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Developing a method using graph theory to quantify student conversations in groups

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Developing a method using graph theory to quantify student conversations in groups

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by

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EPIGRAPH

Education is a social process; education is growth; education is not preparation for life but is life itself.

John Dewey

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ABSTRACT OF THE THESIS

Developing a method using graph theory to quantify student conversations in groups

by

Albert Chai

Master of Science in Biology

University of California San Diego, 2019

Stanley Lo, Chair
James Wilhelm, Co-Chair

Collaborative learning environments are a fundamental basis for evidence-based learning practices to help student learning experiences in STEM. However, current methods for understanding and observing groups are more qualitative than quantitative. This thesis discusses a framework for developing a quantitative method for observing groups using graph theory. The first section describes the theoretical and methodological framework of using graph theory to observe student communication in groups. The second section discusses the development and use of an open source tool in R based on the methodological framework in the first section. The third

section illustrates the use of this quantitative method on a cooperative learning method, the jigsaw classroom.

INTRODUCTION

Collaboration in the classroom has been fundamentally shown to improve student performance, retention, and views on learning (Springer, 1999). Other benefits include improved interpersonal skills (Webb, 1994) and critical thinking and understanding (Bligh, 2000; Webb, 1994). But these benefits are not to be addressed without its disadvantages such as social loafing (Hall, 2012), social exclusion or acceptance (Anderman, 2003; Freeman, 2007), and prior negative experiences towards group work (Forrest, 2003; Hillyard, 2010). While group work Group work on its own is a very dynamic and social process where many different factors can contribute to a student's perception and learning outcome. Examples of these include perception of the utility of group work (Blumenfeld, 1996), number of students in the group (Aggarwal, 2008; Bligh, 2000; Fiechtner, 2016), demographics (Springer, 1999), and personalities (French, 2013).

A fundamental driving point behind this thesis is to identify and develop a quantitative methodology for analyzing student conversations in collaborative group settings. Examining how students converse with one another is crucial to understand what educational interventions are working in the classroom and what needs to be improved for student retention and learning. Typically, conversations are analyzed using discourse analysis defined by Gee, as "the study of language-in-use" (Gee, 2011, p. 8). Discourse analysis is typically used as a way to understand how and why the language is being used in the context that is given (Dunn, 2016) such as student argumentation (Chiu, 2008), classroom identities (Bishop, 2012; Brown, 2005; Kumpulainen, 2017; Wood, 2013), and student dynamics (Ikpeze, 2007; Nystrand, 2003; White, 2003).

Quantitative methods for analyzing group dynamics in educational settings are currently limited. Graph theory was selected as a method of choice to model group dynamics mainly because it allows researchers to better track how students converse with one another. The fundamental basis of graph theory is a set of mathematics that allows one to see how things are related to one another (Scott, 1988; Zweig, 2016). Typically, these graphs are mainly used to observe larger bodies of people, such as the spread of obesity (Christakis, 2007), smoking (Christakis, 2008), happiness (Fowler, 2008) or other types of epidemiological situations (Christakis, 2016). However, not all pieces of information used in traditional graphical settings will apply in smaller educational groups. In the first paper, we strived to identify relevant parameters that can be used in our case. In addition, another benefit for using graph theory is that these graphs can be adapted to hold additional pieces of information such as student demographics. Coupling these graphs with additional information can reveal potential insights about the groups that discourse analysis cannot do on its own. Furthermore, information from discourse analysis can be added to reveal how meaningful an individual contribution really is to the graph. For example, additional of an argument, statement, or question can be coded as a weight into the network for that specific conversation to reveal which students may be engaging in meaningful conversations.

Another one of the goals that came out from this project is to help create an R package that allows researchers to use the parameters specifically that we have identified in the first paper in an easier and streamlined manner. The second paper talks about the basis of the development of the package, functions of the package, and instructions on using the package. R was primarily used as the platform as choice because of its open-source nature, ease to use, and ability of packages used for graphical and statistical analysis (R Core Team, 2018). We based our package on several existing packages including igraph (Csardi, 2006), network (Butts, 2008), and sna (Butts, 2016). Our main goal was to make it easier to streamline the necessary functions without reinventing the wheel nor having to search through all of the necessary existing packages that align with our methodology.

After developing a graph theory framework and tools that can be used to analyze groups, we wanted to observe student behaviors and outcomes in a cooperative learning environment, which is the focus of the third paper in this thesis. The literature states that there are many benefits to cooperative learning, including improved social skills (Johnson, 1998; Manning, 1991), self-esteem (Johnson, 1998; Manning, 1991; Slavin, 1980), and higher academic achievement (Johnson, 1998; Manning, 1991). However, these results can vary based on the type of cooperative learning method, even though all cooperative learning methods have these fundamental benefits: structural interdependence, effective communication, individual accountability, and reflect of one's role in the group (Gilies, 2016; Johnson, 2009; Manning, 1991; Sharan, 2010; Slavin, 1980; Smith, 1996; Tanner, 2003; Watson, 1992). One method that we were interested in observing is the jigsaw technique developed by Elliot Aronson (1979). The jigsaw technique is interesting in its inherent behavioral properties to allow students of various backgrounds to work together to meet an end goal, despite what others may think of each other (Aronson, 1979, 1980, 2000, 2002). However, most studies on the jigsaw technique are mainly conducted using interviews and surveys (Blaney, 1977; Hanze, 2007; Lazarowitz, 1994; Perkins, 2001; Premo, 2018). We are interested in being able to quantify these behaviors and observe how they may relate to the cooperative learning method and academic learning outcomes.

In summary, this thesis consists of a selection of short papers and manuscripts that focus on developing a quantitative methodology for measuring group conversations in educational settings. The first paper discusses the proof of concept of using graph theory to use quantify groups, the second paper talks about the development of an open-source package that allows educational researchers to measure and quantify their groups with R, and the last paper describes using the methodology to quantify a cooperative learning method, specifically the jigsaw classroom.

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Chapter 1: Applying graph theory to examine the dynamics of student discussions in small-group learning

Abstract

Group work in STEM courses is an effective means of improving student outcomes, and many different factors can influence the dynamics of student discussions and ultimately the success of collaboration. The substance and dynamics of group discussions are commonly examined using qualitative methods such as discourse analysis. To complement existing work in the literature, we developed a quantitative methodology that utilizes graph theory to map the progression of talk-turns of discussions within a group. We observed groups of students working with peer facilitators to solve problems in biological sciences, with three iterations of data collection and two major refinements of graph theory calculations. Results include general behaviors based on the turns in which different individuals talk and graph theory parameters to quantify group characteristics. To demonstrate the potential utility of the methodology, we present case studies with distinct patterns: a centralized group where the peer facilitator behaves like an authority figure, a decentralized group where most students talk their fair share of turns, and a larger group with subgroups that have implications for equity, diversity, and inclusion. Together, these results demonstrate that our adaptation of graph theory is a viable quantitative methodology to examine group discussions.

Introduction

Collaboration and small-group discussions form the foundation for many evidence-based instructional practices and are effective means of enhancing student learning in STEM. Learning theories such as constructivism provide broad explanations for the theoretical basis of group discussions (Chi, 2009; Chi, 2014; National Research Council [NRC], 2000). Empirically, group discussions help students develop their cognition such as critical thinking (Bligh, 2000; Gokhale, 1995; Webb, 1982b), problem solving (Heller, 1992), and disciplinary understanding (Freeman, 2014); enhance important skills such as communication (Webb, 1994) and metacognition (Bromme, 2010; Vennman, 2006; Webb, 2003); improve affect such as interest and motivation (Ryan, 2000; Skinner, 1993); and increase completion rates in courses and persistence in STEM majors (Freeman, 2014; Loes, 2017; Tinto, 1997) (Figure 1.1, right).

The effectiveness of discussions depends on how the members of a group interact with one another, and many factors can influence group dynamics (Figure 1.1, left). Some of these factors are related to group composition, including academic preparedness (Hillyard, 2010), gender and race (Springer, 1999), student personalities (French, 2013), and group size (Aggarwal, 2008; Bligh, 2000; Fiechtner, 2016). Other factors involve what students' value and how they behave. Group discussions are only effective when students find the activities useful (Blumenfeld, 1996) and thus are motivated to engage with the activities (Machemer, 2007; Micari, 2010). Similarly, prior experience and attitudes while working in groups (Forrest, 2003; Hillyard, 2010), perception of free-riders (Hall, 2012), and the amount of on-task behavior of other group members (Aggarwal, 2008; Latané, 1979) can influence group dynamics. Community also plays an important role. Positive or negative influences on group dynamics are affected by a strong sense of belonging (Anderman, 2003; Freeman, 2007), intimidation by fellow group members (Micari, 2011), and comparison of one's academic and social standing relative to other group members (Micari, 2014).

In the literature, the substance of group discussions is commonly studied using qualitatively methods, specifically discourse analysis (Figure 1.1, middle). Discourse analysis is defined as "the study of language-in-use" (Gee. 2011, p. 8), which considers how and why certain actions occur and how they become a reality (Dunn, 2016). Typical applications of discourse analysis in this area include understanding student comprehension, knowledge construction, and cognition (Anderson, 2001; Fall, 2000; King, 1994; Kittleson, 2004; Molenaar, 2017; Sfard, 2001a; Webb, 2006); scientific argumentation and the substance of student conversations (Chiu, 2008a; Chiu, 2008b ; Soter, 2008); student participation and communication (Empson, 2003; Sfard, 2001b); collaboration (Premo, 2018; Webb, 2002; Wells, 2006); classroom and student dynamics (Ikpeze, 2007; Nystrand, 2003; White, 2003); and students' emerging STEM identities in the classroom (Bishop, 2012; Brown, 2005; Kumpulainen, 2017; Wood, 2013). However, most of these methodologies capture group discussions only for short durations for in-depth qualitative analyses and have certain limits in tracking how the conversations progress over time in a quantifiable manner.

The dynamics of how students interact and talk with one another in groups is at the crux of many different active-learning strategies, as well as equity and inclusion of all students. To understand how different factors contribute to group dynamics and how different interactions lead to different student outcomes, it is imperative to be able to quantify how students participate and engage in groups (Figure 1.1). By quantifying how students interact and talk with one another in groups, we can identify factors that contribute to how marginalized and minoritized students may or may not be able to engage in groups. Furthermore, understanding the dynamics of student group discussions will help elucidate the mechanisms by which different types of interactions contribute to different student outcomes.

Currently, there are not sufficient quantitative tools to examine the dynamics student group discussions. In this paper, we adapt graph theory to track how students communicate with one another in groups by recording the order in which each participant talks and analyzing these talk-turn patterns in a quantitative manner. Our methodology is developed and tested through three iterations of data collection and two major refinements of the mathematical calculations. Case studies are selected to demonstrate the potential patterns observed and highlight the utility of this methodology in biology education research.

Theoretical framework

There are several learning theories that deal with the fundamental basis of how people learn. We focused on social constructivism because of its relevance to group learning, and we also used cultural-historical activity theory (CHAT) to understand how students interact to make a collaborative group effective. Social constructivism posits that learning is a social process, emphasizing how student interactions in a group or classroom setting contribute to how they learn, think, and converse in their community (Powell, 2009; Adams, 2006; Hirtle, 1996). Vygotsky postulated that people learn by social interactions, and Dewey believed that learners are part of a greater community that teaches and enriches one another (Hirtle, 1996). From Vygotsky and Dewey, it can be said that an open environment where students are able to collaborate with one another is essential for knowledge building (Powell, 2009). This social process of learning forms the foundation of active-learning strategies, which have been shown to be effective across STEM disciplines and settings (Freeman, 2014). Social constructivists strive to provide an open environment for students to share their thoughts freely and to give students democratic control over their learning to foster a sense of deeper inquiry and learning (Adams, 2006; Hirtle, 1996; Davydov, 1995). In this environment, instructors serve as facilitators in the discussions and provide scaffolds for students whenever necessary (Powell, 2009; Adams, 2006; Davydov, 1995). To truly understand learning in the social constructivist view, we need to examine how students interact with one another and with their instructors.

Active engagement with the spoken or written language is an important medium for learning, according to the social constructivist perspective (Hirtle, 1996). When students feel welcomed and their communication styles acknowledged, they are more willing to engage and get more out of the activities done in the classroom (Powell, 2009; Hirtle, 1996). In addition, a welcoming and inclusive environment allows students to freely contribute different perspectives and experiences, which can help enhance student understanding of the subject matter (Powell, 2009; Adams, 2006; Davydov, 1995). However, differences in communication styles can also bring its own set of challenges, which may arise based on how students view other ethnicities and how willing they are to work with others (Powell, 2009; Attwater, 1996). To foster an inclusive classroom, it is imperative to be able to quantify how different students may or may not engage with the group learning environments, so we can understand the potential biases that are present, among other factors that contribute to an effective collaboration in the learning process.

CHAT is another theoretical framework relevant to student learning and especially articulates the connection between what people think and what people do (Nussbummer, 2012; Roth, 2009; Roth, 2004). Specifically, the second-generation CHAT considers how the relationships among people (the subjects), the activity (the object), tools, rules, community, and division of labor can all affect the final outcome (Figure 1.2), and a core idea of CHAT is the interconnectedness of these various components (Nussbummer, 2012; Roth, 2009; Roth, 2004). In the literature, CHAT has been used to observe student-student relationships and student-instructor relationships by examining the division of labor, the learning community, and the unwritten rules guiding these relationships (Nussbummer, 2012). Researchers have also used CHAT to examine how variations in the subject and composition of the community contribute to the learning process by observing how the combined effects of students' demographics, cultural background, and perception to learning connect to the outcome (Roth, 2009). These connections defined across the different components in CHAT cannot necessarily be easily seen directly (Roth, 2012; Roth, 2009), thus necessitating a methodology that can quantify some of these connections.

Methodological framework

We chose graph theory to model the order in which students talk in a group, which we consider a proxy for the dynamics of the discussion. Graph theory utilizes a set of mathematical principles and formulas to examine the relationships among objects of interest (Zweig, 2016). In the simplest form, a graph consists of nodes and edges (Godsil, 2001): Nodes represent the objects of interest, and edges represent the connections between them (Figure 1.3A). In our methodology, we model the participants of the group as nodes. When one participant talks after another, an edge is connected between them, and we define such as an edge as a talk-turn. There are several interpretations for edges. They may track how the discussion turns from one participant to another, who is willing to speak after others, and/or who contributes ideas that could be expanded upon or responded to. An edge does not necessarily suggest that two participants talk directly to each other; in fact, when a participant talks, everyone else in the group may be listening, but only one person talks in the next turn. Thus, an edge only indicates that one participant talks after the other.

Edges have additional important features. First, edges can be weighted, usually to present frequency (Godsil, 2001) (Figure 1.3B). We use edge weight to represent the number of times one participant talks after another participant, capturing the frequency of talk-turns between any two participants. Second, edges may be directed (pointing from one node to another) or undirected (simply connecting two nodes) (Godsil, 2001) (Figure 1.3C). In our methodology, the edges in a directed graph track the sequential order in which participants talk. We used a directed graph rather than an undirected graph because we can track the reciprocation between a pair of nodes; i.e. if one person responds more after another person but not the other way around. On the other hand, an undirected graph only shows that there was a talk turn between the two nodes. Tracking the directionality of conversations is important to understand the equity and inclusion of different students in group learning environments; for example, Webb (1985) found that males are less likely to respond to females' requests in conversations, while females are more likely to reciprocate back.

Graphs can have many mathematical parameters, and we selected relevant parameters to capture information on the dynamics of group discussions (Table 1.1). Degree and density are related parameters dealing with the number of connections that nodes have with one another (Figure 1.3D and 3E); here, these parameters represent how many participants talk after another. Degree is a parameter of individual nodes and measures the number of edges connected a node (Zarafani, 2014). Density is a parameter for the entire graph and is the total number of edges in a graph normalized to the maximum number of possible edges (Borgatti, 2013). Density for a given graph ranges from 0 to 1 in value and is calculated as:

$$\text{Density} = \frac{\text{\# of edges}}{\text{Maximum \# of possible edges}}$$

Nodes with higher degrees indicate participants who engage in talk-turns with or between more people. Graphs with higher density values indicate greater overall diversity in participants talking after one another; in other words, participants are talking after different people more often.

Centrality and centralization are another pair of related parameters for individual nodes and the entire graph respectively (Figure 1.3F and 3G). Centrality captures the notion that some nodes are more important to the connections of edges in a graph than others (Zarafani, 2014). Centrality can be estimated using a variety of methods, which emphasize different interpretations for what an edge means in a graph. Many types of centrality deal with connections of edges beyond two nodes and are often used to examine the flow of information across many people. In this study, we model talk-turns between two participants as the smallest unit of analysis; we also do not imply that information is flowing only from one participant to the next, as everyone in the group can be listening to the information. Thus, degree centrality is the most appropriate because it relies only on the degree of a node or the number of edges connected to a node. Degree centrality for a given node is calculated as:

$$\text{Degree centrality} = \#\text{ edges pointed to a node} + \#\text{ edges pointed out of a node}$$

A node with high degree centrality means that the participant talks between many different people, which is another proxy for active participation. This parameter provides additional information to the frequency of talk-turns (edge weights).

