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Publication Date
1986

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Physics Division

Submitted for publication

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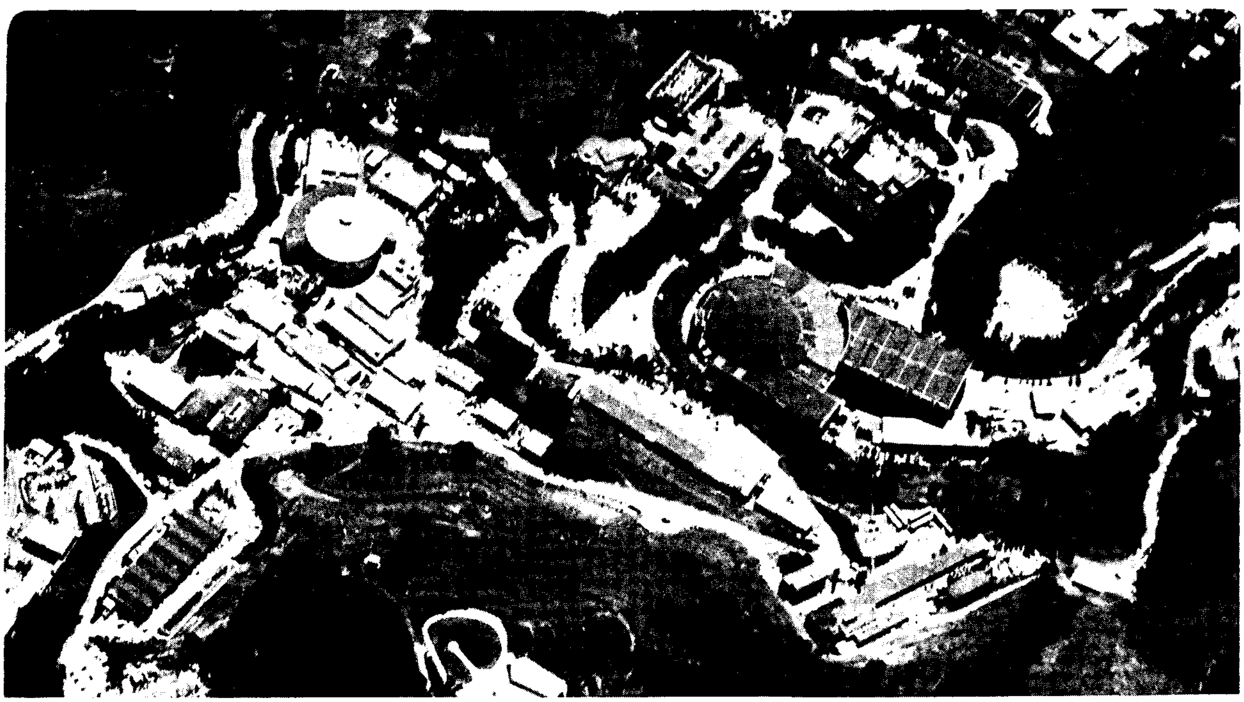

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

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January 1986 LBL-20867

AN UPPER LIMIT ON THE MASSES OF THE CHARGED HIGGS BOSONS IN THE GELMINI-RONCADELLI MODEL*

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ABSTRACT

The Gelmini-Roncadelli model allows neutrino masses via an enlarged Higgs sector. One-loop corrections to $W^{\pm}$ and Z masses from the Higgs potential of this model are calculated, to leading order in heavy Higgs masses. Using the value of the ρ parameter, the masses of the charged scalars are bounded.

*This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under contract DE-AC03-76SF00098.

The model of Gelmini and Roncadelli[1,2] offers an attractive way to add neutrino masses to the standard model. In their scheme, there are no heavy unobserved neutrinos, but rather an enlarged Higgs sector. The purpose of this paper is to show that the masses of some of these unconventional Higgs particles generate radiative corrections to the $W^{\pm}$ and Z masses, which will cause corrections to the ρ parameter. Therefore, the existing measurements of the $W^{\pm}$ and Z masses constrain the masses of some of these new Higgs particles.

