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Abstract

This paper provides a formal model of contingent convertible bonds (CCBs), a new instrument offering potential value as a component of corporate capital structures for all types of firms, as well as being considered for the reform of prudential bank regulation following the recent financial crisis. CCBs are debt instruments that automatically convert to equity if and when the issuing firm or bank reaches a specified level of financial distress. CCBs have the potential to avoid bank bailouts of the type that occurred during the subprime mortgage crisis when banks could not raise sufficient new capital and bank regulators feared the consequences if systemically important banks failed. While qualitative discussions of CCBs are available in the literature, this is the first paper to develop a formal model of their properties. The paper provides analytic propositions concerning CCB attributes and develops implications for structuring CCBs to maximize their general benefits for corporations and their specific benefits for prudential bank regulation.

Keywords: Contingent Convertible Bond, Banking Regulation, Subprime Mortgage Crisis, Structural Model, Corporate Finance

1 Introduction

This paper provides a formal model of contingent convertible bonds (CCBs), a new instrument offering potential value as a component of corporate capital structures for all types of firms, as well as being considered for the reform of prudential bank regulation following the recent financial crisis. CCBs are debt instruments that automatically convert to equity if and when the issuing firm or bank reaches a specified level of financial distress. While qualitative discussions of CCBs

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are available in the literature, this is the first paper to develop a formal model of their properties. The paper provides analytic propositions concerning CCB attributes and develops implications for structuring CCBs to maximize their general benefits for corporations and their specific benefits for prudential bank regulation.

CCBs are receiving attention as a new instrument for prudential banking regulation because they have the potential to avoid the bank bailouts that occurred during the subprime mortgage crisis when banks could not raise sufficient new capital and bank regulators feared the consequences if systemically important banks failed[1]. A more standard proposal for bank regulatory reform is to raise capital requirements since, if set high enough, they can achieve any desired level of bank safety. Very high capital ratios, however, impose significant costs on banks and thereby inhibit financial intermediation; and/or the capital requirements will be circumvented through regulatory arbitrage[2]. There have also been proposals to focus a component of the capital requirements on systemic risk (Adrian and Brunnermeier (2009)), or to prohibit banks outright from risky activities that are not fundamental to their role as financial intermediaries (Volcker (2010)). While these proposals could well improve prudential bank regulation, they do not directly address the issue of how distressed banks can raise new capital in order to preclude the need for government bailouts.

In this setting, CCBs have been proposed by academics (Flannery (2002, 2009a, 2009b), Duffie (2009), Squam Lake Working Group on Financial Regulation (2009), and McDonald (2010)) and endorsed for further study by bank regulators (Bernanke (2009) and Dudley (2009))[3]. In fact,

[1]The bank bailouts during the subprime crisis reflect a failure of the regulatory principles created under the 1991 Federal Deposit Insurance Corporation Improvement Act (FDICIA) and specifically of its requirement that bank regulators take "prompt corrective action" (PCA) in response to declining bank capital ratios. Eisenbeis and Wall (2002) provide a detailed discussion of FDICIA and its PCA requirements. As examples of the PCA requirements, "significantly undercapitalized" banks are to raise new equity or promptly merge into a well capitalized bank, and "critically undercapitalized" banks are to be placed under a receiver within 90 days of attaining that status. The subprime crisis also revealed that subordinated debt holders failed to "discipline" the banks, while the government bank bailouts protected these debt holders from the major losses they would have otherwise faced in a bankruptcy.

[2]High capital requirements limit the use of debt tax shields, impose a high cost on existing shareholders when raising new capital due to the debt overhang problem, and accentuate important principal-agent inefficiencies within the banks; see Kashyap, Rajan, and Stein (2008), Squam Lake Working Group on Financial Regulation (2009), Dudley (2009), and Flannery (2009) for further discussion of these issues.

[3]There have also been proposals for contingent capital instruments that are not bonds. Kashyap, Rajan, and Stein (2008) propose an insurance contract that provides banks with capital when certain triggering events occur and Zingales and Hart (2009) focus on the use of credit swaps. Wall (2009) provides a survey of this evolving literature.

Lloyd's bank issued the first £7 billion ($11.6 billion) CCBs (CoCo bonds)[4] in 2009. CCBs initially enter a bank's capital structure as debt instruments, thus providing the debt-instrument benefits of a tax shield and a control on principal-agent conflicts between bank management and shareholders. If and when the bank reaches the specified degree of financial distress, however, the debt is automatically converted to equity. The conversion recapitalizes he bank without requiring any ex-post action by banks to raise new equity or the government to bail them out. The automatic recapitalization feature of CCBs thus offers a relatively low-cost mechanism to avoid the costs that otherwise arise with the threatened bankruptcy of systemically important banking firms.

The existing CCB proposals-see especially Flannery (2009a) and McDonald (2010)-provide a list of issues that must be settled in formulating any specific plan for implementation:

- The trigger must be designed to avoid accounting manipulation, and the resulting conversion of CCB to equity must be automatic and inviolable. In fact, an accounting trigger in the Lloyd's bank 2009 CCB issue has already raised serious concern; see Duffie (2009). Most proposals instead recommend a trigger based on a market measure of each bank's solvency, such as a minimum ratio of market equity to asset value[5]. This is the case we model. We also analyze the issue of market manipulation of the equity value that may arise with a market-value trigger.

- The CCB to equity conversion terms applied after the trigger is activated must be specified. A key question is how the value of the equity shares received at conversion compare to the value of the converting bonds; see Flannery (2009a and 2009b) and McDonald (2010). We consider the general case in which the ratio of the equity conversion value to the CCB value is a contract parameter (λ) to be chosen. Among other effects, we analyze the impact this contract parameter may have on the incentive to manipulate the market value of the bank's

[4]Source: Financial Times from November 5, 2009.

[5]The Squam Lake Working Group and McDonald proposals require two triggers to be activated before conversion occurs. One trigger is based on each bank's own financial condition while the second trigger is based on an aggregate measure of banking system distress. This means that individual banks can become insolvent prior to CCB conversion if the aggregate trigger is not activated. For this reason, Flannery (2009a) argues for a single, bank-based, trigger. In this paper, we formally model only this single trigger case.

equity shares.

- The CCB contract could impose a dynamic sequence in which specified amounts of CCB convert at different thresholds. Flannery (2009a), furthermore, proposes a regulatory requirement whereby converted CCB must be promptly replaced in a bank's capital structure. While we comment on the possible advantages of such dynamic contract features, our formal model covers only the case of a one-time and complete conversion.

- The adoption of CCBs by banks could be voluntary or a required component of their capital requirements[6]. We consider both possibilities.

The key contribution of the current paper is to provide a formal financial model in which the effects of alternative CCB contract provisions can be analytically evaluated. We develop closed form solutions for CCB value by adapting the Leland (2004) model[7]. Our results apply equally well to the addition of CCBs to the capital structure of corporations generally, as well as for their specific application as a tool for prudential bank regulation. We make three assumptions throughout the paper regarding a firm's use of CCBs:

1) The firm is allowed a tax deduction on its CCB interest payments as long as the security remains outstanding as a bond. This would be the likely case for banks if CCBs were to become a formal and established component of prudential banking regulation. At the same time, this means that the public cost of the CCB tax shield must be included when evaluating the possible role of CCBs for prudential bank regulation. For corporations more generally, we acknowledge that

[6] For example, Flannery (2009a) provides an illustrative example in which banks are required to choose between satisfying their capital requirements by (i) holding equity equal to 6% of an asset aggregate or (ii) holding equity equal to 4% of the asset aggregate and CCBs equal to 4% of the asset aggregate. This suggests a regulatory tradeoff in which 4 percentage points of CCBs are the equivalent of 2 percentage points of equity.

[7] The Leland model has been successful applied in recent studies of other fixed-income debt security innovations, although none analyzes the case of a bond conversion triggered by financial distress. Bhanot and Mello (2006) study corporate debt that includes a rating trigger such that a rating downgrade requires the equity holders to compensate the bondholders with early debt redemption or other benefits. Manso, Strulovici, and Tchistyi (2009) study a class of debt obligations where the required interest payments depend on some measure of the borrower's performance. This could include the extreme case in which the debt interest coupons reach zero at some level of financial distress. This case would provide some of the same benefits in reducing or eliminating bankruptcy costs as provided by CCBs in the current paper.

the tax deductibility of CCB interest payment will likely require further IRS rulings, including possible legal challenges and new legislative actions.

2) In all cases, we assume that adding CCBs to a firm's capital structure has no impact on the level of the firm's asset holdings (A). In other words, we assume the addition of CCBs must take the form of either a CCB for equity swap (with the CCB proceeds paid out as a dividend to equity holders) or as a CCB for straight debt swap (with the CCB proceeds used to retire existing straight debt). The CCB for straight debt swap appears to be the most important case for regulatory applications, while we acknowledge that the future study of assets effects could be important for more general corporate finance applications.

3) Our analysis is carried out under the condition-Condition 1 below-that CCBs must convert to equity at a time prior to any possible default by the firm on its straight debt. This condition constrains the set of feasible CCB contracts that are considered in our analysis. This constraint is a necessary, and sensible, requirement if CCBs are to have the desired property of reducing the bankruptcy costs associated with a bond default.

The following is a summary of our main questions and results. We first consider a firm that has a new opportunity to include CCBs within its existing capital structure in a setting where no regulatory restrictions are imposed on CCB issuance (with the exception of the contract constraints created by Condition 1 as just described).

Q1. Will a firm include CCBs in its capital structure if it is freely allowed to do so?

A1. A firm will always gain from including CCB in its capital structure as a result of the tax shield benefit. This is true whether or not the firm also includes straight debt in its optimal capital structure; in fact, the optimal amount of straight debt is unaffected by the addition of CCB. Given that total assets are unchanged by assumption, in effect the CCB are being swapped for, i.e. replacing, equity in the capital structure. Since the asset-value default threshold on any existing straight debt is unchanged, adding CCB in this manner provides no benefits for regulatory safety, while taxpayers pay the cost of the additional tax shield. The addition of CCB in this form may also magnify the firm's incentive for asset substitution (to expand its asset risk).

We next consider a firm that operates under the regulatory constraint that it may issue CCB only as a part of a swap that retires an equal amount of straight debt. This constraint is implicit or explicit in various proposals to use CCB for prudential bank regulation; see Flannery (2009a). The result depends on whether the firm is creating a *de novo* optimal capital structure or is adding CCB to an already existing capital structure.

Q2. Will a firm add CCBs to a *de novo* optimal capital structure, assuming it faces the regulatory constraint that the CCB can only replace a part of what would have been the optimal amount of straight debt?

A2. A bank creating a *de novo* capital structure under the regulatory constraint will always include at least a small amount of CCB in its optimal capital structure. The reduction in expected bankruptcy costs ensures a net gain, even if the tax shield benefits are reduced, at least for small additions of CCB. The addition of CCBs also has the effect of reducing the incentive for asset substitution. The bottom line is that CCBs in this form provide an unambiguous benefit for regulatory safety.

Q3. Will a firm add CCBs to an existing capital structure, assuming it faces the regulatory constraint that the CCB can only be introduced as part of a swap for a part of the outstanding straight debt?

A3. Assuming the initial amount of straight debt equals or exceeds the optimal amount, the existing equity holders will not voluntary enter into the proposed swap of CCB for straight debt. While the swap may increase the firm's value–the value of reduced bankruptcy costs may exceed any loss of tax shield benefits–the gain accrues only to the holders of the existing straight debt. This is thus a classic debt overhang problem in which the equity holders will not act to enhance the overall firm value. To be clear, this result depends in part on our assumption that the straight debt has the form of a consol with indefinite maturity. If the straight debt has finite maturities, then the CCB could be swapped only for maturing debt, thus reducing the debt overhang cost.

Q4. How can CCBs be designed to provide a useful regulatory instrument for expanding the safety and soundness of banks that are acknowledged to be too big to fail (TBTF)?

A4. We assume a TBTF bank is one for which its straight debt is risk free because the bond holders correctly assume they will protected from any potential insolvency. We also assume a regulatory limitation on the amount of debt such a bank may issue. Under this limitation, a CCB for straight debt swap reduces the value of the government subsidy because it reduces the expected cost of bondholder bailouts. While this has a taxpayer benefit, the equity holders of such a bank would not voluntarily participate in such a swap.

Q5. May CCBs create an incentive for market manipulation?

A5. CCB may potentially create an incentive for either the CCB holders or the bank's equity holders to manipulate the bank's stock price to a lower value in order to force a CCB for equity conversion. The incentive for CCB holders to manipulate the equity price exists only if the ratio of equity conversion value to CCB value (λ in the model) is sufficiently high to make the conversion profitable for the CCB holders. The incentive for bank equity holders to manipulate the equity price exists, comparably, only if the ratio of equity conversion value to CCB value (λ) is sufficiently low to make the forced conversion profitable for the equity holders.

Q6. May restrictions on CCB contract and issuance terms be useful in maximizing the regulatory benefits of bank safety?

A6. The regulatory benefits of CCB issuance will generally depend on the CCB contract and issuance terms. Perhaps most importantly, the regulatory benefits vanish if banks simply substitute CCBs for capital, leaving the amount of straight debt unchanged. It is thus essential to require CCB issuance to substitute for straight debt (and not for equity). In addition, the higher the threshold for the conversion trigger the greater the regulatory safety benefits. The conversion ratio of equity for CCBs may also determine the incentive for CCB holders or equity holders to manipulate the stock price.

The structure of the paper is as follows. Part 2 develops the formal model. Part 3 applies the model to determine the role CCBs play in a bank's optimal capital structure. Part 4 analyzes bank issuance of CCBs when regulators require that the CCBs provide a net addition to bank safety. Part 5 applies the model to the role of CCBs when banks are too big to fail (TBTF). Part 6 provides our discussion of market manipulation involving CCBs. Part 7 investigates the effects of CCBs on asset substitution efficiency. Part 8 provides a summary and policy conclusions.

2 Model

We use the traditional capital structure modeling framework based on Leland (1994). A firm has productive assets that generate after-tax cash flows with the following dynamics

$$\frac{d\delta_t}{\delta_t} = \mu dt + \sigma dB_t^Q, \tag{1}$$

where μ and σ are constant, and B^Q defines a standard Brownian motion under the risk-neutral measure. The risk-free rate, r, is constant and, by assumption, is such that $\mu < r$. The tax rate $\theta \in (0,1)$. Interest payments are tax deductible.

At $\forall\, t$ the market value of assets A_t is defined as the value of all future cash flows. Given (1), that is

$$A_t = E_t^Q\left[\int_t^\infty e^{-r(s-t)}\delta_s ds\right] = \frac{\delta_t}{r-\mu}.$$

The dynamics for A_t are: $dA_t = \mu A_t dt + \sigma A_t dB_t^Q$.

The firm can issue equity and either a straight bond (straight debt) or both a straight bond and a CCB. Both bonds are *consol* type, meaning they are annuities with infinitive maturity. Straight debt pays coupon c_b, continually in time, until default. At default, fraction $\alpha \in [0,1]$ of the firm assets is lost. CCB pays coupon c_c, continually in time, until stopping time $\tau(A_C) = \inf\{s : A_s \leq A_C\}$. At $\tau(A_C)$ CCB *fully* converts into equity - bond holders receive equity valued at its market price in the amount of $\left(\lambda \frac{c_c}{r}\right)$. The coefficient λ is the CCB contract term that determines the ratio of the market value of equity relative to the market value of debt at the point of conversion. With $\lambda = 1$, CCBs convert into a market value of equity equal to the market value of the CCB debt.

With $\lambda < 1$ ($\lambda > 1$), the market value of equity received is at a discount (premium) relative to the market value of the CCB delivered for conversion.

The following Condition 1 is assumed to hold **always**.

Condition 1: c_b, c_c, A_C and λ are such that the firm does not default before or at CCB conversion.

One way to motivate Condition 1 is by considering the banking system. Regulators might look at CCB as a way to cushion individual banks and help them maintain capital ratios above predetermined levels in the event of a financial crisis. Having a bank default before or at conversion would obviate this role for CCB.

There are two results from the existing financial structure literature that will be used later in the paper. First, as in Duffie (2001), for a given constant $K \in (0, A_t)$, the market value of a security that claims one unit of account at the hitting time $\tau(K) = \inf\{s : A_s \leq K\}$ is, at $\forall\, t < \tau(K)$,

$$E_t^Q\left[e^{-r(\tau(K)-t)}\right] = \left(\frac{A_t}{K}\right)^{-\gamma}, \tag{2}$$

where $\gamma = \frac{m+\sqrt{m^2+2r\sigma^2}}{\sigma^2}$ and $m = \mu - \frac{\sigma^2}{2}$.

Second, also as in Duffie (2001), the default-triggering asset level that corresponds to the optimal default time $\tau(A_B)$ for the case when the capital structure of the firm includes only equity and straight debt is, at $\forall\, t < \tau(A_B)$,

$$A_B = \beta(1-\theta)c_b, \tag{3}$$

where $\beta = \frac{\gamma}{r(1+\gamma)}$.

Lemma 1. *Let the capital structure of the firm include equity and straight debt. If at $\forall\, t$ before default the firm decides to issue CCB without changing the existing amount of straight debt, the optimal default boundary A_B will remain the same.*

Proof. We assume that Condition 1 holds. Therefore, there is no default before or at conversion. At conversion CCB holders become equity holders. The value of assets does not change. After conversion the maximum-equity-valuation problem of equity holders (including the ones that became equity holders as the result of conversion) is the same as in the case when the capital structure includes only equity and straight bond. Hence, the same A_B. □

2.1 Closed-Form Solutions

Our goal in this subsection is to derive closed-form solutions for the values of claims associated with the capital structure when the firm issues equity, straight debt and CCB.

At $\forall\, t$ the following budget equation holds:

$$A_t + TB(A_t; c_b, c_c) \quad = \quad W(A_t; c_b, c_c) + U^B(A_t; c_b, c_c) + U^C(A_t; c_b, c_c) + BC(A_t; c_b, c_c), \quad (4)$$

where $TB(A_t; c_b, c_c)$ is the expected present value of tax benefits, $W(A_t; c_b, c_c)$ is the value of equity, $U^B(A_t; c_b, c_c)$ is the value of straight debt, $U^C(A_t; c_b, c_c)$ is the value of CCB and $BC(A_t; c_b, c_c)$ is the expected present value of bankruptcy costs[8].

