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SANTA CRUZ

A HYBRID PID DESIGN FOR ASYMPTOTIC STABILIZATION WITH INTERMITTENT MEASUREMENTS

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Daniel T. Lavell

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The Thesis of Daniel T. Lavell
is approved:

Ricardo Sanfelice, Chair

Gabriel Elkaim

Berk Altin

Tyrus Miller
Vice Provost and Dean of Graduate Studies

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Abstract

A HYBRID PID DESIGN FOR ASYMPTOTIC STABILIZATION WITH INTERMITTENT MEASUREMENTS

by

Daniel T. Lavell

In this paper, we consider the case when implementing a proportional-integral-derivative (PID) controller in the presence of non-periodic and intermittent sensor measurements. Current PID control design methods cannot accurately handle the case when the measurements arrive at isolated and potentially non-periodic time instants. We model the continuous-time plant being control, the impulsive measurement updates and the PID control law adapted to handle the measurement structure using a hybrid systems framework. By way of Lyapunov stability analysis for hybrid systems, we given sufficient conditions and a computationally tractable design method for the PID controller. We conclude by illustrating the results through numerical examples.

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Chapter 1

Introduction

1.1 Background

Proportional-integral-derivative controllers are incredibly popular in engineering applications, see [1, 6, 7]. For continuous-time systems, a PID controller is given by

$$u(t) = K_P e(t) + K_I \int_0^t e(s)ds + K_D \dot{e}(t), \tag{1.1}$$

where $t \geq 0$, u is the input to the system being controlled, e is the error between the state and the reference to be tracked, and K_P, K_I, and K_D are the proportional, integral and derivative parameters (or gains) to be designed, respectively. Several design techniques are available to determine the three parameters in the PID controller to meet design specifications such as rise time, settling time, and overshoot [20, 6]. However, classical designs require continuous (or very fast and periodic) measurements of the output, which may not be practical for all applications, [6, 7]. Namely, when the measurements are available only at non periodic, intermittent, and isolated time instances novel methods for the design of the con-

trol law in (1.1) are needed, and unavoidably, demand the use of hybrid systems tools.

Some design techniques for PID controllers that could have potential for the settling of intermittent, non periodic sampling are available in the literature. It should be noted that techniques that assume a periodic sample rate, see, e.g. [6, 7] are not suitable for the settling of intermittence. In fact, the choice of a constant sampling rate precludes handling non periodic measurements. A multi-rate PID control law is considered in [16] through discretizing the continuous-time dynamics and considering a delayed sensor to input signal dependent on the sample rate. It should be pointed out that first-order reset elements have shown to be advantageous towards the performance of the controller [18]. On the other hand, with the popularization of systems that contain both continuous and discrete dynamics (known as hybrid systems), there are several novel approaches with the potential for the design of PID controllers under intermittence. In [12], the authors consider a continuous-time system and design an event-triggered control law using a Lyapunov-based analysis. In [11], the authors utilize an impulsive systems approach to design a static feedback controller for a continuous-time linear time-invariant system and uses an estimate event-based trigger to update the controller. Studies of hybrid controllers with sporadic measurements have been studied in [5, 13]; however they lack the application to hybrid PID controllers specifically, which is the goal of this paper.

1.2 Contributions

In our work, we consider the case when the plant is a linear time-invariant system, but the output is only measured at, potentially non periodic, isolated time instances. Namely, subsequent measurements can occur any time within a known

bounded window of ordinary time. To cope with this setting, we introduce a hybrid PID control law akin to the continuous-time one in (1.1), that allows for continuous evolution of the state as well as impulsive measurements and control updates. Due to the continuous-time and impulsive dynamics of the closed-loop systems (see Figure 3.1), we utilize the hybrid systems framework in [8] to model the coupling of these systems. Using Lyapunov-based results for uniform global asymptotic stability of compact sets, we provide sufficient conditions on the parameters of the hybrid PID controller to guarantee such stability property. Then, using a polytopic embedding approach we give a tractable computation method for design of the PID parameters. Numerical simulations illustrate the results. Specifically this thesis provides the following contributions:

1. A presentation of a hybrid PID model with intermittent sensor measurements
2. Design techniques for the hybrid PID feedback gains
3. Numerical examples exemplifying the design techniques
4. Analysis of alternative methods for integral and derivative terms
5. Analysis of alternative forms of hybridity in the closed-loop system

1.3 Thesis Outline

The paper is organized as follows. Chapter 2 provides notations and preliminaries on hybrid systems as used throughout this paper. Chapter 3 presents the system under consideration and provides a motivational example. Chapter 4 models the closed-loop system as a hybrid system and gives examples for proportional only, proportional-integral and proportion-derivative control laws. Chapter 5 gives

the main results and design methods. Chapter 6 illustrates the main results and design through numerical examples.

Chapter 2

Notation and Preliminaries on Hybrid Systems

2.1 Notation

For a symmetric $n \times n$ matrix P, P is positive definite if all eigenvalues of P are real and positive. We will denote P being positive definite as $P > 0$. Similarly, a matrix P that is negative definite is denoted by $P < 0$. Given $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$, the pair (x, y) is equivalent to $[x^\top, y^\top]^\top$. The distance from a vector $x \in \mathbb{R}^n$ to a closed set $\mathcal{A} \subset \mathbb{R}^n$ is denoted as $|x|_{\mathcal{A}}$. A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a class-$\mathcal{K}$ function, also written $\alpha \in \mathcal{K}$, if α is zero at zero, continuous, strictly increasing; it is said to belong to class-$\mathcal{K}_\infty$, also written $\alpha \in \mathcal{K}_\infty$, if $\alpha \in \mathcal{K}$ and is unbounded; α is positive definite, also written $\alpha \in \mathcal{PD}$, if $\alpha(s) > 0$ for all $s > 0$ and $\alpha(0) = 0$. A function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a class-$\mathcal{KL}$ function, also written $\beta \in \mathcal{KL}$, if it is nondecreasing in its first argument, nonincreasing in its second argument, $\lim_{r \to 0^+} \beta(r, s) = 0$ for each $s \in \mathbb{R}_{\geq 0}$, and $\lim_{s \to \infty} \beta(r, s) = 0$ for each $r \in \mathbb{R}_{\geq 0}$. Given a function $f : \mathbb{R}^n \to \mathbb{R}^m$, the domain of f is denoted by $\operatorname{dom} f$. Given a set

X, co$X$ represents the convex hull of X.

2.2 Preliminaries on Hybrid Systems

This section introduces the main notions and definitions on hybrid systems used throughout this work. More information on such systems can be found in [8]. A hybrid system $\mathcal{H}$ can be represented in the compact form

$$\mathcal{H} : \begin{cases} \dot{x} & = & f(x) & x \in C, \\ x^+ & \in & G(x) & x \in D, \end{cases} \tag{2.1}$$

where $x \in \mathbb{R}^n$ is the state and the data of the hybrid system, denoted (C, f, D, G), is defined as follows:

- $f : \mathbb{R}^n \to \mathbb{R}^n$ is a single-valued map defining the flow map capturing the continuous dynamics;
- $C \subset \mathbb{R}^n$ defines the flow set on which f is effective;
- $G : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ is a set-valued map defining the jump map and models the discrete behavior;
- $D \subset \mathbb{R}^n$ defines the jump set, which is the set of points from where jumps are allowed.