While centrality is a parameter for individual nodes, centralization is the equivalent parameter for the entire graph and measures if the graph is centered around a particular node (Borgatti, 2013). Similarly, we use degree centralization because it does not involve edges beyond two nodes. Degree centralization for a given graph ranges from 0 to 1 and is calculated as:

$$\text{Centralization} = (\#\text{ of nodes}) \times (\text{ max degree of any node}) - \sum \text{degree centralities}$$

We use degree centralization to determine to what extent a discussion is dominated by its most active participant.

Finally, subgraphs are smaller graphs within graphs (Godsil, 2001). We use subgraphs to determine highly connected subgroups within the larger group of participants based on edges and their relative weights (supplemental materials). High connectedness means that individuals talk more frequently after one another within the subgroup than after participants outside the subgroup. Within the subgroup, participants may be willing to speak after one another or are more likely to contribute ideas among one another that could be expanded upon or responded to.

Comparison to similar frameworks

A similar research methodology, social network analysis, has emerged in recent years in biology and physics education research (Bruun, 2016; Grunspan, 2014). However, social network analysis and graph theory are not the same, even though their names are often used interchangeably in the literature (Zweig, 2016). Graph theory is a branch of mathematics that seeks to understand how different parameters and graphical structures are related to one another (Zweig, 2016), and social network analysis is a specific application of graph theory more focused on relating the properties of the graph to understand the flow of information and social capital, as well as the formation of beliefs and identities, within a group of people (Knaub, 2018). In this paper, we use graph theory to track the talk-turns among participants in small-group discussions rather than the flow of information in a social network (Table 1.1).

Methods

Study context

This study was conducted at a large, private not-for-profit, doctoral university (highest research activity), with an undergraduate profile that is four-year, full-time, primarily residential, more selective, and lower transfer-in, as reported by the Carnegie Classification of Institutions of Higher Education™

(McCormick, 2005). We observed groups of introductory biology students tackling conceptual problems related to their coursework in an optional, peer-led academic program (Drane, 2005; Drane, 2014; Light, 2013). In this program, consistent groups of 5-7 students meet weekly to work with peer facilitators who have previously excelled in the course (Swarat, 2004), and groups were observed in the second half of the academic quarter. This study was approved by the Institutional Review Board (IRB) at Northwestern University.

Data collection

Our methodology was developed through three iterations of data collection based on observations of students solving problems in groups (Figure 1.4). In the first iteration, qualitative notes and memos were written during observations to track the discussions. Partial talk-turn data were included as part of the notes. In the second iteration, the relative physical positions of participants in each group were recorded in hand-drawn diagrams. Each talk-turn between any two participants was drawn as a line between them, and the number of talk-turns was tracked by tally marks. This resulted in undirected data for our graph theory calculations.

The third and final iteration combined both the earlier iterations and also recorded the order of talk-turns. In addition to the hand-drawn diagrams for physical positions, talk-turn data were recorded in a question-or-response format in a spreadsheet, and qualitative notes and memos were written during observations. Each participant was assigned a number based on the initial order in which they first talked in the group. Whenever a participant talked, their number was recorded under either the question or response column, which resulted in directed data for our graph theory calculations. For the purpose of this study, questions were non-rhetorical (Smith, 2013), and responses were defined as utterances that did not contain a question. While we acknowledge that group discussions have complex discourse patterns not captured in the simple format, we wanted to include discourse moves as part of the methodology, so future studies can examine group discussions by combining quality discourse data and our quantitative methodology.

General behaviors

We examined how many questions and responses were provided by each participant (peer facilitator and students) in a group. Questions and responses per hour were calculated using the following formulas, and scatterplots were generated to visualize the talk patterns of participants. These plots especially allowed us to compare the behaviors of peer facilitator vs. students within a group.

$$\text{Questions per hour} = \frac{\text{\# of question turns by a participant}}{\text{time in hours}}$$

$$\text{Responses per hour} = \frac{\text{\# of response turns by a participant}}{\text{time in hours}}$$

To compare across groups, a normalized talk ratio was calculated based on a fair-share number of turns for each participant assuming that all participants in the group talked for equal number of turns. Normalized talk ratio for a given participant was then calculated as the number of talk turns by that participant divided by the fair-share number of turns in the group.

$$\text{Fair-share \# of turns} = \frac{\text{Total \# of talk turns}}{\text{\# of participants (nodes)}}$$

$$\text{Normalized talk ratio} = \frac{\text{\# of talk turns by a participant}}{\text{Fair-share \# of turns}}$$

A participant who talked more than their fair share of turns would have a normalized talk ratio of > 1.0, whereas a participant who talked less than their fair share of turns would have a normalized talk ratio of < 1.0, regardless of the size of the group.

Episode length

From our third iteration of data collection with the question-and-response format, we defined an episode in the discussion as the number of talk-turns from a question to the last response immediately prior to the next question. We reasoned that a question was likely to indicate a new episode, especially in the initiation-reply-evaluation discourse pattern typically observed in a classroom (Macbeth, 2003), while acknowledging that many other scenarios may also occur, e.g. a non-sequitur response that leads to a new and productive direction (or episode) of the discussion. Nonetheless, we wanted to establish and test a robust methodology that can handle episodes, a common feature in discourse analysis, for potential future studies. With this operationalized definition of episodes, we calculated the frequency of episodes in different lengths.

Graph theory parameters

Data were processed and analyzed using a combination of Microsoft Excel (Microsoft Corporation, 2016), NodeXL Basic (Smith, 2010), MATLAB (Mathworks, 2017), and R (R Core Team, 2017). For analysis in NodeXL Basic, data in the question-or-response format were converted into an edge list, which included participant pairs who engaged in talk turns, with corresponding weights for each of the edges. Subgroups were identified using the Girvan-Newman algorithm, a hierarchical method designed for small groups (Girvan, 2002). To automate data processing and to make data analysis more transparent, we developed custom scripts in MATLAB and R. Our MATLAB script takes the talk-turn data in the question-and-response format and generates an edge list and a corresponding weight list for the edges. These two lists serve as inputs for our R script, which uses the igraph package to calculate graph theory parameters that we define in the theoretical framework section (Kolaczyk, 2006). All scripts and the source code (at the time of publication) are available online (supplementary materials).

Case study selection

We use a case-study approach to highlight the potential utility of our methodology. Case studies are especially useful for two purposes: (1) to examine the range and variations that exist within a setting and (2) to probe particular instances that are problematic or unusual (Case, 2011). As such, the strength and value of case studies are not about generalizability; rather, case studies can provide insights as exemplars (Flyvbjerg, 2006). Here, we selected three case studies that demonstrate outcomes in group dynamics that could be observed using our methodology. Two cases were selected to contrast the extremes of talk-turn behaviors observed in discussions, and a third case was selected to highlight the existence of hidden subgroups.

Results

Talk-turn behaviors in groups

We used the question-and-response data to examine at the talk-turn behaviors of individual participants in groups, comparing peer facilitators vs. students and different students vs. one another. From the four groups observed in this iteration of data collection, we identified two extreme patterns (Figure 1.5). First, using the question and response per hour data, we found that the peer facilitators in groups A and B were nearly indistinguishable from students in their respective groups (Figure 1.5, first row). In these groups, the peer facilitators and students engaged in similar number question turns and response turns. For example, in group A, the peer facilitator had 17.3 question turns and 88.0 response turns per hour, compared to 13.3 question turns and 80.7 response turns per hour for the next most active person in the group. On the other hand, in groups C and D, the peer facilitators had distinct behaviors compared to students. These peer facilitators engaged in many more talk turns compared to students in their groups and also had more question turns per hour compared to the peer facilitators in groups A and B. For example, in group D, the peer facilitator had 141.3 question turns and 120.0 response turns per hour, compared to 14.7 question turns and 73.3 response turns per hour for the next most active person in the group.

To compare across groups more easily, we used the normalized talk ratio defined in the methods section (Figure 1.5, second row). Consistent with the question and response per hour data, the peer facilitator in group A had a normalized talk ratio of 1.36, closest to 1 out of all the peer facilitators. In contrast, the peer facilitator in group D had a normalized talk ratio of 3.08, highest among the groups. Group A also had the smallest variation in normalized talk ratios among all participants (SD = 0.40, max = 1.36, min = 0.45, range = 0.91). Group B had a similar variation in normalized talk ratios (SD = 0.57, max = 1.96, min = 0.35, range = 1.61). On the other end of the spectrum, group D had the largest variation in normalized talk ratios (SD = 1.22, max = 3.08, min = 0.11, range = 2.97), followed by group C (SD = 0.75, max = 2.05, min = 0.27, range = 1.79). Within each individual group, the peer facilitators had the highest normalized talk ratios. Across groups, we can infer that in groups A and B, the peer facilitators behaved similarly to the students, whereas in groups C and D, the peer facilitator behaved more like a traditional classroom authority figure.

Groups A and B had longer episode lengths and fewer total number of episodes compared to groups C and D (Figure 1.5, third row). In one extreme, group A had episodes ranging from 2-20 talk turns, with an average of 4.85 episodes per hour; on the other hand, group D had episodes ranging from 1-5 talk turns, with an average of 3.26 episodes per hour. Groups A and B also had lower proportions of episodes with two talk turns at 20% and 30% respectively, compared to groups C and D with 61% and 62% respectively. We found that episode lengths tended to be longer in groups where the peer facilitators and students had similar talk-turn behaviors.

<u>Graph theory analysis</u>

To demonstrate the potential ulitity of our methodology, we present three case studies highlighting a decentralized graph, a centralized graph, and a graph with subgroups (Figure 1.6, Table 1.2). Case #1 (Figure 1.5, group A) is a decentralied graph containing four nodes, with a majority male peer facilitator (node 1), two majority female students (nodes 3 and 4), and one underrepresented minority (URM) male student (node 2), seated physically in a circular format similar to that of a roundtable (Figure 1.6, left). The graph has a total of 12 edges, resulting in a density of 1.00; that is, all possible pairs of participants engaged in at least one talk turn between them. The peer faciliator has a degree centrality value of 6.00, and the network has a centralization value of 0.00, meaning that no one participant is the majority speaker in the group. The two female particpants talked for 55% of turns, and the two male particpants talked for 45% of the turns; both percentages are near the 50% of the fair share between genders based on the number of particpants. The one URM participant talked for 11% of the turns, lower than the 25% of the fair share based on ethnicity.

Case #2 (Figure 1.5, group D) is a centralized graph containing five nodes, with a majority male peer faciliator (node 1), one minority female student (node 5), and three URM male students (nodes 2, 3, and 4), seated physically in a more traditional classroom format with the peer faciltator at the front (Figure 1.6, middle). The graph has a total of eight edges, resulting in a density of 0.40; that is, not all participants engaged in talk-turns with others in the group. The peer faciliator has a degree centrality value of 8.00, and the network has a centralization value of 1.00, meaning that one participant (the peer faciltiator) is the majority speaker in the group. The five male participant (including the peer facilitator) talked for an overwhelming 98% of the turns, the whereas the female minority student talked for only 2% of the conversation; these percentages are in stark contrast to the fair share percentages of 80% and 20% for males and females respectively. The three URM students talked for 18% of the turns, much lower than the 60% of their fair share based on ethnicity.

Case #3 is a larger group with an intermediate pattern between the two extremes. This graph contains eight nodes: a male peer faciliator (node 1), three female participants (nodes 3, 4, and 5), and four male participants (nodes 2, 6, 7, and 8), seated physically in a circular format (Figure 1.6, right). The graph has a total of 17 edges, resulting in a density of 0.61. The peer faciliator has a degree centrality of 6.00, and the network has a centralization value of 0.33. The peer facilitator talked for 28% of the turns (compared to fair share of 12.5%); the three female particpants talked for only 24% of the turns (compared to fair share of 37.5%), whereas the four male participants talked for 47% of the turns (compared to fair share of 50%). Most interestingly, two subgroups were identified using the Girvan-Newman algorithmn (Givan, 2002), even though they may not be immediately obvious from the visual inspection of the graph itself. These two subgroups were dvided by gender, with one subgroup consisting of the female participants and the other subgroup the male participants, suggesting that participants of the same gender were more likely to talk before and after one another.

Discussion

In this study, we adapted graph theory as a methodology to examine the dynamics of discussions by tracking the turns in which students talk in small groups. In our peer-led groups, we identified two major patterns: one where the peer facilitator and students contribute to the discussion relatively evenly and another where the peer facilitator behaves more like a classroom authority figure. Furthermore, in one of our large groups, we observed subgroups divided along gender lines. Our data are consistent with patterns described in previous studies in the existing literature, demonstrating the utility and validity of our methodology.

In the groups where the peer facilitator and students had similar behaviors, we observed higher episode lengths and lower ranges of the normalized talk ratios, lower centralization values, and graph higher densities, all indicators of fairly equal division of labor. These observations are consistent with the peer facilitators guiding discussions to help students build conceptual understanding (Eberlein, 2008; Micari, 2010; Pazos, 2010). In contrast, in the groups where the peer facilitator behaved more like an authority figure, we observed shorter episodes (especially with length = 2), greater ranges of normalized talk ratios, higher centralization values, and lower graph densities. The high proportion of episodes with length of two is consistent with the peer facilitators providing directed instruction in the inquiry-response-evaluation discourse pattern typically observed in a classroom (Macbeth, 2003; Micari, 2010; Pazos, 2010), which is not necessarily aligned with the tenets of social constructivism.

Our data suggest that seating arrangements can be correlated with how participants engage in discussion. For example, in Case #1, students were seated in a circular format facing one another, and the resultant graph has a high density, indicating that students engaged in talk-turns with one another. In Case #2, students were seated in a more traditional classroom structure facing the peer facilitator at the front, and the resultant graph has a low density. These observations suggest that physical arrangements of the classroom (i.e. the tools in CHAT) can influence how different people engage with the activity. These observations are consistent with existing literature: Students in circular seating are more likely able to maximize group interactions; in contrast, the typical classroom seating with students facing the front tends to emphasize the role of the instructor (or peer facilitator) and minimize student-student interactions (Borgatti, 2009; McCorskey, 1978; Wannarka, 2008).

We observed some important patterns related to equity and inclusion in groups, again consistently with existing literature. For example, in Case #3, two subgroups were identified using graph theory methods, and the subgroups were divided by gender, suggesting that there are additional hidden social rules within the group that are guiding or informing the talk patterns among students. In Case #2, there was only one female participant, and she had the lowest talk-turn contribution out of all the participants. In contrast, Case #1 had an equal number of female and male participants, and talk-turn contributions were even across the two genders. According to existing literature, groups with gender balance result in females having a slightly greater influence and relatively equal achievement across genders, whereas when females are in the minority of a group, they tend to have less influence and lower achievements than males (Craig, 1986; Strodtbeck, 1956; Webb, 1982a; Webb, 1984). Another important pattern we noticed is in the number of talk-turn contributions of URM students. In both Case #1 and #2, majority students in the group had higher percentage of talk-turn contributions compared to URM students. Our results are also in line with previous research: URM students are more likely to face intimidation and experience social-comparison effects (Micari, 2011) and tend to have fewer interactions within groups (Cohen, 1972).

Our methodology can serve as an important tool to understand and assess how students participate and engage in group discussions. One potential application is observing the effects of how different combinations of demographics may affect student participation, and this information can then be used to inform how instructors can create more equitable classrooms for all students to engage in meaningful learning. Second, we can use this methodology to examine the effects of class structure on student participation; these could include physical structures such as spatial seating of the classroom and pedagogical structures such as instructor talk (Seidel, 2015). Furthermore, information about student participation can be captured at various time points throughout an academic term to determine the progression for how groups may coalesce over time to create more effective collaboration. For assessment purposes, this methodology can provide a means for instructors to quantify contribution by individual members in a group and provide feedback to students. Ultimately, quantitative information obtained from this methodology can be used to help students learn to collaborate and inform instructors on how to moderate discussions.

Limitations of the study

Our study has a few limitations. First, we had a limited number of groups in our final iteration of data collection and analysis, so we were not able to make generalizable conclusions. However, our goal was simply to establish a quantitative methodology based on graph theory to examine student discussions in small groups. Even from our limited dataset, we were able to observe patterns consistent with various observations the existing literature, indicating the validity of our methodology.

Second, our methodology does not consider if an individual is addressing the entire group vs. a specific person. In a group, it is likely that when an individual speaks, everyone else can be listening. However, it is simply not feasible to determine if each person in the group is listening or not. Furthermore, it is not practical to model this kind of listening information using graph theory. Assuming that everyone is listening, the resultant graph will have edges from the speaker node to all other nodes. Essentially, the graph will be saturated with edges and will likely not provide any useful information. Our methodology tracks the talk-turn behavior of individuals in a group, which can tell us much more information about the dynamics of the discussion.

Third, our methodology tracks only the sequential order of talk-turns and not the content of discussion. One solution to this problem is combining our graph-theory methodology with discourse analysis to incorporate the substance of the discussion into our mathematical model. We had intentionally developed the methodology with this purpose in mind, e.g. including discourse moves and episodes in the data processing and analyzing pipeline. As such, our methodology should be robust enough to handle different kinds of data, including clicker discussions, group work in laboratory setting, and various small-group learning environments, e.g. problem-based learning, peer-led team learning, and process oriented guided inquiry learning.

Some quantitative methods currently exist to analyze patterns of group discussions. For example, a computer-based method has been used to map the content of the conversation to show how participants contribute in the group and to understand how group dynamics can affect learning outcomes (Barros, 2000). Our methodology complements this existing work. We tracked how participants engage in talk-turns, while the previous study focused on how the content of the discussion and the types of contributions may affect learning outcomes (Barros, 2000).