The total value of the firm, $G(A_t; c_b, c_c)$, is the sum of the market values of equity and debt

$$G(A_t; c_b, c_c) \quad = \quad W(A_t; c_b, c_c) + U^B(A_t; c_b, c_c) + U^C(A_t; c_b, c_c). \quad (5)$$

Based on (4), this can be re-written as

$$G(A_t; c_b, c_c) \quad = \quad A_t + TB(A_t; c_b, c_c) - BC(A_t; c_b, c_c). \quad (6)$$

Proposition 1. *Let the capital structure of the firm include equity, straight debt and CCB. Then,*

[8] Regarding our notations, in the future $c_c = 0$ will mean that CCB is *not* used. For instance, $U^B(A_t; c_b, 0)$ is the market value of straight debt at time t for a firm that issued only equity and straight debt with coupon c_b.

for $\forall\, t < \tau(A_C)$

$$
\begin{aligned}
G(A_t;c_b,c_c) &= A_t + \frac{\theta c_b}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \alpha A_B\left(\frac{A_t}{A_B}\right)^{-\gamma},\\
W(A_t;c_b,c_c) &= A_t - \frac{c_b(1-\theta)}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right) - \frac{c_c(1-\theta)}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \\
&\quad A_B\left(\frac{A_t}{A_B}\right)^{-\gamma} - \left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma},\\
U^B(A_t;c_b,c_c) &= \frac{c_b}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \left(\frac{A_t}{A_B}\right)^{-\gamma}(1-\alpha)A_B,
\end{aligned}
$$

$$
\begin{aligned}
U^C(A_t;c_c) &= \frac{c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right) + \left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right),\\
TB(A_t;c_b,c_c) &= \frac{\theta c_b}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right),\\
BC(A_t;c_b,c_c) &= \alpha A_B\left(\frac{A_t}{A_B}\right)^{-\gamma}.
\end{aligned}
$$

The value of straight debt, $U^B(A_t;c_b,c_c)$, and the cost of bankruptcy, $BC(A_t;c_b,c_c)$, are not affected by the presence of CCB[9].

The total value of tax benefits, $TB(A_t;c_b,c_c)$, includes two parts:

1. the benefits associated with straight bond

$$TB^B(A_t;c_b,c_c) = \frac{\theta c_b}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right)$$

2. and the benefits associated with CCB

$$TB^C(A_t;c_b,c_c) = \frac{\theta c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right). \tag{7}$$

[9]Although $U^B(A_t;c_b,c_c)$ and $BC(A_t;c_b,c_c)$ do not depend on c_c, we use it in our notations when the capital structure of the firm includes CCB.

2.2 CCB Parameter Choice under Condition 1

In this subsection we look at how A_C affects the values of different claims associated with the capital structure of the firm. We define the lowest A_C that satisfies Condition 1 given the values of c_c and λ.

Let the capital structure of the firm at time t consist of equity and straight debt. Assume that equity holders are planning to issue CCB *without changing the existing amount of straight debt*. The proceeds will be paid off as dividends. The firm has fixed c_c and λ, and A_C is the only parameter that can still be changed.

Proposition 2. *The effect of A_C on the values of different claims associated with the capital structure of the firm when c_c and λ are fixed is such that*

(a) the total value of the firm, $G(A_t; c_b, c_c)$, increases as A_C decreases

(b) the value of equity, $W(A_t; c_b, c_c)$,

- *increases as A_C decreases for $(\lambda + \theta) > 1$*
- *decreases as A_C decreases for $(\lambda + \theta) < 1$*
- *and remains unaffected by A_C for $(\lambda + \theta) = 1$*

(c) the value of CCB, $U^C(A_t; c_c)$,

- *increases as A_C decreases for $\lambda < 1$*
- *and remains unaffected by A_C for $\lambda = 1$*

(d) the total amount of tax savings, $TB(A_t; c_b, c_c)$, increase as A_C decreases

(e) and, the value of straight debt, $U^B(A_t; c_b, c_c)$, and the cost of bankruptcy, $BC(A_t; c_b, c_c)$, remain unaffected by A_C.

Corollary 1. *Given that c_c and λ are fixed, as A_C varies the change in the total value of the firm, $G(A_t; c_b, c_c)$, equals the change in the total amount of tax savings, $TB(A_t; c_b, c_c)$.*

As indicated in Proposition 2, the value $(\lambda + \theta)$ plays an important role in the analysis, so it is useful to provide an intuitive understanding of it. Three factors combine to determine the net benefit or loss for the existing shareholders at CCB conversion:

1) The equity holders are relieved of the CCB obligation, the market value of which is $\frac{c_c}{r}$.

2) The equity holders lose the CCB tax shield benefit, the present value of which is $\frac{\theta c_c}{r}$.

3) The equity holders suffer a loss of $\frac{\lambda c_c}{r}$ if the conversion ratio of equity for CCB is at premium with $\lambda > 1$ (or discount with $\lambda < 1$).

The condition for the equity holders to suffer a net loss at conversion is thus: $\frac{\theta c_c}{r} + \frac{\lambda c_c}{r} > \frac{c_c}{r}$, which is equivalent to $(\lambda + \theta) > 1$. Comparably, the equity holders receive a net benefit at conversion if $(\lambda + \theta) < 1$, and the net benefit is zero if $(\lambda + \theta) = 1$.

Proposition 2 and its corollary indicate that equity holders will benefit from lowering the CCB conversion threshold A_C (making conversion less imminent) whenever CCB conversion creates a net loss for the equity holders (that is, $(\lambda+\theta) > 1$) and the opposite when CCB conversion creates a net gain for equity holders ($(\lambda + \theta) < 1$). For the same reason, the CCB value increases as A_C decreases as long as $\lambda < 1$. The tax savings always rise as A_C decreases, while the value of the straight debt remains unaffected.

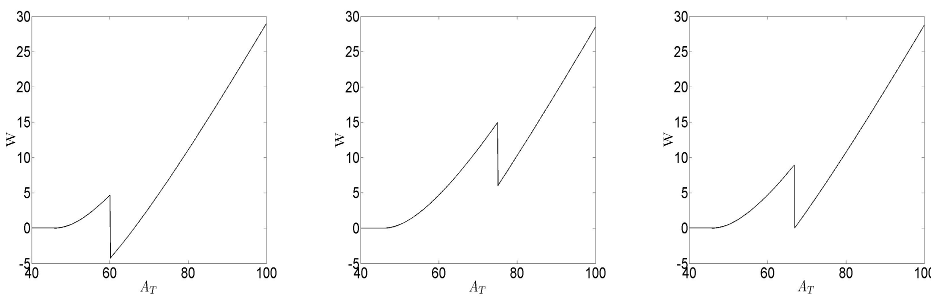
We use the closed-form solution for the value of equity from Proposition 1 and consider several numerical exercises next. The focus is on A_C.

Let $A_t = \$100.0$, $r = 5.00\%$, $\mu = 1.00\%$, $\theta = 35.0\%$ and $\alpha = 50.0\%$. Figure 1 shows how the value of equity, $W(A_T; c_b, c_c)$, depends on future realizations of the value of assets, A_T $(T > t)$. We start with the top three subfigures: (a)-(c). We fix $c_b = \$5.24$, $c_c = \$0.5$ and $\lambda = 0.9$, and consider three different values of A_C: \$60.0, \$75.0 and \$66.9. Based on (3), for all three cases the default-triggering asset level $A_B = \$45.85$.

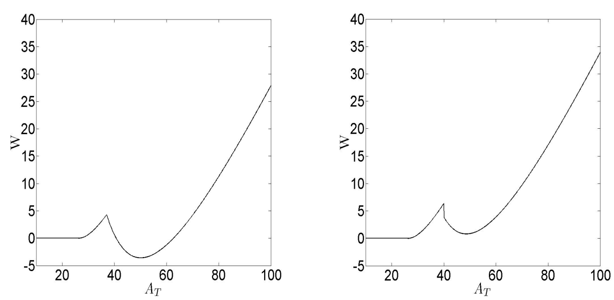
In subfigure (a) $W(A_T; c_b, c_c)$ becomes negative before A_T declines to the conversion-triggering asset level of \$60.0 (and before A_T hits $A_B = \$45.85$). Negative equity for $A_T > A_C$ translates into equity holders defaulting before conversion. This violates Condition 1.

In figure (b) $W(A_T; c_b, c_c)$ is strictly positive before and when A_T hits $A_C = \$75$. Condition 1 is not violated but, according to Proposition 2, equity holders could have set A_C lower in order to increase the total value of the firm.

Finally, in subfigure (c), $W(A_T; c_b, c_c)$ touches zero right at the conversion point and is positive for all $A_T > A_C$. The firm is not going to default before conversion. At conversion, since the value of equity is zero, equity holders are indifferent between defaulting and continuing to hold equity until $\tau(A_B)$. We assume that they continue to hold equity. Condition 1 is not violated. Given $c_b = \$5.24$, $c_c = \$0.5$ and $\lambda = 0.9$, $A_C = \$66.9$ is the lowest conversion-triggering asset level that satisfies Condition 1.



(a) A_C=\$60.0, λ=0.9, c_c=\$0.5, c_b=\$5.24, A_B = \$45.85 (b) A_C=\$75.0, λ=0.9, c_c=\$0.5, c_b=\$5.24, A_B = \$45.85 (c) A_C=\$66.9, λ=0.9, c_c=\$0.5, c_b=\$5.24, A_B = \$45.85



(d) A_C=\$37.0, λ=0, c_c=\$3.0, c_b=\$3.0, A_B = \$26.23 (e) A_C=\$40.0, λ=0.05, c_c=\$2.5, c_b=\$3.0, A_B = \$26.23

Figure 1: **Equity value with different conversion-triggering asset levels.**

It is important to note that in subfigures (a)-(c) $W(A_T; c_b, c_c)$ strictly increases in A_T for $A_T \geq A_C$. One could have found the optimal A_C that satisfies Condition 1 by solving equation

$W(A_C; c_b, c_c) = 0$ for A_C, such that $A_C > A_B$.

We turn to the lower two subfigures. Here, A_C = \$37.0, $\lambda = 0$, c_c = \$3.0 and c_b = \$3.0 for subfigure (d), and A_C = \$40.0, $\lambda = 0.05$, c_c = \$2.5 and c_b = \$3.0 for subfigure (e). One important characteristic that makes these two subfigures different from the ones above is that here for $A_T \geq A_C$ the value of equity as a function of A_T is non-monotone and declines immediately after the A_C values.

In subfigure (d) $W(A_T; c_b, c_c)$ becomes negative before A_T reaches the conversion-triggering asset level of \$37.0. As before, negative equity for $A_T > A_C$ translates into equity holders defaulting before conversion which violates Condition 1.

Note, that, although the value of equity in subfigure (d) is negative for an interval of values of $A_T \geq A_C$, it becomes positive as A_T approach A_C (from the right-hand side). From the point of view of old equity holders (i.e., not including those who become equity holders as the result of conversion) converting CCB into equity translates into getting rid of the obligation to pay c_c. If λ is low (as in subfigure (d)) the cost of eliminating this obligation, $\left(\lambda \frac{c_c}{r}\right)$, is also low and, therefore, as the chance of such an event increases (i.e., A_T get closer to A_C) it translates into higher values of equity. If λ is high (as in subfigure (a)) the cost of eliminating the obligation to pay c_c is also high and, therefore, equity value continues to decline as A_T approaches A_C. The same intuition stands behind the shapes of the equity value functions in the rest of the subfigures.

In subfigure (e) the value of equity is non-monotone but only touches zero for some $A_T > A_C$ without becoming negative. With the assumption that at zero equity holders prefer to continue holding equity to defaulting we get Condition 1 satisfied. Given $c_b = \$3.00$, $c_c = \$2.5$ and $\lambda = 0.05$, $A_C = \$40.0$ is the lowest conversion-triggering asset level that satisfies Condition 1.

Since in subfigure (e) $W(A_T = \$40.0; c_b, c_c) > 0$, we could not have found the above optimal conversion-triggering asset level by solving equation $W(A_C; c_b, c_c) = 0$ for A_C as in the case of subfigure (c). Therefore, in general, we define the lowest level of A_C that satisfies Condition 1 as

$$A_{CL} \quad = \quad \inf\{A_C : W(A_s; c_b, c_c) \geq 0, \forall\ s \geq \tau(A_C)\}. \tag{8}$$

Lemma 2. *If $(\lambda+\theta) > 1$, then the value of equity, $W(A_T; c_b, c_c)$, is a strictly increasing function of A_T for $A_T \geq A_C$. Independent of the values of λ and θ, the lowest asset level A_{CL} that satisfies definition (8) is such that*

$$A_{CL} \in \left(A_B + \lambda \frac{c_c}{r}, \lambda \frac{c_c}{r} + \frac{c_b(1-\theta)}{r} \right).$$

One observation is that, if $(\lambda+\theta) > 1$, the rule for finding A_{CL} that satisfies definition (8) is clear - A_{CL} solves equation $W(A_C; c_b, c_c) = 0$ (subject to $A_C > A_B$). If, however, this condition does not hold, one can only specify a relatively wide interval in which A_{CL} needs to be in.

The second observation is that for $(\lambda+\theta) > 1$, since $W(A_T; c_b, c_c)$ is strictly increasing in A_T, (with no asymmetric information) there is a one-to-one correspondence between equity and assets values for $A_T \geq A_C$. Therefore, the conversion condition for CCB can be formulated in terms of equity values. The debt converts into equity if the value of equity drops to $W_C = W(A_C; c_b, c_c)$. This becomes important since equity prices are observable while asset values are not.

Note that the analytical condition $(\lambda+\theta) > 1$ for the equity value to be strictly increasing in A_T is 'too strong'. Based on the above numerical exercises, the value of equity is non-monotone in A_T only for very small values of λ.

The main economic take-away of this section as a whole is that, if regulators do not want the firm to default before or at conversion, they might need to regulate how CCB parameters c_b, c_c, λ and A_C are set. Our analysis does not exclude the possibility that equity holders might have an incentive to increase the total value of the firm by choosing parameters that violate Condition 1.

3 Optimal Capital Structure

Assume that at time t the firm has no debt but is planning to leverage up by issuing both straight and CCB. The owners (either equity holders or the original owners of the private firm) *fix the* *amount of CCB they plan to issue by setting A_C, c_c and λ first.* Then, they maximize the total value of the firm by finding an optimal amount of straight debt. We look at how the resulting capital structure compares to the optimal capital structure without CCB.

Theorem 1. *The optimal amount of straight debt in a capital structure that includes CCB, equity and straight debt equals the amount of straight debt in the optimal capital structure that includes only equity and straight debt. The coupon on straight debt is the same for both cases*

$$c_b^* = \frac{A_t}{\beta(1-\theta)} \left(\frac{\theta}{r}\right)^{\frac{1}{\gamma}} \left[(\gamma+1)(\frac{\theta}{r} + \alpha\beta(1-\theta))\right]^{-\frac{1}{\gamma}}. \tag{9}$$

Given that the amount of CCB is exogenously low and parameters c_b, c_c, A_C and λ satisfy Condition 1, the above result is intuitive. The firm does not default before or at conversion and, therefore, as in the proof of Lemma 1, after conversion the maximum-equity-valuation problem of equity holder is the same as in the case when the capital structure includes only equity and straight debt. This leads to the same optimal amount of straight debt. Note, that c_b^* depends neither on c_c nor on A_C or λ.

Proposition 3. *If the firm chooses a capital structure that includes CCB, equity and the optimal amount of straight debt then, compared to the case when it chooses the optimal capital structure that includes only equity and straight debt,*

(i) the total value of the firm will be higher by the amount of tax savings associated with c_c

$$G(A_t; c_b^*, c_c) = G(A_t; c_b^*, 0) + TB^C(A_t; c_b^*, c_c)$$

(ii) adjusted for tax benefits, equity will be crowded by CCB one-to-one

$$W(A_t; c_b^*, c_c) = W(A_t; c_b^*, 0) - U^C(A_t; c_b^*, c_c) + TB^C(A_t; c_b^*, c_c) \tag{10}$$

(iii) the total value of tax benefits will be higher by the amount of savings associated with c_c

$$TB(A_t; c_b^*, c_c) = TB(A_t; c_b^*, 0) + TB^C(A_t; c_b^*, c_c)$$

(iv) and, the values of straight debt and bankruptcy costs will be the same

$$\begin{aligned} U^B(A_t; c_b^*, c_c) &= U^B(A_t; c_b^*, 0), \\ BC(A_t; c_b^*, c_c) &= BC(A_t; c_b^*, 0). \end{aligned}$$

The owners of the firm will issue CCB as it increases the total value of the firm by the amount of additional tax savings. The amount of straight debt does not change as CCB is issued on top of the optimal amount of straight debt. Therefore, allowing firms to introduce CCB to their capital structures in the way described above will create extra social costs in the form of additional tax subsidies. The cost of bankruptcy and timing of default will remain the same so the quality of straight debt will not improve.

3.1 Leveraged Firm with Suboptimal Amount of Straight Debt

We started this section with the firm being unlevered. Assume instead that the firm has a capital structure that includes equity and straight debt paying $\tilde{c}_b$ (not necessarily equal to c_b^*) and decides to issue CCB without changing the amount of straight debt. Based to Lemma 1, the default boundary does not change and, therefore, issuing CCB does not affect the values of straight debt and bankruptcy costs. The total value of the firm increases by the value of tax benefits associated with CCB, $G(A_t; \tilde{c}_b, c_c) = G(A_t; \tilde{c}_b, 0) + TB^C(A_t; \tilde{c}_b, c_c)$. And, based on budget equation (5), the new value of equity is

$$W(A_t; \tilde{c}_b, c_c) = W(A_t; \tilde{c}_b, 0) - \left[U^C(A_t; \tilde{c}_b, c_c) - TB^C(A_t; \tilde{c}_b, c_c)\right].$$

These results are similar to the ones from Proposition 3. Equity holders are willing to issue CCB as it increases their overall value. Although they experience a drop in the value of their holdings in the amount of $U^C(A_t;\tilde{c_b},c_c) - TB^C(A_t;\tilde{c_b},c_c)$, they collect dividends in the amount of $U^C(A_t;\tilde{c_b},c_c)$. Extra social costs are created. The quality of straight debt remains the same.

The main economic result of this section as a whole is that allowing firms to issue CCB on top of straight debt would create additional social costs in the form of extra tax subsidies without improving the quality of straight debt.

4 CCB Instead of Straight Debt

In Section 3 CCB was issued *on top* of straight debt. We move now to cases when CCB *replaces* a portion of straight debt that is either to be newly issued (in the optimal amount) by the firm when it has no debt or is already part of a capital structure that includes equity and straight debt (not necessarily in the optimal amount). We study the effect of debt replacement on the values of different claims associated with the capital structure of the firm.

4.1 Initial Choice of Capital Structure Under Regulatory Constraint

We start with the case when CCB replaces a portion of the optimal amount of straight debt that is to be newly issued. Assume that at time t the firm has no debt but is planning to leverage up. Instead of issuing an optimal amount of straight debt, $U^B(A_t, A_B^*; c_b^*, 0)$[10], it has an option to issue both straight and CCB under a regulatory constraint. Regulators fix the amount of straight debt, $U^B(A_t, \bar{A}_B; \bar{c_b}, c_c)$, and the amount of CCB, $U^C(A_t, \bar{A}_B; \bar{c_b}, c_c)$, so that

$$U^B(A_t, \bar{A}_B; \bar{c_b}, c_c) + U^C(A_t, \bar{A}_B; \bar{c_b}, c_c) \quad = \quad U^B(A_t, A_B^*; c_b^*, 0). \tag{11}$$

[10] We slightly change notations in order to keep track of the corresponding default boundaries. For instance, in the case of $U^B(A_t, A_B^*; c_b^*, 0)$ the default boundary is A_B^*.

The total amount of debt equals the optimal amount of straight debt when the capital structure of the firm includes only equity and straight debt.