Solutions ϕ to $\mathcal{H}$ are parameterized by (t, j), where $t \in \mathbb{R}_{\geq 0} := [0, \infty)$ counts ordinary time and $j \in \mathbb{N} := \{0, 1, 2, \dots\}$ counts the number of jumps. The domain dom $\phi \subset \mathbb{R}_{\geq 0} \times \mathbb{N}$ is a hybrid time domain if for every $(T, J) \in$ dom ϕ, the set dom $\phi \cap ([0, T] \times \{0, 1, \dots, J\})$ can be written as the union of sets $\cup_{j=0}^{J}(I_j \times \{j\})$, where $I_j := [t_j, t_{j+1}]$ for a time sequence $0 = t_0 \leq t_1 \leq t_2 \leq \cdots \leq t_{J+1}$. The t_j's with $j > 0$ define the time instants when the state of the hybrid system jumps

and j counts the number of jumps. A solution to $\mathcal{H}$ is called maximal if it cannot be extended, i.e., it is not a truncated version of another solution. It is called complete if its domain is unbounded. A solution is Zeno if it is complete and its domain is bounded in the t direction. A solution is precompact if it is complete and bounded.

A hybrid system $\mathcal{H}$ with data in (2.1) is said to satisfy the *hybrid basic conditions* if the sets C and D are closed, the function $f : \mathbb{R}^n \to \mathbb{R}^n$ is continuous, and the set-valued mapping $G : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ is outer semicontinuous[1], locally bounded relative to D and $D \subset \operatorname{dom} G$. More information can be found in [8].

Definition 2.2.1. *(uniform global asymptotic stability) Let a hybrid system $\mathcal{H}$ be defined on $\mathbb{R}^n$ and $\mathcal{A} \subset \mathbb{R}^n$ be closed. The set $\mathcal{A}$ is said to be*

- *uniformly globally stable (UGS) for $\mathcal{H}$ if there exists $\alpha \in \mathcal{K}_\infty$ such that any solution ϕ to $\mathcal{H}$ satisfies $|\phi(t,j)|_{\mathcal{A}} \leq \alpha(|\phi(0,0)|_{\mathcal{A}})$ for all $(t,j) \in$ dom ϕ;*
- *uniformly globally attractive (UGA) for $\mathcal{H}$ if for each $\varepsilon > 0$ and $r > 0$ there exists $T > 0$ such that, every maximal solution ϕ to $\mathcal{H}$ is complete and if $|\phi(0,0)|_{\mathcal{A}} \leq r$, $(t,j) \in$ dom ϕ and $t + j \geq T$ then $|\phi(t,j)|_{\mathcal{A}} \leq \varepsilon$;*
- *uniformly globally asymptotically stable (UGAS) for $\mathcal{H}$ if it is both UGS and UGA.*

Sufficient conditions for UGAS of a set $\mathcal{A}$ can be found utilizing a Lyapunov function candidate for hybrid systems defined as follows.

Definition 2.2.2. *A function $V : \operatorname{dom} V \to \mathbb{R}$ is said to be a Lyapunov function candidate for the hybrid system $\mathcal{H}$ in (2.1) if the following conditions hold:*

- $\overline{C} \cup D \cup G(D) \subset \operatorname{dom} V$;

[1] A set-valued mapping is outer semicontinuous if its graph G is closed, where the graph of G is defined as $\{(x,y) : y \in G(x)\}$.

- *V is continuously differentiable on an open set containing $\overline{C}$;*

where $\overline{C}$ denotes the closure of C.

Theorem 2.2.3. *Let $\mathcal{H}$ be a hybrid system and let $\mathcal{A} \subset \mathbb{R}^n$ be closed. Suppose that V is a Lyapunov function candidate for $\mathcal{H}$ and there exist α_1, $\alpha_2 \in \mathcal{K}_\infty$, and a continuous $\rho \in \mathcal{PD}$*

$$\alpha_1(|x|_{\mathcal{A}}) \leq V(x) \leq \alpha_2(|x|_{\mathcal{A}}) \quad \forall x \in C \cup D \cup G(D) \tag{2.2a}$$

$$\langle \nabla V(x), f(x) \rangle \leq 0 \quad \forall x \in C \tag{2.2b}$$

$$V(g) - V(x) \leq -\rho(|x|_{\mathcal{A}}) \quad \forall x \in D, g \in G(x) \tag{2.2c}$$

If, for each $r > 0$, there exists $\gamma_r \in \mathcal{K}_\infty$, $N_r \geq 0$ such that for every solution ϕ to $\mathcal{H}$, $|\phi(0,0)|_{\mathcal{A}} \in (0, r], (t, j) \in \operatorname{dom} \phi$, $t + j \geq T$ imply $j \geq \gamma_r(T) - N_r$, then $\mathcal{A}$ is uniformly globally pre-asymptotically stable.

Chapter 3

Motivational Example

Consider a continuous linear time-invariant system with state $z \in \mathbb{R}^n$ and input $u \in \mathbb{R}^m$ given by

$$\dot{z} = Az + Bu \tag{3.1}$$

where A and B are matrices of appropriate dimension. We consider the case when the output of the plant

$$y = Hz \in \mathbb{R}^p$$

is available for the purposes of control at isolated time instances. More precisely, the output y is available to the controller when $t \in \{t_s\}_{s=1}^{\infty}$, where the sequence of times $\{t_s\}_{s=1}^{\infty}$ satisfies

$$\begin{gathered} T_1 \leq t_{s+1} - t_s \leq T_2 \quad \forall s \in \mathbb{N} \setminus \{0\} \\ t_1 \leq T_2 \end{gathered} \tag{3.2}$$

with T_1 and T_2 such that $0 < T_1 \leq T_2$. The parameter T_1 denotes the minimum time for samples while T_2 denotes the maximum time in between samples, which is known in the literature as the maximum allowable transfer interval (MATI); see,

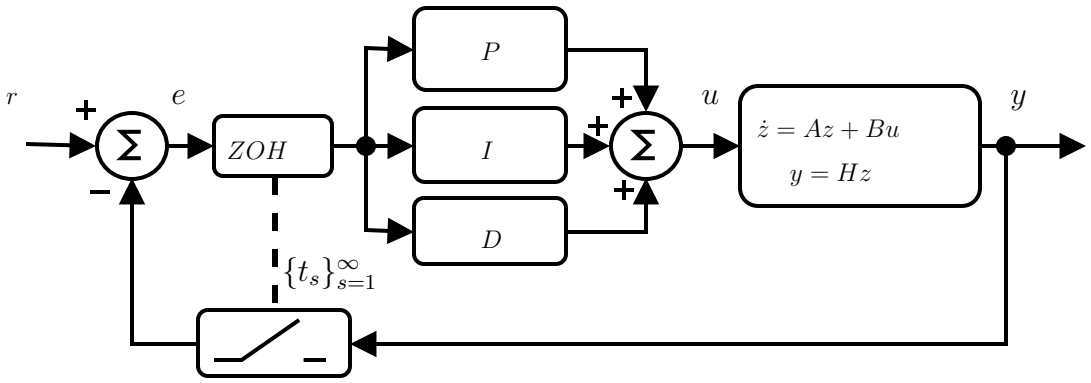

Figure 3.1: Given matrices A and B of appropriate dimension describing the plant dynamics with state z and output y, a control signal u is generated as a sum of the proportional $-K_P e$, integral $-K_I \int_0^t e(s)ds$, and derivative $-K_D \dot{e}$ feedback terms. The samples of the output of the plant and the update of the control signals occur at discrete times given by the sequence $\{t_s\}_{s=1}^{\infty}$.

e.g., [2]. Figure 3.1 depicts a feedback closed-loop system using a PID controller where the output is available at times given by the sequence of points $\{t_s\}_{s=1}^{\infty}$ as indicated by the switch. Note that the closed-loop system includes a reference signal r to be tracked.