Despite these limitations, our methodology can serve as an important tool in examining and understanding group work by capturing the dynamics how students engage in talk-turns in the discussion (Figure 1.1). We observed some interesting and important patterns, such as centralized versus decentralized groups, potential effects of seating arrangements on talk-turn behaviors, and influence of gender and minority status in group contributions, all of which are consistent with other observations in the existing literature. With this methodology, in the future, we will be able to examine how various student characteristics may influence group dynamics in discussions and how differences in talk-turn behaviors may contribute to the success of student outcomes.

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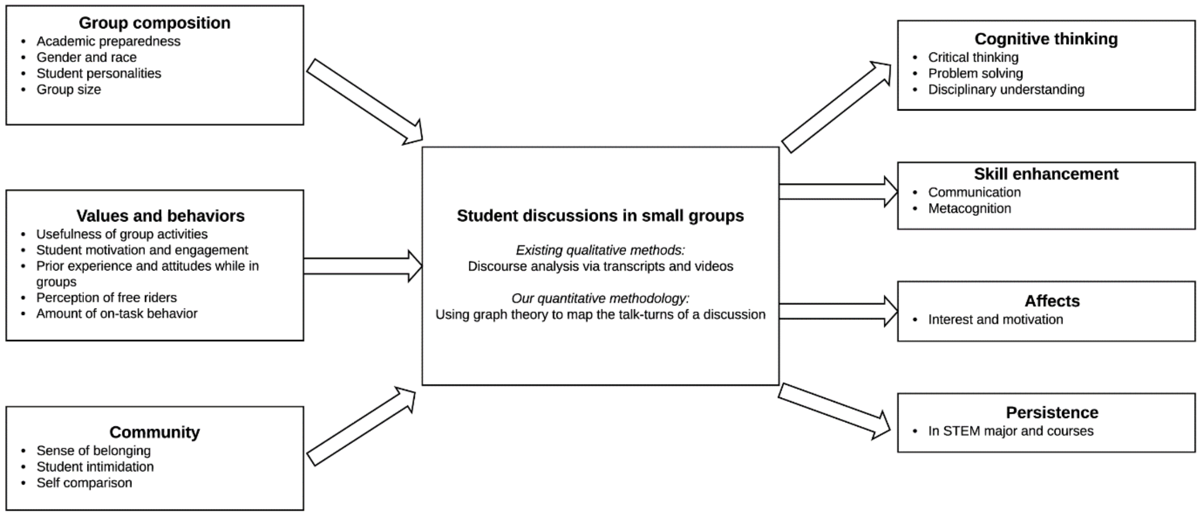


Figure 1.1. Small-group discussions in STEM learning. Student discussions can be influenced by a number of factors, including group composition, sense of belonging, and values and behaviors related to collaborative activities. The dynamics and quality of these discussions can affect student outcomes, such as cognitive learning, development of process skills, affect, and persistence. In existing literature, quality of small-group discussions is typically analyzed by discourse analysis. In this study, we adapt graph theory methodologies to examine the dynamics of these discussions. Citations are available in the body of the text.

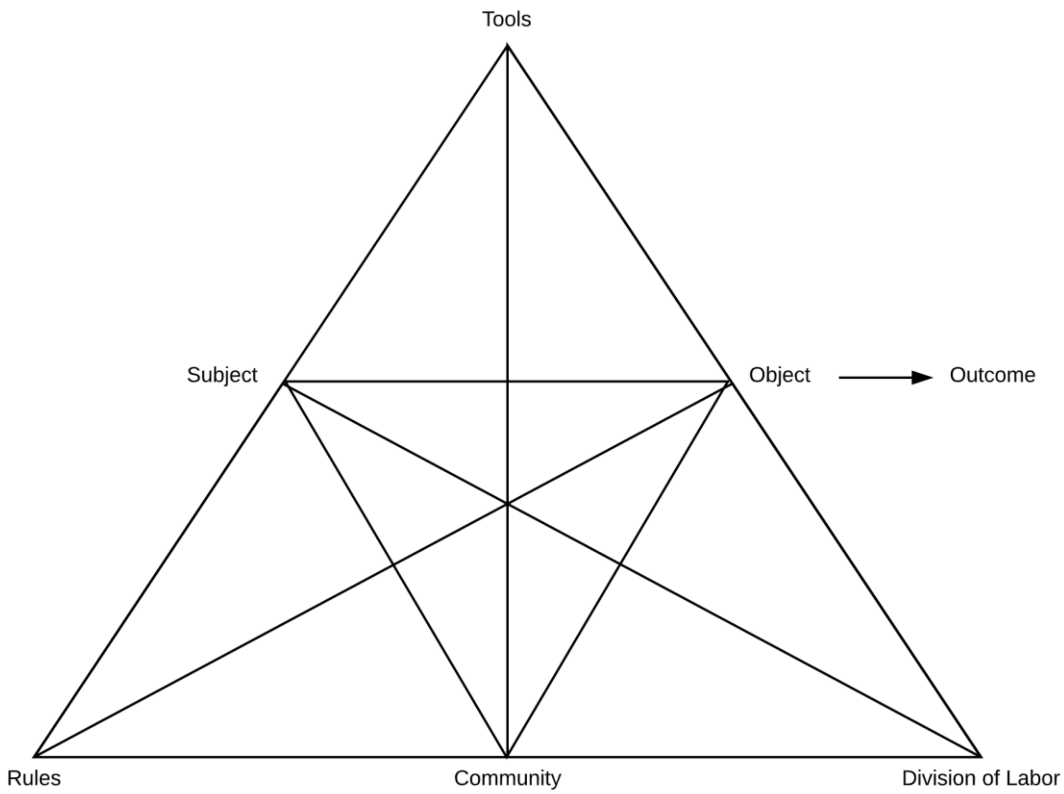
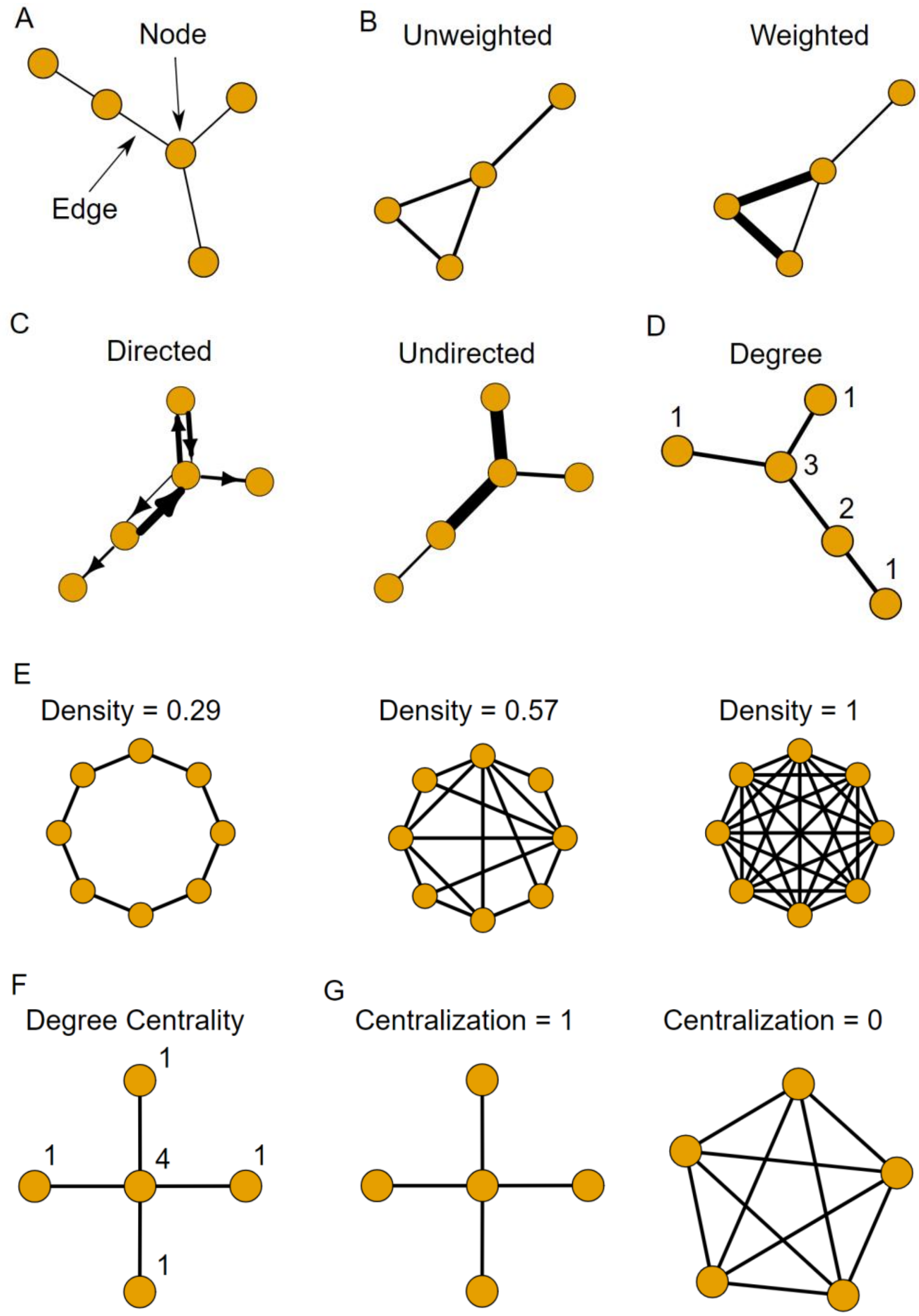


Figure 1.2. Cultural-historical activity theory (CHAT). In this study, we used CHAT to consider how the relationships among students (the subjects), the learning activity (the object), tools, rules, community, and division of labor in small groups can contribute to the final learning outcome. CHAT emphasizes the interconnectedness of these various components. These connections are not always easily observable, thus necessitating a methodology that can quantify some of these connections. Specifically, we developed a methodology based on graph theory to quantify the division of labor, the interactions among students and peer facilitators in small groups (the community), and potentially hidden rules that guide how different students may or may not engage with the activity (the object).

Figure 1.3. Relevant graph theory parameters. (A) In a graph, nodes represent the objects of interest, and edges represent the connections between them. The graph shown here contains 5 nodes and 4 edges. (B) Edges can be weighted, typically to represent frequency of some sort, or unweighted. (C) Graphs can be directed (with edges pointing from one node to another) or undirected (with edges simply connecting two nodes). Directed graphs have more information as they show how much reciprocation is present between a pair of nodes. (D) Degree indicates the number of edges connected to a node. The degree of each node is labeled. (E) Three graphs with their associated densities: As more edges are added, density increases. (F) Degree centralities of a graph with five nodes: The degree centrality is exactly the degree of each node. (G) The centralizations of two graphs that both have five nodes: The left graph has a higher centralization because it is more centralized on Node #1, while the right graph is less centralized.


A
Node
Edge
B
Unweighted
Weighted
C
Directed
Undirected
D
Degree
1
1
3
2
1
E
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Density = 0.57
Density = 1
F
Degree Centrality
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Centralization = 1
Centralization = 0

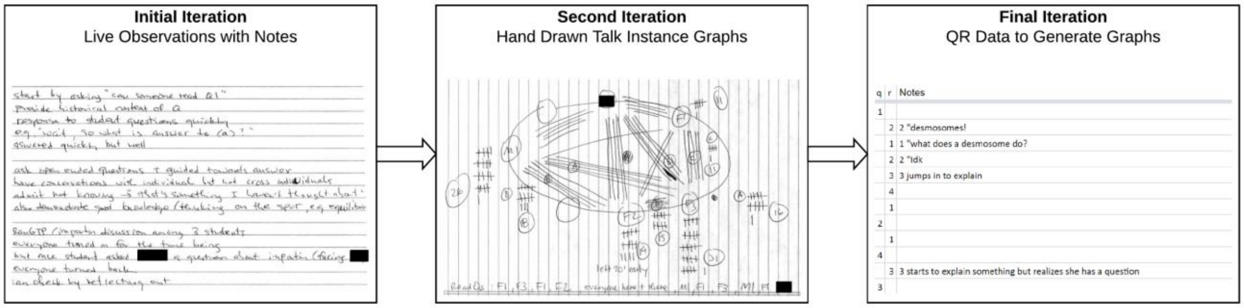

Figure 1.4. Three iterations of data collection. In the first iteration, qualitative memos were recorded in live observations of student groups (n = 8). In the second iteration, hand-drawn graphs depicting dynamics of student conversations were created (n = 3). In the final iteration, talk turns were recorded in a table format along with notes (n =4). Each talk turn was either a question ("q") or response ("r").

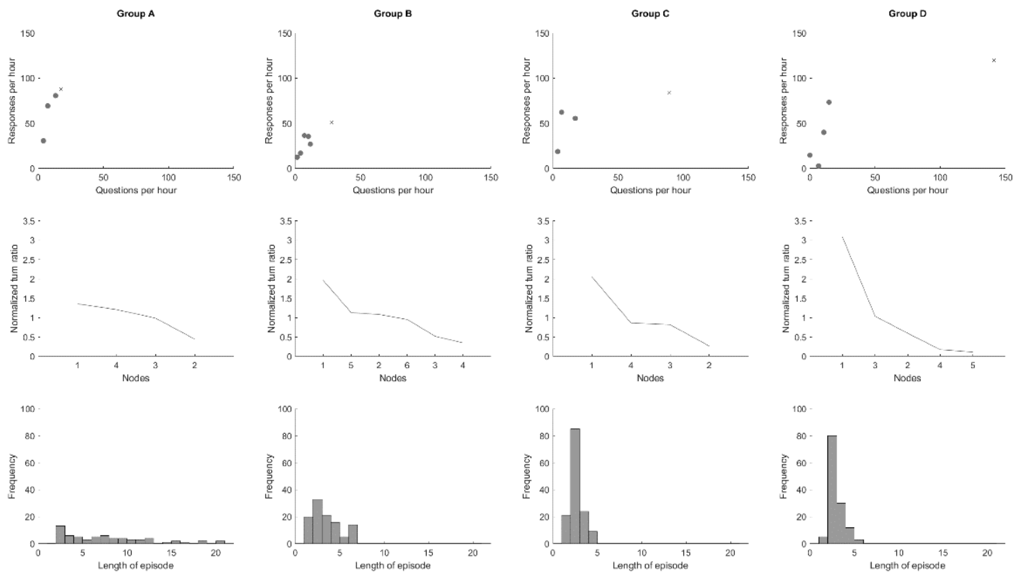

Figure 1.5. Characteristics of individual and group talk-turn behaviors. In the first row, questions and responses per hour are shown in scatter plots. Each point represents a student in the group, with the peer facilitator indicated by X. Four groups (A-D) are arranged based on the average distance of each student from the peer facilitator in the scatter plot to highlight peer facilitator talk behaviors in comparison to student talk behaviors. In the second row, normalized talk turn ratios are plotted in descending order for each individual in the group, with Person #1 being the peer facilitator. Individuals are numbered based on the order in which they first talked. In the third row, histograms represent the distribution of episodes in one recorded session for each group. For the purpose of this study, episodes are defined as the number of talk turns in between two questions.

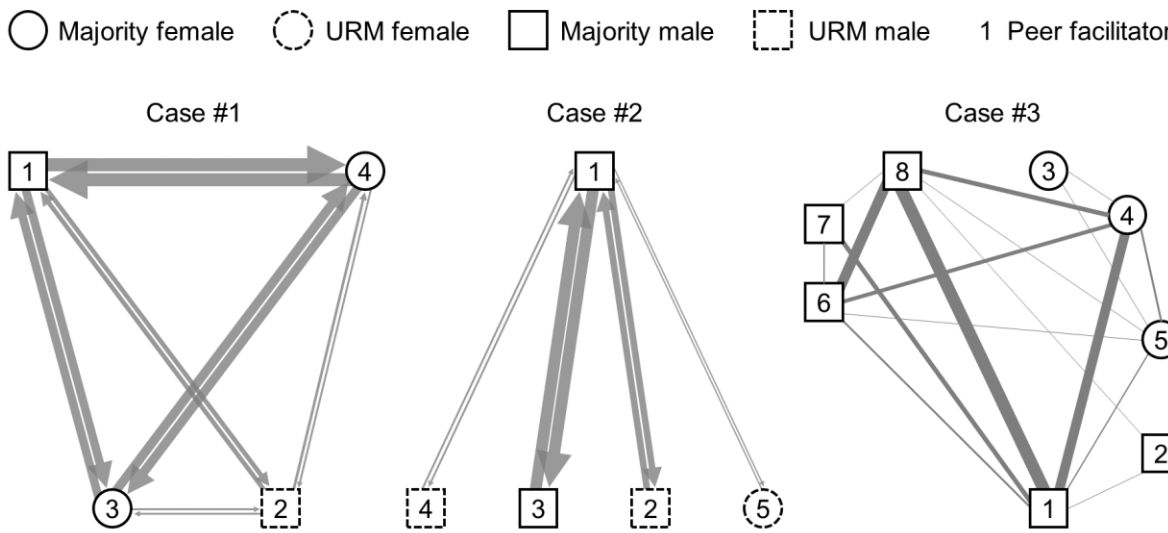

Figure 1.6. Three cases analyzed by our graph theory methodology. In these graphs, each individual and their demographics are presented as a node. Transitions between talk-turns are represented by arrows, and the thickness of arrows indicate the cumulative numbers of transitions. In one extreme, Case #1 is a decentralized group in which the peer facilitator (Person #1) appears to be nearly indistinguishable from students. In the other extreme, Case #2 is a centralized group in which the peer facilitator dominates the talk turns. Case #3 is an intermediate case that also highlights the existence of subgroups, which are divided between the two genders. Note that Case #3 is undirected compared to Case #1 and #2, which are directed.

