relative probability for a pair of nucleons to have relative momentum  $x \hbar k_F$ , where  $k_F$  is the Fermi momentum and has the value  $1.34 \text{ fm}^{-1}$ . For zero temperature,  $T=0$ , it has the familiar form

$$P(x) = 24x^2 \left( 1 - \frac{3}{2}x + \frac{x^3}{2} \right) \quad (7)$$



To determine  $P(x)$  for finite temperature, we assume that the density of the excited nuclear matter is the same as that at  $T=0$ . From the well known Sommerfeld expansion,<sup>18</sup> we can then determine the chemical potential  $\mu$  for the following two limits:

$$\mu \approx \epsilon_F \left\{ 1 - \frac{\pi^2}{12} \left( \frac{T}{\epsilon_F} \right)^2 - \frac{7\pi^2}{960} \left( \frac{T}{\epsilon_F} \right)^4 - \dots \right\} \quad (8a)$$

for  $T \ll \epsilon_F$ , and

$$e^{\mu/T} \approx \frac{4}{3\sqrt{\pi}} \left( \frac{\epsilon_F}{T} \right)^{3/2} \left\{ 1 + \frac{1}{2\sqrt{2}} \frac{4}{3\sqrt{\pi}} \left( \frac{\epsilon_F}{T} \right)^{3/2} - \frac{1}{3\sqrt{3}} \left[ \frac{4}{3\sqrt{\pi}} \left( \frac{\epsilon_F}{T} \right)^{3/2} \right]^2 + \dots \right\} \quad (8b)$$

for  $T \gg \epsilon_F$ . In the above,  $\epsilon_F$  is the Fermi energy and has the value  $\sim 40$  MeV. For  $T$  around  $\epsilon_F$ , we have to determine  $\mu$  consistently from its defining equation. This procedure is more involved numerically and we shall not consider  $T \sim \epsilon_F$  in this paper.

### III. RESULTS

In Fig. 2 we show the numerical results of  $P(x)$  for different values of temperature. We observe that the peak of  $P(x)$  moves to larger values of  $x$  as the temperature increases. At  $T=0$  MeV, it peaks at  $x=0.5$  while at  $T=60$  MeV it peaks at  $x=0.9$ . Also, as the temperature increases, the width of  $P(x)$  becomes wider and the whole function gets flatter. This change of  $P(x)$  with respect to the temperature has effects on the pion absorption cross-section through integrals in Eq. (A.17).

We show in Table I the dominant integrals for different values of the pion momentum and the temperature. In order to remove the energy

dependence we have multiplied these integrals by the energy factors  $\omega D^2$ , where  $\omega$  is the pion energy and  $D$  the propagator of the  $\Delta$ -resonance in Fig. 1. From the table we see that as the temperature increases, those integrals with initial relative angular momentum  $L=0$  decrease while those with  $L=1$  increase. This can be understood qualitatively in the following way. The integrals are limited to the range of  $r$  values determined by the Yukawa functions, which are defined in Eq. (A.9). For the case of pion rescattering, values of  $r$  smaller than 1.4 fm are important. As shown before, when the temperature increases from 0 MeV to 60 MeV, the maximum of the function  $P(x)$  increases from  $x=0.5$  to 0.9. i.e.,  $k_1 = 0.67 \text{ fm}^{-1}$  to  $1.21 \text{ fm}^{-1}$ . Therefore,  $k_1 r$  increases from 0.94 to 1.68 in this temperature region. For these values of  $k_1 r$ , the spherical Bessel functions  $j_0(k_1 r)$  and  $j_1(k_1 r)$  have different behaviors, with the former a decreasing function while the latter an increasing function of the argument. Similar arguments apply to the case of rho-meson rescattering. These explain the features we obtained in Table I.