To illustrate the effect of the measurements of the output not being available continuously, consider a mass-spring system under the effect of viscous friction with unitary friction coefficient where only position can be measured. Therefore, the state $z = (z_1, z_2) \in \mathbb{R} \times \mathbb{R}$ is given by the position z_1 and velocity z_2 of the mass, respectively. Namely, the system in (3.1) has matrices

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{3.3}$$

Suppose the goal is to design a PID controller such that the rise time $t_{rise} \leq 0.2$ seconds, the settling time $t_{settling} \leq 2$ seconds, and the overshoot $M_p \leq 15\%$. When the output y is available continuously, the gains $K_P = 250$, $K_I = 350$, and $K_D = 30$ generate a closed-loop system satisfying the given specifications.

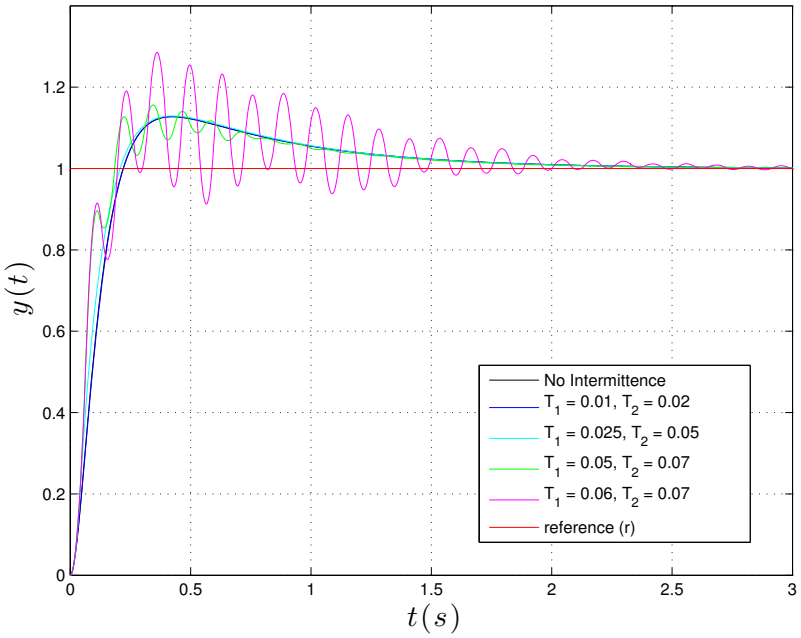

Figure 3.2: Trajectories of a mass-spring system tracking a reference signal $r = 1$ (red). The black trajectory was generated when the closed-loop system had continuous sensor measurements. The remaining trajectories illustrate the degradation of the closed-loop system when sensor measurements arrive sporadically.

The solution in black in Figure 3.2 shows the response of the system with such feedback. However, when using the same feedback gains in the PID controller when the output is measured intermittently at times satisfying (3.2) leads to a degradation of performance of the closed-loop system. In between measurements of the output, the control signal is held constant[1]. Figure 3.2 shows the solutions to the system for the above PID gains with a sample and hold feedback scheme for increasing values of T_1 and T_2. Note that, even for small parameters $T_1 = 0.06$ seconds and $T_2 = 0.07$ seconds (shown in magenta in Figure 3.2), the overshoot increases by 50% compared to the continuous feedback case, and the settling time is well beyond specifications as oscillations are still present beyond 3 seconds. If T_1 and T_2 are large enough, then there is no guarantee the solutions will converge at all.

One example of this architecture being used in practice can be seen directly in the Hybrid Systems Lab where experiments are done using a motion capture system to obtain output measurements. In such a system communication protocols for the motion capture system impose values on T_1 and T_2.

[1]Details about integral and derivative action implementations between output measurements is explained in Chapter 4 and then further explored in Chapter 7.

Chapter 4

Hybrid Modeling

In this section, we present a modeling approach of the PID controller in (1.1) where the measurements occur at times given by (3.2). Due to the continuous dynamics of the plant in (3.1), the intermittent sensor measurements communicating at times given by (3.2), and the control law in (1.1) (yet to be designed), the system naturally has both continuous and discrete behaviors. Therefore, we model the closed-loop systems using the hybrid systems framework presented in [8]. In this paper, for simplicity, we will consider the case when the reference signal is zero, but the results and ideas can be extended to the case when the reference is generated by an exosystem; e.g., as in [15].

The output of the plant is measured at impulsive times satisfying (3.2). To generate all possible such sequences, we define a timer state, denoted by $\tau \in [0, T_2]$, which decreases continuously in ordinary time and, when it reaches zero, it jumps impulsively to a point in the interval $[T_1, T_2]$. The timer can be modeled as an autonomous hybrid inclusion given by

$$\begin{cases} \dot{\tau} & = -1 & \tau \in [0, T_2] \\ \tau^+ & \in [T_1, T_2] & \tau = 0 \end{cases} \tag{4.1}$$

Such a timer defines a hybrid system where the jump times t_j, for all j such that (t_j, j) is in the domain of solutions of the timer system, satisfy (3.2); for more details on the use of such timers, [5, 13].

Next, we introduce each component of the PID controller which will be assigned to the input u in (3.1). Namely, the PID controller has three components: the proportional component, v_P; the integral component, v_I; and derivative component, v_D. Then, with a slight abuse of notation, we denote the output of the controller by the state u, which evolves according to zero-order hold dynamics. Namely, during the intervals of time between successive measurement updates, u is held constant, and, when the controller receives a new measurement, we update it with the components of the controller. Namely, the controller is given by the following dynamics:

$$\begin{cases} \dot{u} \ = 0 & \tau \in [0, T_2] \\ u^+ = v_P + v_I + v_D & \tau = 0 \end{cases} \tag{4.2}$$

where v_P, v_I, and v_D will be defined explicitly below.

Following the construction in (1.1), the contribution of the proportional component v_P of the PID is proportional to the measurement received. It follows that, at jumps, the component v_P is given by $v_P = -K_P y = -K_P H z$.