The same amount of CCB can be issued with different coupons and conversion-triggering asset levels. The firm, for instance, can pick A_C and find c_c by solving (11) as shown below.

$$\begin{aligned}
U^C(A_t, \bar{A}_B; \bar{c_b}, c_c) &= U^B(A_t, A_B^*; c_b^*, 0) - U^B(A_t, \bar{A}_B; \bar{c_b}, c_c) \\
\frac{c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) + \left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda \frac{c_c}{r}\right) &= U^B(A_t, A_B^*; c_b^*, 0) - U^B(A_t, \bar{A}_B; \bar{c_b}, c_c) \\
c_c &= \frac{U^B(A_t, A_B^*; c_b^*, 0) - U^B(A_t, \bar{A}_B; \bar{c_b}, c_c)}{\frac{1}{r}\left(1 - (1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)}. \qquad (12)
\end{aligned}$$

We investigate if the firm would prefer the optimal capital structure that includes only equity and straight debt to the one that includes equity, straight debt and CCB but is subject to regulatory constraint (11).

Proposition 4. *If instead of the optimal capital structure that includes equity and straight debt an unlevered firm chooses its capital structure based on regulatory constraint (11) then*

(a) the change in the total value of the firm will equal the difference in the corresponding values of equity

$$\begin{aligned}
&G(A_t, \bar{A}_B; \bar{c_b}, c_c) - G(A_t, A_B^*; c_b^*, 0) = W(A_t, \bar{A}_B; \bar{c_b}, c_c) - W(A_t, A_B^*; c_b^*, 0) = \\
&(\theta + \alpha - \theta\alpha)\left(\left(\frac{A_t}{A_B^*}\right)^{-\gamma} A_B^* - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} \bar{A}_B\right) - \theta\left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda \frac{c_c}{r}\right) \qquad (13)
\end{aligned}$$

(b) if coupon c_c is relatively small, the total value of the firm will be higher: $\exists\ \bar{c}_1$ such that $G(A_t, \bar{A}_B; \bar{c_b}, c_c) > G(A_t, A_B^*; c_b^*, 0)$ for $\forall\ c_c \in (0, \bar{c}_1)$

(c) and, the cost of bankruptcy will be lower, $BC(A_t, \bar{A}_B; \bar{c_b}) < BC(A_t, A_B^*; c_b^*)$.

The key result is that the owners of the firm gain from replacing a small amount ($c_c \in (0, \bar{c}_1)$) of straight debt with CCB. The intuition is that the tax savings associated with coupon payments decrease due to $\tau(A_C) < \tau(A_B^*)$, but the firm benefits from reducing its bankruptcy costs due to a smaller amount of straight debt after the replacement. For small amounts of CCB the benefits exceed the lost tax savings.

The amount of CCB that the firm can issue is set by regulators exogenously, via constraint (11). Therefore, for the firm to be willing to replace straight debt with CCB, regulators need know how to set the constraint so that c_c does not exceed $\bar{c}_1$.

We continue with analyzing the effects of issuing CCB instead of straight debt numerically. We denote $G(A_t, \bar{A}_B; \bar{c_b}, c_c) - G(A_t, A_B^*; c_b^*, 0)$ by ΔG and use (13) to show how the total value of the firm changes depending on how much of the optimal amount of straight debt is being replaced with CCB.

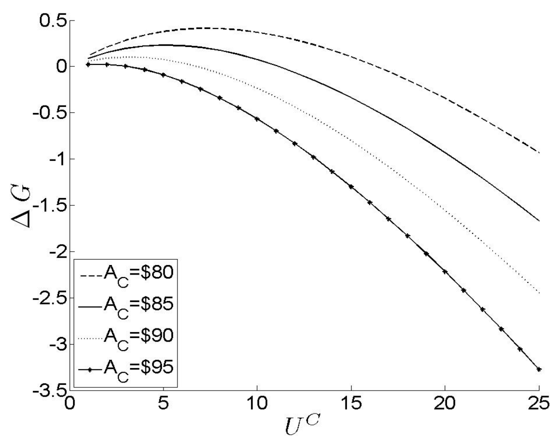


Figure 2: ΔG**.** ($A_t = \$100$, $r = 0.05$, $\mu = 0.01$, $\sigma = 15\%$, $\theta = 35\%$, $\alpha = 50\%$, $\lambda = 1$; $c_b^* = \$5.24$ and $U^B(A_t, A_B^*; c_b^*, 0) = \$88.36$.)

Consider an unlevered firm with parameters in the description of Figure 2. The optimal amount of straight debt that can be issued without the regulatory constraint is $U^B(A_t, A_B^*; c_b^*, 0) =$ \$88.36. The amount of straight debt that is being replaced subject to constraint (11) ranges from \$1 to \$25. As mentioned above, the same amount of CCB can be issued with different conversion-triggering asset levels. For each value of U^C we consider four A_C values: \$80, \$85, \$90, and \$95.

There are three main observations based on Figure 2. First, only a portion of the optimal amount of straight debt can be replaced with CCB without lowering the total value of the firm. (This is consistent with part (b) of Proposition 4.) As U^C gets above (roughly) \$20, ΔG becomes negative and keeps decreasing for all A_C values. Losses in tax benefits due to significant reductions in the amount of straight debt lead to lower total values of the firm.

Second, lower A_C values result in higher total values of the firm. (This is consistent with part (a) of Proposition 2.) Curves that correspond to lower A_C values lie strictly above the ones that correspond to higher A_C values. Lower A_C values translate into later conversions and lead to higher tax savings associated with coupon c_c.

Finally, changes in the total value of the firm are non-monotonic in U^C. For lower A_C values they first increase and then decrease. By gradually replacing straight debt with CCB starting with very small amounts, the firm reduces its bankruptcy costs and increases its total value. But, as the amount of CCB keeps increasing, reduced tax savings start dominating the benefits of lower bankruptcy costs which causes the total value of the firm to go down.

In summary, the firm in the above example would prefer a capital structure based on regulatory constraint (11) to the optimal capital structure that includes only equity and straight debt. The amount of CCB, however, would have to be relatively small.

The main economic result of this section is that letting unlevered firms replace straight debt with CCB in their new, leveraged capital structures creates benefits without additional costs. Total firm values increase and bankruptcy costs decrease. The total amount of debt in the economy remains the same, so there are no extra costs in the form of additional tax subsidies. The only requirement is that regulators need to know how to set their constraint so that firms are incentivize to issue CCB.

4.2 Partially Replacing Existing Straight Debt

We continue with the case when CCB replaces a portion of already existing (not necessarily in the optimal amount) straight debt. Assume that at time t the capital structure of the firm consists of equity and straight debt paying coupon $\hat{c_b}$ (not necessarily equal to c_b^*). The firm wants to issue CCB and swap it for a portion of straight debt in order to reduce $\hat{c_b}$ to $\bar{c_b}$, where $\bar{c_b} < \hat{c_b}$. Once the announcement is made, the market value of straight debt, that is still paying $\hat{c_b}$, will rise from $U^B(A_t, \hat{A}_B; \hat{c_b}, 0)$ to $U^B(A_t, \bar{A}_B; \hat{c_b}, 0)$ to reflect a lower default boundary due to a lesser amount of straight debt after the swap. For the straight debt holders to be indifferent between exchanging their holdings for CCB and continuing to hold straight debt the following budget equation should be true

$$U^B(A_t, \bar{A}_B; \hat{c_b}, 0) \quad = \quad U^C(A_t, \bar{A}_B; \bar{c_b}, c_c) + U^B(A_t, \bar{A}_B; \bar{c_b}, c_c). \tag{14}$$

The value of existing straight debt *post* announcement should equal the value of CCB plus the value of straight debt that remains after the swap.

Coupon $\bar{c_b}$ is set exogenously and, as before, the same amount of CCB can be issued with different coupons and conversion-triggering asset levels. The firm, for example, could pick $\bar{c_b}$ and A_C first and then find c_c by solving (14) as shown below.

$$\begin{aligned}
U^C(A_t, \bar{A}_B; \bar{c_b}, c_c) &= U^B(A_t, \bar{A}_B; \hat{c_b}, 0) - U^B(A_t, \bar{A}_B; \bar{c_b}, c_c) \\
\frac{c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) + \left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda \frac{c_c}{r}\right) &= \frac{\hat{c_b}}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) + \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B - \\
&\quad \frac{\bar{c_b}}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B \\
\frac{c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) + \left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda \frac{c_c}{r}\right) &= \frac{(\hat{c_b} - \bar{c_b})}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) \qquad (15) \\
c_c &= \frac{(\hat{c_b} - \bar{c_b})\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)}{1 - (1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}}
\end{aligned}$$

We try to understand if equity holders would be willing to replace some of the existing straight debt with CCB and what effect this replacement would have on the total value of the firm.

Proposition 5. *If a leveraged firm with a capital structure that includes equity and straight debt replaces a portion of straight debt with CCB then*

(i) the value of equity decreases, $W(A_t, \bar{A}_B; \bar{c_b}, c_c) - W(A_t, \hat{A}_B; \hat{c_b}, 0) < 0$

(ii) the change in the total value of the firm is such that

$$
\begin{aligned}
(a)\; G(A_t, \bar{A}_B; \bar{c_b}, c_c) - G(A_t, \hat{A}_B; \hat{c_b}, 0) &= \frac{\hat{c_b}\theta}{r}\left(\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) + \alpha \hat{A}_B \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \\
&\quad \alpha \bar{A}_B \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \theta \left(\frac{A_t}{A_C}\right)^{-\gamma} \left(\lambda \frac{c_c}{r}\right) \qquad (16) \\
(b)\; G(A_t, \bar{A}_B; \bar{c_b}, c_c) - G(A_t, \hat{A}_B; \hat{c_b}, 0) &> W(A_t, \bar{A}_B; \bar{c_b}, c_c) - W(A_t, \hat{A}_B; \hat{c_b}, 0)
\end{aligned}
$$

(c) if $\hat{c_b} \geq c_b^*$ *then* $\exists\, \bar{\bar{c}}_1$ *such that* $G(A_t, \bar{A}_B; \bar{c_b}, c_c) > G(A_t, \hat{A}_B; \hat{c_b}, 0)$ *for* $\lambda \geq 2 - \left(\frac{A_t}{A_C}\right)^{\gamma}$ *and* $c_c \in (0, \bar{\bar{c}}_1)$

(iii) and, the cost of bankruptcy decreases, $BC(A_t, \bar{A}_B; \bar{c_b}) < BC(A_t, \hat{A}_B; \hat{c_b})$.

If the firm is leveraged optimally or over-leveraged compared to its optimal capital structure ($\hat{c_b} \geq c_b^*$) it could benefit in terms of its *total value* from replacing a certain amount ($c_c \in (0, \bar{\bar{c}}_1)$) of straight debt with CCB (issued with $\lambda \geq 2 - \left(\frac{A_t}{A_C}\right)^{\gamma}$[11]). There are two things at play here. First, as before, replacing straight debt with CCB pushes the tax savings down, but the firm benefits from reducing the cost of bankruptcy. For certain amounts of CCB the benefits will dominate the lost tax savings.

Second, although debt becomes less risky due to $\bar{A}_B < \hat{A}_B$ the total amount of debt increases by the difference between the value of straight debt post announcement, $U^B(A_t, \bar{A}_B; \hat{c_b}, 0)$, and the value of straight debt pre announcement, $U^B(A_t, \hat{A}_B; \hat{c_b}, 0)$. By increasing the total amount of debt while reducing the cost of bankruptcy the firm benefits from relatively higher (compared

[11]Note, that $2 - \left(\frac{A_t}{A_C}\right)^{\gamma} \leq 1$ for $A_t \geq A_C$.

to the case when the total amount of debt did not change as in Section 4.1) tax savings[12]. The presence of these relative benefits is independent of the amount of CCB. The new tax savings for the firm, though, might (if not compensated by the reduction in tax savings due to the use of CCB) translate into additional social costs in the form of extra tax subsidies. As the amount of CCB debt increases ($c_c > \bar{\bar{c}}_1$) lost tax benefits become larger and can turn all the gains, including the ones from lower bankruptcy costs and the additional tax savings, into losses.

Although the total value of the firm could increase, equity holders will not replace voluntarily any amount of existing straight debt with CCB as their value decreases. All the potential gains in the total value of the firm plus a portion the value of equity are passed on to debt holders. The observed effect is due to debt overhang inefficiency.

We analyze the results of Proposition 5 numerically. We plot values of $G(A_t, \bar{A}_B; \bar{c_b}, c_c) - G(A_t, \hat{A}_B; \hat{c_b}, 0)$, denoted by $\Delta\hat{G}$ and computed based on (16), for a range of CCB values and several conversion-triggering asset levels.

We use the firm from Section 4.1. The assumption is that it has issued straight debt in the optimal amount and now is replacing some of this debt with CCB. Coupon $\hat{c_b}$ is set equal to c_b^* and values of the rest of the parameters, including the market value of assets, are exactly the same as in Section 4.1 (all shown in the description of Figure 3).

Figures 2 and 3 are comparable. Figure 2 shows how the total value of the firm changes depending on the amount of CCB the firm uses when it leverages up for the first time based on regulatory constraint (11). Figure 3 shows how the total value of the firm changes depending on the same amounts of CCB when the firm is already leveraged and replaces straight debt with CCB so that market constraint (14) holds. In both cases the firm uses CCB to replace portions of the same (optimal) amount of straight debt.

The general logic of the observations based on Figure 2 applies to Figure 3, so we are not going to repeat the related discussions from Section 4.1.

[12]Based on this, one would expect that, everything else being equal, $\bar{\bar{c}}_1 > \bar{c}_1$.

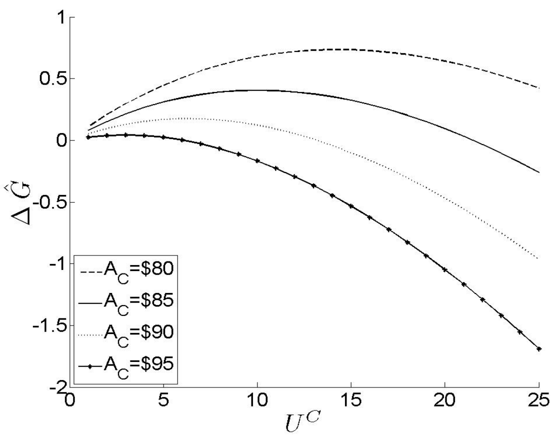


Figure 3: $\Delta\hat{G}$. ($A_t = \$100$, $r = 0.05$, $\mu = 0.01$, $\sigma = 15\%$, $\theta = 35\%$, $\alpha = 50\%$, $\lambda = 1$; $\hat{c_b} = c_b^* = \$5.24$ and $U^B(A_t, \hat{A}_B; \hat{c_b}, 0) = U^B(A_t, A_B^*; c_b^*, 0) = \$88.36$.)

There is an important difference between the two figures, though. Notice that, if plotted together, the curves from Figure 3 would lie above their counterparts from Figure 2. Changes in total values of the firm for the current exercise exceed the corresponding changes in total values for the exercise in Section 4.1. This is due to relatively larger total amounts of debt post replacement and, correspondingly, higher tax savings.

In summary, in the above exercise all the benefits for the firm and the economy overall from replacing straight debt with CCB discussed in Section 4.1 remain. However, the realization of these benefits increases the total amount of debt which translates into more value for the firm but requires additional tax subsidies.

The key economic result of Section 4.2 is that if the firm decided to partially replaces existing straight debt with CCB the total value of the firm would increase and bankruptcy costs together with the total amount of risky straight debt would decrease. No extensive regulation would be required. Equity holders, however, will never initiate this kind of debt replacement on their own due to debt overhang inefficiency.

5 TBTF Firms

In this section we look at firms that are 'too big' for the government to let them fail, as bankruptcy of such firms might cause major disruptions in the overall financial system/economy. At the point of bankruptcy of a TBTF firm the government might assume control over its assets and take over its obligation to make payments to debt holders.

From modeling prospective, a firm reaches bankruptcy when the value of assets, A_t, hits the default boundary level, A_B. At that point, if the government decides to step in to prevent bankruptcy, it would obtain assets worth A_B and an obligation to pay c_b forever worth $\frac{c_b}{r}$. Therefore, the value of government subsidy at the time of bankruptcy is $\frac{c_b}{r} - A_B$. Given (2), at any time t before bankruptcy, the value of subsidy is[13]

$$S(A_t; c_b, c_c) \quad = \quad \left(\frac{c_b}{r} - A_B\right)\left(\frac{A_t}{A_B}\right)^{-\gamma}. \tag{17}$$

Optimal time to default $\tau(A_B)$ solves the maximum-equity-valuation problem of equity holders. Government subsidy kicks in at time $\tau(A_B)$ and covers only straight debt obligations. Therefore, it affects neither the timing of default nor the value of A_B or the value of equity.

Also, based on Lemma 1, the time of default and the value of assets at the time of default do not depend on whether the capital structure of the firm includes equity and straight debt or equity, (the same amount of) straight debt and CCB. Therefore, the value of subsidy is the same whether straight debt is used on not:

$$S(A_t; c_b, c_c) \quad = \quad S(A_t; c_b, 0). \tag{18}$$

Proposition 6. *Let a firm have a capital structure that includes equity, straight debt and CCB. If at* $\forall\, t$ *the government issues a guarantee for the straight debt of the firm, then*

[13]We return to our initial notations. The default boundary is tied to the corresponding coupon on straight debt based on (3).

1. *the total value of the firm will increase and will become strictly increasing in* c_b

$$G(A_t; c_b, c_c) = A_t + \frac{\theta c_b}{r}\left(1 - \left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) + \left(\frac{c_b}{r} - A_B\right)\left(\frac{A_t}{A_B}\right)^{-\gamma} \quad (19)$$

2. *straight debt will become risk-free,* $U^B(A_t; c_b, c_c) = \frac{c_b}{r}$

3. *bankruptcy will be eliminated,* $BC(A_t; c_b) = 0$

4. *and, the values of equity, tax benefits and CCB will not change.*

5.1 Straight Debt Instead of CCB: Initial Choice of Capital Structure

Assume that a TBTF firm is currently unlevered but is considering leveraging up. It could be owned by either equity holders deciding to issue debt or the original owners deciding to issue equity and debt. We try to understand how the value of government subsidy would depend on whether the firm chooses to issue straight debt or both straight and CCB under a regulatory limit on how much debt the firm is allowed to issue.

Equation (19) gives the closed-form solution for the value of the firm when it issues equity, straight debt and CCB. It is easy to see that there is a similar solution for the case when the firm issues only equity and straight debt. In both cases the total value of the firm is strictly increasing in c_b. There is no more trade-off between tax benefits associated with debt and losses due to bankruptcy, as bankruptcy, from the point of view of the total value of the firm, has been eliminated. Equity holders or the original owners of the firm would try to issue as much debt as possible (and collect an amount of tax benefits as large as possible) and default immediately after the issuance. Knowing this, the government could set limits on how much debt a TBTF firm could issue. It could set a maximum straight debt coupon c_b^g for the capital structure that includes only equity and straight debt. It could also give the firm an alternative to issue straight debt with coupon $\bar{c}_b$ and CCB with coupon c_c such that the following regulatory constraint holds

$$U^B(A_t; c_b^g, 0) = U^C(A_t; \bar{c}_b, c_c) + U^B(A_t; \bar{c}_b, c_c). \quad (20)$$

In the presence of government guarantee straight debt is risk-free. Therefore, based on this and Proposition 1, equation(20) can be re-written as

$$\frac{c_b^g}{r} = \frac{c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)+\lambda\frac{c_c}{r}\left(\frac{A_t}{A_C}\right)^{-\gamma}+\frac{\bar{c_b}}{r}.$$

This leads to

$$\bar{c_b} = c_b^g - c_c\left(1-(1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}\right). \tag{21}$$

Note, that $c_b^g > \bar{c_b}$. Also, the higher is c_c the lower is $\bar{c_b}$. The bigger the portion of CCB the smaller the amount of the remaining straight debt.