In Table II, we show the absorption cross-section per nucleon at different temperatures for different pion momenta. We observe that it increases with the temperature up to 30% from  $T=0$  MeV to  $T=60$  MeV. The changes are not appreciable. Considering the approximation 1) which we make in the calculation, we actually overestimate the absorption cross section for very high temperature. If we could include the Pauli principle more accurately, the pion absorption cross section in nuclear matter would be more insensitive to the temperature of the nuclear matter. Using the average absorption cross-section per nucleon,  $\sim 5 \text{ fm}^2$ , we obtained a mean free path of 1 fm for pion absorptions in nuclear matter, if the normal

nuclear matter density is used. This value of the pion absorption mean free path is consistent with that determined in Ref. 14.

#### IV. CONCLUSIONS

From the above analysis, we conclude that the pion absorption cross section in excited nuclear matter has similar values to normal nuclear matter. The mean free path of pion absorptions in nuclear matter is determined to be 1 fm, irrespective of the temperature of the nuclear matter. Therefore, the observed pions in high-energy heavy-ion collisions are from the surface of the source. This fact also implies that there are enhancements of high-energy protons in these collisions as a result of pion absorptions.

#### Acknowledgments

This work was performed under the auspices of the Nuclear Science Division of the U.S. Department of Energy.

APPENDIX

In this Appendix, we shall give the mathematical details of evaluating the absorption process schematically described in Fig. 1. Most of the following material can be found in Ref. 16. We include these details for completeness.

To evaluate the amplitude in Fig. 1, we use the following meson-nucleon effective interaction Lagrangians:

$$L_{\pi NN} = \frac{f}{m_\pi} \chi^+ \xi^+ \vec{\sigma} \cdot \vec{\nabla} \phi \cdot \vec{\tau} \xi \chi \quad (\text{A.1})$$

$$L_{\rho NN} = \frac{g_\rho}{2m} (1 + \kappa) \chi^+ \xi^+ (\vec{\sigma} \times \vec{\nabla}) \cdot (\vec{\rho} \cdot \vec{\tau}) \xi \chi \quad (\text{A.2})$$

$$L_{\pi N\Delta} = \frac{f_\Delta}{m_\pi} \chi^+ \xi^+ \cdot \vec{\nabla} \phi \xi \chi + \text{h.c.} \quad (\text{A.3})$$

$$L_{\rho N\Delta} = \frac{g_{\rho\Delta}}{2m} \chi^+ \xi^+ \cdot (\vec{\nabla} \times \vec{\rho}) \xi \chi + \text{h.c.} \quad (\text{A.4})$$

Here  $\xi$  and  $\chi$  are the nucleon spinor and isospinor, and  $\vec{\xi}$  and  $\vec{\chi}$  the  $\Delta$  vector-spinor and vector-isospinor. The pion isovector field operator is denoted by  $\phi$  and the rho-meson vector-isovector field operator by  $\vec{\rho}$ . The mass of the nucleon and pion are denoted by  $m$  and  $m_\pi$  respectively. The coupling constants are taken to be  $f^2/4\pi = 0.081$ ,  $f_\Delta^2/4\pi = 0.32$ ,  $g_\rho^2/4\pi = 0.55$ , and  $\kappa = 6.6$ . The  $\rho$ - $\Delta$  coupling is  $g_{\rho\Delta} = 6\sqrt{2}/5 g_\rho (1 + \kappa)$  from static quark model.

The two-body  $\pi^+$  absorption operator  $T_q(\vec{r}, 1'2')$  defined in Eq. (4) for pion or rho-meson rescattering is then given by

$$\begin{aligned}
 T_{\tilde{q}}^{\pi, \rho}(\tilde{r}; 1' 2') &= \frac{f_{\Delta}}{\sqrt{2\omega} m_{\pi}} \frac{1}{\omega_R - \omega - \frac{1}{2} i\Gamma_{\Delta}} \left\{ e^{\frac{1}{2} i\tilde{q} \cdot \tilde{r}} V_{12}^{\pi, \rho}(\tilde{r}) (S_1^+ \cdot \tilde{q}) T_{1+}^+ \right. \\
 &\quad \left. + e^{-i \frac{1}{2} \tilde{q} \cdot \tilde{r}} V_{21}^{\pi, \rho}(\tilde{r}) (S_2^+ \cdot \tilde{q}) T_{2+}^+ \right\} \quad (A.5)
 \end{aligned}$$