In classical state-space control design, an integral controller requires the introduction of an auxiliary state which 'memorizes' the evolution of the error between the state and reference [6, 7]. More specifically, in the continuous-time case, the integral component in (1.1) is the integral of the error over time. For the case of intermittent measurements, we introduce two states: a memory state m_s and an integral state z_I. We use $z_I \in \mathbb{R}^p$ as the state storing an approximation of the running total integral. The memory state $m_s \in \mathbb{R}^p$ is used to store the most recent

measurement of the output. The memory state is updated when a new output measurement is available, which is when the timer τ is equal to zero. Between sensor measurements, the integral state z_I evolves according to $\dot{z}_I = m_s$ while the memory state m_s remains constant. The integral control law is then implemented as

$$v_I = -K_I z_I \tag{4.3}$$

To implement the derivative action v_D, first, consider the case when only the derivative term in (1.1) is present. This leads to

$$\begin{aligned} v_D &= -K_D \dot{y} \\ &= -K_D H(Az + Bv_D) \\ \Rightarrow v_D &= -(I + K_D HB)^{-1} K_D HAz \end{aligned} \tag{4.4}$$

where we assume that $I + K_D HB$ is invertible. With this expression, the proportional and integral actions can be incorporated too. It follows that

$$v_D = -(I + K_D HB)^{-1} K_D H(Az - BK_P Hz - BK_I z_I) \tag{4.5}$$

To write the resulting hybrid closed-loop system combining the three control actions, we define the state of the hybrid system $\mathcal{H}$ as $x = (x_1, x_2)$, where $x_1 = (z, z_I, u, m_s)$ and $x_2 = \tau$. The resulting closed-loop system with plant as in (3.1), PID controller output in (4.2), and timer in (4.1) has data (C, f, D, G) given by

$$\begin{aligned} f(x) &:= \begin{bmatrix} A_f x_1 \\ -1 \end{bmatrix} \qquad \forall x \in C := \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^m \times \mathbb{R}^p \times [0, T_2] \\ G(x) &:= \begin{bmatrix} A_g x_1 \\ [T_1, T_2] \end{bmatrix} \qquad \forall x \in D := \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^m \times \mathbb{R}^p \times \{0\}\,. \end{aligned} \tag{4.6}$$

The matrices A_f and A_g are given by

$$A_f = \begin{bmatrix} A & 0 & B & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A_g = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ -\tilde{K}_P - \tilde{K}_D & -\tilde{K}_I & 0 & 0 \\ H & 0 & 0 & 0 \end{bmatrix} \tag{4.7}$$

where

$$\begin{aligned} \tilde{K}_P &= K_P H - (I + K_D HB)^{-1} K_D HBK_P H \\ \tilde{K}_I &= K_I - (I + K_D HB)^{-1} K_D HBK_I \\ \tilde{K}_D &= (I + K_D HB)^{-1} K_D HA \end{aligned} \tag{4.8}$$

Note that the definitions of $\tilde{K}_P$, $\tilde{K}_I$, and $\tilde{K}_D$ depend on (K_P, K_D), (K_I, K_D), and K_D respectively[1]. We will treat $\tilde{K}_P$, $\tilde{K}_I$, and $\tilde{K}_D$ as our design parameters.

Given the hybrid closed-loop system in (4.6), and parameters $0 < T_1 \leq T_2$, our goal is to design the parameters of the PID controller such that the compact set

$$\begin{aligned} \mathcal{A} = \{(z, z_I, u, m_s, \tau) \in \mathbb{R}^{n+p+m+p} \times [0, T_2] : \\ z = z_I = u = m_s = 0, \tau \in [0, T_2]\} \end{aligned} \tag{4.9}$$

is uniformly globally asymptotically stable. Next, we showcase three special cases of the PID controller which simplify it and find wide application.

4.1 Proportional Control Case

In the case when the control law only contains the proportional component, the states z_I and m_s in (4.6) can be removed since there is no need to approximate integration. The state $x = (x_1, x_2)$ of the closed-loop system is $x_1 = (z, u)$ and

[1]For the scalar case, the expressions for $\tilde{K}_P$ and $\tilde{K}_I$ are so that K_P and K_I can be chosen to yield desired values of $\tilde{K}_P$ and $\tilde{K}_I$, even though K_D plays a role in their definition.

$x_2 = \tau$. The flow map, flow set, jump map, and jump set are still given as in (4.6) but with obvious changes on dimensions.

The matrices in (4.7) reduce to

$$A_f = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}, \quad A_g = \begin{bmatrix} I & 0 \\ -\tilde{K}_P & 0 \end{bmatrix} \tag{4.10}$$

with $\tilde{K}_P = K_P H$. In this case, the desired set to stabilize is

$$\mathcal{A} = \left\{ (z, u, \tau) \in \mathbb{R}^{n+m+1} : z = u = 0, \tau \in [0, T_2] \right\} \tag{4.11}$$

4.2 Proportional-Integral Control Case

The model in (4.6) for only proportional-integral (PI) control still requires the memory states m_s and z_I used to approximate integration between sampling events. The state $x = (x_1, x_2)$ of the closed-loop system has $x_1 = (z, z_I, u, m_s)$ and $x_2 = \tau$. Definitions of A_f and A_g follow directly from (4.7) with the derivative gain $K_D = 0$, resulting in

$$A_f = \begin{bmatrix} A & 0 & B & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A_g = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ -\tilde{K}_P & -\tilde{K}_I & 0 & 0 \\ H & 0 & 0 & 0 \end{bmatrix} \tag{4.12}$$

with $\tilde{K}_P = K_P H$ and $\tilde{K}_I = K_I$.

The flow map, flow set, jump map, and jump set are still given as in (4.6) and the set to stabilize remains as in (4.9).

4.3 Proportional-Derivative Control Case

In the case of proportional-derivative (PD) only control, the state $x = (x_1, x_2)$ of the model (4.6) simplifies to $x_1 = (z, u)$ and $x_2 = \tau$, as the integration states z_I and m_s are no longer needed. The matrices A_f and A_g reduce to

$$A_f = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}, A_g = \begin{bmatrix} I & 0 \\ -\tilde{K}_P - \tilde{K}_D & 0 \end{bmatrix} \tag{4.13}$$

where the gains $\tilde{K}_P$ and $\tilde{K}_D$ are defined in (4.8), while the definitions of the data of (4.6) remain the same, modulo changes of dimensions. The set to stabilize is given in (4.11).

In the next section, we provide tools for the design of the gains in (4.8).

Chapter 5

Results

5.1 Stability Results

The following result gives sufficient conditions for uniform global asymptotic stability of the set $\mathcal{A}$ in (4.9) for the hybrid system in (4.6) in terms of matrix inequalities. The result hold for matrices A_f and A_g in (4.7) but also apply to the special cases in Chapters 4.A, 4.B, and 4.C after obvious changes in dimensions. Recall that uniform global asymptotic stability is defined in Definition 2.2.1.

Following [5], we establish UGAS of $\mathcal{A}$ using a Lyapunov based analysis. Namely, we consider the following Lyapunov function candidate:

$$V(x) = W(\exp(A_f \tau) x_1) \tag{5.1}$$

where $W(s) = s^\top P s$, and P is a symmetric positive definite matrix. Note that (5.1) is a Lyapunov function candidate according to Definition 2.2.2, namely, that V is continuously differentiable everywhere and the set $\overline{C} \cup D \cup G(D)$ is contained in the domain of V.