We can use (3) to re-write (17) as

$$S(A_t; c_b, c_c) = \left(\frac{c_b}{r} - c_b(1-\theta)\beta\right)\left(\frac{A_t}{c_b(1-\theta)\beta}\right)^{-\gamma} = c_b\left(\frac{1}{r}-(1-\theta)\beta\right)\left(\frac{c_b(1-\theta)\beta}{A_t}\right)^{\gamma}.$$

Given that $r < 1$, $\theta < 1$ and $\beta < 1$, $S(A_t; c_b, c_c)$ is an increasing function in c_b. Therefore, based on (18) and the fact that $c_b^g > \bar{c_b}$

$$S(A_t; c_b^g, 0) > S(A_t; \bar{c_b}, c_c). \tag{22}$$

The above arguments can be summarized in a proposition.

Proposition 7. *Replacing straight debt with CCB in a new capital structure of an unlevered, TBTF firm deciding to issue debt and subject to a regulatory limit on how much debt it is allowed to issue reduces the expected present value of government subsidy.*

Government subsidy eliminates bankruptcy costs, $BC(A_t; \hat{c}_b, 0) = 0$, and, based on equation (6), the total value of the firm for the case when it does not issue CCB is

$$G(A_t; c_b^g, 0) = A_t + TB(A_t; c_b^g, 0) + S(A_t; c_b^g, 0).$$

Equivalently, the total value of the firm when it does issue CCB is

$$G(A_t; \bar{c}_b, c_c) \quad = \quad A_t + TB(A_t; \bar{c}_b, c_c) + S(A_t; \bar{c}_b, c_c).$$

Consider the difference

$$G(A_t; c_b^g, 0) - G(A_t; \bar{c}_b, c_c) \quad = \quad TB(A_t; c_b^g, 0) - TB(A_t; \bar{c}_b, c_c) + S(A_t; c_b^g, 0) - S(A_t; \bar{c}_b, c_c) \tag{23}$$

Based on equation (22), the last difference on the right-hand side of the above equation is strictly positive, $S(A_t; c_b^g, 0) - S(A_t; \bar{c}_b, c_c) > 0$. The reduction in the value of government subsidy if the firm issues CCB is expected to either dominate or magnify the change in the value of tax benefits.

The above arguments can be summarized in a proposition.

Proposition 8. *Equity holders of an unleveraged TBTF firm will loose in value if they issue CCB instead of straight debt,* $G(A_t; c_b^g, 0) > G(A_t; \bar{c}_b, c_c)$.

5.2 Straight Debt Instead of CCB: Partially Replacing Existing Straight Debt

Assume that a TBTF firm has a capital structure that includes equity and straight debt (paying coupon $\hat{c}_b$). We try to analyze the effect of replacing a portion of its current debt with CCB on the value of equity.

In the presence of government subsidy at $\forall\ t$ the following budget equation holds

$$A_t + TB(A_t; \hat{c}_b, 0) + S(A_t; \hat{c}_b, 0) \quad = \quad W(A_t; \hat{c}_b, 0) + U^B(A_t; \hat{c}_b, 0). \tag{24}$$

There are no bankruptcy costs, $BC(A_t; \hat{c}_b, 0) = 0$, and debt is risk-free, $U^B(A_t; \hat{c}_b, 0) = \frac{\hat{c}_b}{r}$.

The firm is to replace a portion of its current straight debt with CCB paying c_c. The remaining straight debt will be paying coupon $\bar{c}_b$, such that $\bar{c}_b < \hat{c}_b$. The government guarantee remains in place, so straight debt will still be risk-free, $U^B(A_t; \bar{c_b}, c_c) = \frac{\bar{c_b}}{r}$. As before, straight debt holders should be indifferent between exchanging their holdings for CCB and continuing to hold straight debt

$$U^B(A_t; \hat{c_b}, 0) = U^C(A_t; \bar{c_b}, c_c) + U^B(A_t; \bar{c_b}, c_c). \tag{25}$$

Equation (25) is equivalent to equation (14) in Section 4.2. The key difference, though, is that after a TBTF firm announces its decision to replace straight debt with CCB the value of existing straight debt does not change. Debt is risk-free and, therefore, contrary to what we had before, the announcement does not affect its default boundary.

After the firm replaces a portion of its straight debt with CCB for $\forall\ t$ the following budget equation will hold

$$A_t + TB(A_t; \bar{c_b}, c_c) + S(A_t; \bar{c_b}, c_c) = W(A_t; \bar{c_b}, c_c) + U^B(A_t; \bar{c_b}, c_c) + U^C(A_t; \bar{c_b}, c_c). \tag{26}$$

Given (24), (25) and (26),

$$W(A_t; \hat{c_b}, 0) - W(A_t; \bar{c_b}, c_c) = TB(A_t; \hat{c_b}, 0) - TB(A_t; \bar{c_b}, c_c) + S(A_t; \hat{c_b}, 0) - S(A_t; \bar{c_b}, c_c). \tag{27}$$

Since $\hat{c_b} > \bar{c_b}$, based on equation (22), $S(A_t; \hat{c_b}, 0) - S(A_t; \bar{c_b}, c_c) > 0$. As before, the reduction in the value of government subsidy due to replacing straight debt with CCB is expected to either dominate or magnify the change in the value of tax benefits.

We proved the following proposition.

Proposition 9. *Equity holders of a TBTF firm will loose in value if they replace a portion of existing straight debt with CCB,* $W(A_t; \hat{c_b}, 0) > W(A_t; \bar{c_b}, c_c)$.

Note that, contrary to what we observed in Section 4.2, straight debt holders do not gain anything. Since debt is risk-free, the decision of the firm to replace straight debt with contingent capital debt does not affect its value. The reduction in the value of equity is mainly due to the reduction in government subsidy. There are no explicit debt overhang inefficiency effects.

The main economic result of this section is that TBTF firms will oppose issuing CCB as by doing so they will lose a portion of government subsidy.

6 Multiple Equilibrium Equity Prices and Market Manipulations

Our goal in this section is to present two issues related to having firms use CCB. We show the existence of multiple equilibrium equity prices and look at incentives of market participants to manipulate the equity market. We distinguish between manipulations by CCB holders and the ones by equity holders.

Throughout this entire section we assume that the value of equity is strictly increasing in the value of assets before and at conversion. This allows us to formulate an alternative conversion rule for CCB in terms of equity value - CCB converts into equity when equity value drops to $W_C = W(A_C; c_b, c_c)$.

6.1 Two Equilibrium Equity Prices

We start with showing existence of two different equilibrium prices for the equity of a firm that issues straight debt, equity and CCB.

As it is captured in Figure 4, consider three time instances t, t_+ and t_{++}, where $t < t_+ < t_{++}$.

At time t the market value of assets, A_t, is uncertain. The uncertainty is resolved at t_{++} when A_t takes either the value of A_H with probability p or the value of A_L with probability $(1-p)$. A_H, A_L and A_C are such that $A_L < A_C < A_H$.

The sequence of possible events is as follows. The market value of equity, W_t, is observed at time t. If $W_t \leq W_C$, then at time t_+ CCB converts into equity. Otherwise, there is no conversion

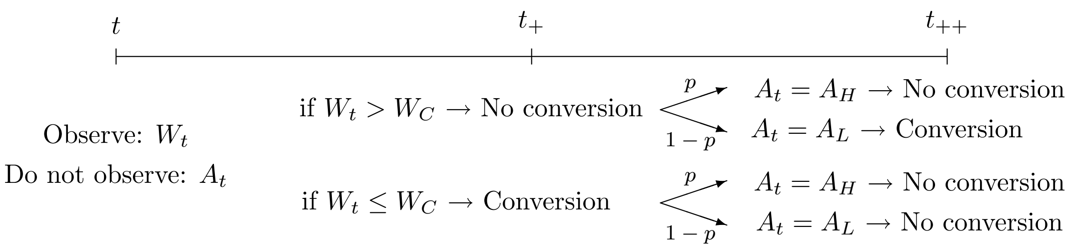


Figure 4: **Equity price, asset value and CCB conversion decisions.**

at t_+. The realized market value of assets, A_t, is observed at t_{++}. If the realization is A_H and there was no conversion at t_+ CCB converts into equity. Otherwise, there is no conversion at t_{++}.

Consider the case when there is no conversion (i.e., $W_t > W_C$) at time t_+. If the realization of A_t at time t_{++} is A_H, then the value of old equity at time t_{++} is $W(A_H; c_b, c_c)$. This value reflects the fact that, due to $A_C < A_H$, A_H does not trigger conversion at t_{++}. There are no new equity holders.

On the other hand, if the realization of A_t at time t_{++} is A_L, then the value of old equity at time t_{++} is $W(A_L; c_b, 0) - \lambda \frac{c_c}{r}$. In this case, due to $A_L < A_C$, A_L does trigger conversion at t_{++}. The value of old equity equals the value of total (old and new) equity post conversion minus the value of new equity issued to replace CCB.

We denote the value of old (observed) equity at time t_+ when there is no conversion at t_+ by $\bar{W}_t$. It can be calculated as the expected value of old equity at time t_{++}:

$$\bar{W}_t \quad = \quad pW(A_H; c_b, c_c) + (1-p)\left(W(A_L; c_b, 0) - \lambda \frac{c_c}{r}\right). \tag{28}$$

Now consider the case when at time t_+ CCB does convert into equity (i.e., $W_t \leq W_C$). Independent of what the realization of asset value at time t_{++} is there will be no conversion at that time.

We denote the value of old (observed) equity at time t_+ when there is conversion at t_+ by $\hat{W}_t$. Then, the value of total (old and new) equity post conversion at time t_+ is $\hat{W}_t + \lambda \frac{c_c}{r}$. The values of total equity at time t_{++} for A_H and A_L are $W(A_H; c_b, 0)$ and $W(A_L; c_b, 0)$, correspondingly. And, the value of total equity at time t_+ is its expected value at time t_{++}:

$$\hat{W}_t + \lambda \frac{c_c}{r} = pW(A_H; c_b, 0) + (1-p)W(A_L; c_b, 0).$$

This leads to

$$\hat{W}_t = pW(A_H; c_b, 0) + (1-p)W(A_L; c_b, 0) - \lambda \frac{c_c}{r}. \tag{29}$$

Based on (28), (29) and Proposition 1, the difference in the observed values of equity for the two cases is

$$\begin{aligned}
\bar{W}_t - \hat{W}_t &= p\left(W(A_H; c_b, c_c) - W(A_H; c_b, 0)\right) - \frac{\lambda c_c}{r} - (1-p)\frac{\lambda c_c}{r} \\
&= p\left(-\frac{c_c(1-\theta)}{r}\left(1 - \left(\frac{A_H}{A_C}\right)^{-\gamma}\right) - \frac{\lambda c_c}{r}\left(\frac{A_H}{A_C}\right)^{-\gamma} + \frac{\lambda c_c}{r}\right) \\
&= p\frac{c_c}{r}\left(1 - \left(\frac{A_H}{A_C}\right)^{-\gamma}\right)(\lambda + \theta - 1).
\end{aligned}$$

If $(\lambda + \theta) > 1$, $\bar{W}_t > \hat{W}_t$.

We proved the following theorem.

Theorem 2. *If $(\lambda + \theta) > 1$, then there could be two market equilibria*

1. *a relatively high equity price at time t and no conversion of CCB into equity at time t_+*

2. *and, a relatively low equity price at time t and conversion of CCB into equity at time t_+.*

Note that the proof above is based on the assumption that W_C is such that $\hat{W}_t < W_C < \bar{W}_t$. Since $\bar{W}_t$ is strictly higher than $\hat{W}_t$, we can find values of W_C that satisfy the above condition by picking certain values for c_c, p, etc.[14]

[14] One can try to specify more explicitly the corresponding values (or ranges of values) for these parameters by using (28), (29) and Proposition 1.

The intuition behind Theorem 2 is as follows. Early conversion leads to a guaranteed loss of tax benefits associated with CCB. This leads to a lower value of equity, $\hat{W}_t$. If, on the other hand, the debt is not converted before the uncertainty about the value of assets is resolved, the tax benefits are lost only with probability $(1 - p)$. This corresponds to a higher value of equity, $\bar{W}_t$.

6.2 Equity Market Manipulations

We continue with analyzing the conditions under which market participants might be willing to manipulate the equity market.

6.2.1 Manipulation by CCB Holders

We start with the case when CCB holders are behind the manipulation.

The motivation is as follows. Market participants might be able to make a profit by buying CCB when the stock price of the firm is above the conversion-triggering level W_C, driving the price down (by spreading negative news, short selling equity, etc.) in order to trigger conversion and then selling the equity obtained as the result of conversion when the price corrects.

Assume that at time t the market value of assets, A_t, is uncertain. At some future time the uncertainty is resolved and A_t can take the value of A_H with probability p or A_L with probability $(1-p)$. p and $(1-p)$ reflect correct beliefs about realizations A_H and A_L. Also, A_H and A_L are such that, when the true value of A_t is realized, only A_L triggers conversion (i.e., $A_L < A_C < A_H$).

As before, we also assume that the market value of equity is observed before the uncertainty about A_t is resolved.

We model CCB holders driving the equity price down as them manipulating the market into believing that the probability of A_t reaching A_H is p' and the probability of A_t reaching A_L is $(1-p')$, where $p' < p$.

Assume that, if the market believes that the probability of A_t reaching A_H is p (and the probability of A_t reaching A_L is $(1-p)$), the value of equity is above W_C and, therefore, there is no conversion. On the other hand, if the market believes that the probability is p', CCB does convert into equity.

In expectation *post* manipulation (i.e., after the stock price is driven down, CCB is converted into equity and the correct belief p is restored) the value of equity is

$$\tilde{W}_t = pW(A_H; c_b, 0) + (1-p)W(A_L; c_b, 0).$$

When the market is manipulated, at the point of conversion the value of equity is

$$\tilde{\tilde{W}}_t = p'W(A_H; c_b, 0) + (1-p')W(A_L; c_b, 0).$$

CCB holders receive equity in the amount of $\lambda\frac{c_c}{r}$. As the market belief corrects, the value of equity changes from $\tilde{\tilde{W}}$ to $\tilde{W}_t$. Therefore, the expected value of the payoff to (former) CCB holders after the market corrects is

$$\Pi'_t = \lambda\frac{c_c}{r}\frac{pW(A_H; c_b, 0) - (1-p)W(A_L; c_b, 0)}{p'W(A_H; c_b, 0) - (1-p')W(A_L; c_b, 0)}.$$

If there is no manipulation that triggers conversion, the expected value of the payoffs to CCB holders is

$$\Pi_t = pU^C(A_H; c_b, c_c) + (1-p)\lambda\frac{c_c}{r}.$$

Consider the difference in these two values

$$\Pi'_t - \Pi_t = \lambda\frac{c_c}{r}\frac{(p-p')(W(A_H; c_b, 0) - W(A_L; c_b, 0))}{p'W(A_H; c_b, 0) - (1-p')W(A_L; c_b, 0)} - p\left(U^C(A_H; c_b, c_c) - \lambda\frac{c_c}{r}\right).$$

By using the closed-form solution for $U^C(A_H; c_b, c_c)$ from Proposition 1 and rearranging terms, we get

$$\begin{aligned}\Pi_t' - \Pi_t &= \lambda \frac{c_c}{r} \frac{(p-p')(W(A_H;c_b,0) - W(A_L;c_b,0))}{p'W(A_H;c_b,0) - (1-p')W(A_L;c_b,0)} - p(1-\lambda)\frac{c_c}{r}\left(1 - \left(\frac{A_H}{A_C}\right)^{-\gamma}\right) \\ &= \lambda \frac{c_c}{r} \frac{p-p'}{p' + \frac{W(A_L;c_b,0)}{W(A_H;c_b,0)-W(A_L;c_b,0)}} - p(1-\lambda)\frac{c_c}{r}\left(1 - \left(\frac{A_H}{A_C}\right)^{-\gamma}\right). \qquad (30)\end{aligned}$$

It's easy to see that if $\lambda = 0$, based on (30), $\Pi_t' < \Pi_t$. CCB holders do not have an incentive to manipulate the market.

Also from (30), $\Pi_t' - \Pi_t$ is strictly increasing in λ and the value of λ for which the difference in the two payoffs is zero is

$$\lambda^* = \frac{p\left(1 - \left(\frac{A_H}{A_C}\right)\right)^{-\gamma}}{\frac{p-p'}{p' + \frac{W(A_L;c_b,0)}{W(A_H;c_b,0)-W(A_L;c_b,0)}} + p\left(1 - \left(\frac{A_H}{A_C}\right)\right)^{-\gamma}}. \qquad (31)$$

Clearly, $\lambda^* > 0$. Also, since $\frac{p-p'}{p' + \frac{W(A_L;c_b,0)}{W(A_H;c_b,0)-W(A_L;c_b,0)}} > 0$, $\lambda^* < 1$.

All the above arguments could be summarized in a theorem.

Theorem 3. *$\exists \lambda^* \in (0,1)$ such that $\Pi_t - \Pi_t' = 0$ and*

1. *if $\lambda \leq \lambda^*$ CCB holders will not manipulate the equity market*
2. *and, if $\lambda > \lambda^*$ CCB holders will manipulate the equity market.*

The intuition for why small values of λ should prevent manipulation is as follows. At conversion CCB holders give up a stream of future coupon payments for the value of $\lambda \frac{c_c}{r}$. Small values of λ mean that, even after we account for the appreciation of equity post conversion, the value CCB holders receive is too small compared to the value of future coupon payments they need to give up. Therefore, CCB holders will not try to force conversion.

Based on equation (31), there two major drivers behind the value of λ^*. The first one is the distance between probabilities p and p'. The bigger is the difference $(p - p')$ the lower is λ^*. The interpretation is that the easier it is to manipulate the market the lower should the conversion ration be in order to avoid manipulation.

The second driver is the difference between equity values for asset realizations A_H and A_L. Here, again, the bigger is the difference $(W(A_H; c_b, 0) - W(A_L; c_b, 0))$ the lower is λ^*. The higher is the uncertainty about the value of assets the lower should the conversion ratio be.

6.2.2 Manipulation by Equity Holders

We turn to the case when equity holders might attempt to manipulate the market.

The motivation is that equity holders might increase the value of their holdings by driving the equity price down to W_C by making the market believe in poor prospects for the firm, triggering conversion, and then correcting the market belief. The potential value increase is due to being able to cheaply get rid of the obligation to pay c_c.