and

$$V_{12}^{\pi}(\tilde{r}) = \frac{1}{3} \frac{f_{\Delta} f}{4\pi m_{\pi}} \left\{ Y_0(m_{\pi}^* r) S_1 \cdot \sigma_2 + Y_2(m_{\pi}^* r) S_{12}^*(\tilde{r}) \right\} T_1 \cdot \tau_2 \quad (A.6)$$

$$V_{12}^{\rho}(\tilde{r}) = \frac{1}{3} \frac{g_{\rho} g_{\rho\Delta}}{4\pi m^2} m_{\rho}^3 \left\{ 2Y_0(m_{\rho}^* r) S_1 \cdot \sigma_2 - Y_2(m_{\rho}^* r) S_{12}^*(\tilde{r}) \right\} T_1 \cdot \tau_2 \quad (A.7)$$

$$S_{12}^* = 3S_1 \cdot \hat{r} \sigma_2 \cdot \hat{r} - S_1 \cdot \sigma_2 \quad (A.8)$$

$$Y_0(x) = \frac{e^{-x}}{x}, \quad Y_2(x) = \left( 1 + \frac{3}{x} + \frac{3}{x^2} \right) \frac{e^{-x}}{x} \quad (A.9)$$

with S and T the transition spin and isospin operators which connect nucleons and isobars. The effective pion and rho-meson masses are defined respectively to be  $m_{\pi}^* = \sqrt{m_{\pi}^2 - \omega^2/4}$  and  $m_{\rho}^* = \sqrt{m_{\rho}^2 - \omega^2/4}$  in terms of the pion energy  $\omega$ . The position of the isobar resonance and its width are

$$\omega_R = m_{\Delta} - m + \frac{q^2}{2m_{\Delta}} \quad (A.10)$$

and

$$\Gamma_{\Delta} = \frac{2}{3} \frac{f_{\Delta}^2}{4\pi} \frac{q^3}{m_{\pi}^2}$$

respectively, where the mass  $m_{\Delta}$  of  $\Delta$  is 1232 MeV.

After summing over the initial and final spin and isospin states and carrying out partial wave expansions, we obtain the pion absorption

cross-section per nucleon

$$\begin{aligned}
 \sigma = & \frac{4k_f^4 m q^2}{3\pi m_\pi^6} * \left\{ \sum_L^- \sum_{L'}^+ \sum_\ell^+ [24 \mathcal{K}^{LL'\ell} + 18 \mathcal{K}^{LL'\ell}] \right. \\
 & + \sum_{\ell'}^+ (24 m^{LL'\ell\ell'} - 9\sqrt{70} \mathcal{N}^{LL'\ell\ell'}) + \sum_L^- \sum_{L'}^- \sum_\ell^- [8 \mathcal{K}^{LL'\ell} \\
 & + 22 \mathcal{K}^{LL'\ell} + \sum_{\ell'}^- (16 m^{LL'\ell\ell'} - 5\sqrt{70} \mathcal{N}^{LL'\ell\ell'})] \\
 & + \sum_L^+ \sum_{L'}^- \sum_\ell^- [8 \mathcal{K}^{LL'\ell} + 22 \mathcal{K}^{LL'\ell} + \sum_{\ell'}^- (16 m^{LL'\ell\ell'} - 5\sqrt{70} \mathcal{N}^{LL'\ell\ell'})] \\
 & \left. + \sum_L^+ \sum_{L'}^+ \sum_\ell^+ [18 \mathcal{K}^{LL'\ell} - \sum_{\ell'}^+ 9\sqrt{70} \mathcal{N}^{LL'\ell\ell'}] \right\} \quad (A.11)
 \end{aligned}$$

where the (+) and (-) in the summations denote even and odd respectively.