The Higgs sector of the GR model has, in addition to the ordinary doublet $\phi = (\varphi_0, \varphi_-)$, a complex triplet $\chi = (\chi_0, \chi_-, \chi_{--})$. Since the χ multiplet has hypercharge -1, there is only one allowed coupling to fermions

$$g_{LL'}\bar{\Psi}_L\sigma^i\tilde{\Psi}^c_{L'}\chi^i + \text{h.c.}$$

where

$$\Psi_L = \begin{pmatrix} \nu_L \\ \ell^-_L \end{pmatrix}$$

and

$$\tilde{\Psi}^c_{L'} = \begin{pmatrix} -\ell^{-\,c}_{L'} \\ \nu_{L'}{}^c \end{pmatrix}$$

are the lepton spinors for the three generations. There are no couplings to quarks. This term conserves lepton number if the field χ is given a lepton number -2. If χ gets a vacuum expectation value, then the vacuum has lepton number, lepton number is spontaneously broken, and the neutrinos get a Majorana mass.

As in ref [2], the most general potential for ϕ, χ preserving both the gauge and lepton symmetries can be written

$$\begin{aligned} V(\phi,\chi) &= \lambda_1(\phi^\dagger\phi - \frac{1}{2}{v_2}^2)^2 + \lambda_2(\chi^\dagger\chi - \frac{1}{2}{v_3}^2)^2 \\ &+ \lambda_3(\phi^\dagger\phi - \frac{1}{2}{v_2}^2 + \chi^\dagger\chi - \frac{1}{2}{v_3}^2)^2 \\ &+ \lambda_4(\frac{1}{2}\phi^\dagger\phi\chi^\dagger\chi - \frac{1}{2}(\phi^\dagger\sigma^i\phi)(\chi^\dagger\tau^i\chi)) \\ &+ \lambda_5(\frac{1}{2}\chi^\dagger\chi\chi^\dagger\chi - \frac{1}{2}(\chi^\dagger\tau^i\chi)(\chi^\dagger\tau^i\chi)) \end{aligned}$$

where σ^i are the Pauli matrices and τ^i are the normalized $SU(2)$ matrices for spin 1. If

$$\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 > 0 \qquad \lambda_4 > 0 \qquad \lambda_5 > 0$$

then the potential is bounded below, and the vev's will be $\langle\phi\rangle = (v_2/\sqrt{2}, 0)$ and $\langle\chi\rangle = (v_3/\sqrt{2}, 0, 0)$. The three Goldstone bosons which become the longitudinal components of the $W^\pm$ and Z are respectively

$$\begin{aligned} G_- &= \frac{1}{\sqrt{v_2{}^2 + 2v_3{}^2}}\left(v_2\varphi_- + \sqrt{2}v_3\chi_-\right) \\ G_0 &= \frac{1}{2\sqrt{v_2{}^2 + 4v_3{}^2}}\left(v_2\frac{1}{i\sqrt{2}}(\varphi_0 - \varphi_0{}^*) + 2v_3\frac{1}{i\sqrt{2}}(\chi_0 - \chi_0{}^*)\right) \end{aligned}$$

The physical Higgs spectrum includes the doubly charged boson χ_{--} which gets a mass $m_{\chi_{--}}{}^2 = \lambda_4 v_2{}^2 + 2\lambda_5 v_3{}^2$. There is a singly charged particle

$$M_- = \frac{1}{\sqrt{v_2{}^2 + 2v_3{}^2}}(v_2\chi_- - \sqrt{2}v_3\varphi_-)$$

which has a mass $m_{M_-}{}^2 = \lambda_4(v_2{}^2 + 2v_3{}^2)/2$. The theory also contains a massless Majoron

$$M_0 = \frac{1}{2\sqrt{v_2{}^2 + 4v_3{}^2}}\left(v_2\frac{1}{i\sqrt{2}}(\chi_0 - \chi_0{}^*) - 2v_3\frac{1}{i\sqrt{2}}(\varphi_0 - \varphi_0{}^*)\right)$$

which is the Goldstone boson of the spontaneously broken lepton symmetry. The two other Higgs degrees of freedom $(\varphi_0 + \varphi_0{}^*)/\sqrt{2} - v_2$ and $(\chi_0 + \chi_0{}^*)/\sqrt{2} - v_3$ have a mass squared matrix

$$\begin{pmatrix} 2(\lambda_1 + \lambda_3)v_2{}^2 & \lambda_3 v_2 v_3 \\ \lambda_3 v_2 v_3 & 2(\lambda_2 + \lambda_3)v_3{}^2 \end{pmatrix}$$

The two eigenvectors of this matrix shall be referred to as H_h and H_l, for "heavy" and "light".