Table 1.1. Graph theory parameters used in the development of our methodology.

Parameter	Definition	Our methodology	Social network analysis
Node	An object of interest	Student or facilitator	Person
Edge	A connector between two nodes	Talk-turn between two individuals	Flow of information between two people
Direction	Defines which node points to another using the edge	Indicates which individual talks after the other	Indicates which person has ties to the other
Weight	A number associated with an edge	Frequency of a talk-turn between two individuals	Frequency of information flow between two people
Degree	Number of edges connected to a node	Number of people an individual talk before/after	Number of people an individual has ties to
Density	Number of edges divided by number of possible edges	Talk-turns occurring among different individuals	Interactions occurring among different people
Centrality	A number for the importance of a given node in the graph	Amount of talk-turn contribution of an individual	Amount of influence of each person
Centralization	A number for the importance of the central node	Dependence of a group on its most active individual	Dependence of a network on its most active person
Subgraph	A smaller graph within a graph	Individuals who talk after each other more	Individuals who have closer ties to each other

Table 1.2. Summary of graph theory parameters of the selected cases.

Parameter	Case #1: Decentralized	Case #2: Centralized	Case #3: Intermediate
Nodes	4	5	8
Edges	12	8	17
Density	1.00	0.40	0.61
Centrality (peer facilitator)	6	8	6
Centralization	0.00	1.00	0.33
Subgroups	None	None	2

Chapter 1, in full, has been accepted for publication in CBE-Life Science Education. Chai, Albert; Le, Joshua P.; Lee, Andrew S.; Lo, Stanley M. The thesis author was the primary investigator and author of this paper. The thesis author has contributed to the analysis and preparation of the manuscript sections: introduction, methods, results, and conclusion.

Chapter 2: discourseGT: Analyze group patterns using graph theory in educational settings

Abstract

Student collaborations in the classroom are an important concept in evidence-based learning. However, most methods for analyzing student collaboration are more qualitative. In our package, discourseGT, we introduce a quantitative method for analyzing student conversation pattterns. We utilize graph theory to help understand these patterns and pick the most relevant network parameters for an educational context. The package produces graphs of the collaboration and relevant statistics from our methods. discourseGT will help provide the necessary quantative framework to analyze student conversational dynamics.

Introduction

Group work is an important phenomenon in the classroom. It allows students to develop critical thinking skills, work with their fellow peers, and develop a better understanding of the material (Springer, 1999). However, with group learning, there are various different factors such as group composition (Springer, 1999), student attitudes towards group work (Forrest, 2003; Hillyard, 2010), and sense of belonging (Anderman, 2003; Freeman, 2007). These factors can influence how students behave in group settings, which can affect how students achieve learning outcomes.

Most studies that observe student groups in the classroom have largely been qualitative. Typical qualitative methods for analysis include discourse analysis, video observations, and structured interviews. However, there have not been many sufficient tools to be able to observe students quantitatively. In our package, we propose a method to observe students using graph theory to track how the conversations progress in a group discussion (Chai, 2019). With this method, we are interested in tracking the order of the conversation, which alone may provide valuable insight to how conversations progress. Coupled with other demographic information, such as ethnicity, gender, GPA, and academic standing, our tool could help provide more valuable information to how student characteristics can affect student learning.

Current implementations of network analysis, such as those in igraph (Csardi, 2006), network (Butts, 2008), and sna (Butts, 2016), focus more primarily on larger scale networks, such as social media networks (Jones, 2017), epidemiological networks (Christakis, 2011), and political networks (Hobbs, 2016). However, in educational settings, most groups are much smaller than these networks and parameters that are relevant for larger networks are not necessarily applicable to smaller scale networks (Lou, 2001). Most of these tools available are very general and do not provide the necessary functionality that educational researchers focus on.

We have decided to base the package on the R programming language (R Core Team, 2016) because of its open-source nature and extensibility of packages. Our package does not rebuild all network components from scratch, but rather we build upon the existing network packages.

Relevant Graphical Parameters

Based on a methodological and theoretical review of the literature as found in our previous paper in Chai (2019), we have selected a subset of graphical parameters that are relevant in educational group settings. These are the parameters of focus in this package.

Table 1: Parameters identified for analysis

Graphical Parameter	Graphical Definition	Our Definition	Social network analysis
Node	An object of interest	Student or facilitator	Person
Edge	A connector between two nodes	Talk-turn between two individuals	Flow of information between two people
Direction	Defines which node points to another using the edge	Indicates which individual talks after the other	Indicates which person has ties to the other
Weight	A number associated with an edge	Frequency of a talk-turn between two individuals	Frequency of information flow between two people
Degree	Number of edges connected to a node	Number of people an individual talk before/after	Number of people an individual has ties to
Density	Number of edges divided by number of possible edges	Talk-turns occurring among different individuals	Interactions occurring among different people
Centrality	A number for the importance of a given node in the graph	Amount of talk-turn contribution of an individual	Amount of influence of each person
Centralization	A number for the importance of the central node	Dependence of a group on its most active individual	Dependence of a network on its most active person
Subgraph	A smaller graph within a graph	Individuals how talk after each other more	Individuals who have closer ties to each other

Obtaining the Package and Licensing

The package can be obtained through the Comprehensive R Archive Network (CRAN). Users can type:

```
install.packages('discourseGT')

## Installing package into '/home/albertchai/R/x86_64-pc-linux-gnu-library/3.
5'
## (as 'lib' is unspecified)
```

```r
library(discourseGT)
```

This package is MIT Licensed. However, please cite the package if you choose to use it in your research. Citation information can be obtained by typing `citation("discourseGT")` in R. For the examples shown, we will be using an observed classroom data set recorded using our initial method (Chai, 2019) along with a comprehensive demographics data set.

Getting Started

The functions of this package are designed to be as modular as possible, meaning that you can run the analysis functions that you are interested in. In Figure 1, we describe the basic workflow of the discourseGT. The workflow is as follows:

1. Prepare the raw data in the necessary 2 column format (question-and-response) with q,r as the header
2. Tabulate the edges and weights from the raw data
3. Set the network properties, such as if the network is directed and/or weighted, and if self-interactions are allowed.
4. Run the analysis of interest
5. Export results from analysis

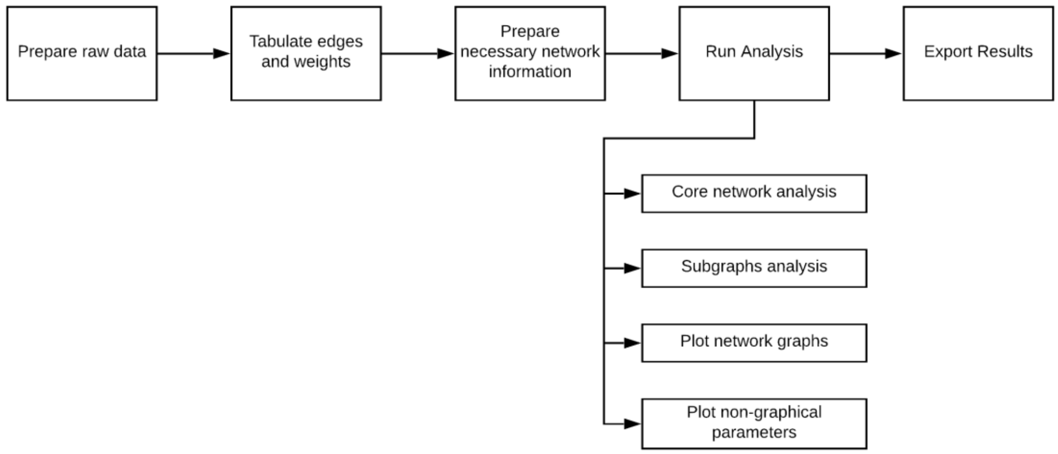

Figure 2.1. The Typical workflow of the discourseGT. The typical workflow of the discourseGT. Prepare raw data, tabulate the edges and weights, set the network properties such as directional, weighted, etc., run the analysis of interest, and export needed results.

Prepping the data

The original data must be in a 2 column format, preferably with header names q and r. The q and r labeling can be arbitrary depending on the case, but the function still requires it. The raw data can be prepared using any spreadsheet software or text editor of your choice. It is recommended that the raw data is saved as a comma-separated (CSV) file.

```
case4 <- read.csv('data/case4_raw.csv')
head(case4)

##    q  r
## 1  1 NA
## 2 NA  2
## 3  1 NA
## 4 NA  2
## 5  1 NA
## 6 NA  2
```

Once you have finished preparing your raw data for import, use the `read.csv()` function to import the file into R. Double check that your data has been imported properly into the environment. You should have 2 variables in your data frame object, similar to what is presented above.

Preparing the data objects

Now that the data is in the environment, use the `tabulate_edges()` command to produce an weighted edge list. By default, the weight of an edge is defined as the number of times an edge has occurred between two nodes. Weights can be redefined based on other available criteria such as the length of each conversation or the content of the edge, but they will have to be done manually.

```
# Calculate the weighted edge list.
tabNet <- tabulate_edges(case4, iscsvfile = FALSE)
# Checking the weighted edge list generated
head(tabNet$master)
```

```
##   source target weight
## 1      1      1     73
## 2      2      1     38
## 3      3      1     66
## 4      4      1     11
## 5      5      1      7
## 6      1      2     38
```

Next, we need to prepare the weighted edge lists for analysis. We will need to run the `prepareNetworks()` function to do this. With this function, we are giving the basic information on how we want to treat our networks.

The function requires the following information:

- The object file that stores the weight and edge list
- The title of your project, default will be blank
- Is your network directed (as opposed to undirected)? Default: `TRUE`
- Are self interactions permitted? Default: `FALSE`
- Is the network weighted? Default: `TRUE`

```
prepNet <- prepareNetworks(tabNet,
                           project_title = "Case 4",
                           directedNet = TRUE,
                           selfInteract = FALSE,
                           weightedGraph = TRUE)
```

Once we have set how we want our networks to be treated, we can begin running the necessary network analysis.

Running network analysis

One benefit of the `discourseGT` package is that the analysis functions are designed to be as modular as possible, meaning that you can run only the analysis that you are interested in. In this section, we will cover what each analysis has to offer and what is best suited for your purpose. All of the functions below will require the object from the `prepareNetworks()` function. To save the output of the function for exporting, assign the function to an object in the environment.

The core network parameters function

The `coreNetAnalysis()` function will count the number of edges, nodes, and weighted edges, calculate the average network degree, modularity, and centrality, and determine which nodes are articulation points and if a node has communicated with at least one other person in the network based on the methodology specified in our framework.

```
coreNet <- coreNetAnalysis(prepNet)
```

The subgroups function

The `subgroupsNetAnalysis()` function will observe how many potential subgroups are possible in the network. This will return the likely number of subgroups present in the network and the possible combinations for the members in those subgroups based on our methodology.

```
subNet <- subgroupsNetAnalysis(prepNet)
```

Generating summaries

The summary function generates a quick summary sheet of all of the parameters that were executed. The function will ask you to map each corresponding option with the proper stored object in the R environment. The function requires the initial configuration data that was generated from the `prepareNetworks()` function. The remaining options, `coreNetAnalysisData` and `subgroupNetAnalysisData` are optional. If you want to be able to export the summary data, you need to assign the function to a R object. Otherwise, if you want to view the summary data in the console, you do not need to assign it to an object and set the display flag to `TRUE`.

```
summaryData <- summaryNet(netintconfigData = prepNet,
                          coreNetAnalysisData = coreNet,
                          subgroupsNetAnalysisData = subNet,
                          display = TRUE)

## discourseGT R Package - Production
## Package Version: [1] '1.0.0'
## Network Results - Project Summary
##
## ---------------PROJECT DETAILS---------------
## Name of Project:  Case 4
## Summary Results Generated On: [1] "2019-05-13 23:09:11 PDT"
##
## ---------------NETWORK CONFIGURATION---------------
## Weighted Graph:  TRUE
## Self-Interactions Allowed:  FALSE
## Graph Directed:  TRUE
##
## ---------------CORE PARAMETERS ANALYSIS---------------
## Number of Edges:  8
## Number of Nodes:  5
## Weighted Edges:  317
## Graph Adjacency Matrix:
## 5 x 5 sparse Matrix of class "dgCMatrix"
##    1  2  3  4 5
## 1  . 38 66 11 7
## 2 38  .  .  . .
## 3 66  .  .  . .
## 4 11  .  .  . .
## 5  7  .  .  . .
##
## Network Density: 0.4
## Average Degree:  3.2
## Strong/Weak Interactions:
## 1 2 3 4 5
## 1 1 1 1 1
##
## Unrestricted Modularity:  NA
##
## ---------------NETWORK CENTRALITY---------------
## Degree Centrality:
## $res
## [1] 8 2 2 2 2
```

```
##
## $centralization
## [1] 1
##
## $theoretical_max
## [1] 24
##
##
## Articulation Points List:
## + 1/5 vertex, named, from 2ea9d66:
## [1] 1
##
##
## ---------------SUBGROUPS AND MODULARITY---------------
## Overview of Possible Cliques:
##
## 1 2
## 5 4
##
## Maximal Cliques Possible:
## [1] 4
##
## Clique Member Lists:
## [[1]]
## + 1/5 vertex, named, from 42a2281:
## [1] 1
##
## [[2]]
## + 1/5 vertex, named, from 42a2281:
## [1] 5
##
## [[3]]
## + 2/5 vertices, named, from 42a2281:
## [1] 1 5
##
## [[4]]
## + 1/5 vertex, named, from 42a2281:
## [1] 4
##
## [[5]]
## + 2/5 vertices, named, from 42a2281:
## [1] 1 4
##
## [[6]]
## + 1/5 vertex, named, from 42a2281:
## [1] 3
##
## [[7]]
## + 2/5 vertices, named, from 42a2281:
## [1] 1 3
```

```
##
## [[8]]
## + 1/5 vertex, named, from 42a2281:
## [1] 2
##
## [[9]]
## + 2/5 vertices, named, from 42a2281:
## [1] 1 2
##
##
## Group Core Members:
##  1  2  3  4  5
## 66 38 66 11  7
##
## Graph Symmetry of Members:
## $mut
## [1] 4
##
## $asym
## [1] 0
##
## $null
## [1] 6
##
##
## Graph Connectedness Census:
##
## 5
## 1
##
## Neighborhood List for Each Adjacent Node:
## [[1]]
## + 5/5 vertices, named, from 42a2281:
## [1] 1 2 3 4 5
##
## [[2]]
## + 2/5 vertices, named, from 42a2281:
## [1] 2 1
##
## [[3]]
## + 2/5 vertices, named, from 42a2281:
## [1] 3 1
##
## [[4]]
## + 2/5 vertices, named, from 42a2281:
## [1] 4 1
##
## [[5]]
## + 2/5 vertices, named, from 42a2281:
## [1] 5 1
```

```
##
##
## Transitivity/Clustering Coefficients:
## Local Transitivity values:
## [1] 0 0 0 0 0
## Global Transitivity values:
## [1] 0
##
##
## ---------DISCLAIMER AND WARRANTY OF PROVIDED RESULTS AND CODE---------
## Results from Code:
## The researcher(s) are primary responsible for the
##       interpretation of the results presented here with the script.
##       The authors accept no liability for any errors that
##       may result in the procesing or the interpretation of
##       your results. However, if you do encounter errors in
##       the package that shouldn't have happened, please let us
##       know
##
## Code Warranty:
## MIT License
## Copyright (c) 2018 Albert Chai and Andrew S. Lee
##
## Permission is hereby granted, free of charge, to any person obtaining
##       a copy of this software and associated documentation files
##       (the 'Software'), to deal in the Software without restriction,
##       including without limitation the rights to use, copy, modify,
##       merge, publish, distribute, sublicense, and/or sell copies of
##       the Software, and to permit persons to whom the Software is
##       furnished to do so, subject to the following conditions:
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## The above copyright notice and this permission notice shall be
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## THE SOFTWARE IS PROVIDED 'AS IS', WITHOUT WARRANTY OF ANY KIND, EXPRESS OR
##       IMPLIED, INCLUDING BUT NOT LIMITED TO THE WARRANTIES OF
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T.
##       IN NO EVENT SHALL THE AUTHORS OR COPYRIGHT HOLDERS BE LIABLE FOR
##       ANY CLAIM, DAMAGES OR OTHER LIABILITY, WHETHER IN AN ACTION OF
##       CONTRACT, TORT OR OTHERWISE, ARISING FROM, OUT OF OR IN CONNECTION
##       WITH THE SOFTWARE OR THE USE OR OTHER DEALINGS IN THE SOFTWARE.
```

Plotting networks

For a more visual representation of the networks, the `plotNetworks()` function can be used to plot the network. The function will plot the nodes and edges based on a project properties selection from the `prepareNetworks()` function. The default network projection is Fruchterman Reingold. The function supports mapping one attribute on the network, such as gender or ethnicity. The attributes can be retrieved from a data frame object, vector, or list in the environment. A label can be given to attribute being mapped as well. Mapping node labels are supported along with adjusting the size for each particular node. However, mapping an attribute is not required in this function. It will require the object from the `prepareNetworks()` function to create the graph. If you want to export the graph, assign the function to an object in the environment.