Theorem 5.1.1. *Let T_1 and T_2 be positive scalars such that $T_1 \leq T_2$. Suppose*

there exist matrices $\tilde{K}_P$, $\tilde{K}_I$, and $\tilde{K}_D$, and a positive definite symmetric matrix P satisfying

$$\Gamma(\nu)^\top P\Gamma(\nu) - P < 0 \qquad \forall \nu \in [T_1, T_2] \tag{5.2}$$

where $\Gamma(\nu) = \exp(A_f\nu)A_g$, and the matrices A_f and A_g are given in (4.7)*. Then, the set $\mathcal{A}$ in* (4.9) *is uniformly globally asymptotically stable for the hybrid system $\mathcal{H}$ with data as in* (4.6)*.*

Proof. First, note that, by construction, the intervals of flow time between jumps is upper bounded by T_2 which implies that

$$t \leq (j+1)T_2. \tag{5.3}$$

where $T_2 > 0$. Consider the function $W(s) = s^\top Ps$ for each $s \in \mathbb{R}^n$ where $P = P^\top > 0$. In light of continuity of (5.2), we have that there must exist an arbitrarily small scalar $\epsilon > 0$ satisfying

$$\begin{aligned} W(\Gamma(\nu)x_1) - W(x_1) &= x_1^\top(\Gamma(\nu)^\top P\Gamma(\nu) - P)x_1 \\ &\leq -\epsilon\,|x_1|^2 \end{aligned}$$

for each $\nu \in [T_1, T_2]$.

Due to the construction of the set $\mathcal{A}$ in (4.9), it follows that the distance of the state x to the set $\mathcal{A}$ satisfies $|x|_{\mathcal{A}}^2 = |x_1|^2$. Now, consider the Lyapunov function candidate in (5.1). It follows that this candidate satisfies the conditions in Definition 2.2.2. Moreover, V is bounded by

$$\underline{c}\,|x|_{\mathcal{A}}^2 \leq V(x) \leq \bar{c}\,|x|_{\mathcal{A}}^2 \quad \forall x \in C \cup D \cup G(D) \tag{5.4}$$

where

$$\begin{aligned}\overline{c} &= \max_{\tau\in[0,T_2]} \lambda_{\max}(\exp(A_f^\top \tau) P \exp(A_f \tau)) \\ \underline{c} &= \min_{\tau\in[0,T_2]} \lambda_{\min}(\exp(A_f^\top \tau) P \exp(A_f \tau))\end{aligned} \tag{5.5}$$

Next, we consider the change in V during flows. Namely, for each $x \in C$, the change in V is given by,

$$\langle \nabla V(x), f(x) \rangle = 0$$

where we used the property that transition matrices commute, i.e., $A_f \exp(A_f\tau) = \exp(A_f\tau)A_f$.

Next, we consider the case of jumps, namely, for each $x \in D$. For such cases, we have that the timer is such that $\tau = 0$, which triggers an update in the state. We have that the timer state τ is updated to a point in the interval $[T_1, T_2]$ and the x_1 state is updated as $A_g x_1$. Then, for such points $x \in D$, $g = (g_{x_1}, \nu) \in G(x)$, the change in V is given by

$$\begin{aligned}V(g) - V(x) &= W(\Gamma(\nu)x_1) - W(x_1) \\ &\leq -\epsilon |x_1|^2 = -\epsilon \, |x|_{\mathcal{A}}^2 \,.\end{aligned}$$

where $\nu \in [T_1, T_2]$. Note that all maximal solutions are complete (such a property follows from Proposition 6.10 in [8]), and, in light of (5.3), for every solution satisfying $t + j \geq T$ the inequality (5.3) yields $j \geq \frac{T}{T_2+1} - \frac{T_2}{T_2+1}$. It follows from Theorem 2.2.3 with $\gamma_r(T) = \frac{T}{T_2+1}$ and $N_r = \frac{T_2}{T_2+1}$ therein, the set $\mathcal{A}$ is uniformly globally asymptotically stable for the hybrid system defined in (4.6). □

Remark 5.1.2. *Numerically, the matrix P can be difficult to find. We will use*

the convex optimization solver CVX, see [10], to solve for P under the constraints that P must be positive definite symmetric matrix and the matrix inequality (5.2) *holds. If the gains of the PID controller are chosen a priori then* (5.2) *reduces to a linear matrix inequality, however when the gains are being designed,* (5.2) *contains nonlinear terms and may be difficult to solve. Next, we will provide a systematic approach to overcoming this problem by linearizing the nonlinear matrix inequality using a polytopic embedding approach as in [5] to solve for the controller gains, the matrix P.*

Remark 5.1.3. *While the set $\mathcal{A}$ in* (4.9) *is compact with u and z_I equal to zero, the conditions in Theorem 5.1.1 extend to the tracking case in which case u and z_I converge to their steady states.*

Following Proposition 1 in [5], we use the projection lemma and Schur's complement to show an equivalent form for equation (5.2).

Theorem 5.1.4. *Let T_1 and T_2 be positive scalars such that $T_1 \leq T_2$. Given the matrices A, B, and H defining the plant dynamics and output, the matrices A_f and A_g in* (4.7)*, the matrix P and function Γ satisfy* (5.2) *if and only if there exists a matrix $F \in \mathbb{R}^{n\times n}$ such that for every $\nu \in [T_1, T_2]$*

$$\begin{bmatrix} -(F+F^\top) & FA_g & \exp(A_f^\top \nu)P \\ * & -P & 0 \\ * & * & -P \end{bmatrix} < 0 \tag{5.6}$$

Proof. First, let us set

$$Z(\nu) = \begin{bmatrix} \exp(A_f^\top \nu) P \exp(A_f \nu) & 0 \\ 0 & -P \end{bmatrix}$$
$$S = \begin{bmatrix} A_g \\ I \end{bmatrix} \qquad Y = \begin{bmatrix} 0 \\ I \end{bmatrix} \tag{5.7}$$

Then, condition (5.2) can be rewritten as

$$S^\top Z(\nu) S < 0 \qquad \forall \nu \in [T_1, T_2] \tag{5.8}$$

The positive definiteness of P can be equivalently expressed as

$$Y^\top Z(\nu) Y < 0 \qquad \forall \nu \in [T_1, T_2] \tag{5.9}$$

Now, using the projection lemma [14], inequalities (5.8) and (5.9) hold true if and only if there exists a matrix F such that

$$\begin{bmatrix} \exp(A_f^\top \nu) P \exp(A_f \nu) - (F + F^\top) & F A_g \\ * & -P \end{bmatrix} < 0 \tag{5.10}$$

for all $\nu \in [T_1, T_2]$. By Schur's complement [19], one can obtain

$$\begin{bmatrix} -(F + F^\top) & F A_g & \exp(A_f^\top \nu) \\ * & -P & 0 \\ * & * & -P^{-1} \end{bmatrix} < 0 \tag{5.11}$$

By pre-and-post multiplying by

$$\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & P \end{bmatrix} \tag{5.12}$$

we obtain the inequality in (5.6). □

Theorem 5.1.4 gives an equivalent form of (5.2) that is linear with respect to P, F, and A_g. However, this condition still needs to be checked for infinitely many values of $\nu \in [T_1, T_2]$. One method of dealing with this is to embed $\exp(A_f\nu)$ into the interval $[T_1, T_2]$. That is, find matrices $\{X_1, X_2, \ldots, X_w\}$ such that $\exp(A_f\nu) \subset \text{co}\{X_1, X_2, \ldots, X_w\}$ when $\nu \in [T_1, T_2]$. Choosing w gives a finite number of inequalities that imply condition (5.6).