In the above, we have used the following expressions:

$$\mathcal{K}^{LL'\ell} = [L][L'][\ell] \int_0^\infty dx P(x) x' \begin{pmatrix} L' & L & \ell \\ 0 & 0 & 0 \end{pmatrix} |H^{LL'\ell}(k_{F x'}, k_{F x})|^2 \quad (A.12)$$

$$\begin{aligned}
 \mathcal{K}^{LL'\ell} = & [L][L'][\ell] \int_0^\infty dx P(x) x' |K^{LL'\ell}(k_{F x'}, k_{F x})|^2 \sum_C [C] \\
 & * \begin{pmatrix} 2 & \ell & C \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{pmatrix} L & L' & C \\ 0 & 0 & 0 \end{pmatrix}^2 \quad (A.13)
 \end{aligned}$$

$$\begin{aligned}
 m^{LL'\ell\ell'} = & (-)^{\ell-\ell'/2} [L][L'][\ell][\ell'] \begin{pmatrix} \ell & \ell' & 2 \\ 0 & 0 & 0 \end{pmatrix}^2 \int_0^\infty dx P(x) x' \\
 & \times \text{Re} \left\{ \begin{pmatrix} L & L' & \ell' \\ 0 & 0 & 0 \end{pmatrix}^2 [H^{LL'\ell'}(k_{F x'}, k_{F x})]^* K^{LL'\ell}(k_{F x'}, k_{F x}) \right. \\
 & \left. + \begin{pmatrix} L & L' & \ell \\ 0 & 0 & 0 \end{pmatrix}^2 H^{LL'\ell}(k_{F x'}, k_{F x}) [K^{LL'\ell'}(k_{F x'}, k_{F x})]^* \right\} \quad (A.14)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{N}^{LL'\ell\ell'} &= (-)^{\ell-\ell'/2} [L][L'][\ell][\ell'] \begin{pmatrix} \ell & \ell' & 2 \\ 0 & 0 & 0 \end{pmatrix} \int_0^\infty dx P(x) x' \\
 &\times \operatorname{Re} \left\{ K^{LL'\ell}(k_F x', k_F x) [K^{LL'\ell'}(k_F x', k_F x)]^* \right\} \sum_C (-)^C [C] \\
 &\times \begin{pmatrix} 2 & \ell & C \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & \ell' & C \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L' & L & C \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{Bmatrix} 2 & 2 & 2 \\ C & \ell & \ell' \end{Bmatrix} \quad (\text{A.15})
 \end{aligned}$$

We use the abbreviation  $[L] \equiv 2L+1$ . The quantity  $x'$  is given by, through energy conservation,  $x' = [x^2 + \omega m(1 - q^2/4\omega m)/k_F^2]^{1/2}$ . The function  $P(x)$ , which is the relative probability for a pair of nucleons with relative momentum  $x \hbar k_F$ , is given by

$$P(x) = 24x^2 \left( \frac{32\pi k_F^3}{3} \right)^{-1} \int d^3 \underline{k} f\left(\left|\frac{\underline{K}}{2} + \underline{k}\right|\right) f\left(\left|\frac{\underline{K}}{2} - \underline{k}\right|\right) \quad (\text{A.16})$$