Notice that the massless field M_0 is comprised partly of the doublet field ϕ, and therefore it has couplings to quarks. Since the mixing of ϕ in M_0 is proportional to the vev of χ, one can bound v_3 by demanding that the new long range interactions mediated by the Majoron be weak. The best limit is from the evolution of stellar objects[3,4] from which one deduces that v_3 is not more than a few MeV. That v_3 is so small compared to $v_2 \approx 250$GeV implies that the fields M_0, M_-, and H_l are almost entirely χ, while the field H_h is almost entirely ϕ.

An interesting feature of this model is that it does not preserve the relation

$$\rho \equiv \frac{M_W{}^2}{M_Z{}^2\cos^2(\theta_W)} = 1$$

This is already apparent at tree level, because

$$M_W{}^2 = \frac{g^2}{4}(v_2{}^2 + 2v_3{}^2) \qquad M_Z{}^2 = \frac{g^2 + g'^2}{4}(v_2{}^2 + 4v_3{}^2)$$

which implies

$$\rho_{\text{tree}} = 1 - \frac{2v_3{}^2}{v_2{}^2 + 4v_3{}^2}$$

Since the value of v_3 is so strongly bounded, there is no conflict with the best experimental value[5] $\rho = 1.02 \pm 0.02$.

The smallness of v_3 in no way ensures that the radiative corrections to ρ are negligible. In the standard model, these corrections are bounded by the existence of a custodial $SU(2)$ symmetry[6] under which the gauge bosons W^i transform as a triplet and the Higgs field ϕ transforms as a doublet. The only terms in the lagrangian of the standard model which violate this symmetry are the electromagnetic couplings of the $W^\pm$ and the Yukawa couplings of the fermions to the Higgs. The fact that the fermions generate corrections to ρ can be used to put limits on their masses[7].

One might ask what becomes of the custodial $SU(2)$ in the GR model. The neutrino mass, λ_3, and λ_5 terms are all consistent with the custodial symmetry if the field χ transforms as a triplet. The λ_4 term, however, is not. This means that, to the extent that this term is present, it will generate corrections to ρ. The λ_4 term is responsible for the masses of the particles M_- and χ_{--}, because, as was noted above,

$$\begin{aligned} m_{\chi_{--}}{}^2 &= \lambda_4 v_2{}^2 + O(v_3{}^2) \\ m_{M_-}{}^2 &= \frac{1}{2}\lambda_4 v_2{}^2 + O(v_3{}^2) \end{aligned}$$

Therefore, the radiative corrections to ρ will increase as the mass of these particles increases.

The mass of these particles cannot be made too small, because they mediate lepton number violating processes such as neutrinoless double beta decay, and $\mu^- \to e^-e^-e^+$. As was noted in ref [4], the fact that M_- was not observed at PETRA means that its mass must be at least 21GeV, and therefore $m_{\chi_{--}} > 30$GeV.

Following ref [7], one expects that for large λ_4, the most important corrections to the tree level $W^\pm$ and Z propagators will be of the form

$$\Delta^{\mu\nu}(p) = \Delta^{\mu\nu}_{\text{tree}}(p)\left(1 + O\left(g^2 + \frac{m^2}{M_W{}^2}\right)\right)$$

where m is a Higgs mass scale that depends on λ_4. If one assumes that this Higgs mass scale is larger than the gauge boson mass, one need retain only terms enhanced by a power of m and can drop terms which are only as large as $g^2 \ln(m^2/M_W{}^2)$ relative to terms like $g^2 m^2/M_W{}^2$. It is useful to work in the Landau gauge, because it decouples the gauge from the Higgs degrees of freedom, allowing easy recognition of exactly which diagrams contribute effects enhanced by the Higgs mass. It is important to note in this connection that the Landau gauge condition is unique in that it is invariant under renormalization, and therefore the masses of the non-physical Higgs particles will remain zero even at one loop.