Corollary 5.1.5. *Let T_1 and T_2 be positive scalars such that $T_1 \leq T_2$. Let matrices $\{X_1, X_2, \ldots, X_w\}$ be given satisfying*

$$\exp(A_f[T_1, T_2]) \subset co\{X_1, X_2, \ldots, X_w\}.$$

If there exist matrices J and F, and a positive definite symmetric matrix P such that, for each $i \in \{1, 2, \ldots, w\}$

$$\begin{bmatrix} -(F + F^\top) & J & X_iP \\ \star & -P & 0 \\ \star & \star & -P \end{bmatrix} < 0 \tag{5.13}$$

where

$$J = \begin{bmatrix} J_{11} & J_{12} & 0 & 0 \\ J_{21} & J_{22} & 0 & 0 \\ J_{31} & J_{32} & 0 & 0 \\ J_{41} & J_{42} & 0 & 0 \end{bmatrix} \tag{5.14}$$

such that

$$\begin{bmatrix} F_{11} - F_{13}K_{PD} + F_{14}C & F_{12} - F_{13}\tilde{K}_I \\ F_{21} - F_{23}K_{PD} + F_{24}C & F_{22} + F_{23}\tilde{K}_I \\ F_{31} - F_{33}K_{PD} + F_{34}C & F_{32} - F_{33}\tilde{K}_I \\ F_{41} - F_{43}K_{PD} + F_{44}C & F_{42} - F_{43}\tilde{K}_I \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \\ J_{31} & J_{32} \\ J_{41} & J_{42} \end{bmatrix} \tag{5.15}$$

where $K_{PD} = \tilde{K}_P + \tilde{K}_D$ *then the matrices* P *and* $FA_g = J$ *satisfy condition* (5.2)[1].

Corollary 5.1.5 allows us to linearize (5.2) when P, $\tilde{K}_P$, $\tilde{K}_I$, and $\tilde{K}_D$ are unknown. In doing so, we are able to find the value of A_g that satisfies the stability criteria.

[1]Note that there are multiple options for constraining F and J according to (5.15), for instance consider $F_{23} = F_{33} = F_{43} = 0$ and $F_{13} = I$, then, $\tilde{K}_{DP} = J_{11} - F_{11} - F_{14}C$ and $\tilde{K}_I = J_{12} - F_{12}$.

Chapter 6

Numerical Examples

We present numerical examples for the proposed controller designs. There are simulated using the Hybrid Equations (HyEQ) Toolbox in Matlab [17][1].

Example 6.0.1. *Consider the mass-spring damper system with matrices in* (3.3) *and state* $x = (x_1, x_2)$ *where* $x_1 = (z, z_I, u, m_s)$ *and* $x_2 = \tau$. *We choose proper values of* $\tilde{K}_P$, $\tilde{K}_I$, *and* $\tilde{K_D}$ *and show a* P *matrix satisfying* (5.2). *This applies directly to the results in Theorem 5.1.1. Let* $\tilde{K}_P = 10$, $\tilde{K}_I = 4$, *and* $\tilde{K}_D = 4$, *and define matrices* A_f *and* A_g *as in* (4.7). *The time bounds* T_1 *and* T_2 *are chosen as* $T_1 = 0.1$ *seconds and* $T_2 = 0.25$ *seconds. Using CVX [10], we solve for* P *while enforcing the condition in* (5.2) *and that* P *needs to be symmetric positive definite matrix. The state response of the closed-loop system is shown in Figure 6.1. Under intermittent output measurements, we are able to guarantee UGAS. Figure 6.1 also shows the control input to the system over time where the value of the control signal* u *is held constant between output measurements. Simulation results validate the stability results presented in this paper.*

Using the convex optimization solver CVX [10], the matrix P *is found such*

[1]All MATLAB code for simulations presented in this paper are maintained in a GitHub repository at `https://github.com/HybridSystemsLab/HybridPID.git`.

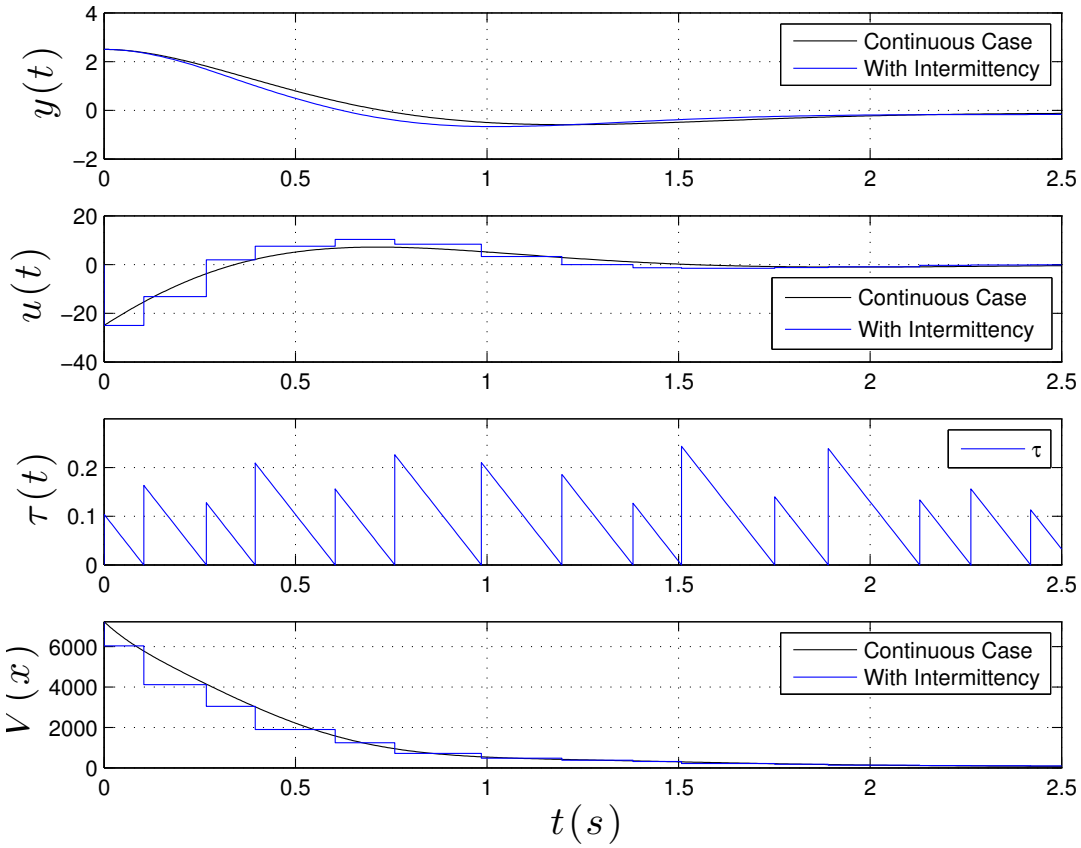

Figure 6.1: The position of the mass from a mass-spring-damper system when using a PID controller and for a given reference signal $r = 0$ is shown in the first plot. The control signal generated from the PID controller with a sample and hold mechanism in the feedback loop is depicted in the second graph. The timer triggering the sporadic events is seen in the third plot. The Lyapunov function along a solution is shown in the bottom plot.