The two functions  $H^{LL'\ell}$  and  $K^{LL'\ell'}$  are defined respectively by

$$\begin{Bmatrix} H^{LL'\ell}(k_f, k_i) \\ K^{LL'\ell}(k_f, k_i) \end{Bmatrix} = \int_0^\infty dr r^2 j_{L'}(k_f r) j_\ell\left(\frac{qr}{2}\right) \begin{Bmatrix} Y_0^*(r) \\ Y_2^*(r) \end{Bmatrix} j_L(k_i r) \quad (\text{A.17})$$

with

$$\begin{aligned}
 Y_0^*(r) &= \xi_\pi Y_0(m_\pi^* r) + 2\xi_\rho Y_0(m_\rho^* r) \\
 Y_2^*(r) &= \xi_\pi Y_2(m_\pi^* r) - \xi_\rho Y_2(m_\rho^* r)
 \end{aligned} \quad (\text{A.18})$$

where

$$\xi_{\pi} = \frac{4}{27} f \left( \frac{f_{\Delta}^2}{4\pi} \right) \left( \frac{m_{\pi}^*}{m_{\pi}} \right)^3 \frac{1}{D \sqrt{\omega}}$$

and

(A.19)

$$\xi_{\rho} = \frac{8}{75} f \left( \frac{g_{\rho}^2}{4\pi} \right) \left( \frac{m_{\rho}^*}{m^2 m_{\pi}} \right) \frac{1}{D \sqrt{\omega}} (1 + \kappa)^2$$

Hadronic form factors in the vertex can be straightforwardly included. In the monopole forms, we make the following replacements for the coupling constants

$$f \rightarrow f \frac{\Lambda_{\pi}^2 - m_{\pi}^{*2}}{\Lambda_{\pi}^2 - Q^2}$$

and

(A.20)

$$g \rightarrow g \frac{\Lambda_{\rho}^2 - m_{\rho}^{*2}}{\Lambda_{\rho}^2 - Q^2}$$

where  $Q$  is the momentum of the rescattered pion or rho-meson. These lead to modified expressions for  $Y_0$  and  $Y_2$  as shown in Ref. 16. The values of  $\Lambda_{\pi}$  and  $\Lambda_{\rho}$  are both taken to be 1.2 GeV in our calculations.



REFERENCES

1. S.Nagamiya et al., Lawrence Berkeley Laboratory Report LBL-6770 (1977)
2. K.Nakai et al., Proceedings of the Int. Conf. on Nuclear Structure, Tokyo (1977).
3. S.Y.Fung et al., Phys. Rev. Lett. 40, 292 (1977).
4. J.I.Kapusta, Phys. Rev. C16, 1493 (1977).
5. J.Gosset, J.I.Kapusta, and G.D.Westfall, Phys. Rev. C18, 844 (1978).
6. M.Gyulassy and S.K.Kauffman, Phys. Rev. Lett. 40, 298 (1978).
7. J.P.Vary, Phys. Rev. Lett. 40, 295 (1978).
8. H.B.Mathis and Meng Ta-chung, Phys. Rev. C18, 952 (1978).
9. Y.Afek, G.Berlad, A.Dar, and G.Eilam, Phys. Rev. Lett. 41, 849 (1978).
10. F.B.Yano and S.E.Koonin, Phys. Lett. B78, 556 (1978).
11. S.Y.Fung et al., Phys. Rev. Lett. 41, 1592 (1978).
12. J.Hüfner, Phys. Lett. C21, 1 (1975).
13. E.Belotti, D.Cavalli, and C.Matteuzzi, Nuovo Cim. 18A, 75 (1973).
14. J.N.Ginocchio, Phys. Rev. C17, 195 (1978).
15. G.F.Bertsch and D.O.Riska, Phys. Rev. C18, 317 (1978).
16. C.M.Ko and D.O.Riska, MSU preprint (1978), and Nucl. Phys. (in press).
17. F.Hachenberg, J.Hüfner, and H.J.Pirner, Phys. Lett. 66B, 425 (1977).
18. K.Huang, Statistical Mechanics (John Wiley & Sons, New York, 1965).

TABLE I. Dominant integrals in units of  $m_\pi^{-6}$ , with  $m_\pi$  the inverse pion Compton wave length, for pion momentum  $q = 0.8, 1.0$  and  $1.2 \text{ fm}^{-1}$  at different temperatures.