Suppose that the truncated 1PI bubble for the gauge boson is

$$\Pi^{\mu\nu} = iA(p^2)g^{\mu\nu} + iB(p^2)p^\mu p^\nu$$

The renormalized mass of the gauge particle is given by

$$M_R{}^2 = M_0{}^2 + A(0) + M_0{}^2 A'(0)$$

where the subscript R denotes renormalized quantites, and the subscript 0 denotes bare quantities. The physical ρ parameter is defined by

$$\rho \equiv \frac{M_{W\,R}{}^2}{M_{Z\,R}{}^2 \cos^2(\theta_{W\,R})}$$

and so therefore

$$\rho = 1 + \frac{A_W(0)}{M_{W\,0}{}^2} + A'_W(0) - \frac{\cos^2(\theta_{W\,0})A_Z(0)}{M_{W\,0}{}^2} - A'_Z(0) + \frac{\cos^2(\theta_{W\,R}) - \cos^2(\theta_{W\,0})}{\cos^2(\theta_{W\,0})}$$

In the Landau gauge, diagrams similar to those in figures 1 and 2 are not enhanced by a factor of m and so can be neglected; one need only evaluate diagrams like those shown in figures 3 and 4. Notice that the seagull diagram in figure 3 makes no contribution to $A'(0)$, while the loop of figure 4 makes contributions to $A'(0)$ which are only of order $\ln(m^2/M_W{}^2)$. Therefore, the terms $A'(0)$ may be neglected.

For compactness, the mass eigenstate fields of the Higgs particles will be written as

$$\begin{aligned} G_- &= \alpha_{G_-}\varphi_- + \beta_{G_-}\chi_- \\ G_0 &= \alpha_{G_0}\frac{1}{i\sqrt{2}}(\varphi_0 - \varphi_0{}^*) + \beta_{G_0}\frac{1}{i\sqrt{2}}(\chi_0 - \chi_0{}^*) \end{aligned}$$

for the Goldstone boson fields, and similarly for M_-, M_0, H_h, and H_l. The α's and β's are as given above. In order to use dimensional regularization, one must replace all d^4p by $d^{4-2\epsilon}p$ and introduce the arbitrary parameter μ with dimension of mass. Then, defining,

$$\begin{aligned} x &= (1/\epsilon) + (1-\gamma) + \ln(4\pi\mu^2) \\ h(m_1, m_2) &= \frac{1}{m_1{}^2 - m_2{}^2}(m_1{}^4\ln(m_1{}^2) - m_2{}^4\ln(m_2{}^2) - \frac{1}{2}m_1{}^4 + \frac{1}{2}m_2{}^4) \\ h_1(i) &= \frac{1}{2}\left(\frac{1}{2}\alpha_i{}^2 + \beta_i{}^2 m_i{}^2\right)(x - \ln(m_i)) \\ h_2(i,j) &= \left(\frac{1}{2}\alpha_i\alpha_j + \frac{1}{\sqrt{2}}\beta_i\beta_j\right)^2 (x(m_i{}^2 + m_j{}^2) - h(m_i, m_j)) \\ h_3(i) &= \left(\frac{1}{4}\alpha_i{}^2 + \beta_i{}^2\right) m_i{}^2(x - \ln(m_i{}^2)) \\ h_4(i,j) &= \left(\frac{1}{2}\alpha_i\alpha_j + \beta_i\beta_j\right)^2 (x(m_i{}^2 + m_j{}^2) - h(m_i, m_j)) \end{aligned}$$

where γ is Euler's constant, one finds

$$\begin{aligned} A_W(0) = {} & \frac{g^2}{16\pi^2}\times \\ & \Big[-h_1(G_0) - h_1(M_0) - h_1(H_h) - h_1(H_l) \\ & - \left(\left(\frac{1}{2}\alpha_{G_-}{}^2 + 2\beta_{G_-}{}^2\right)(x - \ln(m_{G_-}))\right) \\ & - \left(\left(\frac{1}{2}\alpha_{M_-}{}^2 + 2\beta_{M_-}{}^2\right)(x - \ln(m_{M_-}))\right) \\ & - (m_{\chi_{--}}{}^2(x - \ln(m_{\chi_{--}}))) \\ & + \left((\beta_{G_-}{}^2)(x(m_{\chi_{--}}{}^2 + m_{G_-}{}^2) - h(m_{\chi_{--}}, m_{G_-}))\right) \\ & + \left((\beta_{M_-}{}^2)(x(m_{\chi_{--}}{}^2 + m_{M_-}{}^2) - h(m_{\chi_{--}}, m_{M_-}))\right) \\ & + h_2(G_-, H_h) + h_2(G_-, H_l) + h_2(M_-, H_h) + h_2(M_-, H_l) \\ & + h_2(G_-, G_0) + h_2(G_-, M_0) + h_2(M_-, G_0) + h_2(M_-, M_0)\Big] \end{aligned}$$