The value of (old) equity holders before they attempt to manipulate the market is $W(A_t; c_b, c_c)$. At the point of conversion the value of equity is $W(A_C; c_b, 0)$. As the market belief corrects, the value of equity rises to $W(A_t; c_b, 0)$. The new value of (old) equity holders equals the difference between $W(A_t; c_b, 0)$ and the value of (new) equity that belongs to former CCB holders, $\left(\lambda \frac{c_c}{r}\right) \frac{W(A_t; c_b, 0)}{W(A_C; c_b, 0)}$. (Old) Equity holders will not manipulate the market if

$$W(A_t; c_b, c_c) - \left[W(A_t; c_b, 0) - \lambda \frac{c_c}{r} \frac{W(A_t; c_b, 0)}{W(A_C; c_b, 0)}\right] \quad \geq \quad 0.$$

Based on Proposition 1

$$\begin{aligned} W(A_t; c_b, c_c) - \left[W(A_t; c_b, 0) - \lambda \frac{c_c}{r} \frac{W(A_t; c_b, 0)}{W(A_C; c_b, 0)}\right] \quad &= \quad -\frac{c_c(1-\theta)}{r} \left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \\ &\frac{\lambda c_c}{r} \left(\frac{A_t}{A_C}\right)^{-\gamma} + \frac{\lambda c_c}{r} \frac{W(A_t; c_b, 0)}{W(A_C; c_b, 0)}. \end{aligned} \tag{32}$$

Clearly, when $\lambda = 0$ equity holders will manipulate the market as the right-hand side of equation (32) is negative.

For $\theta = 0$ and $\lambda = 1$ the right-hand side of equation (32) is strictly positive:

$$-\frac{c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)-\frac{c_c}{r}\left(\frac{A_t}{A_C}\right)^{-\gamma}+\frac{c_c}{r}\frac{W(A_t;c_b,0)}{W(A_C;c_b,0)} \quad = \quad \frac{c_c}{r}\left(\frac{W(A_t;c_b,0)}{W(A_C;c_b,0)}-1\right)>0.$$

As θ increases the right-hand side of equation 32 only becomes larger. Therefore, for any feasible θ, if $\lambda = 1$, the difference in the equity values is going to be positive. This means that no market manipulations will be taking place.

It is also clear that the right-hand side of equation (32) is strictly increasing in λ and the value of λ for which the difference in the equity values is zero is

$$\lambda^{**} \quad = \quad \frac{(1-\theta)\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)}{\frac{W(A_t;c_b,0)}{W(A_C;c_b,0)}-\left(\frac{A_t}{A_C}\right)^{-\gamma}}. \tag{33}$$

For values of λ higher or equal to λ^{**} the right-hand side of equation (32) is going to be non-negative and equity holders will not have an incentive to manipulate the market. On the other hand, for values of λ lower than λ^{**} they will manipulate the market. Note, that λ^{**} is a decreasing function of the asset value A_t. Therefore, for the above observation to be true for any realization of A_t before or at conversion, we choose the highest

$$\lambda^{**} \quad = \quad \frac{(1-\theta)\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)}{\frac{W(A_C;c_b,0)}{W(A_C;c_b,0)}-\left(\frac{A_t}{A_C}\right)^{-\gamma}} = (1-\theta). \tag{34}$$

We proved the following theorem.

Theorem 4. *If $\lambda \geq (1-\theta)$ equity holders will not manipulate the equity market, and if $\lambda < (1-\theta)$ equity holders will manipulate the equity market.*

The intuition for why in order to disincentivize equity holders to manipulate the market λ should be set above a certain level is straight forwards - when λ values are above λ^{**} for equity holders the cost of getting rid of the obligation to pay c_c is 'too' high.

The main conclusion of this section as a whole is that if $\lambda = 1 - \theta$ and $(1 - \theta) < \lambda^*$ then there will be a single equilibrium equity price and neither equity holders nor CCB holders will attempt to manipulate the equity market.

7 CCB and Asset Substitution Inefficiency

In this section we investigate if including CCB in the capital structure of a firm would make equity owners more willing to switch to riskier technologies by choosing higher asset volatility parameters. There are three cases. First, we consider including CCB in a capital structure as part of a CCB for equity swap that leaves the amount of straight debt unchanged as discussed in Section 3. Second, we return to the assumptions of Section 4.1 and look at the case when CCB is included in a de novo capital structure as a CCB for straight debt swap under regulatory constraint (11). Third, we consider adding CCB to an existing capital structure as a CCB swap for existing straight debt under constraint (14), following the assumptions of Section 4.2.

7.1 CCB Introduced as a CCB for Equity Swap in a De Novo Capital Structure

We start by returning to the conditions of Section 3, where we found that an unconstrained firm would add CCB to a de novo capital structure as part of a CCB for equity swap. We now consider whether adding CCB to the capital structure of a firm in this manner creates an incentive for the firm to change the riskiness of its technology (i.e. by changing the parameter σ). We have the following result.

Proposition 10. *Assume that the firm has set its capital structure, which includes CCB, equity, and the optimal amount of straight debt. If $(\lambda + \theta) > 1$ ($(\lambda + \theta) < 1$) then, compared to the case of the optimal capital structure that includes only equity and straight debt, dollar gains of equity holders from switching to riskier technologies will be lower (higher).*

The intuition for Proposition 10 follows from understanding that a higher asset volatility increases the likelihood that the CCB trigger will be reached. From our discussion above, conversion is costly for existing shareholders when $(\lambda + \theta) > 1$, and thus the equity holders in a firm with such a CCB contract will prefer safer technologies. In contrast, CCB conversion is profitable for the existing shareholders when $(\lambda + \theta) < 1$ and the equity holders in a firm with such a CCB contract would then prefer the higher risk technologies.

The applicability of Proposition 10 also requires that the dollar gain in equity value be the proper criterion for assessing asset substitution effects. This is an issue because, for the case in this section, the firm adds CCB to its capital structure through a CCB for equity swap that leaves the CCB firm with a lower equity value than a comparable firm with only straight debt. As long as the firm's shareholders participate proportionately in the CCB for equity swap, the dollar gain in equity remains the proper criterion. On the other hand, if a set of shareholders were to gain a larger proportionate share of the firm as a result of the CCB for equity swap, they could make the asset substitution decision based on the percentage increase in their equity value. This might be true, in particular, for managers with stock options who would be unable to participate in the CCB for equity swap. We now use two numerical exercises to demonstrate that a percentage change in equity value criterion expands the conditions under which the incentive for asset substitution is greater for a CCB firm.

Consider an unlevered firm at time t with the market value of assets $A_t = \$100$ and a constant instantaneous volatility of changes in the value of assets firm $\sigma = 15.0\%$. The firm sets an optimal capital structure by leveraging up. Four cases are considered. In the base case the firm issues no CCB. In the remaining three cases the firm issues CCB with $A_C = \$70$, $c_c = \$0.5$ and three different conversion values: $\lambda = 0$, $\lambda = 0.65$, and $\lambda = 1$. Since we set $\theta = 0.35$, the three λ cases correspond to $(\lambda + \theta) < 1$, $(\lambda + \theta) = 1$, and $(\lambda + \theta) > 1$, respectively. Straight debt is issued with coupon $c_b^* = \$5.24$ based on (9). Values of the rest of the parameters are shown in the description of Figure 5.

Once the straight debt and CCB are issued, equity holders might attempt to raise the value

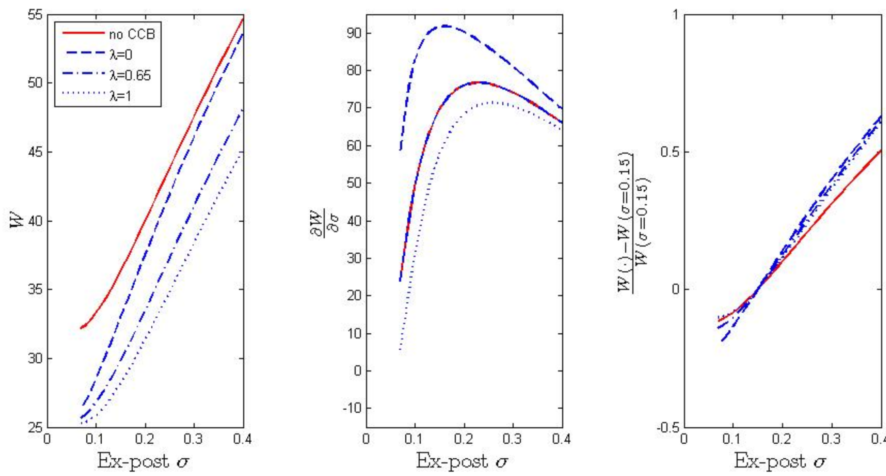


Figure 5: **Ex-post equity values: single firm.** (A_t = \$100, (initial) σ = 15.0%, c_b^* = \$5.24, A_C = \$70, c_c = \$0.5, $r = 0.05$, $\mu = 0.01$, $\theta = 35\%$, and $\alpha = 50\%$.)

of their holdings by increasing or decreasing σ. Figure 5 helps analyze how the value of equity depends on changes in volatility. The subfigure on the left-hand side graphs the values of equity as functions of ex-post σ values. The subfigure in the middle graphs the slopes of the functions on the left-hand side. Both are consistent with the results of Proposition 10. In particular, in the middle panel, the sensitivity of the equity value to σ is inversely related to the λ value of the respective curves. In addition, the curve for $\lambda = 0.65$ (that is, $(\lambda + \theta) = 1$) coincides exactly with the curve for no CCB. The plot on the right-hand side of Figure 5 shows the values of $\frac{W(\sigma)-W(\sigma=0.15)}{W(\sigma=0.15)}$. The key observation is that for ex-post σ values (roughly) above 20%, independent of λ and θ, relative changes in the value of equity in the presence of CCB are higher than the ones for the base case. The conclusion is that equity holders are generally inclined to increase the volatility ex-post if CCB is used.

Our second exercise looks at a range of firms (each represented by a different initial σ) and only local changes in volatility. This contrasts with the first exercise in which we considered a single firm for a range of different ex-post σ outcomes. We let the values of A_t, A_C, r, c_c, μ, θ and

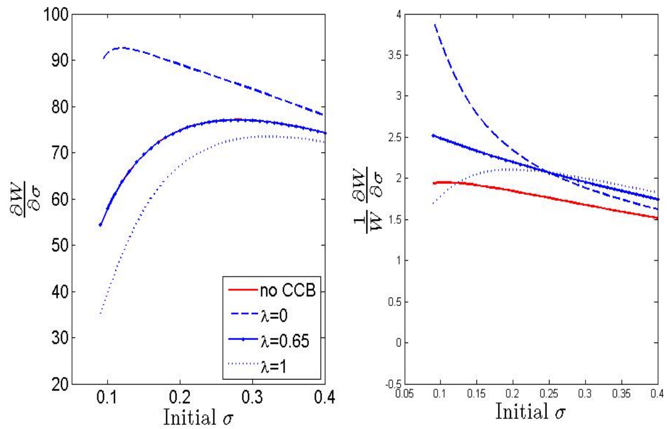


Figure 6: **Local rate of change and percentage change in ex-post equity values: multiple firms.** ($A_t = \$100$, individual initial σ values, $c_b^* = \$5.24$, $A_C = \$70$, $c_c = \$0.5$, $r = 0.05$, $\mu = 0.01$, $\theta = 35\%$, and $\alpha = 50\%$.)

α be the same as before and the same across all firms. Similar to the first exercise, we consider a base case - when firms issue only straight debt and equity - and three more cases - when the firm includes CCB in its capital structure and we evaluate three different λ values: 0 ($(\lambda+\theta) < 1$), 0.65 ($(\lambda + \theta) = 1$), and 1 ($(\lambda + \theta) > 1$).

Figure 6 plots ex-post values of $\frac{\partial W}{\partial \sigma}$ (left panel) and $\frac{1}{W}\frac{\partial W}{\partial \sigma}$ (right panel) for each σ (that is, each firm) and for no CCB and three λ cases. The left panel shows, again, that the sensitivity of equity to σ is higher the lower the λ value and that the curve with $\lambda = 0.65$ coincides exactly with the no CCB curve. The right panel shows that the percentage change in equity value with respect to σ is always higher for the capital structures with CCB except for the lower end of $\lambda = 1$ curve. This suggests that in the presence of CCB equity holders will generally switch to riskier technologies for the above choice of parameters across all initial σ values (i.e., across all firms). This reinforces the conclusion that if CCB is used the effect of asset substitution inefficiency might be magnified.

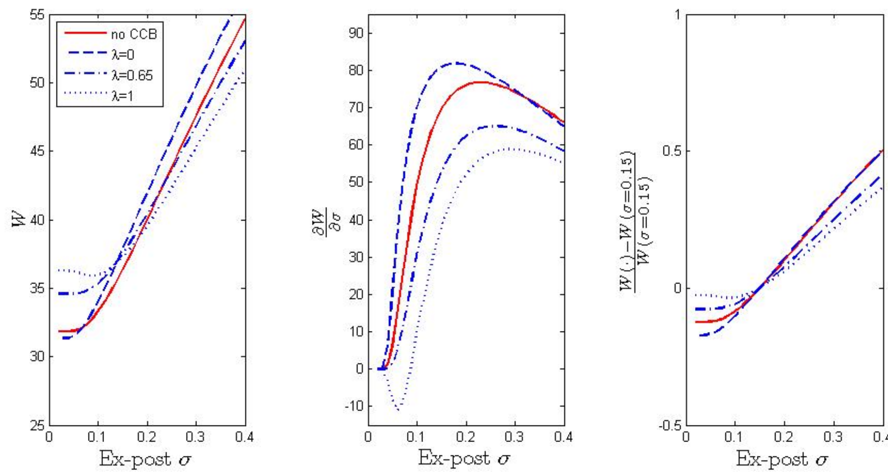


Figure 7: **Ex-post equity values: single firm.** (A_t = \$100, initial σ = 15.0%, c_b^* = \$5.24, A_C = \$70, c_c = \$0.5, $r = 0.05$, $\mu = 0.01$, $\theta = 35\%$, and $\alpha = 50\%$.)

7.2 Asset Substitution Under Regulatory Constraint

We continue with the question of asset substitution inefficiency, but now for a firm that introduces CCB to its capital structure by swapping CCB for straight debt following regulatory constraint (11). We repeat the two exercises from Section 7.1. Everything remains the same, including the amounts of CCB and the optimal amounts of straight debt that are issued. The only difference is that now CCB replaces a portion of straight debt based on (11) instead of being added to a capital structure that also includes straight debt.

Figure 7 helps to analyze how the value of equity changes depending on what level is set to ex-post. The values are shown for a single company (represented by the initial σ = 15.0%) and a range of ex-post σ values. As in Figure 5, the level of the graphs in Figure 7 are always inversely related to the λ value. The curve with no CCB, however, is now higher, so the addition of CCB now systematically reduces the incentive for asset substitution except for some instances with the extreme value of $\lambda = 0$. The intuition is that compared with the base case of no CCB, the firm now has less straight debt, the effect of which is to reduce the incentive for asset substitution.

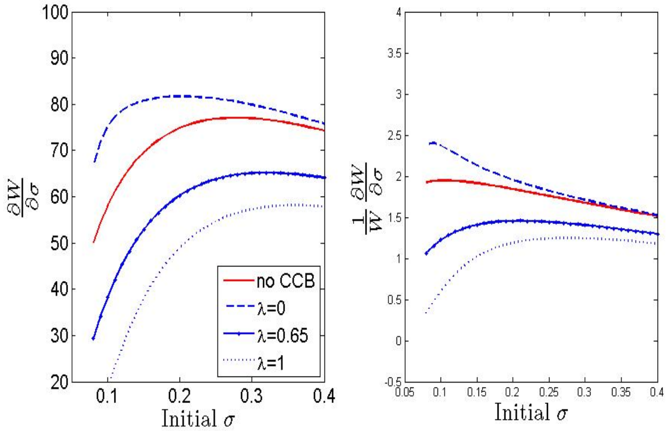


Figure 8: **Local rate of change and percentage change in ex-post equity values: multiple firms.** ($A_t = \$100$, individual initial σ values, $c_b^* = \$5.24$, $A_C = \$70$, $c_c = \$0.5$, $r = 0.05$, $\mu = 0.01$, $\theta = 35\%$, and $\alpha = 50\%$.)

Figure 8 shows how equity values respond in relative terms to local changes in volatility for a range of firms (each represented by a different initial σ). These graphs confirm the conclusion of Figure 7, namely that a swap of CCB for straight debt reduces the incentive for asset substitution except for some extreme cases with $\lambda = 0$. Thus quite generally, we find that introducing CCB to the capital structures of a firm based on regulatory constraint (11) reduces the effect of asset substitution inefficiency. This is different from what we saw in Section 7.1.

7.3 Asset Substitution After Swapping CCB into an Existing Capital Structure

We continue with the question of asset substitution, but now for a firm that swaps CCB for straight debt in an already existing capital structure. We repeat the two exercises from Sections 7.1 and 7.2. Coupon $\hat{c}_b$ is set equal to c_b^* so the amount of straight debt is optimal and the same as before. Everything else, including the amounts of CCB, is also the same. The only difference is that CCB replaces straight debt that has already been issued so that the swap constraint (14) holds. The

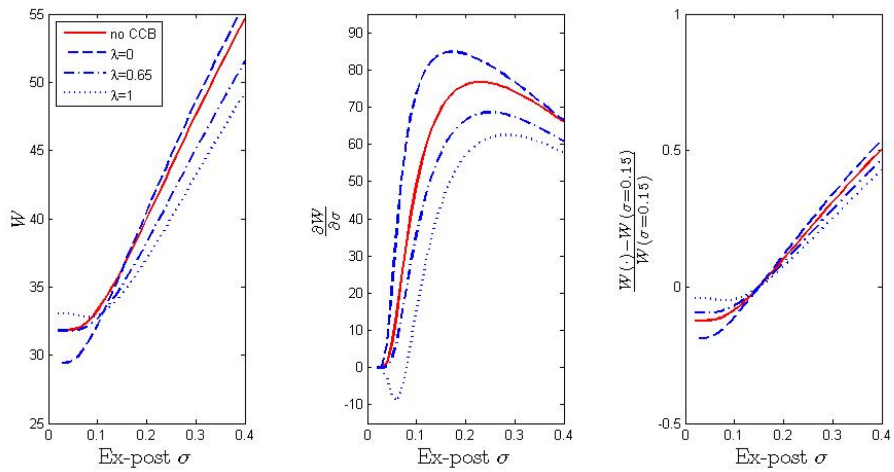


Figure 9: **Ex-post equity values: single firm.** (A_t = \$100, initial σ = 15.0%, $\hat{c_b}$ = \$5.24, A_C = \$70, c_c = \$0.5, $r = 0.05$, $\mu = 0.01$, $\theta = 35\%$, and $\alpha = 50\%$.)

CCB swap for straight debt now has the additional effect, however, of increasing the value of the straight debt, so that this firm will have a somewhat greater value of outstanding straight debt than in Section 7.2.

Figures 9 and 10 show the same experiment as illustrated in Figures 7 and 8. Indeed, the graphs are very similar, and the conclusion is again that the introduction of CCB in the capital structure under the condition of a CCB for debt swap decreases the incentive for asset substitution. Careful examination, however, indicates that the no CCB curve is slightly lower in Figures 9 and 10, indicating that the disincentive for asset substitution is slightly weaker in this case. The intuition is that the value of the straight debt in the capital structure is slightly greater in this case, which slightly magnifies the incentive for asset substitution.

The main take-away of this section is that issuing CCB as part of a CCB for equity swap, keeping the amount of straight debt unchanged, could magnify the effect of asset substitution inefficiency. On the other hand, issuing CCB as part of a CCB for straight debt swap could lead to reduced asset substitution.