T (MeV)	q ( $\text{fm}^{-1}$ )	$\omega D^2 \mathcal{K}_{110}$	$\omega D^2 \mathcal{K}_{011}$	$\omega D^2 \mathcal{K}_{130}$	$\omega D^2 \mathcal{K}_{031}$	$\omega D^2 \mathcal{K}_{020}$
0	0.8	2.09(-4)	1.11(-4)	1.08(-4)	6.33(-5)	3.85(-4)
	1.0	1.98(-4)	1.67(-4)	9.46(-5)	9.49(-5)	3.15(-4)
	1.2	1.80(-4)	2.30(-4)	8.05(-5)	1.26(-4)	2.50(-4)
20	0.8	4.06(-4)	1.22(-4)	1.45(-4)	5.07(-5)	3.22(-4)
	1.0	3.85(-4)	1.82(-4)	1.30(-4)	7.73(-5)	2.65(-4)
	1.2	3.60(-4)	2.49(-4)	1.13(-4)	1.05(-4)	2.10(-4)
60	0.8	6.66(-4)	8.50(-5)	1.21(-4)	2.05(-5)	1.41(-4)
	1.0	6.29(-4)	1.27(-4)	1.12(-4)	3.22(-5)	1.17(-4)
	1.2	6.19(-4)	1.82(-4)	1.02(-4)	4.55(-5)	9.31(-5)

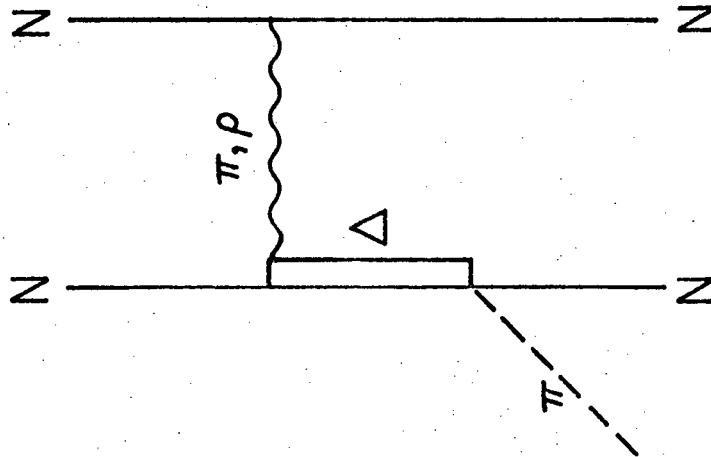
TABLE II. Absorption cross section in units of  $\text{fm}^2$  for pion momentum  $q = 0.8, 1.0$  and  $1.2 \text{ fm}^{-1}$  at different temperatures.

T (MeV)	q ( $\text{fm}^{-1}$ )	$\sigma$ ( $\text{fm}^2$ )
0	0.8	2.54
	1.0	4.71
	1.2	4.99
20	0.8	3.23
	1.0	6.04
	1.2	6.40
60	0.8	3.53
	1.0	6.63
	1.2	7.32