that (5.2) *is satisfied and* P *is a positive-definite symmetric matrix. Using the* P *matrix found, the Lyapunov function in* (5.1) *can be evaluated along solutions to the system.*

Example 6.0.2. *Consider designing a PI controller as in Chapter 4.B for the mass-spring system with matrices in* (3.3) *and now the ability to observe both position* z_1 *and velocity* z_2 *when the output is measured, that is now* $H = \begin{bmatrix} 1 & 1 \end{bmatrix}$. *Given a constant reference signal, a PI controller should have the steady state*

error $e_{ss} = 0$. *We choose appropriate values of* $\tilde{K}_P$ *and* $\tilde{K}_I$ *to show that with sporadic output measurements triggered at times satisfying* (3.2)*, the steady state error of the closed-loop system with PI control is zero.*

Choose $\tilde{K}_P = 2$ *and* $\tilde{K}_I = 1$ *and define* A_f *and* A_g *as in* (4.12). *Figure 6.2 compares the state response and associated input signal for the system given a unit step input* $r = 1$. *The initial state of the system is* $(z_1, z_2) = (0, 0)$ *and the time bounds are chosen as* $T_1 = 0.4$ *and* $T_2 = 0.8$. *With continuous output measurements, the steady state error* $e_{ss} = 0$ *and the closed-loop system has settling time* $t_{settling} = 6.5$ *seconds. When the output of the system is measured sporadically, the steady state error* e_{ss} *remains zero, however, the closed-loop system has a longer settling time of* $t_{settling} = 7.2$ *seconds.*

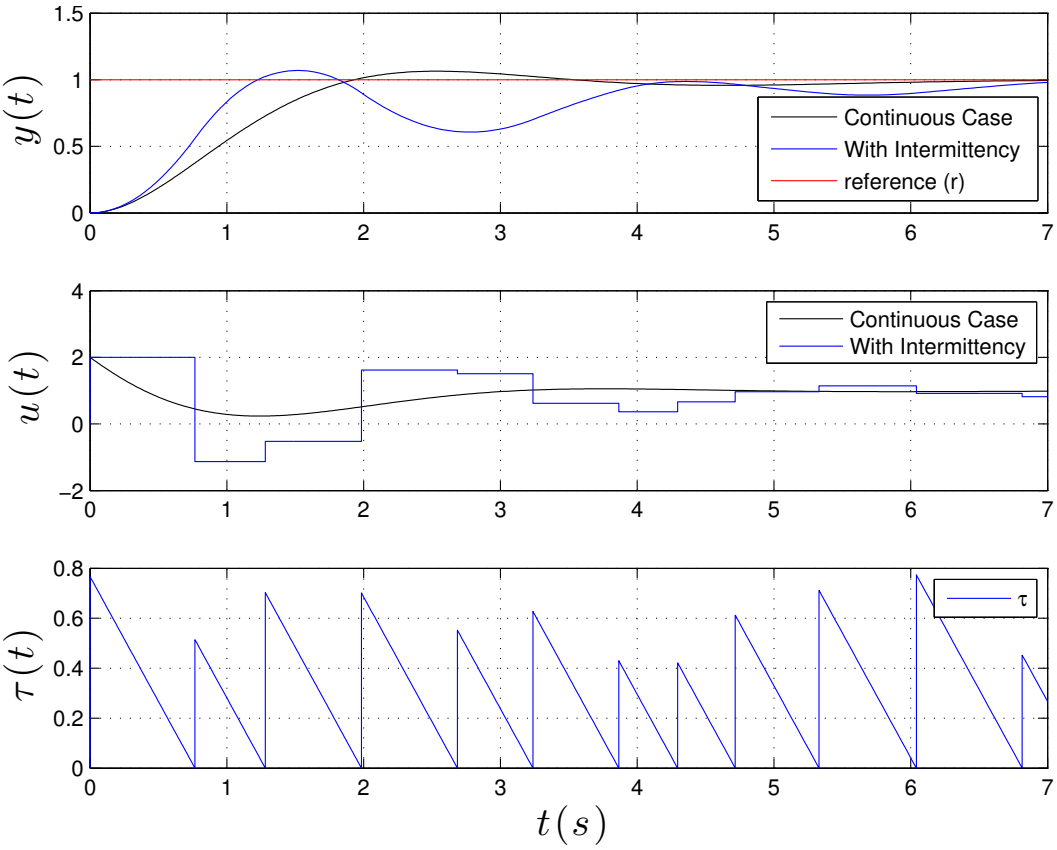

Figure 6.2: The closed-loop state response for PI controller given constant reference signal $r = 1$ in the top plot shows solutions for the mass-spring system when outputs are measured continuously (black) and sporadically (blue). The middle plot illustrates the associated control signals generated by the PI controller in both cases. The bottom plot shows values of the timer τ used to trigger the sporadic output measurements.

Chapter 7

Discussion

In this chapter we discuss some key alterations to the aforementioned design techniques. Specifically we discuss alternative forms of estimating the integral and derivative terms focused more on sampled approximations. Additionally, we briefly present two more forms of hybridity in a PID controller. That is, allowing the controller to change gains at jumps, and when the integral error is reset at zero-crossings.

7.1 Sampled Integral and Derivative Implementation

In Chapter 5, we focused on a modeling approach that approximates the integral and derivate terms using continuous-time dynamics. These terms can also be approximated discretely. To do so, we introduce a timer $\tau_I \in [0, T_2]$ with continuous and discrete dynamics

$$\dot{\tau}_I = 1, \qquad \tau_I^+ = 0 \tag{7.1}$$

respectively, as a means of recording the time between successive jumps. Using the memory state m_s and the new timer τ_I, it follows that the integration action can be approximated on jumps as $z_I^+ = z_I + m_s\tau_I$. Then, as in (4.3), the contribution of the integration to the control law is given by $v_I = -K_I z_I$.

Similarly, the derivative action can be approximated by, for instance, using the backward Euler integration method. The derivative component is approximated as

$$v_D = -K_D H\left(\frac{z - m_s}{\tau_I}\right) \tag{7.2}$$

The resulting closed-loop system with $x = (x_1, x_2)$ where $x_1 = (z, z_I, u, m_s)$ and $x_2 = (\tau, \tau_I)$ is given as

$$\begin{aligned} f(x) &:= \begin{bmatrix} A_f x_1 \\ -1 \\ 1 \end{bmatrix} && \forall x \in C \\ G(x) &:= \begin{bmatrix} A_g x_1 + B_g \frac{x_1}{\max\{\epsilon, \tau_I\}} \\ [T_1, T_2] \\ 0 \end{bmatrix} && \forall x \in D \end{aligned} \tag{7.3}$$

where $\epsilon > 0$ is a small parameter preventing division by zero, and from the conditions above

$$\begin{aligned} C &:= \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^m \times \mathbb{R}^p \times [0, T_2] \times [0, T_2] \\ D &:= \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^m \times \mathbb{R}^p \times \{0\} \times [0, T_2] \end{aligned} \tag{7.4}$$

define the flow set and jump set. The matrices A_f and A_g are given by

$$A_f = \begin{bmatrix} A & 0 & B & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_g = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -K_D H & 0 & 0 & K_D H \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$A_g = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & \tau_I \\ -K_P & -K_I & 0 & 0 \\ H & 0 & 0 & 0 \end{bmatrix} \tag{7.5}$$

where the gains K_P and K_I no longer have a dependency with the derivative gain K_D. This method presents difficulties in analytically proving convergence due to the nonlinear mappings. However, it can be seen in simulation that the PID feedback gains can be found such that the closed-loop system converges.