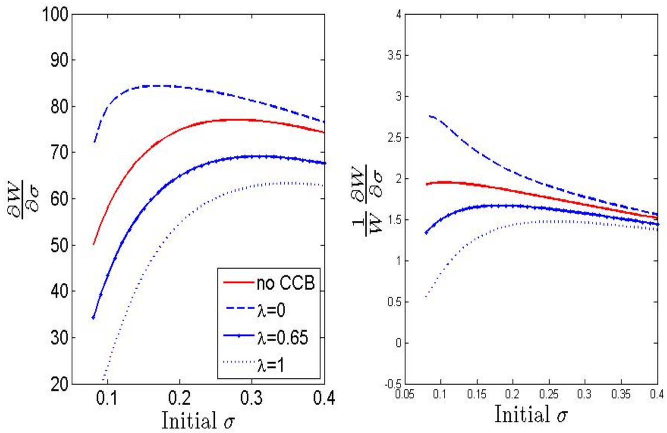


Figure 10: **Local rate of change and percentage change in ex-post equity values: multiple firms.** ($A_t = \$100$, individual initial σ values, $\hat{c_b} = \$5.24$, $A_C = \$70$, $c_c = \$0.5$, $r = 0.05$, $\mu = 0.01$, $\theta = 35\%$, and $\alpha = 50\%$.)

8 Summary and Policy Conclusions

This paper has provided a formal model of CCBs. The results of the formal model are summarized in Tables 1 and 2. Table 1 summarizes the primary effects of CCB issuance on firm and equity value as a function of the firm's capital structure status and any imposed constraints. Table 2 provides our primary results showing how the conversion ratio λ affects the incentives for CCB holders and equity holders to manipulate the firm's stock price in order to trigger conversion.

In terms of prudential bank regulation, we have shown that CCBs provide a new instrument that allows banks or firms to recapitalize in an automatic and dependable fashion whenever their capital reaches a distressed level. In other words, CCBs generally have the potential to provide most of the tax shield benefits of straight debt while providing the same protection as equity capital against bankruptcy costs. For CCBs to be effective in this role, however, it is important that the banks be required to substitute CCBs for straight debt, and not for equity, in their capital structure. The regulatory benefits of CCBs for bank safety also are greater the higher the trigger at which conversion occurs.

Table 1: Effects of CCB issuance on the capital structure of the firm*

Firm	Constraint	Firm Value	Equity Holders' Value	Default Risk	Asset Substitution	Tax Savings	Other Effects	Firm Decision
Unleveraged	Sufficiently small amount of CCB	↑	↑	↔	↑	↑	n/c	Issue CCB on top of optimal amount of SD
Leveraged with SD	Sufficiently small amount of CCB	↑	↑	↔	↑	↑	n/c	Issue CCB on top of existing amount of SD
Unleveraged	Total amount of debt is fixed	↑	↑	↓	↓	∼	n/c	Replace some SD with CCB
Leveraged	Total amount of debt is fixed	↑	↓	↓	↓	∼	Debt overhang	Do not issue CCB
TBTF (Leveraged/ Unleveraged)	Total amount of debt is fixed	↓	↓	↓	n/c	∼	Reduced government subsidy	Do not issue CCB

*SD: straight debt; TBTB: Too-big-to-fail; n/c: not considered; ↑: increase; ↓: decrease; ↔: no change; ∼: no effect or insignificant increase/decrease

Table 2: Incentives of CCB holders and equity holders to manipulate the stock market

Conversion Ratio	**Action**	**Intuition**
$0 < \lambda^* < \lambda$	CCB holders want to drive the stock price down to trigger conversion	If λ is high CCB holders receive a large amount of undervalued equity at conversion
$\lambda \leq \lambda^*$	CCB holders do not want to trigger conversion	If λ is low CCB holders are poorly compensated at conversion
$\lambda < 1 - \theta$	Equity holders want to drive the stock price down to trigger conversion	If λ is low equity holders can cheaply get rid of the obligation to pay c_c
$1 - \theta \leq \lambda$	Equity holders do not want to trigger conversion	If λ is high conversion is costly to equity holders

We conclude with comments on important topics for future research. We first comment on three extensions that would generalize assumptions in the current paper. One useful extension would fully determine the firm's optimal capital structure in the presence of CCBs. In particular, our analysis has been static in the sense that we assume the firm's entire CCB issue is converted into equity at a single point when the trigger is activated. We suspect, however, that the CCB benefits would expand further if the bonds could be converted in a sequence of triggers and/or that banks committed to issue new CCBs as soon as the existing bonds were converted. A second factor is that our analysis has assumed that both the CCBs and straight debt have an unlimited maturity in the fashion of a consol. We expect that an analysis with finite maturity bonds would find lower debt overhang costs of swapping CCBs for straight debt. A third extension would allow the geometric Brownian motion of asset dynamics to include jumps. This would have an impact on the valuation of all claims in the model. A related assumption is that we have not allowed the firm to use CCBs to purchase additional assets. While we do not expect this will change our basic results, this should be confirmed.

We conclude with two topics for future research concerning the use of CCBs for prudential bank regulation. As one topic, Flannery (2009a) has suggested that banks be presented with the choice of raising their capital ratio by a given amount or of raising their capital ratio by a smaller amount as long as it is combined with a specified amount of CCBs. The regulatory parameters in such a menu determine the tradeoff between regulatory benefits and bank costs. A calibrated version of our model could potentially measure the terms of this tradeoff . A second regulatory topic concerns the amount of tax shield benefit allowed CCB. As just one example, it would be useful to explore the effects of allowing full deduction for interest payments that correspond to the coupon on similar straight bank debt, but to exclude any part of the CCB coupon that represents compensation for the conversion risk.

Appendix: Proofs

Proof of Proposition 1. Except for the solutions for $G(A_t; c_b, c_c)$ and $W(A_t; c_b, c_c)$ all the other ones are derived as present values of the corresponding cash flows. Equation (2) is applied repeatedly with different values for K. Proposition 1 is used implicitly.

$$\begin{aligned}
U^B(A_t; c_b, c_c) &= E_t^Q\left[\int_t^{\tau(A_B)} e^{-r(s-t)} c_b ds + e^{-r(\tau(A_B)-t)}(1-\alpha)A_B\right] \\
&= E_t^Q\left[\frac{c_b}{r}\left(1 - e^{-r(\tau(A_B)-t)}\right) + e^{-r(\tau(A_B)-t)}(1-\alpha)A_B\right] \\
&= \frac{c_b}{r}\left(1 - \left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \left(\frac{A_t}{A_B}\right)^{-\gamma}(1-\alpha)A_B. \\
U^C(A_t; c_c) &= E_t^Q\left[\int_t^{\tau(A_C)} e^{-r(s-t)} c_c ds + e^{-r(\tau(A_C)-t)} L\right] \\
&= E_t^Q\left[\frac{c_c}{r}\left(1 - e^{-r(\tau(A_C)-t)}\right) + e^{-r(\tau(A_C)-t)} L\right] \\
&= \frac{c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) + \left(\frac{A_t}{A_C}\right)^{-\gamma} L. \\
TB(A_t; c_b, c_c) &= E_t^Q\left[\int_t^{\tau(A_B)} e^{-r(s-t)} \theta c_b ds + \int_t^{\tau(A_C)} e^{-r(u-t)} \theta c_c du\right] \\
&= \frac{\theta c_b}{r}\left(1 - \left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right). \\
BC(A_t; c_b) &= E^Q\left[\int_0^{\tau(A_B)} e^{-r(s-t)} \alpha A_B ds\right] = \alpha A_B \left(\frac{A_t}{A_B}\right)^{-\gamma}.
\end{aligned}$$

Based on the budget equation (4)

$$W(A_t; c_b, c_c) = A_t + TB(A_t; c_b, c_c) - U^B(A_t; c_b, c_c) - U^C(A_t; c_c) - BC(A_t; c_b).$$

Therefore,

$$\begin{aligned} W(A_t; c_b, c_c) &= A_t + \frac{\theta c_b}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right) \\ &\quad -\frac{c_b}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right) - \left(\frac{A_t}{A_B}\right)^{-\gamma}(1-\alpha)A_B \\ &\quad -\frac{c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \left(\frac{A_t}{A_C}\right)^{-\gamma} L - \alpha A_B \left(\frac{A_t}{A_B}\right)^{-\gamma} \\ &= A_t + \frac{c_b(\theta-1)}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \frac{c_c(\theta-1)}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \\ &\quad A_B\left(\frac{A_t}{A_B}\right)^{-\gamma} - L\left(\frac{A_t}{A_C}\right)^{-\gamma}. \end{aligned}$$

Finally, based on (5)

$$G(A_t; c_b, c_c) = A_t + \frac{\theta c_b}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \alpha A_B\left(\frac{A_t}{A_B}\right)^{-\gamma}.$$

□

Proof of Proposition 2. Based on Proposition 1

$$\frac{\partial G(A_t; c_b, c_c)}{\partial A_C} = -\frac{\theta c_c}{r}\gamma\frac{1}{A_C}\left(\frac{A_t}{A_C}\right)^{-\gamma} < 0$$

and

$$\frac{\partial TB(A_t; c_b, c_c)}{\partial A_C} = -\frac{\theta c_c}{r}\frac{1}{A_C}\left(\frac{A_t}{A_C}\right)^{-\gamma} < 0.$$

Since by design $\lambda \leq 1$,

$$\frac{\partial U^C(A_t; c_c)}{\partial A_C} = -\frac{c_c}{r}\gamma\frac{1}{A_C}\left(\frac{A_t}{A_C}\right)^{-\gamma} + \lambda\frac{c_c}{r}\gamma\frac{1}{A_C}\left(\frac{A_t}{A_C}\right)^{-\gamma} = \frac{c_c}{r}\gamma\frac{1}{A_C}\left(\frac{A_t}{A_C}\right)^{-\gamma}(\lambda-1) \leq 0.$$

Finally,

$$\begin{aligned}\frac{\partial W(A_t;c_b,c_c)}{\partial A_C} &= -\frac{c_c(1-\theta)}{r}\gamma\frac{1}{A_C}\left(\frac{A_t}{A_C}\right)^{-\gamma} - \lambda\frac{c_c}{r}\gamma\frac{1}{A_C}\left(\frac{A_t}{A_C}\right)^{-\gamma}\\ &= \frac{c_c}{r}\gamma\frac{1}{A_C}\left(\frac{A_t}{A_C}\right)^{-\gamma}((1-\theta)-\lambda).\end{aligned}$$

If $\lambda > (1-\theta)$, the value of equity decreases as A_C increases; if $\lambda < (1-\theta)$, the value of equity and A_C move in the same direction; and, if $\lambda = (1-\theta)$, the value of equity remains unaffected by changes in A_C.

The last statement of the proposition is trivial. In Proposition 1 A_C does not enter the solutions for the values of any of the remaining claims. □

Proof of Corollary 1. Consider two conversion-triggering asset levels $\bar{A}_C$ and $\bar{\bar{A}}_C$. Parameters c_c and λ are the same for both cases and Condition 1 holds. Based on Proposition 1

$$G(A_t,\bar{A}_C;c_b,c_c) - G(A_t,\bar{\bar{A}}_C;c_b,c_c) = \frac{\theta c_c}{r}\left(1-\left(\frac{A_t}{\bar{A}_C}\right)^{-\gamma}\right) - \frac{\theta c_c}{r}\left(1-\left(\frac{A_t}{\bar{\bar{A}}_C}\right)^{-\gamma}\right) \quad (A.1)$$

and

$$TB(A_t,\bar{A}_C;c_b,c_c) - TB(A_t,\bar{\bar{A}}_C;c_b,c_c) = \frac{\theta c_c}{r}\left(1-\left(\frac{A_t}{\bar{A}_C}\right)^{-\gamma}\right) - \frac{\theta c_c}{r}\left(1-\left(\frac{A_t}{\bar{\bar{A}}_C}\right)^{-\gamma}\right) \quad (A.2)$$

The right-hand sides of equations (A.1) and (A.2) are the same. □

Proof of Lemma 2. Based on Proposition 1

$$\begin{aligned}\frac{\partial W(A_t;c_b,c_c)}{\partial A_t} &= 1 + \frac{c_b(1-\theta)}{r}(-\gamma)\frac{1}{A_t}\left(\frac{A_t}{A_B}\right)^{-\gamma} + \frac{c_c(1-\theta)}{r}(-\gamma)\frac{1}{A_t}\left(\frac{A_t}{A_C}\right)^{-\gamma} - \\ &\quad (-\gamma)A_B\frac{1}{A_t}\left(\frac{A_t}{A_B}\right)^{-\gamma} - \frac{\lambda c_c}{r}(-\gamma)\frac{1}{A_t}\left(\frac{A_t}{A_C}\right)^{-\gamma}.\end{aligned}$$

Since by design $A_T \geq A_B$ it follows that

$$\begin{aligned}
\frac{\partial W(A_T; c_b, c_c)}{\partial A_T} &\geq 1 + \frac{c_b(1-\theta)}{r}(-\gamma)\frac{1}{A_B} + \frac{c_c(1-\theta)}{r}(-\gamma)\frac{1}{A_T}\left(\frac{A_T}{A_C}\right)^{-\gamma} - \\
&\quad (-\gamma) - \frac{\lambda c_c}{r}(-\gamma)\frac{1}{A_T}\left(\frac{A_T}{A_C}\right)^{-\gamma} \\
&= 1 - \gamma\left(\frac{c_b(1-\theta)}{r}\frac{1}{A_B} - 1\right) - \frac{c_c(1-\theta)\gamma}{r}\frac{1}{A_T}\left(\frac{A_T}{A_C}\right)^{-\gamma} + \frac{\lambda c_c \gamma}{r}\frac{1}{A_T}\left(\frac{A_T}{A_C}\right)^{-\gamma} \\
&= 1 - \gamma\left(\frac{1}{r\beta} - 1\right) - \frac{c_c\gamma}{r}\frac{1}{A_T}\left(\frac{A_T}{A_C}\right)^{-\gamma}(1-\theta-\lambda) \\
&= 1 - \gamma\left(\frac{\gamma+1}{\gamma} - 1\right) - \frac{c_c\gamma}{r}\frac{1}{A_T}\left(\frac{A_T}{A_C}\right)^{-\gamma}(1-\theta-\lambda) \\
&= -\frac{c_c\gamma}{r}\frac{1}{A_T}\left(\frac{A_T}{A_C}\right)^{-\gamma}(1-\theta-\lambda).
\end{aligned}$$

Finally, if $(\lambda+\theta) > 1$, then $\frac{\partial W(A_T; c_b, c_c)}{\partial A_T} > 0$. Note that the closed-form solution for $W(A_T; c_b, c_c)$ that we used above is for asset values above or at conversion-level A_C. This is the implicit assumptions of Proposition 1. Therefore, the above result holds for $A_T \geq A_C$. This completes the proof of the first statement.

Based on definition (8), $W(A_{CL}; c_b, c_c) = 0$. Given the results of Proposition 1, this leads to

$$\begin{aligned}
A_{CL} &- \frac{c_b(1-\theta)}{r}\left(1 - \left(\frac{A_{CL}}{A_B}\right)^{-\gamma}\right) - \frac{c_c(1-\theta)}{r}\left(1 - \left(\frac{A_{CL}}{A_{CL}}\right)^{-\gamma}\right) - \\
&\quad A_B\left(\frac{A_{CL}}{A_B}\right)^{-\gamma} - \left(\lambda\frac{c_c}{r}\right)\left(\frac{A_{CL}}{A_{CL}}\right)^{-\gamma} = 0 \\
A_{CL} &= \frac{c_b(1-\theta)}{r}\left(1 - \left(\frac{A_{CL}}{A_B}\right)^{-\gamma}\right) + A_B\left(\frac{A_{CL}}{A_B}\right)^{-\gamma} + \left(\lambda\frac{c_c}{r}\right). \qquad \text{(A.3)}
\end{aligned}$$

The right-hand side in (A.3) equals

$$\begin{aligned}
\frac{c_b(1-\theta)}{r} + \left(A_B - \frac{c_b(1-\theta)}{r}\right)\left(\frac{A_{CL}}{A_B}\right)^{-\gamma} + \left(\lambda\frac{c_c}{r}\right) &= \\
\frac{c_b(1-\theta)}{r} + \left(\frac{c_b(1-\theta)}{r}\beta - \frac{c_b(1-\theta)}{r}\right)\left(\frac{A_{CL}}{A_B}\right)^{-\gamma} + \left(\lambda\frac{c_c}{r}\right) &= \\
\frac{c_b(1-\theta)}{r} + \frac{c_b(1-\theta)}{r}(\beta-1)\left(\frac{A_{CL}}{A_B}\right)^{-\gamma} + \left(\lambda\frac{c_c}{r}\right) &< \frac{c_b(1-\theta)}{r} + \left(\lambda\frac{c_c}{r}\right).
\end{aligned}$$

The last inequality is based on $\beta < 1$. This proves that $A_{CL} < \frac{c_b(1-\theta)}{r} + \left(\lambda\frac{c_c}{r}\right)$.

As for the left-hand side, based on (A.3)

$$\begin{aligned}
A_{CL} &= \frac{c_b(1-\theta)}{r}\left(1-\left(\frac{A_{CL}}{A_B}\right)^{-\gamma}\right) - A_B + A_B\left(\frac{A_{CL}}{A_B}\right)^{-\gamma} + A_B + \left(\lambda\frac{c_c}{r}\right) \\
&= \left(\frac{c_b(1-\theta)}{r} - A_B\right)\left(1-\left(\frac{A_{CL}}{A_B}\right)^{-\gamma}\right) + A_B + \left(\lambda\frac{c_c}{r}\right) > A_B + \left(\lambda\frac{c_c}{r}\right).
\end{aligned}$$

This completes the proof of the second statement. □

Proof of Theorem 1. The capital structure that maximizes the market value, $G(A_0; c_b, c_c)$, received by the *initial owners* from the sale of a straight bond, equity and CCB is based on the straight debt coupon c_b^* that solves

$$\max_{c_b \geq 0} G(A_0; c_b, c_c) \equiv \max_{c_b \geq 0}\left[A_0 + TB(A_0; c_b, c_c) - BC(A_0; c_b)\right].$$

Since A_0 at $t = 0$ is constant, we get a new maximization problem:

$$\max_{c_b \geq 0}\left[TB(A_0; c_b, c_c) - BC(A_0; c_b)\right].$$

From Proposition 1 and (3)

$$\max_{c_b \geq 0}\left[\frac{\theta c_b}{r}\left(1-\left(\frac{A_0}{\beta(1-\theta)c_b}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1-\left(\frac{A_0}{A_C}\right)^{-\gamma}\right) - \alpha\beta(1-\theta)c_b\left(\frac{A_0}{\beta(1-\theta)c_b}\right)^{-\gamma}\right].$$

FOCs:

$$\left(\frac{\theta c_b}{r}\left(1-\left(\frac{\beta(1-\theta)}{A_0}\right)^\gamma c_b^\gamma\right)+\frac{\theta c_c}{r}\left(1-\left(\frac{A_C}{A_0}\right)^\gamma\right)-\alpha\frac{(\beta(1-\theta))^{\gamma+1}}{A_0^\gamma}c_b^{\gamma+1}\right)'=0$$
$$\left(\frac{\theta c_b}{r}-\frac{\theta}{r}\left(\frac{\beta(1-\theta)}{A_0}\right)^\gamma c_b^{\gamma+1}+\frac{\theta c_c}{r}\left(1-\left(\frac{A_C}{A_0}\right)^\gamma\right)-\alpha\frac{(\beta(1-\theta))^{\gamma+1}}{A_0^\gamma}c_b^{\gamma+1}\right)'=0$$
$$\frac{\theta}{r}-\frac{\theta(\gamma+1)}{r}\left(\frac{\beta(1-\theta)}{A_0}\right)^\gamma c_b^\gamma-\alpha(\gamma+1)\frac{(\beta(1-\theta))^{\gamma+1}}{A_0^\gamma}c_b^\gamma=0$$
$$c_b=\left[\frac{\theta}{r}(\gamma+1)^{-1}\left(\frac{\beta(1-\theta)}{A_0}\right)^{-\gamma}\left(\frac{\theta}{r}+\alpha\beta(1-\theta)\right)^{-1}\right]^{\frac{1}{\gamma}}.$$

Finally,

$$c_b^*(A_0;c_c) \quad = \quad \frac{A_0}{\beta(1-\theta)}\left(\frac{\theta}{r}\right)^{\frac{1}{\gamma}}\left[(\gamma+1)(\frac{\theta}{r}+\alpha\beta(1-\theta))\right]^{-\frac{1}{\gamma}}.$$

We can see that the optimal coupon on the straight debt does not depend on any characteristics of CCB. One can repeat the above calculations for the case when the capital structure includes only equity and straight debt to show formally that $c_b^*(A_0;c_c)=c_b^*(A_0;0)$. □

Proof of Proposition 3. We start with the last statement and work our way up to the first one in a consecutive order.