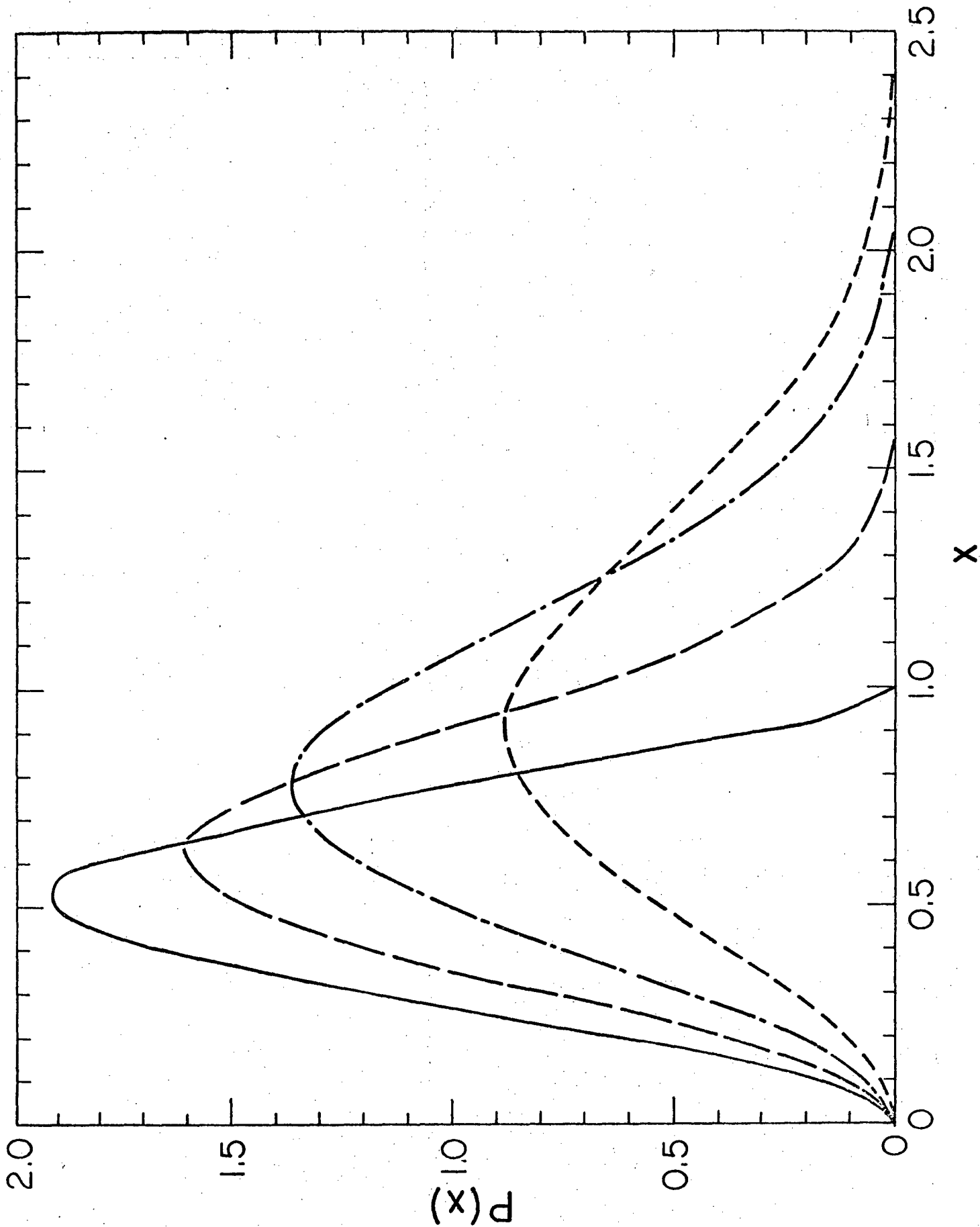
FIGURE CAPTIONS

Fig. 1. Pion and  $\rho$ -meson rescattering through  $\Delta$  resonant intermediate state.

Fig. 2. Relative probability function  $P(x)$  as a function of the relative momentum  $x$  in units of the Fermi momentum  $k_F$  for different temperatures. The curves are represented as: solid curve for  $T=0$  MeV, long-dashed curve for  $T=20$  MeV, long-dashed-dotted curve for  $T=40$  MeV, and short-dashed curve for  $T=60$  MeV.



XBL 791 -39



This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

TECHNICAL INFORMATION DEPARTMENT  
LAWRENCE BERKELEY LABORATORY  
UNIVERSITY OF CALIFORNIA  
BERKELEY, CALIFORNIA 94720