In this example, we will plot the network without specifying an attribute.

```
plotNetworks(prepNet)

## $g2plot
```

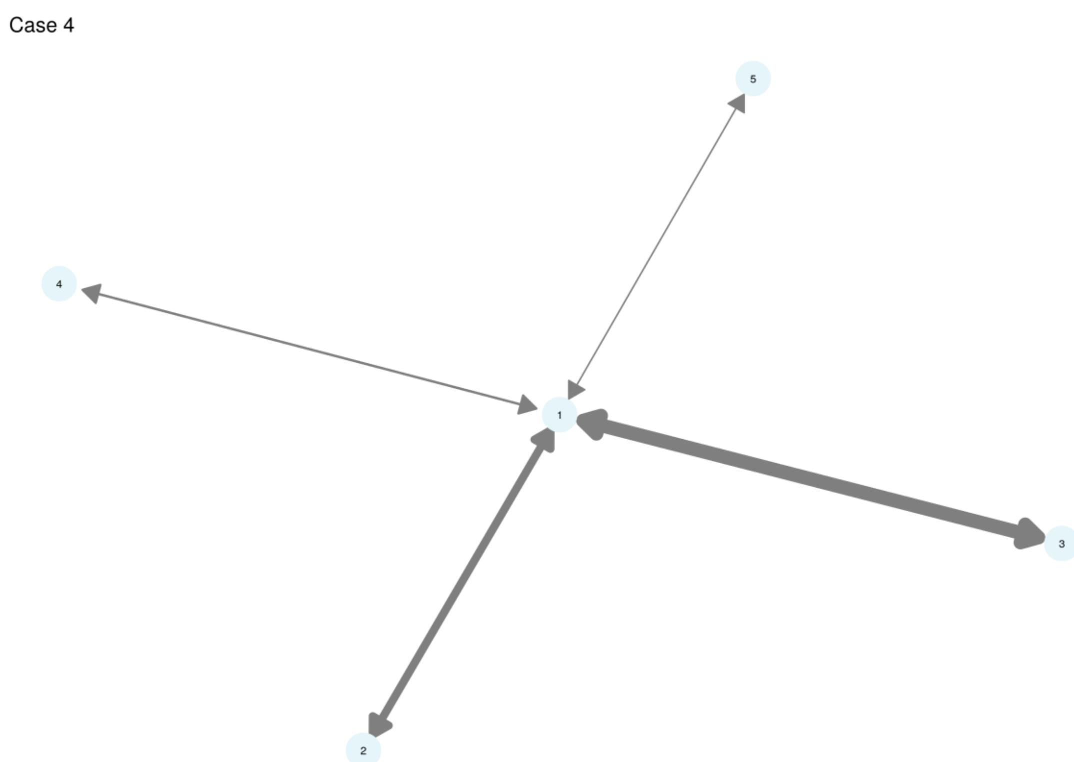

Figure 2.2 Network plot without attribute data.

```
##
## $saveDataVar
## [1] 1
```

Now, if we specify an attribute in the network plots, we need to make sure that our attribute data frame is prepared correctly.

For example, in this attributeData data frame, it contains information about student demographics.

```
##   node gender        ethnicity current_gpa first_generation stem_major
## 1    1 female            white        3.56               no        yes
## 2    2   male            white        3.26              yes         no
## 3    3 female            asian        3.46               no        yes
## 4    4   male african_american        3.60              yes        yes
## 5    5   male           latino        3.59              yes        yes
##               major course_reason class_level number_prior_ap residency
## 1    bioengineering         major      junior               0        CA
## 2 political_science            ge      senior               2        CA
## 3           biology         major   sophomore               3        CA
## 4         chemistry      elective      junior               4        WA
## 5       mathematics         major    freshman               5        CA
##   sat_score
## 1      1323
## 2      1449
## 3      1228
## 4      1494
## 5      1263
```

Once the attributeData data frame is loaded into the environment, we can map the attributes into the `plotNetworks()` function. In this graph, we want to map gender information on our graphs.

```
plotNetworks(prepNet,
              attribute = attributeData$gender,
              attribute.label = "Gender",
              attribute.node.labels = attributeData$node,
              attribute.nodesize = 20)

## $g2plot
```

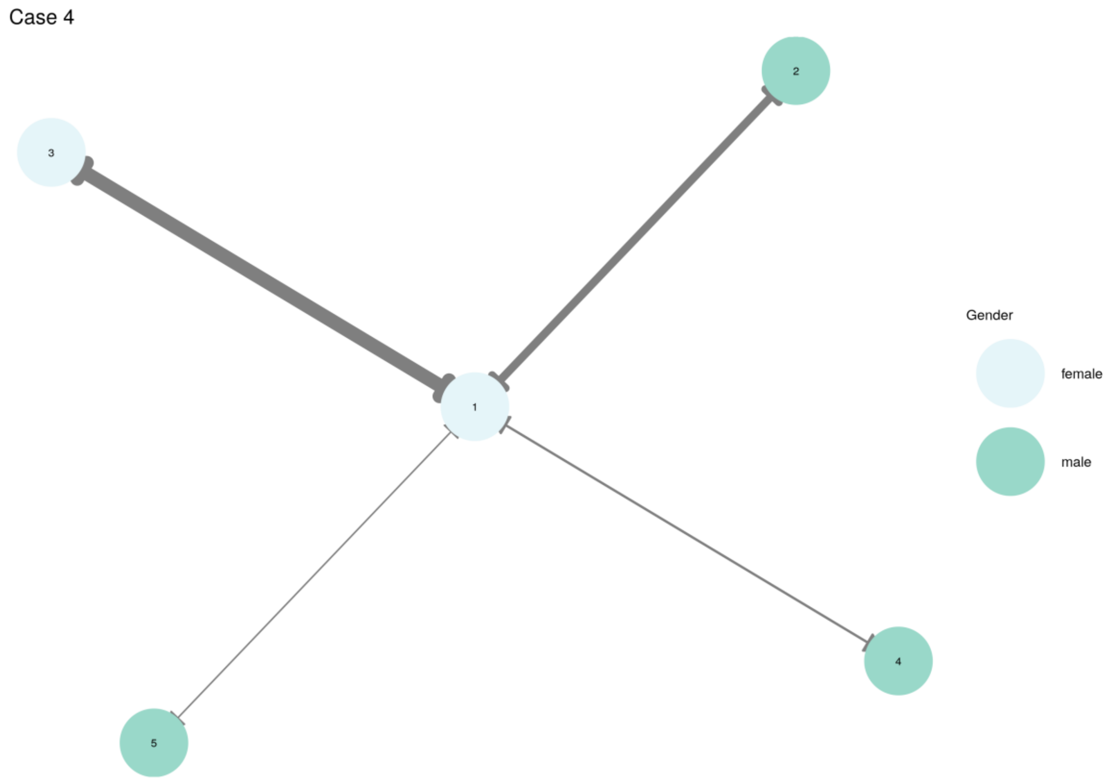

Figure 2.3 Network plot with attribute data.

```
##
## $saveDataVar
## [1] 1
```

Now, if we want to plot 2 attributes in one graph, we can use the `plotNetworks2()` function.

The `plotNetworks2()` function will allow mapping of 2 attributes in the network. The difference between `plotNetworks()` and `plotNetworks2()` is that `plotNetworks()` does not require an attribute, however `plotNetworks2()` requires all 2 attributes to be specified.

In this network, we are interested in gender and ethnicity being plotted on the graph. Attribute 1 is mapped to the gender column in the `attributeData` data frame while attribute 2 is mapped to ethnicity.

```
plotNetworks2(prepNet,
              attribute1 = attributeData$gender,
              attribute1.label = "Gender",
              attribute2 = attributeData$ethnicity,
              attribute2.label = "Ethnicity",
```

```
                attribute.node.labels = attributeData$node,
                attribute.nodesize = 20)
```

```
## $g2plot
```

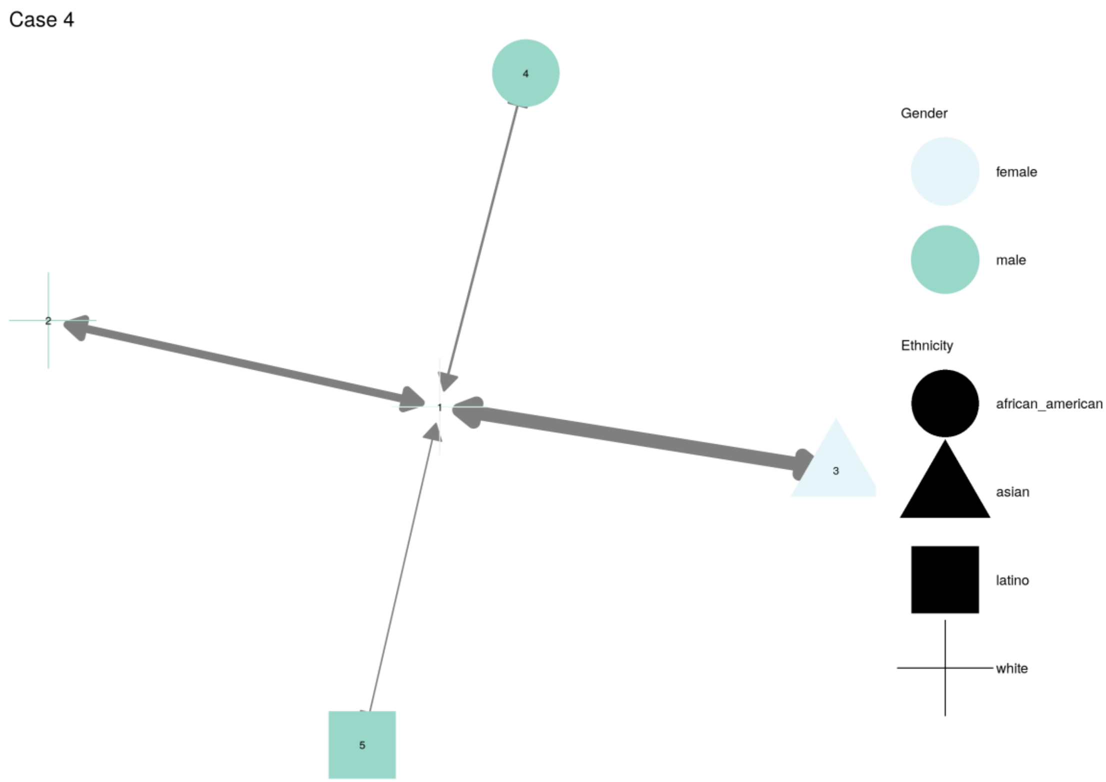

Figure 2.4 Network plot with 2 types of attribute data points.

```
##
## $saveDataVar
## [1] 2
```

Non-Graphical Parameters

The Non-Graphical Parameters are viewed as another exploratory method to viewing how the conversation in the network is progressing. The `plotNGTData()` will return 5 objects:

- 3 graphs:
  - questions per hour versus responses per hour scatter plot
  - number of episodes histogram
  - normalized turn ratio line graph

- combined graph of all of the 3 graphs above
- a data frame containing the normalized turn ratio for each member in the group, the number of questions per hour, responses per hour, raw questions, responses, and total counts for each participant

The `plotNGTData()` function will need the length of the conversation in minutes and the raw question-and-response data.

The first plot, the questions per hour versus responses per hour scatter plot, is calculated by determining the amount of questions and responses a particular node has contributed in that group, then dividing it by the length of the conversation by hours.

$$\text{Questions per hour} = \frac{\text{number of question turns by participant}}{\text{time in hours}}$$

$$\text{Responses per hour} = \frac{\text{number of response turns by participant}}{\text{time in hours}}$$

The second plot, number of episodes histogram, is determined by the length from question-to-question.

The third plot, normalized turn ratio line graph, is generated by first determining the fair share

$$\text{Fair share number of turns} = \frac{\text{total turns}}{\text{number of nodes}}$$

then calculating each individuals actual turn ratio by

$$\text{Normalized talk ratio} = \frac{\text{Number of talk turns by a participant}}{\text{Fair share number of turns}}$$

```
plotNGTData(case4, convoMinutes = 90, iscsvfile = FALSE)

## $ngt_stats
##   participant questions responses questions_hour responses_hour
## 1           1       106        90      70.666667      60.000000
## 2           2         8        30       5.333333      20.000000
## 3           3        11        55       7.333333      36.666667
```

```
## 4            5          5          2       3.333333       1.333333
## 5            4          0         11       0.000000       7.333333
##   total_count normalized_turn_ratio
## 1         196             3.0817610
## 2          38             0.5974843
## 3          66             1.0377358
## 4           7             0.1100629
## 5          11             0.1729560
##
## $episodes_plot
```

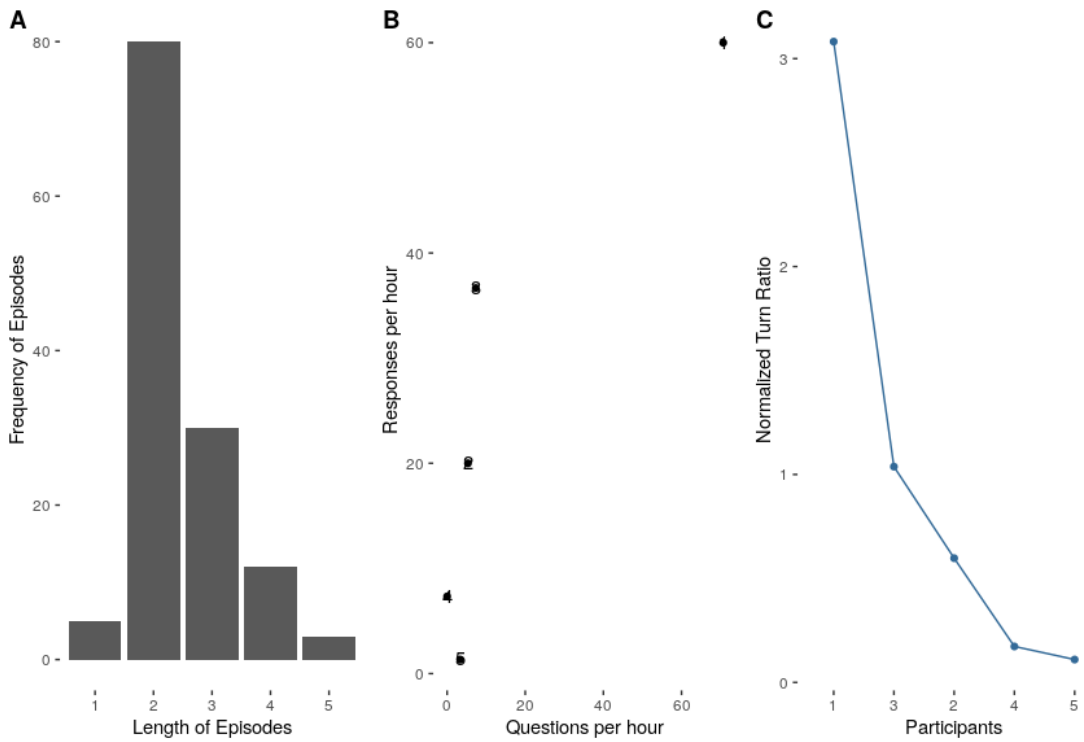

Figure 2.5. Normalized turn ratio plots. A) Histogram of episode lengths, B) Questions per hour versus response per hour, C) normalized turn ratios for each participant in graph.

```
##
## $saveDataVar
## [1] 3
```

Exporting Data

Analysis data can be exported using the `writeData()` command. The `writeData` command will export the data of any of the following objects that have been given. It will accept any of the following modular objects from the `plotNetwork` or `plotNetworks2`, non-graphical parameters plots, weighted edge lists, and the summary generation object. You will need to specify a location on your computer for the data to be exported. Images will automatically export as .tiff at 300 DPI and summary data will be provided as a text file.

```
# Export the summary data to the disk
writeData("Case 4", summaryData, dirpath = tempdir())

## Results have been exported to disk!
## Your results have been exported as:  /tmp/RtmpoFKy7X/Case 4.txt

# Export the network graph with 2 attributes to disk
writeData("Case 4", netplot, dirpath = tempdir())

## Saving 5 x 4 in image
```

Conclusion

In this paper, we have introduced the basics of our `discourseGT` and the basic methodology behind the package. We have demonstrated the basic workflow of this package through an example with data collected from our original paper. Our motivation for creating this package is to allow educational researchers to levy a quantitative method to analyze their groups while only using the most relevant parameters. In future releases of this package, we plan on adding different types of weights that researchers can calculate based on their interest and easily add to their graphs.

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Chapter 2, in part, is currently being prepared for submission for publication of the material. Chai, Albert; Lee, Andrew S.; Lo, Stanley M. The thesis author was the primary investigator and author of this material. The thesis author has contributed to the code development and preparation of the manuscript.