One can closely follow the math in Appendix A to derive results similar to the ones in Proposition 1 for the case when the firm uses only straight debt and equity[15]. Item (*iv*) will follow.

As for item (*iii*), based on Proposition 1 and (7)

$$\begin{aligned} TB(A_0;c_b^*,c_c) &= \frac{\theta c_b^*}{r}\left(1-\left(\frac{A_0}{A_B}\right)^{-\gamma}\right)+\frac{\theta c_c}{r}\left(1-\left(\frac{A_0}{A_C}\right)^{-\gamma}\right) \\ &= TB(A_0;c_b^*,0)+TB^C(A_0;c_b^*,c_c). \end{aligned}$$

$A_C<A_0$ (by design) and $\gamma>0$ (as shown above), so $TB^C(A_0;c_b^*,c_c)>0$. The tax savings

[15]These results have been derived in previous literature (Leland (1994)). One can also use Proposition 1 to try to recognize what the corresponding values should be when CCB is not used.

increase in c_c and decline in A_C. Higher c_c raises the deductibles which leads to higher savings. But, higher levels of A_C trigger sooner conversions and, therefore, reduce the amount of time during which the firm can deduct c_c.

By re-grouping the terms in the formula for the value of equity from Proposition 1, we get

$$
\begin{aligned}
W(A_0; c_b^*, c_c) &= A_0 - \frac{c_b^*(1-\theta)}{r}\left(1-\left(\frac{A_0}{A_B}\right)^{-\gamma}\right) - A_B\left(\frac{A_0}{A_B}\right)^{-\gamma} - \\
&\quad \left[\frac{c_c(1-\theta)}{r}\left(1-\left(\frac{A_0}{A_C}\right)^{-\gamma}\right) + \left(\lambda\frac{c_c}{r}\right)\left(\frac{A_0}{A_C}\right)^{-\gamma}\right] + \\
&\quad \frac{\theta c_c}{r}\left(1-\left(\frac{A_0}{A_C}\right)^{-\gamma}\right) \\
&= W(A_0; c_b^*, 0) - U^C(A_0; c_c) + TB^C(A_0; c_b^*, c_c)
\end{aligned}
$$

as in item (ii).

Finally, based on (5),

$$
\begin{aligned}
G(A_0; c_b^*, c_c) &= A_0 + TB(A_0; c_b^*, c_c) - BC(A_t, c_b) \\
&= \left[A_0 + \frac{\theta c_b}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right) - \alpha A_B\left(\frac{A_t}{A_B}\right)^{-\gamma}\right] + \\
&\quad \frac{\theta c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right), \\
&= G(A_0; c_b^*, 0) + TB^C(A_0; c_b^*, c_c).
\end{aligned}
$$

This proves item (i). □

Proof of Proposition 10. By differentiating both sides of (10) relative to σ, we get

$$
\frac{\partial W(A_0; c_b^*, c_c)}{\partial \sigma} = \frac{\partial W(A_0; c_b^*, 0)}{\partial \sigma} - \frac{\partial\left[U^C(A_0; c_c) - TB^C(A_0; c_b^*, c_c)\right]}{\partial \sigma} \tag{A.4}
$$

Based on Proposition 1 and (7)

$$\begin{aligned}U^C(A_0;c_c)-TB^C(A_0;c_b^*,c_c) &= \frac{c_c}{r}\left(1-\left(\frac{A_0}{A_C}\right)^{-\gamma}\right)+\left(\frac{A_0}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right)- \\ &\quad \frac{\theta c_c}{r}\left(1-\left(\frac{A_0}{A_C}\right)^{-\gamma}\right) \\ &= \frac{c_c}{r}(1-\theta)\left(1-\left(\frac{A_0}{A_C}\right)^{-\gamma}\right)+\left(\frac{A_0}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right) \\ &= \frac{c_c}{r}(1-\theta)+\frac{c_c}{r}(\lambda+\theta-1)\left(\frac{A_0}{A_C}\right)^{-\gamma}\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{\partial\left[U^C(A_0;c_c)-TB^C(A_0;c_b^*,c_c)\right]}{\partial\sigma} &= \frac{\partial\left[\frac{c_c}{r}(1-\theta)+\frac{c_c}{r}(\lambda+\theta-1)\left(\frac{A_0}{A_C}\right)^{-\gamma}\right]}{\partial\sigma} \\ &= \frac{c_c}{r}(\lambda+\theta-1)\frac{\partial\left[\left(\frac{A_0}{A_C}\right)^{-\gamma}\right]}{\partial\sigma} \\ &= \frac{c_c}{r}(\lambda+\theta-1)\frac{\partial\left[e^{-\gamma\log\left(\frac{A_0}{A_C}\right)}\right]}{\partial\sigma} \\ &= \frac{c_c}{r}(\lambda+\theta-1)e^{-\gamma\log\left(\frac{A_0}{A_C}\right)}\left(-\log\left(\frac{A_0}{A_C}\right)\right)\frac{\partial\gamma}{\partial\sigma} \\ &= -\frac{c_c}{r}(\lambda+\theta-1)e^{-\gamma\log\left(\frac{A_0}{A_C}\right)}\log\left(\frac{A_0}{A_C}\right)\frac{\partial\gamma}{\partial\sigma}.\end{aligned}$$

By plugging this into (A.4), we get

$$\frac{\partial W(A_0;c_b^*,c_c)}{\partial\sigma} = \frac{\partial W(A_0;c_b^*,0)}{\partial\sigma}+\frac{c_c}{r}(\lambda+\theta-1)e^{-\gamma\log\left(\frac{A_0}{A_C}\right)}\log\left(\frac{A_0}{A_C}\right)\frac{\partial\gamma}{\partial\sigma}. \qquad \text{(A.5)}$$

It is easy to show that for $\sigma>0$: $\frac{\partial\gamma}{\partial\sigma}<0$[16]. If $(\lambda+\theta)>1$, then the last term in (10) is strictly positive and $\frac{\partial W(A_0;c_b^*,c_c)}{\partial\sigma}<\frac{\partial W(A_0;c_b^*,0)}{\partial\sigma}$. If $(\lambda+\theta)>1$, then it is strictly negative and $\frac{\partial W(A_0;c_b^*,c_c)}{\partial\sigma}>\frac{\partial W(A_0;c_b^*,0)}{\partial\sigma}$. The interpretation is that, if $(\lambda+\theta)>1$, then the equity value holders

[16]Show that or $\sigma>0$: $\frac{\partial\gamma}{\partial\sigma}<0$

of the firm that issued CCB, straight debt and equity compared to the equity holders of the firm that issued only straight debt and equity will gain less from switching to riskier technologies. On the other hand, if $(\lambda + \theta) > 1$, the presence of CCB makes the shareholders gain more from taking extra risk. $\square$

Proof of Proposition 4. Based on budget equation (5) and regulatory constraint (11)

$$\begin{aligned} G(A_0, \bar{A}_B; \bar{c_b}, c_c) - G(A_0, A_B^*; c_b^*, 0) &= W(A_0, \bar{A}_B; \bar{c}, c_c) + U^C(A_0, \bar{A}_B; \bar{c_b}, c_c) + \\ &\quad U^B(A_0, \bar{A}_B; \bar{c_b}, c_c) - W(A_0, A_B^*; c_b^*, 0) - \\ &\quad U^B(A_0, A_B^*; c_b^*, 0) \\ &= W(A_0, \bar{A}_B; \bar{c_b}, c_c) - W(A_0, A_B^*; c_b^*, 0). \end{aligned}$$

The change in the total value of the firm equals the change in the value of equity.

Denote $W(A_0, \bar{A}_B; \bar{c_b}, c_c) - W(A_0, A_B; c_b^*, 0)$ by ΔW. Based on budget equation (4)

$$\begin{aligned} \Delta W &= A_0 + TB(A_0, \bar{A}_B; \bar{c_b}, c_c) - U^B(A_0, \bar{A}_B; \bar{c_b}, c_c) - U^C(A_0, \bar{A}_B; c_c) - BC(A_t, \bar{A}_B; \bar{c_b}) - \\ &\quad A_0 - TB(A_0, A_B^*; c_b^*, 0) + U^B(A_0, A_B^*; c_b^*, 0) + BC(A_0, A_B^*; c_b^*) \\ &= TB(A_0, \bar{A}_B; \bar{c_b}, c_c) - TB(A_0, A_B^*; c_b^*, 0) + BC(A_0, A_B^*; c_b^*) - BC(A_t, \bar{A}_B; \bar{c_b}). \end{aligned}$$

Next, based on closed-form solutions from Proposition 1

$$\begin{aligned} \Delta W &= \frac{\theta \bar{c_b}}{r}\left(1 - \left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1 - \left(\frac{A_0}{A_C}\right)^{-\gamma}\right) - \frac{\theta c_b^*}{r}\left(1 - \left(\frac{A_0}{A_B^*}\right)^{-\gamma}\right) + \\ &\quad \alpha A_B^* \left(\frac{A_0}{A_B^*}\right)^{-\gamma} - \alpha \bar{A}_B \left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}. \end{aligned}$$

Add and subtract terms and use Proposition 1 again

$$
\begin{aligned}
\Delta W &= \theta\{\frac{\bar{c_b}}{r}\left(1-\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}\right)+\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B+\frac{c_c}{r}\left(1-\left(\frac{A_0}{A_C}\right)^{-\gamma}\right)+ \\
&\left(\frac{A_0}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right)-\frac{c_b^*}{r}\left(1-\left(\frac{A_0}{A_B^*}\right)^{-\gamma}\right)-\left(\frac{A_0}{A_B^*}\right)^{-\gamma}(1-\alpha)A_B^*- \\
&\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B-\left(\frac{A_0}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right)+\left(\frac{A_0}{A_B^*}\right)^{-\gamma}(1-\alpha)A_B^*\}+ \\
&\alpha A_B^*\left(\frac{A_0}{A_B^*}\right)^{-\gamma}-\alpha\bar{A}_B\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma} \\
&= \theta\{U^B(A_0,\bar{A}_B;\bar{c_b},c_c)+U^C(A_0,\bar{A}_B;c_b,c_c)-U^B(A_0,A_B^*;c_b^*,0)- \\
&\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B-\left(\frac{A_0}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right)+\left(\frac{A_0}{A_B^*}\right)^{-\gamma}(1-\alpha)A_B^*\}+ \\
&\alpha A_B^*\left(\frac{A_0}{A_B^*}\right)^{-\gamma}-\alpha\bar{A}_B\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}.
\end{aligned}
$$

Based on (11) and by re-grouping terms

$$
\begin{aligned}
\Delta W &= -\theta\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B-\theta\left(\frac{A_0}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right)+\theta\left(\frac{A_0}{A_B^*}\right)^{-\gamma}(1-\alpha)A_B^*+ \\
&\alpha A_B^*\left(\frac{A_0}{A_B^*}\right)^{-\gamma}-\alpha\bar{A}_B\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}.
\end{aligned}
$$

Finally,

$$
\begin{aligned}
\Delta W &= -(\theta(1-\alpha)+\alpha)\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}\bar{A}_B+(\theta(1-\alpha)+\alpha)\left(\frac{A_0}{A_B^*}\right)^{-\gamma}A_B^*- \\
&\theta\left(\frac{A_0}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right) \\
&= (\theta+\alpha-\theta\alpha)(\left(\frac{A_0}{A_B^*}\right)^{-\gamma}A_B^*-\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}\bar{A}_B)-\theta\left(\frac{A_0}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right),
\end{aligned}
$$

which completes the proof of the first part of the proposition.

Denote $G(A_0, \bar{A}_B; \bar{c_b}, c_c) - G(A_0, A_B^*; c_b^*, 0)$ by ΔG. We know that $\Delta G = \Delta W$ and, therefore,

$$\Delta G = (\theta + \alpha - \theta\alpha)(\left(\frac{A_0}{A_B^*}\right)^{-\gamma} A_B^* - \left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma} \bar{A}_B) - \theta \left(\frac{A_0}{A_C}\right)^{-\gamma} \left(\lambda \frac{c_c}{r}\right).$$

This leads to

$$\frac{\partial \Delta G}{\partial \bar{c_b}} = -(\theta + \alpha - \theta\alpha)(\gamma + 1) \left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma} \frac{\partial \bar{A}_B}{\partial \bar{c_b}} - \theta \left(\frac{A_0}{A_C}\right)^{-\gamma} \frac{\lambda}{r} \frac{\partial c_c}{\partial \bar{c_b}}. \tag{A.6}$$

Based on (12)

$$\begin{aligned}
\frac{\partial c_c}{\partial \bar{c_b}} &= \frac{-\frac{\partial U^B(A_t, \bar{A}_B; \bar{c_b}, 1)}{\partial \bar{c_b}}}{\frac{1}{r}\left(1 - (1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)} \\
&= -\frac{\frac{1}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \frac{\bar{c_b}}{r}\gamma\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} \frac{1}{\bar{A}_B}\frac{\partial \bar{A}_B}{\partial \bar{c_b}} + (1-\alpha)(\gamma+1)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\frac{\partial \bar{A}_B}{\partial \bar{c_b}}}{\frac{1}{r}\left(1 - (1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)}. \qquad \text{(A.7)}
\end{aligned}$$

Given that $\bar{A}_B = \beta(1-\theta)\bar{c_b}$ and $\frac{\partial \bar{A}_B}{\partial \bar{c_b}} = \beta(1-\theta)$ and by plugging (A.7) into (A.6), we get

$$\begin{aligned}
\frac{\partial \Delta G}{\partial \bar{c_b}} &= -(\theta + \alpha - \theta\alpha)(\gamma + 1)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} \beta(1-\theta) + \\
&\quad \theta\left(\frac{A_t}{A_C}\right)^{-\gamma} \frac{\lambda}{r} \frac{\frac{1}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \frac{1}{r}\gamma\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} + (1-\alpha)(\gamma+1)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\beta(1-\theta)}{\frac{1}{r}\left(1 - (1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)} \\
&= -(\theta + \alpha - \theta\alpha)(\gamma + 1)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} \beta(1-\theta) + \\
&\quad \frac{\theta\lambda\left[\frac{1}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \frac{1}{r}\gamma\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} + (1-\alpha)(\gamma+1)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\beta(1-\theta)\right]}{\left(\frac{A_t}{A_C}\right)^{\gamma} - 1 + \lambda}.
\end{aligned}$$

Since $\frac{\lambda}{\left(\frac{A_t}{A_C}\right)^{\gamma}-1+\lambda} \leq 1$

$$\begin{aligned}
\frac{\partial \Delta G}{\partial \bar{c_b}} &\leq -(\theta+\alpha-\theta\alpha)(\gamma+1)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\beta(1-\theta)+ \\
&\quad \theta\left[\frac{1}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)-\frac{1}{r}\gamma\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}+(1-\alpha)(\gamma+1)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\beta(1-\theta)\right] \\
&= -\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\left[(\gamma+1)\beta(1-\theta)\alpha+\frac{\theta\gamma}{r}\right]+\frac{\theta}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) \\
&= -\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\left[(\gamma+1)\left(\alpha\beta(1-\theta)+\frac{\theta}{r}\right)\right]+\frac{\theta}{r}.
\end{aligned}$$

Based on the above

$$\left.\frac{\partial \Delta G}{\partial \bar{c_b}}\right|_{\bar{c_b}=c_b^*} \leq -\left(\frac{A_t}{A_B^*}\right)^{-\gamma}\left[(\gamma+1)\left(\alpha\beta(1-\theta)+\frac{\theta}{r}\right)\right]+\frac{\theta}{r}. \tag{A.8}$$

From (9)

$$\begin{aligned}
\frac{\beta(1-\theta)c_b^*}{A_t} &= \left(\frac{\theta}{r}\right)^{\frac{1}{\gamma}}\left[(\gamma+1)(\frac{\theta}{r}+\alpha\beta(1-\theta))\right]^{-\frac{1}{\gamma}} \\
\left(\frac{A_t}{A_B^*}\right)^{-\gamma} &= \left(\frac{\theta}{r}\right)\left[(\gamma+1)(\frac{\theta}{r}+\alpha\beta(1-\theta))\right]^{-1}
\end{aligned}$$

By using this in (A.8)

$$\left.\frac{\partial \Delta G}{\partial \bar{c_b}}\right|_{\bar{c_b}=c_b^*} \leq -\frac{\theta}{r}+\frac{\theta}{r}=0.$$

This means that $\exists\, \bar{c}_1$ such that, for $\forall\, c_c \in (0,\bar{c}_1)$, $\Delta G \leq 0$. Given that $G(A_0, A_B^*; c_b^*, 0)$ is fixed, for $\forall\, c_c \in (0,\bar{c}_1)$, $G(A_0, \bar{A}_B; \bar{c_b}, c_c) \geq G(A_0, A_B^*; c_b^*, 0)$. This proves the second part of the proposition.