and

$$\begin{aligned} A_Z(0) = {} & \frac{g\cos(\theta_W) + g'\cos(\theta_W)}{16\pi^2}\times \\ & \Big[-h_3(G_0) - h_3(M_0) - h_3(H_h) - h_3(H_l) \\ & - \left(\left(\frac{1}{2}\alpha_{G_-}\beta_{G_-}\right) m_{G_-}{}^2(x - \ln(m_{G_-}{}^2))\right) \\ & - \left(\left(\frac{1}{2}\alpha_{M_-}\beta_{M_-}\right) m_{M_-}{}^2(x - \ln(m_{M_-}{}^2))\right) \\ & + h_4(G_0, H_h) + h_4(G_0, H_l) + h_4(M_0, H_h) + h_4(M_0, H_l) \\ & + \left(2\left(\frac{1}{2}\alpha_{G_-}\alpha_{M_-}\right)(x(m_{G_-}{}^2 + m_{M_-}{}^2) - h(m_{G_-}, m_{M_-}))\right)\Big] \end{aligned}$$

The α's and β's given above obey

$$\begin{aligned} {\alpha_i}^2 + {\beta_i}^2 &= 1 \\ \alpha_j \alpha_k + \beta_j \beta_k &= 0 \end{aligned}$$

where

$$\begin{aligned} i &= \{G_0, G_-, M_0, M_-, H_h, H_l\} \\ (j,k) &= \{(G_0, M_0), (G_-, M_-), (H_h, H_l)\} \end{aligned}$$

This allows one to show that the dependence on x cancels in both $A_W(0)$ and $A_Z(0)$. Therefore, to order $g^2m^2/{M_W}^2$ there are only finite renormalizations to the $W^\pm$ and Z masses.

This cancellation of the infinities is not accidental; it is a manifestation of the gauge invariance of the unbroken theory. In the GR model, as well as in all similar versions of the $SU(2) \times U(1)$ model, gauge invariance is broken softly, by the vev of one or more Higgs fields. Therefore, all infinities in the $W^\pm$ and Z masses must vanish when the vev vanishes. The masses of the Higgs particles are fixed by terms in the Lagrangian independent of the vev, so an infinity proportional to the mass of a Higgs particle would not vanish in the limit of no vev. Therefore, there can be no infinite contributions of $O(g^2m^2/{M_W}^2)$ to M_W or M_Z. This can be compared with the situation in refs [7,8], where the fermions gave infinite contributions to the $W^\pm$ and Z masses. Any infinity proportional to a fermion mass is in turn proportional to the vev, and therefore will disappear when the vev vanishes.

Appearing in the expression for ρ is the correction to $\cos^2(\theta_W)$. To evaluate it, one looks for effects which would change the mixing between the W^3 and B mesons at one loop, or in other words, a diagram which mixes the photon and the Z. Potentially, the diagrams like 3 and 4 with a photon on one external leg and a Z on the other could produce such a mixing. One finds, however, that these bubbles contribute nothing of order $m^2/{M_W}^2$, and, therefore, that there is no correction to $\cos^2(\theta_W)$ to this order.

To simplify evaluation of ρ, note that α_G , β_M , α_{H_h}, and β_{H_l} are all $O(1)$, but β_G, α_M, β_{H_h}, and α_{H_l} are all $O(v_3/v_2)$. Dropping all the latter terms and retaining only the former, one finds

$$\rho = 1 + \frac{g^2}{16\pi^2 {M_W}^2}\Big[\, f(m_{M_-}, m_{\chi_{--}}) + \frac{1}{4} f(m_{G_-}, m_{H_h}) + \frac{1}{2} f(m_{M_-}, m_{H_l})$$

$$+ \frac{1}{4} f(m_{G_-}, m_{G_0}) + \frac{1}{2} f(m_{M_-}, m_{M_0}) - \frac{1}{4} f(m_{G_0}, m_{H_h}) - f(m_{M_0}, m_{H_l}) + O\left(\frac{v_3}{v_2}\right)\Big]$$

where

$$f(m_1, m_2) = \frac{1}{2}({m_1}^2 + {m_2}^2) + \frac{{m_1}^2 {m_2}^2}{{m_1}^2 - {m_2}^2} \ln\left(\frac{{m_2}^2}{{m_1}^2}\right)$$

This expression should be compared with the one given in ref [7], where the fermion corrections to ρ were considered. ρ here has the same functional dependence on the mass of the particles (through f) as in that case.