7.2 Dynamic Gains

Aside from intermittent sensor measurements, other forms of hybridity may be added to the closed-loop system (4.6). To improve performance, the gains of the PID controller could be updated at certain events. Continuing with the mass-spring system with matrices in (3.3), we have one set of gains $K_P = 250$, $K_I = 350$, and $K_D = 30$ which under sufficiently small intermittent sensor measurements responds to a unit step input seen in Figure 3.2. This response has relatively good overshoot compared to the rise time and settling time. To improve the rise time and settling time of this response, consider using a more aggressive set of gains when the state of the system is far from the reference signal. Choosing

$K_P = 300$, $K_I = 200$, and $K_D = 10$ results in a response with $t_{rise} \leq 0.1$ seconds, settling time $t_{settling} = 2.45$ seconds, and overshoot $M_p = 75\%$, shown in Figure 7.1. This set of feedback gains leads to oscillations not seen in the first set. Here we show that a switch can be made between the sets of feedback gains to improve performance of the closed-loop system.

To ensure the switch between PID controller gains is made at the appropriate time, add a logic state $q \in \{0, 1\}$ and hysteresis parameters $\epsilon_1 > \epsilon_0 > 0$. Denote $K_0 = \begin{bmatrix} 300 & 200 & 10 \end{bmatrix}$ and $K_1 = \begin{bmatrix} 250 & 350 & 30 \end{bmatrix}$ as the PID gains associated to the possible values of q. The logic supervising chooses gains K_0 when $q = 0$ and K_1 when $q = 1$. The switches of q are triggered according to the error $e = y - r$ and the hysteresis bands defined by ϵ_0 and ϵ_1: when $|e| \leq \epsilon_0$ and $q = 0$, switch to $q = 1$; when $|e| \geq \epsilon_1$ and $q = 1$, switch to $q = 0$. Using this additional mechanism, the performance of the system is improved dramatically by maintaining the faster rise time of the controller with gains K_0 while mitigating overshoot and settling time with the controller with gains K_1.

7.3 Clegg Integrator/Reset

It can be advantageous to reset the error of the integrator as shown in the [18]. This technique can be applied to the setting discussed in this thesis. Namely, consider applying the properties of the Clegg Integrator to the state z_I storing the running total integral value. That is, when the error being integrated changes sign, reset z_I to zero.

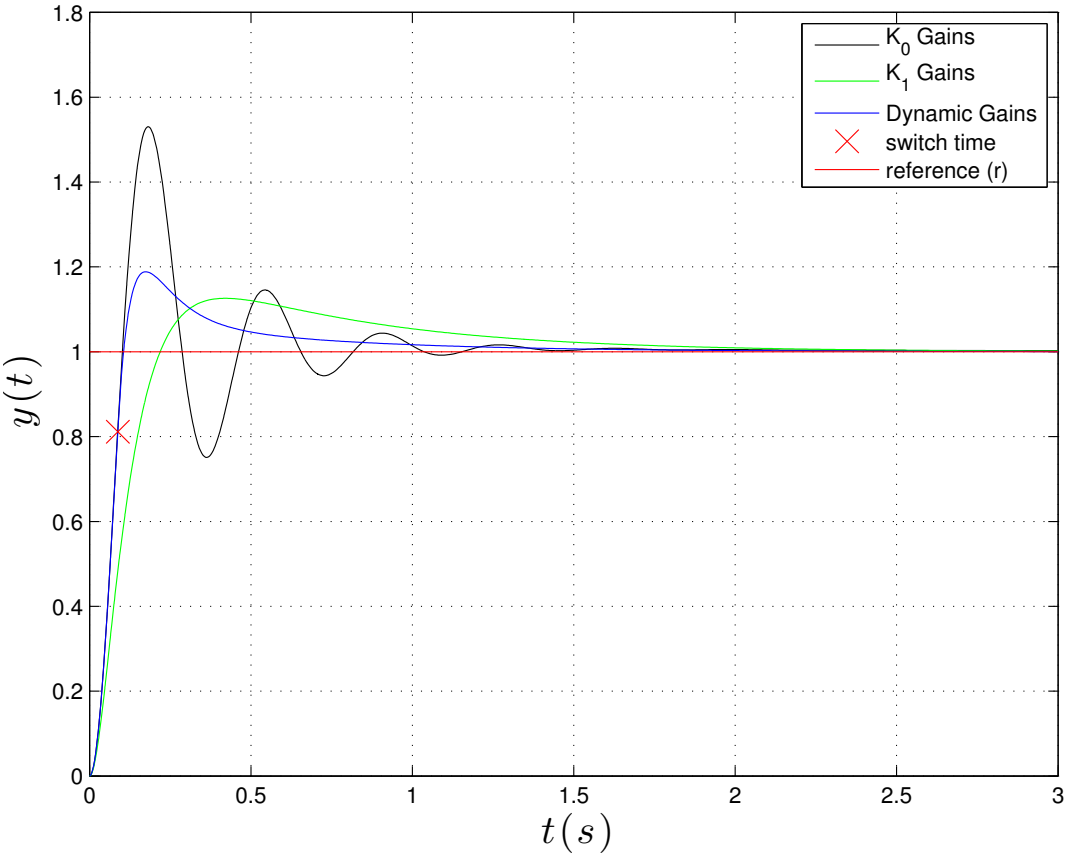

Figure 7.1: Using a supervisory controller, the performance of the closed-loop system can be improved if the gains of the PID controller are switched at the appropriate time. The trajectory in red was generated using PID gains K_0 while the trajectory in black was generated using PID gains K_1. The trajectory in blue was generated by switching between gains K_0 and K_1 with hysteresis band values $\epsilon_0 = 0.2$ and $\epsilon_1 = 0.5$.

Chapter 8

Conclusion

We have shown a systematic approach to designing a PID controller where the measurements occur at intermittent instances. By modeling the closed-loop system using the hybrid inclusion framework, we provided sufficient conditions for uniform global asymptotic stability for the set of interest and give a detailed approach for design following a polytopic embedding approach. We provided numerical examples and simulation using the Hybrid Equations Toolbox. Lastly, we gave a brief discussion about an alternative approach to modeling PID controllers and the potential of utilizing dynamic gains using a hybrid systems approach. Future work for this research is to investigate dynamic gain scheduling to maximize convergence rate while minimizing overshoot. An interesting problem arises when performance specifications also change with time. This research could provide foundation to answering those questions.

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