Chapter 3: Quantifying cooperative learning: Measuring the effects of the jigsaw classroom with graph theory

Abstract

Cooperative learning has been gaining interest among educators for its potential to increase student motivation, provide better learning experience, and learning outcomes. One cooperative learning method that this study focused on is the Jigsaw Classroom. We sought to quantify cooperative learning techniques using graph theory in our previously developed method in Chai (2019). In addition, we wanted to see the effects of jigsaw learning on student attitudes and behaviors with academic learning outcomes. There were 112 students that participated in the cooperative learning classroom. Results show that there is no significance among communication patterns throughout the semester, and no dramatic shifts in the various phases of the jigsaw. In addition, there are no significant differences in student achievement among different behavioral outcomes in the jigsaw groups. Implications of the method show that the jigsaw classroom is sufficient for improving student integration among various ethnicities, genders, and academic ability level. However, there is still a weak correlation between academic achievement and the jigsaw classroom. Future directions of the study include more cooperative learning styles conducted on college student populations and implications for university pedagogies of teaching.

Introduction

Cooperative learning has been gaining interest for its potential to create better learning experiences and higher academic achievement for students. Its emphasis is to promote a more social environment to learning rather than isolated individualistic experiences (Johnson, 1998; 2009). Cooperative learning is defined as a group of students working together to achieve a common goal (Cohen, 1994; Gillies, 2016; Johnson, 1998; Johnson, 2009; Manning, 1991; Schul, 2011; Sharan, 2010; Slavin, 1980). Students can only achieve their own individual goal if the group achieves their goal, ensuring that it is in the group's best interest to assist each other (Johnson, 1998; Manning, 1991). Some of the benefits of cooperative learning include increased academic grades (Johnson, 1998; Manning, 1991), improved social skills (Johnson, 1998; Manning, 1991), and better student self-esteem (Johnson, 1998; Manning, 1991; Slavin, 1980). Direct increases in student self-esteem can create more positive attitudes towards school, more friends in school, and impedes segregation between "winners" and "losers" of the classroom (Manning, 1991). However, simply putting students in groups and having them do group work is not cooperative learning (Johnson, 1998). For cooperative learning to be effective, the group activity must be structured in a way to promote interdependence and that each student plays a critical role in completing the task, ability to communicate with one another their ideas and viewpoints in an articulate manner, be able to resolve conflict constructively, individual accountability for their role, and reflection of one's role and contribution to the group (Gilies, 2016; Johnson, 2009; Manning, 1991; Sharan, 2010; Slavin, 1980; Smith, 1996; Tanner, 2003; Watson, 1992). There are many different types of cooperative learning methods such as Jigsaw (Aronson, 1979), Team-Games Tournament (DeVries, 1978), Student Teams-Achievement Divisions (Slavin, 1978), and Small-Group Teaching (Sharan, 1976). However, we are interested in observing the relationship between student interdependency and learning outcomes. One such method that captures our interest is the jigsaw method developed by Elliot Aronson (1979).

The Jigsaw method aims to make every participant in the group an important member of the group (Aronson, 1979; 1980; 2000; 2002) and it emphasizes student mastery of the material and focuses on student motivation (Sharan, 2010). Each member of the group has information about one component of the assignment which cannot be completed without that member's contribution. The basic structure of the jigsaw method is as follows [Figure 1]. Students are divided into small groups of five or six students. Each member is responsible for one part of the assignment in which they will become the expert of the material. In the first phase, students across all groups who are responsible for covering that component of the assignment meet and discuss it, becoming the expert of that section. Then after, they return to their original jigsaw groups to guide their fellow students through that section. The other students in the original groups must rely on that student for that part since they will not have access to information on that portion of the assignment. With this structure, it is to everyone's benefit to cooperate with one another since they all have a common goal to complete the assignment (Aronson, 1979; 1980; 2000; 2002). Some of the benefits of the jigsaw method are that it helps encourage student engagement and feeling empathy for one another (Aronson, 2000). It also helps make individuals feel more valued in the group since everyone must rely on that person since for that information like parsing together a puzzle (Aronson, 1979; 2000).

Prior literature has shown that the fundamental basis of the jigsaw method is focused on student behaviors and attitudes towards learning and their peers than academic performance. The origins of the jigsaw method came after the ruling of *Brown v. Board of Education*, in which students in public schools are desegregating and Aronson sought the need to integrate students without increasing racial tensions further while he was in Austin, Texas (Aronson, 1980). There was a great need for a technique to integrate students of various races because it was common for students to associate with themselves with members of their own races (Aronson, 2014). In addition, racial tensions along with a very competitive environment where students are competition against one another for the best grades, the teacher's attention, and resources increased the need for a cooperative method to reduce segregation among one another (Aronson, 2014). The jigsaw method was successful for integrating students of different ethnicities. An example case is students of White and African Americans reported having greater zeal for school while Mexican Americans reported having lower zeal (Aronson, 1980). Aronson (1980) speculates that Mexican Americans reported lower enthusiasm for schooling because of a language barrier and they are not super comfortable in participating in a group setting.

Consistent with Aronson's theoretical framework, some studies were able to replicate his method. Students in the jigsaw groups reported having higher self-esteem (Anderson, 2001; Blaney, 1977; Lazarowitz, 1994; Walker, 1998; Williams, 2004), became less competitive and more cooperative (Anderson, 2001; Blaney, 1977), had better attitudes towards each other in the group and friendships formed (Anderson, 2001; Blaney, 1977; Hanze, 2007; Lazarowitz, 1994; Premo, 2018; Rego, 2005; Walker, 1998), increased desire to learn (Blaney, 1977; Hanze, 2007; Rego, 2005; Williams, 2004), and improved communication skills/increased communication (Premo, 2018; Rego, 2005). The jigsaw groups also helped increase student involvement for those in minority groups (Blaney, 1977) and for those who did not have much experience (Premo, 2018), consistent with Aronson's findings. However, despite the potential benefits of the jigsaw learning method, some current literature reports mixed results on the jigsaw method in terms of student perceptions and academic achievement. Some studies report no positive effects on student perception of schooling (Moskowitz, 1985) or lack of significant increase in academic performance in the majority population but some improvement in minority populations (Hanze, 2007; Lucker, 1976). However, Moskowitz (1985) also reported that the method was not correctly deployed and in Lucker (1976) found increases in performance mainly in the minority populations when they were paired with highly skilled Whites. In addition, the jigsaw method may cause students to have a negative view of school because they may not be very open to the experience simply because of the high degree of social demands that the student must give or may rather complete the assignments on their own pace than rather with the group pace (Perkins, 2001).

Currently, most studies that observe cooperative learning measure outcomes using student surveys or interviews (Blaney, 1977; Hanze, 2007; Lazarowitz, 1994; Perkins, 2001; Premo, 2018). These surveys or interviews observe how the students mainly experience while working in groups. However, these measures are often subjective and varies from student to student and experience to experience. In addition, other factors, such as prior experiences (Colbeck, 2000) can influence how students perceive the cooperative learning method and influence their survey responses or interviews. We are interested in quantifying cooperative learning in a less subjective manner that does not fully require the use of survey or interview data. This quantification of groups would rely on how well the students converse in these groups and how much productivity is generated from group discussions.

In our earlier paper in Chai et al. (2019), we discuss the current state of observing group work and laid out a methodological framework using graph theory to quantify these group dynamics. Graph theory is a useful complement because of its ability to be able to have multiple pieces of information about each member of the group. In applying graph theory to cooperative learning, we can track how each student behaves from one situation to another situation on an individual level and on a group level. Our main interest variables are density, degree centrality, and centralization for quantification. Density as we defined it as a measure of how much students are conversing with one another. Centralization is a measure if the groups are centered around a specific person, aka the person is the center of attention. Degree centrality is a measure of an individual’s activity within the group relative to other members in the group.

In this study, we want to observe the effects of cooperative learning on a group level, but also on an individual level. We are aiming to answer the following questions:

1. Student behaviors while working in groups
    a. Is there a shift in communication patterns when students change phases in the jigsaw groups?
    b. How does student communication patterns change throughout the semester?
2. Student achievement outcomes while working in groups
    a. Does student communication behavior predict grades?

Methods

Study context and Data Collection

This follow up study is based on an earlier study conducted in Premo (2018). Participants in the study were from an undergraduate introductory-level biology course at a large public research university. There were 483 enrolled undergraduate students, but 112 students that consented to participate in the experimental groups. The remainder students did not consent in the experimental group or were in the control group. Sections were executed simultaneously to reduce variance at the time of the study. Students were tasked in completing activities regarding the SEA-PHAGES laboratory. Three iterations of data collection were conducted, each with 2 phases. There were also a quiz following each iteration for a total of three quizzes. Topics that were discussed in each of the iterations are phage-bacteria interactions (iteration 1, week 2), experimental troubleshooting (iteration 2, week 4), and experimental replication (iteration 3, week 6). In phase 1 of the groups, students in groups of 4 were tasked to answer a single question to become the "expert" of that question to the best of their ability. In phase 2, new groups of 4 were formed with at least one "expert" of each of the questions. These students in turn became specialized in their respective question, creating an interdependence in order to complete all the questions. A variation from the traditional jigsaw technique is that an additional phase was added where all the groups come together as a class to discuss the best answers to the questions. This study was approved by the Washington State University Internal Review Board (IRB Protocol # 15680-003).

Graphical parameters processing

Data was processed using R version 3.5.1 (R Core Team, 2018). We utilized the discourseGT R package that we have previously developed (Chai, 2019) to extract the density, centralization, degree centrality, number of nodes, edges, and weighted edges for each group. The talk order of each student in each group was recorded for each phase and iteration. Episodes are defined by the length of the talk turns the group spent discussing a question.

Modeling

For question 1, we ran OLS regression models using Stata SE 15.1 (Stata Corp, 2018) observing the effects of density and centralization on iteration and phase of the jigsaw groups. We ran 2 models, a crude model with just the density or centralization against the phase or iteration, and a full adjusted model with the crude model plus covariates (Gender, Ethnicity, and Class standing). We created indicators for the covariates based on the majority parameter, such as majority female, majority male or equal for gender. We defined majority as greater than 50% of the group composition. Groups that had 50% - 50% composition were defined as equal for that parameter. For question 2, we checked for students that have participated in all iterations and phases of the study. However, there was not a sufficient sample size among using all three iterations of the data, so we used only iterations 1 and 2 to get a larger N. We used two metrics to observe student communication behaviors, degree centrality and normalized turn ratios. We calculated the degree centrality values and normalized turn ratios using the functions in the discourseGT R package, respectively. Each parameter was then scaled accordingly and clustered using a K-means clustering algorithm with the base R package. Another method that was considered was Multilevel Modeling (MLM); however, because we did not meet the sufficient requirements to run such an analysis, such as sufficient sample size (Hox, 1994; Woltman, 2012), running a MLM would be inappropriate. The K-means algorithm had recommended 4-5 clusters, but because the clusters were overlapping at 3 clusters, we decided to use 2 clusters instead since they were distinct enough (Cormack, 1971; Steinley, 2006). After identifying the clusters for each student, we performed an OLS regression using Stata SE 15.1 with normalized overall grades and the average of their quiz 2 and quiz 4 grades.

Results

<u>Sample population</u>

The sample size of the population includes 112 students. 72% of the population is White, 12% Latinx, 3.5% African American, and 9% Asian. 75% of the students are female and 25% of students are male. 75% of the students are also underclassmen. 12% of the population have never taken an undergraduate science course. The average normalized lab score is 92.3% with a standard deviation of 6.08%. Descriptives of the student population can be found in Table 1.

<u>Graphical processing</u>

A selection of groups processed with the discourseGT package are shown in Figure 2. Three iterations along with their individual phases (phase 1 and 2) are shown.

<u>Student behaviors while working in groups</u>

Question 1a: Is there a dramatic shift in communication patterns when students change phases in the jigsaw groups?

To observe the effects in patterns in between phases, we used density and centralization network parameters with an OLS regression model. There were 68 groups used in this comparison with no missing covariates. There were no significant differences in the phases or covariates with either density or centralization. Beta estimates for the model are available in Table 3.

Question 1b: How does student communication patterns change throughout the semester?

For observing communication patterns throughout the semester, we used density and centralization network parameters with an OLS regression model. There were 39 available groups that had no missing covariates. We only used phase 1 data from all three iterations because those groups remained the same throughout the entire study. We did not see any significant differences in between the iterations of the model for either centralization or density. However, while using centralization as the outcome, we saw being male as a significant covariate in the full adjusted model. For density, we saw being in the minority class as a significant covariate in the full adjusted model. Beta estimates for the model are available in Table 2.

Student achievement outcomes while working in groups

Question 2: Does student communication behavior predict grades?

We used two different parameters to predict student communication patterns, degree centrality and normalized turn ratio. Degree centrality is an unweighted network parameter which only records the first occurrence interaction between the person and other nodes in the network. Normalized turn ratio is non-graphical parameter, but it calculates the fair share of turns for each person in the network and divides that by the number of turns that the actual person has taken. This in turn allows us to apply a weight to the parameter for each person.

Using degree centrality, we only have 22 participants that had all iteration 1 and 2 instances along with phase 1 and 2 data (all 4 data points) along with no missing covariates. Using k-means clustering, we identified 2 clusters. Cluster plots are available in Figure 3. Cluster 1, the average degree centrality normalized value is 0.222 and in cluster 2, the average value is 0.274. Demographics of each of the clusters are displayed in Table 4. Students in cluster 1, N = 8, reported having a lower degree centrality norm, but they were composed of sophomores, juniors, and seniors, students having more experience in college. Their average scores were also higher, with lab quiz 2 at 1.94 and lab quiz 4 at 1.88 (lab scores were out of 2 points total) and normalized final lab score of 92%. Students in cluster 2 had a higher degree centrality norm than cluster 1 but had lower test scores. Students in cluster 2, N = 14, were composed of mainly freshmen and sophomores. The average lab quiz 2 score was 1.68, lab quiz 4 was 1.77, and normalized lab score at 86%. After creating the clusters, we ran an OLS regression against their normalized class average scores and the average of quiz 2 and 4 scores. We only used the average of quiz 2 and 4 scores because we only included only iterations 1 and 2 in the initial clustering. We did not see any significant differences in between the clusters with either normalized scores or averaged quiz grades as the dependent variable. However, Latinx was a significant ethnicity covariate in the normalized class score in the full adjusted model. Beta estimates of the model are found in Table 5.

Using normalized turn ratios, we only have 21 participants that had all iteration 1 and 2 instances with phase 1 and 2 data (all 4 data points) along with no missing covariates. We identified 2 clusters using K-means clustering, similar to using degree centrality. Cluster plots are available in Figure 3. In cluster 1, N= 8, the average normalized turn ratio is 0.850 and in cluster 2 is 1.00. Demographics of each of the clusters are displayed in Table 4. Students in cluster 1 also reported having higher test scores and composed of sophomore and juniors, with average quiz grades of 1.93 and 1.86 for quiz 2 and quiz 4, respectively, and a normalized lab score of 91%. Students in cluster 2, N= 13, are composed of freshmen and sophomores with an average quiz scores of 1.68 and 1.77 for quiz 2 and quiz 4, respectively, and a normalized lab score of 86%. Students in cluster 1 reported having an average normalized turn ratio of 0.85 but higher test scores and students in cluster 2 had an equal average normalized turn ratio of 1.00 but lower test scores. After determining the clusters, we ran an OLS regression against normalized lab scores and the average of quiz 2 and 4 scores. There was no significant difference in between clusters in either dependent variable. However, in the normalized dependent variable in the full model, Latinx was a significant ethnicity covariate. Beta estimates of the model are in Table 6.

Discussion

Student participation in jigsaw groups reported lack of significance in improvement in communication patterns over time. Three interactions in participating the jigsaw technique is not sufficient enough to elicit student communication improvements. Most cooperative learning methods are primarily used as supplements to instruction and are not used as frequently nor properly (Slavin, 1983). However, current literature that explores friendships in students have shown that student perceptions to academics can be perceived by their peers (Berndt, 1999). Student friendships can be positively influenced by the activities that their peers are also doing (Berndt, 1999). While in our case, the students in these groups may or may not be friends, the implications of observing these communication patterns may provide ground to how involved the students are in the course matter and their major in college.