As for the bankruptcy costs, based on (4), $A_B^* = \beta(1-\theta)c_b^*$ and $\bar{A}_B = \beta(1-\theta)\bar{c_b}$. Given the closed-form solutions from Proposition 1

$$\begin{aligned} BC(A_0,\bar{A}_B;\bar{c_b}) - BC(A_0,A_B^*;c_b^*) &= \alpha\bar{A}_B\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma} - \alpha A_B^*\left(\frac{A_0}{A_B^*}\right)^{-\gamma} \\ &= \alpha\bar{c_b}\beta(1-\theta)\left(\frac{\bar{c_b}\beta(1-\theta)}{A_0}\right)^{\gamma} - \\ &\quad \alpha c_b^*\beta(1-\theta)\left(\frac{c_b^*\beta(1-\theta)}{A_0}\right)^{\gamma} \\ &= (\bar{c_b}^{\gamma+1} - (c_b^*)^{\gamma+1})\alpha\frac{(\beta(1-\theta))^{\gamma+1}}{A_0^{\gamma}}. \end{aligned}$$

Since $\bar{c_b} < c_b^*$, the last term is strictly negative. Therefore, $BC(A_0,\bar{A}_B;\bar{c_b}) < BC(A_0,A_B^*;c_b^*)$. □

Proof of Proposition 5. Denote $W(A_t,\bar{A}_B;\bar{c_b},c_c) - W(A_t,\hat{A}_B;\hat{c_b},0)$ by $\Delta\hat{W}$. When the capital structure of the firm includes only equity and straight debt the closed-form solution for the value of equity is

$$W(A_t,\hat{A}_B;\hat{c_b},0) = A_t - \frac{\hat{c_b}(1-\theta)}{r}\left(1-\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) - \hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}. \qquad \text{(A.9)}$$

Based on (A.9) and the closed-form solution for $W(A_t,\bar{A}_B;\bar{c_b},c_c)$ from Proposition 1

$$\begin{aligned} \Delta\hat{W} &= A_t - \frac{\bar{c_b}(1-\theta)}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \frac{c_c(1-\theta)}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \\ &\quad \left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma} - A_t + \frac{\hat{c_b}(1-\theta)}{r}\left(1-\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) + \hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} \\ &= \frac{(\hat{c_b}-\bar{c_b})(1-\theta)}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \frac{c_c(1-\theta)}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \\ &\quad \left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma} + \frac{\hat{c_b}(1-\theta)}{r}\left(\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) + \hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}. \end{aligned}$$

Multiply both sides of (15) by $(1-\theta)$ and use the result to reduce the first three terms after the equal sign above to get

$$
\begin{aligned}
\Delta\hat{W} &= -\theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma} + \frac{\hat{c}_b(1-\theta)}{r}\left(\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) + \\
& \hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}. \qquad \text{(A.10)}
\end{aligned}
$$

Continue with showing that $\Delta\hat{W} < 0$. From (A.10)

$$
\begin{aligned}
\Delta\hat{W} &= \frac{\hat{c}_b(1-\theta)}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \\
& \left(\frac{\hat{c}_b(1-\theta)}{r}\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) - \theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma} \\
&= -(H(\hat{A}_B) - H(\bar{A}_B)) - \theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}, \qquad \text{(A.11)}
\end{aligned}
$$

where $H(X) \equiv \frac{\hat{c}_b(1-\theta)}{r}\left(\frac{A_t}{X}\right)^{-\gamma} - X\left(\frac{A_t}{X}\right)^{-\gamma}$. $H(X)$ is such that

$$
\begin{aligned}
H'(X) &= \gamma\frac{\hat{c}_b(1-\theta)}{r}\left(\frac{A_t}{X}\right)^{-\gamma}\frac{1}{X} - (1+\gamma)\left(\frac{A_t}{X}\right)^{-\gamma} \\
&= \left(\frac{A_t}{X}\right)^{-\gamma}\frac{1}{X}\left(\gamma\frac{\hat{c}_b}{r}(1-\theta) - (1+\gamma)X\right).
\end{aligned}
$$

Since $\hat{A}_B = \frac{\gamma(1-\theta)\hat{c}_b}{r(1+\gamma)}$

$$
\begin{aligned}
H'(\hat{A}_B) &= \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\frac{1}{\hat{A}_B}\left(\gamma\frac{\hat{c}_b}{r}(1-\theta) - (1+\gamma)\frac{\gamma(1-\theta)\hat{c}_b}{r(1+\gamma)}\right) \\
&= \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\frac{1}{\hat{A}_B}\left(\gamma\frac{\hat{c}_b}{r}(1-\theta) - (1-\theta)\gamma\frac{\hat{c}_b}{r}\right) \\
&= 0.
\end{aligned}
$$

It it also clear from above that if $0 < X < \hat{A}$, then $H'(X) > 0$. $H(X)$ is an increasing function of X on $(0, \hat{A})$. Since $0 < \bar{A} < \hat{A}_B$, $H(\hat{A}_B) > H(\bar{A}_B)$ and, based on (A.11), $\Delta\hat{W} < 0$. The value of equity always decreases. This proves item (i).

Denote $G(A_t, \bar{A}_B; \bar{c_b}, c_c) - G(A_t, \hat{A}_B; \hat{c_b}, 0)$ by $\Delta\hat{G}$. When the capital structure of the firm includes only equity and straight debt the closed-form solution for the total value of the firm is

$$G(A_t, \hat{A}_B; \hat{c_b}, 0) \quad = \quad A_t + \frac{\hat{c_b}\theta}{r}\left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) - \alpha\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}. \tag{A.12}$$

Given (A.12) and the closed-form solution for $G(A_t, \bar{A}_B; \bar{c_b}, c_c)$ from Proposition 1

$$\begin{aligned}
\Delta\hat{G} \quad &= \quad A_t + \frac{\theta\bar{c_b}}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \alpha\bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \\
&\quad\quad A_t - \frac{\hat{c_b}\theta}{r}\left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) + \alpha\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} \\
&= \quad \frac{\theta\bar{c_b}}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \alpha\bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \\
&\quad\quad \frac{\hat{c_b}\theta}{r}\left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) + \alpha\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}.
\end{aligned}$$

Multiply both sides of (15) by θ and replace $\frac{\theta c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right)$ above to get

$$\begin{aligned}
\Delta\hat{G} \quad &= \quad \frac{\theta\bar{c_b}}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) + \frac{\theta(\hat{c_b} - \bar{c_b})}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \theta\left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right) - \\
&\quad\quad \frac{\hat{c_b}\theta}{r}\left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) + \alpha\left(\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) \\
&= \quad \frac{\hat{c_b}\theta}{r}\left(\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) + \alpha\left(\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \\
&\quad\quad \theta\left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right).
\end{aligned}$$

This proves (16).

We continue with showing that $\Delta\hat{G} > \Delta\hat{W}$. Based on (A.10) and (16)

$$\begin{aligned}
\Delta\hat{G} - \Delta\hat{W} &= \frac{\hat{c}_b}{r}\left(\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - (1-\alpha)\left(\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) \\
&= \frac{\hat{c}_b}{r}\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - (1-\alpha)\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \\
&\quad \left(\frac{\hat{c}_b}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - (1-\alpha)\bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) \\
&= F(\hat{A}_B) - F(\bar{A}_B), \qquad \text{(A.13)}
\end{aligned}$$

where $F(X) \equiv \frac{\hat{c}_b}{r}\left(\frac{A_t}{X}\right)^{-\gamma} - (1-\alpha)X\left(\frac{A_t}{X}\right)^{-\gamma}$. $F(X)$ is such that

$$\begin{aligned}
F'(X) &= \gamma\frac{\hat{c}_b}{r}\left(\frac{A_t}{X}\right)^{-\gamma}\frac{1}{X} - (1-\alpha)(1+\gamma)\left(\frac{A_t}{X}\right)^{-\gamma} \\
&= \left(\frac{A_t}{X}\right)^{-\gamma}\frac{1}{X}\left(\gamma\frac{\hat{c}_b}{r} - (1-\alpha)(1+\gamma)X\right).
\end{aligned}$$

Note, that $\hat{A}_B = \frac{\gamma(1-\theta)\hat{c}_b}{r(1+\gamma)}$ and, therefore,

$$\begin{aligned}
F'(\hat{A}_B) &= \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\frac{1}{\hat{A}_B}\left(\gamma\frac{\hat{c}_b}{r} - (1-\alpha)(1+\gamma)\frac{\gamma(1-\theta)\hat{c}_b}{r(1+\gamma)}\right) \\
&= \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\frac{1}{\hat{A}_B}\left(\gamma\frac{\hat{c}_b}{r} - (1-\alpha)(1-\theta)\gamma\frac{\hat{c}_b}{r}\right) \\
&= \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\frac{1}{\hat{A}_B}\gamma\frac{\hat{c}_b}{r}\left(1 - (1-\alpha)(1-\theta)\right).
\end{aligned}$$

By assumption, $\alpha \in [0,1]$ and $\theta \in (0,1)$, so $(1-(1-\alpha)(1-\theta)) > 0$. It follows that $F'(\hat{A}_B) > 0$ for all $0 < X \leq \hat{A}_B$, and, since $0 < \bar{A}_B < \hat{A}_B$, $F(\hat{A}_B) > F(\bar{A}_B)$. Finally, based on (A.13), $\Delta\hat{G} > \Delta\hat{W}$.

We continue with proving the last statement of item (ii).

$$\begin{aligned}\frac{\partial \Delta\hat{G}}{\partial \bar{c_b}} &= \left(-\gamma\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\frac{1}{\hat{A}_B}\frac{\hat{c_b}\theta}{r} - \alpha(1+\gamma)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)\frac{\partial \bar{A}_B}{\partial \bar{c_b}} - \\ &\theta\left(\frac{A_t}{A_C}\right)^{-\gamma}\frac{1}{r}\frac{-\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - (\hat{c_b}-\bar{c_b})\gamma\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\frac{1}{\bar{A}_B}\frac{\partial \bar{A}_B}{\partial \bar{c_b}}}{1-(1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}}\end{aligned}$$

where $\frac{\partial \bar{A}_B}{\partial \bar{c_b}} = \beta(1-\theta)$. Based on the above

$$\begin{aligned}\left.\frac{\partial \Delta\hat{G}}{\partial \bar{c_b}}\right|_{\bar{c_b}=\hat{c_b}} &= -\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\left(\frac{\gamma}{\hat{A}_B}\frac{\bar{c_b}\theta}{r} + \alpha(1+\gamma)\right)\beta(1-\theta) + \\ &\frac{\theta}{r}\left(\frac{A_t}{A_C}\right)^{-\gamma}\frac{1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}}{1-(1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}} \\ &= -\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\left(\frac{\gamma\theta}{r} + \alpha(1+\gamma)\beta(1-\theta)\right) + \\ &\frac{\theta}{r}\left(\frac{A_t}{A_C}\right)^{-\gamma}\frac{1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}}{1-(1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}} \\ &= -\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\left(\frac{\gamma\theta}{r} + \alpha(1+\gamma)\beta(1-\theta)\right) + \\ &\frac{\theta}{r}\frac{1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}}{\left(\frac{A_t}{A_C}\right)^{\gamma}-(1-\lambda)}.\end{aligned}$$

For $\left(\frac{A_t}{A_C}\right)^{\gamma} - (1-\lambda) \geq 1$ or $\lambda \geq 2 - \left(\frac{A_t}{A_C}\right)^{\gamma}$

$$\begin{aligned}\left.\frac{\partial \Delta\hat{G}}{\partial \bar{c_b}}\right|_{\bar{c_b}=\hat{c_b}} &\leq -\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\left(\frac{\gamma\theta}{r} + \alpha(1+\gamma)\beta(1-\theta)\right) + \frac{\theta}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) \\ &= -\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}(1+\gamma)\left(\frac{\theta}{r} + \alpha\beta(1-\theta)\right) + \frac{\theta}{r}. \end{aligned} \tag{A.14}$$

Next, assume that $\hat{c_b} \geq c_b^*$. Then, based on (9)

$$\begin{aligned}
\hat{c_b} &\geq \frac{A_t}{\beta(1-\theta)} \left(\frac{\theta}{r}\right)^{\frac{1}{\gamma}} \left[(\gamma+1)(\frac{\theta}{r} + \alpha\beta(1-\theta))\right]^{-\frac{1}{\gamma}} \\
\frac{\beta(1-\theta)\hat{c_b}}{A_t} &\geq \left(\frac{\theta}{r}\right)^{\frac{1}{\gamma}} \left[(\gamma+1)(\frac{\theta}{r} + \alpha\beta(1-\theta))\right]^{-\frac{1}{\gamma}}.
\end{aligned}$$

Based on (3)

$$\begin{aligned}
\frac{\hat{A}_B}{A_t} &\geq \left(\frac{\theta}{r}\right)^{\frac{1}{\gamma}} \left[(\gamma+1)(\frac{\theta}{r} + \alpha\beta(1-\theta))\right]^{-\frac{1}{\gamma}} \\
\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} &\geq \frac{\theta}{r} \left[(\gamma+1)(\frac{\theta}{r} + \alpha\beta(1-\theta))\right]^{-1} \\
-\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} &\leq -\frac{\theta}{r} \left[(\gamma+1)(\frac{\theta}{r} + \alpha\beta(1-\theta))\right]^{-1}. \qquad \text{(A.15)}
\end{aligned}$$

By using (A.15) in (A.14), we get

$$\left.\frac{\partial \Delta \hat{G}}{\partial \bar{c_b}}\right|_{\bar{c_b}=\hat{c_b}} \leq -\frac{\theta}{r} + \frac{\theta}{r} = 0.$$

This means that $\exists\ \bar{\bar{c}}_1$ such that, for $\forall\ c_c \in (0, \bar{\bar{c}}_1)$, $\Delta\hat{G} \leq 0$. Given that $G(A_0, A_B^*; c_b^*, 0)$ is fixed, for $\forall\ c_c \in (0, \bar{\bar{c}}_1)$, $G(A_0, \bar{A}_B; \bar{c_b}, c_c) \geq G(A_0, A_B^*; c_b^*, 0)$.

Finally, we prove item (iii) of the proposition. Since $\hat{c_b} > \bar{c_b}$, based on (4), the optimal default-triggering boundary drops from $\hat{A}_B = \beta(1-\theta)\hat{c_b}$ to $\bar{A}_B = \beta(1-\theta)\bar{c_b}$. Given this and the closed-form solution for the cost of bankruptcy from Proposition 1

$$\begin{aligned}
BC(A_t, \bar{A}_B; \bar{c_b}) - BC(A_t, \hat{A}_B; \hat{c_b}) &= \alpha\bar{A}_B \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \alpha\hat{A}_B \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} \\
&= \alpha\bar{c_b}\beta(1-\theta)\left(\frac{\bar{c_b}\beta(1-\theta)}{A_t}\right)^{\gamma} - \alpha\hat{c_b}\beta(1-\theta)\left(\frac{\hat{c_b}\beta(1-\theta)}{A_t}\right)^{\gamma} \\
&= (\bar{c_b}^{\gamma+1} - (\hat{c_b})^{\gamma+1})\alpha\frac{(\beta(1-\theta))^{\gamma+1}}{A_t^{\gamma}}.
\end{aligned}$$

Since $\bar{c_b} < \hat{c_b}$, the last term above is strictly negative. Therefore, $BC(A_t, \bar{A}_B; \bar{c_b}) < BC(A_t, \hat{A}_B; \hat{c_b})$.

$\square$

Proof of Proposition 6. By definition, government subsidy prevents the firm from going into default. Therefore, $BC(A_t; c_b) = 0$.

Also by definition, at time $\tau(A_B)$ the government obtains the obligation to pay c_b to straight debt holders forever. This leads to $U^B(A_t; c_b, c_c) = \frac{c_b}{r}$.

Government subsidy has no effect on A_C, $\tau(A_C)$, A_B or $\tau(A_B)$. Therefore, the values of equity, CCB and tax benefits do not change.

Based on equation (6), Proposition 1 and given that $BC(A_t; c_b) = 0$

$$G(A_t; c_b, c_c) = A_t + \frac{\theta c_b}{r}\left(1 - \left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) + \left(\frac{c_b}{r} - A_B\right)\left(\frac{A_t}{A_B}\right)^{-\gamma}.$$

Based on Proposition 1, $G(A_t; c_b, c_c)$ above is higher than the total value of the firm before the guarantee was issued by $\left[\left(\frac{c_b}{r} - A_B\right)\left(\frac{A_t}{A_B}\right)^{-\gamma} + \alpha A_B\left(\frac{A_t}{A_B}\right)^{-\gamma}\right]$.

$\square$

Proof of Proposition 9. We denote $W(A_t; \hat{c_b}, 0) - W(A_t; \bar{c_b}, c_c)$ by $\Delta\tilde{W}$.

When the capital structure of the firm includes only equity and straight debt the closed-form solution for the value of tax benefits is

$$TB(A_t; \hat{c_b}, 0) = \frac{\theta \hat{c_b}}{r}\left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right). \quad \text{(A.16)}$$

Given (27), (A.16), the closed-form solution for the value of tax benefits from Proposition 1, and equation (17) for the value of government subsidy

$$\begin{aligned}\Delta\tilde{W} = {} & \frac{\theta \hat{c_b}}{r}\left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) - \frac{\theta \bar{c_b}}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \frac{\theta c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) + \\ & \left(\frac{\hat{c_b}}{r} - \hat{A}_B\right)\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \left(\frac{\bar{c_b}}{r} - \bar{A}_B\right)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}. \quad \text{(A.17)}\end{aligned}$$

By multiplying both side of equation (25) by θ and using the closed-form solutions for the values of straight and CCB, we get

$$\begin{aligned}
\frac{\hat{c}_b\theta}{r}\left(1-\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right)+\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}(1-\alpha)\hat{A}_B\theta \quad &= \quad \frac{\bar{c}_b\theta}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)+\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B\theta+ \\
&\quad \frac{c_c\theta}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)+\theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}.
\end{aligned}$$

By rearranging terms

$$\begin{aligned}
&\frac{\hat{c}_b\theta}{r}\left(1-\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right)-\frac{\bar{c}_b\theta}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)-\frac{c_c\theta}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)=\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B\theta+ \\
&\theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}-\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}(1-\alpha)\hat{A}_B\theta. \qquad \text{(A.18)}
\end{aligned}$$

Now we can use (A.18) in (A.17) to get

$$\begin{aligned}
\Delta\tilde{W} \quad &= \quad \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B\theta+\theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}-\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}(1-\alpha)\hat{A}_B\theta+ \\
&\quad \left(\frac{\hat{c}_b}{r}-\hat{A}_B\right)\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}-\left(\frac{\bar{c}_b}{r}-\bar{A}_B\right)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} \\
&= \quad \theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}+\left(\frac{\hat{c}_b}{r}\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}-\frac{\bar{c}_b}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)+\left((1-\alpha)\theta+1\right)\bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}- \\
&\quad \left((1-\alpha)\theta+1\right)\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}.
\end{aligned}$$

Given (3) and $\beta = \frac{\gamma}{r(1+\gamma)}$,

$$\begin{aligned}
\Delta\tilde{W} &= \theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma} + \left(\frac{\hat{c_b}}{r}\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \frac{\bar{c_b}}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \\
&\quad ((1-\alpha)\theta+1)(1-\theta)\frac{\gamma}{(1+\gamma)}\left(\frac{\hat{c_b}}{r}\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \frac{\bar{c_b}}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) \\
&= \theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma} + \\
&\quad \left(1-((1-\alpha)\theta+1)(1-\theta)\frac{\gamma}{(1+\gamma)}\right)\left(\frac{\hat{c_b}}{r}\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \frac{\bar{c_b}}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right). \qquad \text{(A.19)}
\end{aligned}$$

In (A.19) the first term on the right-hand side is positive, $\theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma} > 0$. Also, for $\hat{c_b} > \bar{c_b}$, $\left(\frac{\hat{c_b}}{r}\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \frac{\bar{c_b}}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) > 0$. Finally,

$$1-((1-\alpha)\theta+1)(1-\theta)\frac{\gamma}{(1+\gamma)} > 1-(\theta+1)(1-\theta)\frac{\gamma}{(1+\gamma)} = 1-(1-\theta^2)\frac{\gamma}{(1+\gamma)} > 0.$$

All the terms on the right-hand side of equation (A.19) are positive. Therefore, $\Delta\tilde{W} > 0$. □

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