One may now use the relations for the masses of the Higgs particles given above: $m_{\chi_{--}} = \sqrt{2} m_{\chi_-} + O(v_3)$, $m_{H_l} = O(v_3)$, and $m_{G_0} = m_{G_-} = m_{M_0} = 0$. This yields

$$\rho = 1 + \frac{g^2}{8\pi^2}\left(\frac{m_{M_-}}{M_W}\right)^2 (1 - \ln 2) = 1 + 2.6 \times 10^{-3}\left(\frac{m_{M_-}}{100\text{GeV}}\right)^2$$

Notice that the masses of all the Higgs particles except for M_- and χ_{--} have cancelled out. This is precisely what was expected, as these particles were the only ones which got their mass from the λ_4 term in the potential.

If one demands that the value for ρ agree to within one standard deviation of the experimental value, then

$$m_{M_-} < 400\text{GeV}$$

and

$$m_{\chi_{--}} < 560\text{GeV}$$

If the GR model is correct, these particles must be light enough to be produced at the SSC.

The production of heavy doubly charged Higgs bosons by gauge boson fusion (fig 5) was discussed by Georgi and Machachek[8]. As they point out, the $\chi_{++}W^-W^-$ vertex is proportional to the vev of χ_0. In the GR model v_3 is very small, and therefore W^-W^- fusion will not be an appreciable source of doubly charged bosons. On the other hand, the coupling $\chi_{++}\chi_{--}\gamma$ is not dependent on the vev of the triplet. If it is light enough, the doubly charged boson may be observed at the SSC, in pair production via the Drell-Yan process.

Note Added: After completion of this work I became aware of a similar calculation done for the case of two Higgs doublets (D. Toussaint, Phys. Rev. **D18**, 1626 (1978)).

Acknowledgments

I wish to thank Robert Cahn, Michael Dugan, and Ian Hinchliffe for helpful discussions. I wish to thank Michael Chanowitz for helpful discussions and advice. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under contract DE-AC03-76SF00098.

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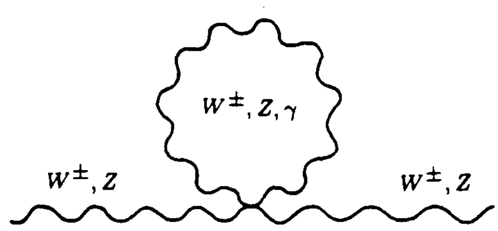

Figure 1a

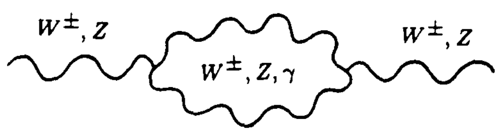

Figure 1b

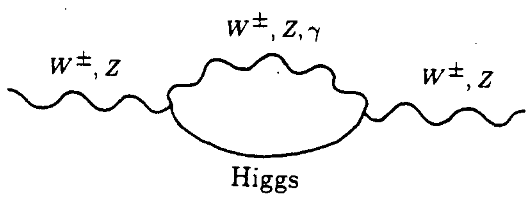

Figure 2

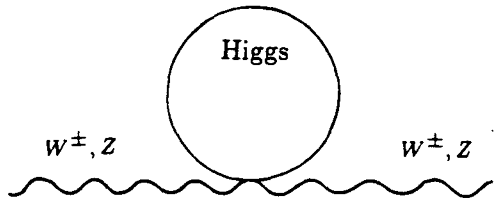

Figure 3

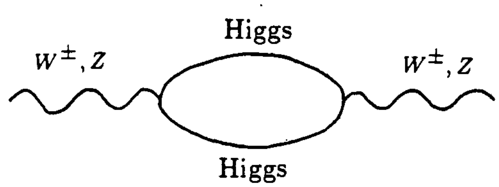

Figure 4

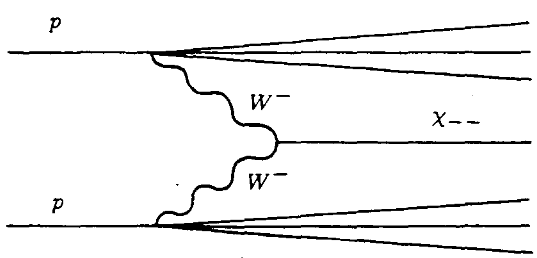

Figure 5

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