There was also not a dramatic shift in communication patterns when students change into their reassigned groups. We believe that this assumption holds true because of the postulate that there is a common goal that all group members must achieve (Slavin, 1983; 1987). There was no significant difference in the centralization value of the group nor density among both phases among all iterations. The structure of the group work is essential to the cooperativity of students as a lack of structure will cause participants to not cooperate as well (Fehr, 2004; Fowler, 2010; Nowak, 2006; Ohtuski, 2006). The structured nature of the jigsaw groups creates a social norm for students to cooperate with one another even though there might be some participants that do not want to cooperate. Without this social norm or structured cooperation in the jigsaw technique, students would not likely to cooperate with one another and would prefer to defect than cooperate (Fehr, 2004; Fowler, 2010; Nowak, 2006; Ohtuski, 2006). This norm in the structured nature of the jigsaw groups also give reason for students to cooperate more than defect, as evidenced from various game theory experiments (Fehr, 2004; Fowler, 2010; Nowak, 2006; Ohtuski, 2006). This also shows that the jigsaw method is effective in the integration and inclusion of students of different backgrounds. OLS regressions did not find any significant covariates in gender, ethnicity, nor class standing in the full adjusted models for both centralization and density network parameters. Our analysis supports the current literature on the jigsaw technique supportive of student integration (Eilks, 200; Maceiras, 2011; Smith, 1991). In addition, it can also be supported by the original purpose of the jigsaw technique as a method for integration after the *Brown v. Board of Education* decision as well (Aronson, 2002; Slavin, 1985)

With student behaviors and learning outcomes with the jigsaw technique, we did not see any correlations between the two. Students using both degree centrality and normalized turn ratio reported having an inversed relationship with behavior and outcomes. In one cluster, members had a lower degree centrality and normalized turn ratio but had higher test scores, and in the other cluster, members had higher degree centrality and normalized turn ratios but lower test scores. We postulate for this lack of correlation in part due two reasons, the influences of novices teaching other students and to of a lack of a shared group incentive in the jigsaw technique. An experiment done by Berger et al. (2015) has shown that the quality of how students teach each other effects their perception of the material. If students have lack of experience teaching other students the material, especially if they are the ones that everyone relies on for that part of the jigsaw material, then the other students may not understand the material as well

(Berger, 2015). In addition, the topics of the material may also play a role as well with more difficult topics resulting in lower correlations with performance and jigsaw groups (Berger, 2015). With the incentive structure of the jigsaw groups, students are still tested individually rather than have a collective incentive to do well together (Slavin, 1983; 1989; 1996; 2011). This missing component in what is needed for a cooperative technique to be fully effective, can reduce student learning outcomes even though that there is some cooperation among students through a shared common goal (Slavin, 1983; 1989; 1996; 2011). A shared common goal is not alone enough to incentive students to do well. Previous studies have shown that jigsaw groups are effective in improving student behavioral outcomes, such as student views on learning, their peers, and the material, it does not have much significant effect on academic learning outcomes (Moskowitz, 1985). Additional literature has also postulated the successful nature of adding the critical incentive component in cooperative learning (Thomas, 1986). The original intent of the jigsaw learning method is more focused on behavioral outcomes than academic achievement outcomes (Slavin, 1985).

There are some limitations with the current study. There were not enough students by the time of the iteration 3 portion of the study, so we could not do a full clustering analysis with all 6 time points of data for question 2. In addition, the limited sample size available to do such clustering was also limited as well. A lack of statistical power may contribute to only the formation of two clusters where the optimal number of clusters was 4-5 with the algorithm. However, the increasing the statistical power may also change the number of optimal clusters as well. This was consistent for both using a weighted and unweighted metric. In addition, there was only three iterations of data collection with the jigsaw technique. Therefore, we were not able to fully grasp the development of student's behavior overtime in a full semester. We cannot draw any definitive conclusions about whether the jigsaw technique is effective or ineffective in improving peer attitudes while in group settings.

However, there are some strengths of this study. Most current studies on the jigsaw technique has been performed on grade and middle school students (Slavin, 1987; 1990) rather than at the high school (Eilks, 2005) and college level (Fraser, 1977; Smith, 1991). Cooperative learning techniques are more targeted towards elementary and middle school students and collaborative learning more towards higher education (Bruffee, 1995; Matthews, 1995). Collaborative learning differs from cooperative learning by educators having a more “hands-off” role in student groups, such as intervening in groups or evaluating the quality of group processes, and encourages dissent in social processes (Bruffee, 1985). Collaborative learning also does not remove the competitive element as cooperative learning does, rather it shifts it from individual competition to group competition (Bruffee, 1985). This study adds to the growing literature on the parts of cooperative learning techniques that are effective in higher education and does present the need for more studies to be conducted in such setting. Notwithstanding the current limitations, some future directions that can be done to address more in thorough the jigsaw technique and cooperative learning, in general, in a college classroom include longer duration of student participation in the jigsaw classroom, other cooperative learning methods such as Student Teams-Achievement Divisions (STAD) (Slavin, 1978) that incorporate a shared group incentive like a group grade. However, even beyond expanding to the knowledge of the literature of cooperative learning in higher education, it would also require university intervention to make cooperative learning a more used technique in the classroom. Current norms in college classrooms view group work as an inefficient process of learning mainly because of the large content of material required to be covered in the time frame and current rote memorization processes are a more efficient use of instruction time (Moskowitz, 1983; Phipps, 2001). However, the benefits cooperative learning is something that should not be ignored in current instruction and should be considered as a part of instruction.

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Table 3.1. Student demographics of the Jigsaw Study. N = 112

Demographics	**Percentage**
Ethnicity	
Black	3.57
Asian	8.93
Latinx	11.61
Multiracial	2.68
White	72.32
Gender	
Female	75.00
Male	25.00
Class Standing	
Freshman	23.21
Sophomore	51.79
Junior	19.64
Senior	4.46
Non-Degree Undergraduate	0.89
Number of Prior Undergraduate Science Courses	
0 courses	11.61
1-2 courses	21.43
3-5 courses	49.11
6-10 courses	16.07
>10 courses	1.79
Grades	**Mean (SD)**
Lab Quiz 2	1.757 (0.336)
Lab Quiz 4	1.757 (0.457)
Lab Quiz 6	1.158 (0.521)
Normalized Lab Score	0.923 (0.0608)

Table 3.2. Regression estimates among phases and network parameters centralization and density for observing effects of dramatic shifts among phases.

	Centralization		Density	
	Crude	Full	Crude	Full
	b/se	b/se	b/se	b/se
Phase (Ref: Phase 1)				
Phase 2	0.019 (0.059)	0.013 (0.063)	-0.008 (0.045)	-0.008 (0.048)
Gender (Ref: Female)				
Male		0.208 (0.151)		-0.139 (0.115)
Equal		-0.071 (0.091)		0.059 (0.069)
Ethnicity (Ref: Majority)				
Minority		0.003 (0.147)		-0.103 (0.112)
Equal		0.023 (0.113)		-0.050 (0.086)
Class Standing (Ref: Underclassmen)				
Upperclassmen		0.005 (0.118)		-0.032 (0.090)
Equal		-0.029 (0.112)		0.044 (0.085)
Constant	0.273*** (0.039)	0.276*** (0.048)	0.794*** (0.030)	0.800*** (0.036)
Number of Groups	68.000	68.000	68.000	68.000

Table 3.3. Regression estimates among iterations and network parameters centralization and density for observing effects among student communication patterns throughout the semester.

	Centralization		Density	
	Crude	Full	Crude	Full
	b/se	b/se	b/se	b/se
Iteration (Ref: Iteration 1)				
Iteration 2	-0.033 (0.080)	-0.016 (0.079)	0.050 (0.062)	0.076 (0.061)
Iteration 3	0.054 (0.117)	0.140 (0.112)	-0.105 (0.090)	-0.150 (0.087)
Gender (Ref: Female)				
Male		0.585* (0.221)		-0.223 (0.171)
Equal		-0.166 (0.118)		0.112 (0.091)
Ethnicity (Ref: Majority)				
Minority		0.192 (0.164)		-0.340* (0.127)
Equal		0.100 (0.126)		-0.106 (0.098)
Class Standing (Ref: Underclassmen)				
Upperclassmen		-0.109 (0.117)		0.027 (0.091)
Equal		0.044 (0.114)		-0.013 (0.088)
Constant	0.279*** (0.053)	0.249*** (0.058)	0.789*** (0.041)	0.806*** (0.045)
Number of Groups	39.000	39.000	39.000	39.000

Table 3.4. Student demographics for each cluster class by degree centrality and normalized turn ratio observing academic performance and student behaviors in groups.

	Degree Centrality			Normalized Turn Ratio		
	Cluster 1	**Cluster 2**	**Total**	**Cluster 1**	**Cluster 2**	**Total**
Degree Centrality Normalized	0.2219	0.2736	0.2548	NA	NA	NA
Normalized Turn Ratio	NA	NA	NA	0.8503	1.009	0.9567
	%					
Ethnicity						
Black	12.50	0.00	4.55	14.29	0.00	4.76
Asian	12.50	0.00	4.55	NA	NA	NA
Latinx	0.00	14.29	9.09	0.00	14.29	9.52
White	75.00	85.71	81.82	85.71	85.71	85.71
Gender						
Female	87.50	71.43	77.27	85.71	71.43	76.19
Male	12.50	28.57	22.73	14.29	28.57	23.81
Class Standing						
Freshman	0.00	64.29	40.91	0.00	64.29	42.86
Sophomore	50.00	35.71	40.91	57.14	35.71	42.86
Junior	37.50	0.00	13.64	42.86	0.00	14.29
Senior	12.50	0.00	4.55	NA	NA	NA
Number of prior science courses						
0 courses	0.00	7.14	4.55	0.00	7.14	4.76
1-2 courses	37.50	28.57	31.82	42.86	28.57	33.33
3-5 courses	50.00	64.29	59.09	42.86	64.29	57.14
6-10 courses	12.50	0.00	4.55	14.29	0.00	4.76
	Mean (SD)					
Grades						
Lab quiz 2	1.94(0.18)	1.68(0.36)	1.77(0.33)	1.93(0.19)	1.68(0.36)	1.76(0.33)
Lab quiz 4	1.88(0.23)	1.77(0.46)	1.81(0.39)	1.86(0.24)	1.77(0.46)	1.80(0.40)
Lab quiz 6	0.94(0.42)	1.38(0.62)	1.22(0.58)	0.89(0.43)	1.38(0.62)	1.21(0.60)
Normalized lab score	0.92(0.04)	0.86(0.09)	0.88(0.08)	0.91(0.03)	0.86(0.09)	0.88(0.08)

Table 3.5. Regression estimates observing clusters generated from student behaviors using degree centrality by normalized lab grade and average of quiz grades 2 and 4.

	Normalized		Quiz Avg 2/4	
	Crude	Full	Crude	Full
	b/se	b/se	b/se	b/se
Cluster (Ref: Cluster 1)	-0.056 (0.036)	0.001 (0.053)	-0.183 (0.145)	-0.358 (0.280)
Ethnicity (Ref: White)				
African American		-0.055 (0.133)		0.021 (0.703)
Asian		0.125 (0.098)		-0.219 (0.519)
Latinx		-0.189** (0.057)		0.096 (0.303)
Class Standing (Ref: Freshmen)				
Sophomore		0.031 (0.047)		-0.164 (0.251)
Junior		0.039 (0.076)		-0.191 (0.403)
Senior		0.000 (.)		0.000 (.)
Gender (Ref: Female)				
Male		0.007 (0.052)		0.035 (0.275)
Number of UG Sci Courses (Ref: 0 courses)				
1-2 courses		-0.055 (0.079)		-0.221 (0.418)
3-5 courses		-0.056 (0.081)		0.111 (0.428)
6-10 courses		0.000 (.)		0.000 (.)
Constant	0.916*** (0.028)	0.925*** (0.086)	1.906*** (0.116)	2.108*** (0.458)
Number of Students	22.000	22.000	22.000	22.000

Table 3.6. Regression estimates observing clusters generated from student behaviors using normalized turn ratio by normalized lab grade and average of quiz grades 2 and 4.

	Normalized		Quiz Avg 2/4	
	Crude	Full	Crude	Full
	b/se	b/se	b/se	b/se
Cluster (Ref: Cluster 1)	-0.045 (0.037)	0.001 (0.053)	-0.170 (0.155)	-0.358 (0.280)
Ethnicity (Ref: White)				
African American		-0.055 (0.133)		0.021 (0.703)
Latinx		-0.189** (0.057)		0.096 (0.303)
Class Standing (Ref: Freshmen)				
Sophomore		0.031 (0.047)		-0.164 (0.251)
Junior		0.039 (0.076)		-0.191 (0.403)
Gender (Ref: Female)				
Male		0.007 (0.052)		0.035 (0.275)
Number of UG Sci Courses (Ref: 0 courses)				
1-2 courses		-0.055 (0.079)		-0.221 (0.418)
3-5 courses		-0.056 (0.081)		0.111 (0.428)
6-10 courses		0.000 (.)		0.000 (.)
Constant	0.905*** (0.030)	0.925*** (0.086)	1.893*** (0.127)	2.108*** (0.458)
Number of Students	21.000	21.000	21.000	21.000

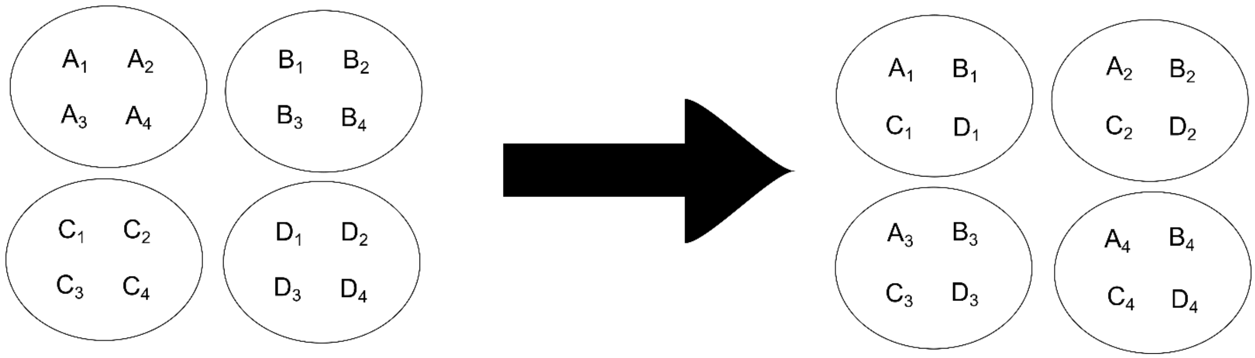

Figure 3.1. An overview of the jigsaw classroom. Students in the first phase, denoted by the left of the arrow meet to discuss one question and become an expert on that question. Students in the initial groups will be responsible for teaching that question to remainder of the students in their new groups. In the second phase, denoted by the right of the arrow, groups are shuffled where each group has an expert for each question from the first phase.

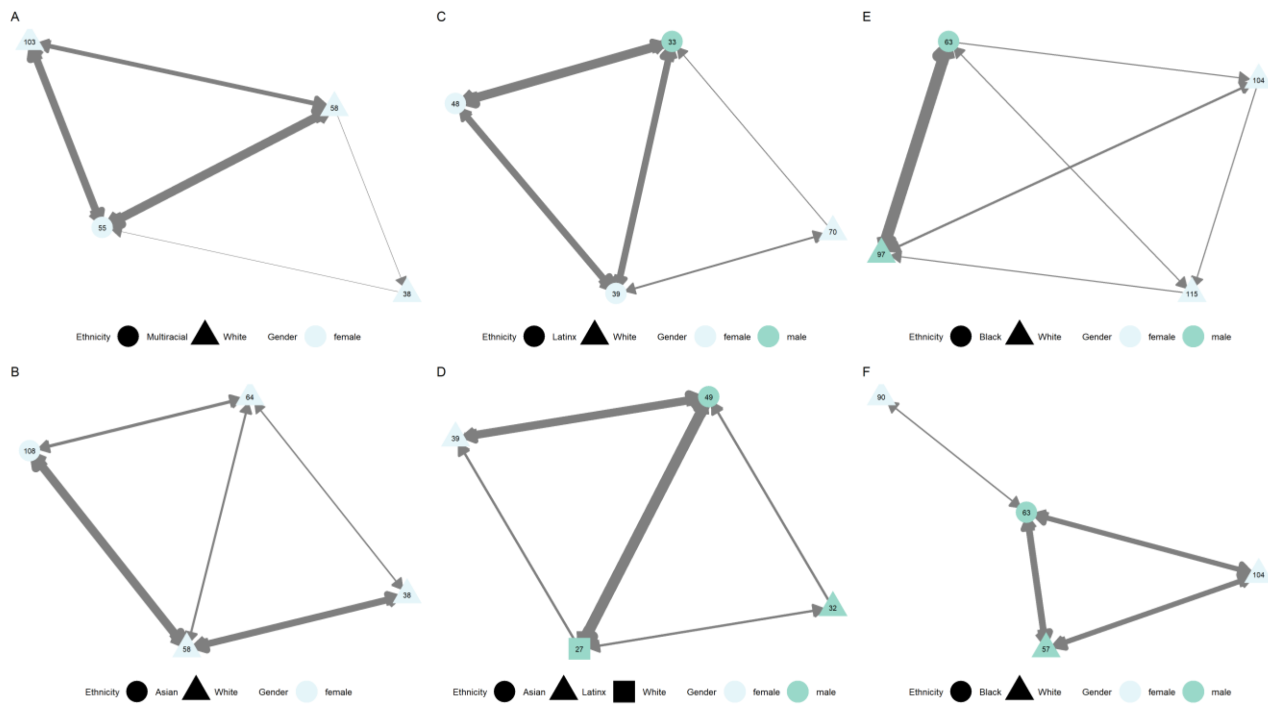

Figure 3.2. Graphical representation of a select number of groups by each iteration and by phase. Graphs A and B are of iteration 1, C and D of iteration 2, and E and F of iteration 3. Graphs A, C, and E are phase 1 groups, while graphs B, D, and F are phase 2 groups.

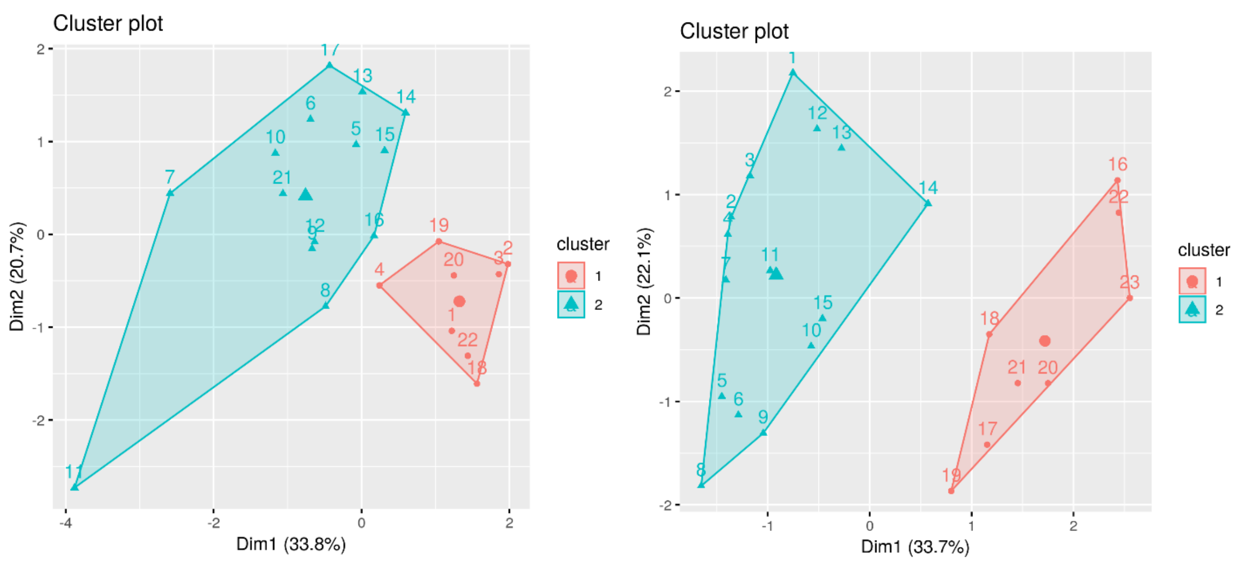

Figure 3.3. Clustering plots for grouping student behaviors in groups. The cluster plot on the left are using degree centrality to cluster students while the plot on the right uses normalized turn ratio to cluster students.

Chapter 3, in part, is currently being prepared for submission for publication of the material. Chai, Albert; Lee, Andrew S., Premo, Joshua T., Cavagnetto, Andy R., Lo, Stanley M. The thesis author was the primary investigator and author of this material. The thesis author has contributed to the analysis and preparation of the manuscript sections: introduction, methods, results, and conclusion.

CONCLUSION

Graph theory can be an effective method to track how student converse in group settings. In the first paper, a methodological and theoretical framework using graph theory and the fundamental basis of how students learn in group settings was developed. In addition, a case with students at a supplemental learning instruction has reveal several different types of group behavioral patterns. In the second paper, an open-source tool based on the first paper methodological framework was developed. In the third paper, we test our methodological framework in a cooperative learning method, specifically the jigsaw method. The jigsaw method was effective in integrating students of various backgrounds together but has various effects on academic achievement.

In the first paper, a theoretical and methodological framework were discussed in implementing graph theory to track student conversations in group settings. Implications from that study in proof of concept cases have shown 2 different types of groups on the spectrum. One case is where the groups are all connected, resembling an all connected graph. The other case is where members only talk to the peer facilitator, but not to each other. Implications of these graphs have revealed that the positioning of the classroom (Borgatti, 2009; McCorskey, 1978; Wannarka, 2008), along with the gender and race composition (Cohen, 1972; Craig, 1986; Strodtbeck, 1956; Webb, 1984) can play a role in how students converse in a group setting. However, limitations in this chapter include that the weights of each of the graphs are based on the number of occurrences a pair of nodes had conversed right after each other rather than the content. The implications of this limitation include that a person may have a small weight but may say very insightful thoughts whereas someone with a large weight may have said confirmatory statements, such as “okay”, or off-topic statements.

The second paper discusses the development of an open-source implementation of the methodological framework. The package is based from other existing network analysis packages, such as igraph (Csardi, 2006), sna (Butts, 2016), and network (Butts, 2008), but streamlines the necessary functions from those packages aligned with our methodology to provide educational researchers a place where everything they need to run their analysis.

In the last paper, we wanted to test this methodology on another type of group learning method, so we utilized the jigsaw cooperative learning method. The jigsaw method is more focused on behavioral outcomes rather than academic outcomes as a method that was idealized after the *Brown v. Board of Education* decision. There were no correlations between behavioral outcomes and academic achievements, and there were no differences in the different phases of the groups. Even though the groups had different student compositions, from gender, ethnicity, and prior academic experiences, there were no differences among the groups. This supports the original intent of the jigsaw method to integrate students of various backgrounds as everyone plays an important role in the groups success and holds a critical piece of the puzzle towards that success (Aronson, 1979). Reasons for the lack of correlations between behaviors and academic performance in the jigsaw have been theorized in the lack of shared group incentives that all cooperative learning methods should entail for it to be effective (Slavin ,1983). However, we were not able to make any definitive conclusions about the length of exposure primarily because there were only three instances when the data was collected, and current literature studies have cited instances of at least 10 times that the jigsaw was performed in the classroom. While with its limitations, this study has helped contribute to the growth of using cooperative learning in higher education, whereas typically used in elementary and middle school students.

Across the two studies that have been conducted, the methodology has been successful at quantifying student behaviors in group settings. We tested this methodology in two different settings, supplemental instruction groups and jigsaw cooperative learning groups. Findings in both settings have been consistent on what has been also observed in the literature as well, respectively. Some of the current limitations across all these studies mainly include a limited sample size and conditions. We cannot say that these are representative of their sample populations. However, the trends that are observed are similar to what has been observed in the current literature.

Future works may include replicating these conditions in larger populations over a longer period, such as the jigsaw groups with larger durations or using another type of cooperative learning method to observe student learning outcomes. Having a sufficiently large number of groups would help increase the statistical significance of the given results and reveal various different types of patterns across groups. The current patterns that we currently observed are based on the demographic composition of these groups and the student's ability, but other types of patterns can emerge if factoring how individual motivations, such as value of group work and prior experiences, can affect the overall group dynamics. In addition, sense of belonging, culturally and/or socially, can also yield different patterns to how individuals contribute to the group conversations. Overall, this methodology can be used to track how students are conversing with one another in a group setting, which in turn can be a useful tool for instructors and teaching assistants to make their student group learning experiences more inclusive.

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APPENDIX

Subgraphs and special subgraphs

A subgraph is a smaller graph within a graph. Below are some special subgraphs useful for determining highly connected groups.

Neighborhood of a node: The immediately connected nodes

The neighborhood of a node is the set of nodes that are connected to it by a single edge (Godsil, 2001).

K-core: Subgraph with nodes of degree at least K

The K-core of a graph is the subgraph including as many nodes as possible where each node has degree at least K (Borgatti, 2013) (Figure S1).

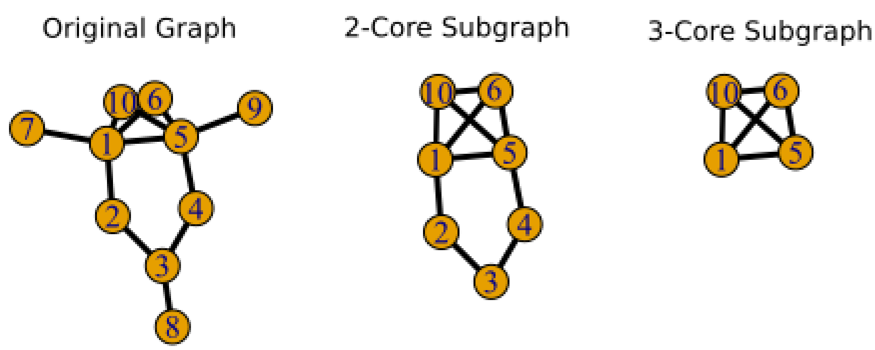


Figure S1. A graph with its 2-core and 3-core subgraphs. As the k in k-core increases, there are fewer vertices that satisfy the degree requirement. Each vertex in the k-core subgraph has degree at least k among the vertices in the subgraph.

Directed graphs, weakly and strongly connected

A directed graph is weakly connected if replacing all of its directed edges with undirected edges results in a connected graph (Godsil, 2001). A weakly connected component is a subgraph of a directed graph that is weakly connected (Figure S2).

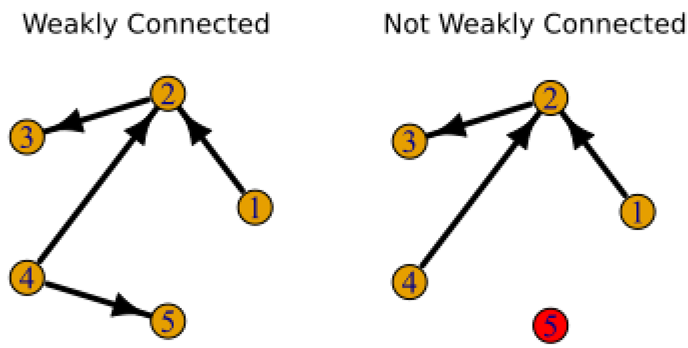


Figure S2. Two graphs, one weakly connected and the other not weakly connected. The right graph is not weakly connected because converting the directed edges to undirected does not result in a connected graph (node 5 is still not connected).

A directed graph is strongly connected if there is a directed path between any two vertices (Godsil, 2001).

A strongly connected component is a subgraph of a directed graph that is strongly connected (Figure S3).

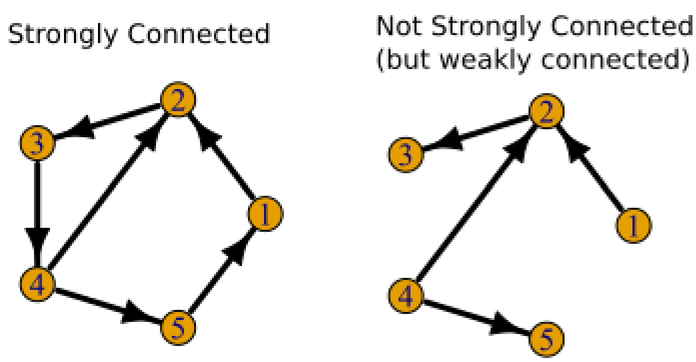


Figure S3. Two graphs, one strongly connected and the other not strongly connected. The right graph is not strongly connected because there is not a directed path from 3 to 2, from 2 to 1, 5 to 4, etc. Both graphs, however, are weakly connected.

<u>Clique: A fully connected subgraph</u>

A clique is a subgraph that is fully connected, meaning that all edges that could possibly exist do exist (Godsil, 2001) (Figure S4).

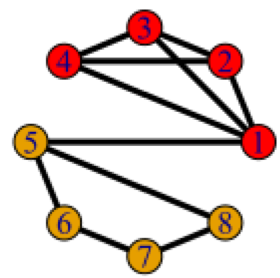


Figure S4. A graph with a clique (nodes 1, 2, 3, and 4). Nodes 5, 6, 7, and 8 are not included in the clique because nodes in a clique must all be directly connected to each other.

Graph metrics

In this section, we define a variety of metrics one can calculate about graphs, namely the parameters output by the R script mentioned in Methodology section.

Order and size: Counting

The order of a graph is the number of vertices is has, i.e. the number of elements in the vertex set V. Similarly, the size of a graph is the number of edges a it has, i.e. the number of elements in the edge set E. The order is sometimes denoted |V|, and size is denoted |E|.

Modularity: Graph divisions

Modularity is a measure of how well one can separate a graph into distinct groups. It is used to identify densely connected community structures within the entire graph. To calculate modularity, one must have a graph and a group assignment to each vertex already in mind; in other words, modularity is specific to a graph as well as how one picks the groups.

A graph and group assignment with high modularity (close to 1) indicates that there are many edges within the groups that were picked and fewer edges connecting groups. A graph and group assignment with low modularity (close to -1) indicates the opposite: fewer edges within groups and more edges connecting groups.

The modularity of a graph and group assignment is the sum over all groups of the fraction of edges within the given group minus the expected fraction of edges within the given group. Another way to visualize it is to let M be a list of all group numbers, then the modularity is given by the following formula:

$$Modularity = \sum_{m \in M} \left(\frac{\#\ of\ edges\ within\ group\ m}{\#\ of\ edges} - \frac{\#\ of\ possible\ directed\ edges\ within\ group\ m}{\#\ of\ possible\ directed\ edges\ in\ the\ graph} \right)$$

If we pick a particular group number (e.g. m = group 1), then the first fraction is "edge density" within group 1. The second fraction acts as a normalizing term; it accounts for the "edge density" within group 1 that is expected to occur if all edges in the graph were given random edge assignments. In Figure S5, the same graph is used to show the effect of different group assignments.

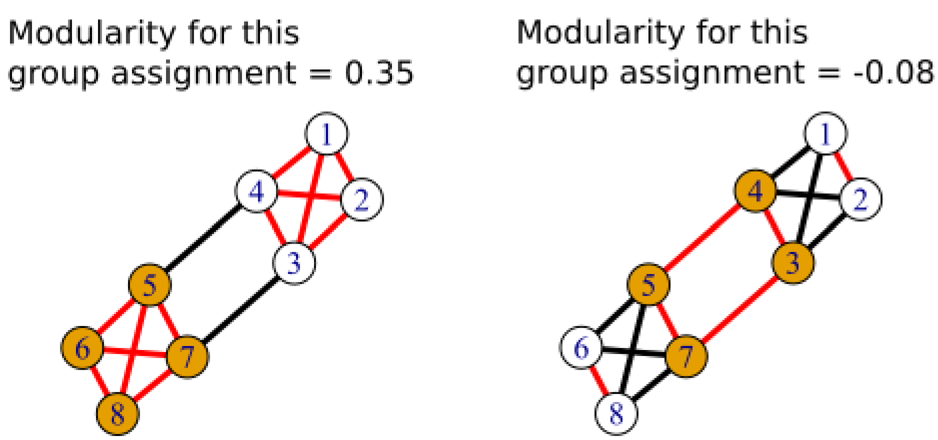


Figure S5. The same graph with different group assignments and their associated modularities. Edges colored red are within the group, and edges colored black are between groups. Group assignments that have more edges within the group and fewer edges between groups have higher modularity.

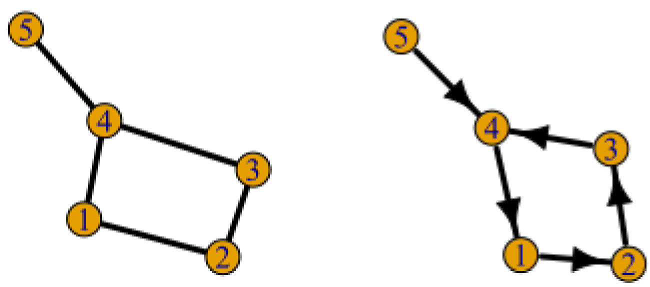
Graph representation and visualization

This section focuses on how to represent a graph (in code) and different ways graphs can be drawn for clarity.

Adjacency matrix: A graph representation

The adjacency matrix for a graph is a n by n (also written n x n) matrix where n is the number of vertices in the vertex set. Each number (or entry) in the matrix--take for example the number in the ith row and jth column--corresponds to whether or not there is an edge from vertex i to vertex j (Godsil, 2001).

Note that the adjacency matrix of an undirected graph is symmetric, meaning it is the same when flipped across the diagonal that goes from the top-left corner to the bottom-right corner. On the other hand, the adjacency matrix for a directed graph is not necessarily symmetric. See Figure S6 for examples of adjacency matrices for undirected and directed graphs.



Adjacency Matrix:

	1	2	3	4	5
1	0	1	0	1	0
2	1	0	1	0	0
3	0	1	0	1	0
4	1	0	1	0	1
5	0	0	0	1	0

Adjacency Matrix:

	1	2	3	4	5
1	0	1	0	0	0
2	0	0	1	0	0
3	0	0	0	1	0
4	1	0	0	0	1
5	0	0	0	1	0

Figure S6. Adjacency matrices for two graphs, undirected and directed. The undirected graph has a symmetric adjacency matrix while the directed graph does not.

Projections: Different ways to draw a graph

Our R script allows users to display the graphs created in various ways, which are covered in this section.

The Fruchterman Reingold and Kamada Kawai projections are force-directed layouts, meaning they use attractive and repulsive forces inspired by physics to place vertices and edges (Wild, 2016).

The Reingold Tilford projection clearly encodes the depth of a graph, which may be needed if path lengths are of interest (Kaufmann, 2003).

The Bipartite projection attempts to separate the graph into two rows of vertices with edges crossing in between rows, but not within a row (Wild, 2016).

Link to code and usage

The code used to run the analysis and usage instructions can be found at: https://github.com/ucsd-lo-group/social-network-analysis/tree/gen1

Before running the code, you need one record of talk-turns in the question-and-response format described in the methods section.

To run the code, download the GitHub repository using the link above. There is a button labeled "Clone or download" that allows you to download a zip file of all the code. You must choose a location on your computer to save it and unzip the file.

Next, run the MATLAB code. Open MATLAB and set the working directory to the location of the unzipped code folder. Now you should be able to run "rawdatamatrixprocessor" in the console. It will guide you through the process of tabulating the talk-turns.

Next, run the R code. Open RStudio and set the working directory to the location of the unzipped code folder. Next, open the "start.R" script and click the "Source" button at the top. This script will guide you through analyzing the data and outputting statistics.

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