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Portfolio Choice Analysis in a Multi-Country Macro Model

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Abstract

This paper examines portfolio choice in a dynamic stochastic general equilibrium model with trade and financial linkages across 43 countries. I conduct comparative statics analysis with this structural model to disentangle potential mechanisms of global financial allocation, including risk hedging, risk diversification, risk sharing, and financial friction. For asset home bias, the model predicts that risk hedging is less essential in a multi-country than in a two-country setting. For bilateral asset positions, the model implies that variations in financial friction and asset covariance are major determinants of observed cross-country portfolios. Meanwhile, bilateral financial linkages strongly covary with trade linkages across countries. Comparative statics suggests that this covariance is mainly driven by the high correlation of frictions across the two channels of globalization.

JEL Codes: F30, F41, F62

Keywords: Portfolio choice in DSGE framework, Asset home bias in open economy macro models, Macro implications of finance and trade

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1 Introduction

Global financial allocation is an important topic in open economy macro with profound policy implications. Understanding the allocation requires developing multi-country general equilibrium models to quantify different mechanisms of portfolio choice. However, there have been few quantitative studies in the international macro literature due to the difficulty of solving a portfolio choice problem, especially in a setting with numerous economies in incomplete markets. This paper fills this gap and examines asset allocation in a DSGE model with frictional financial and trade linkages across many countries.

I examine mechanisms of cross-country portfolio choice including those proposed in the asset home bias literature (see [Coeurdacier and Rey \(2013\)](#) for a survey). Many home bias papers assume two symmetric countries in complete markets where analytical solutions are obtainable. Nonetheless, financial frictions in the form of trading costs, capital controls, information asymmetry, and tax differentials are major barriers to asset allocation (see, for example, [Lewis \(1999\)](#) and [Portes and Rey \(2005\)](#)). I allow for the existence of financial frictions when solving the portfolio choice problem. Furthermore, the model covers 43 countries which trade with each other in both asset and goods markets subject to frictions. When deciding on financial allocation, investors not only choose between domestic and foreign assets, but also assign weights to each of the foreign assets in their portfolios. The derived portfolios, influenced by cross-country covariances of macro and financial variables, enable general equilibrium analysis of both asset home bias and bilateral asset positions. I use this structural framework to evaluate potential determinants of global financial allocation, which include risk sharing affected by financial frictions, risk hedging against labor income and real exchange rate risks, risk diversification shaped by the asset covariance structure, and the variation of bilateral financial frictions in international markets.

To solve for portfolios with many countries subject to financial frictions, I follow the solution method developed by [Devereux and Sutherland \(2011\)](#) (similar to [Tille and van Wincoop \(2010\)](#)). They combine a second-order approximation of the Euler equations with a first-order approximation of other model equations to determine a steady-state (zero-order) portfolio. It is computationally challenging to perform higher-order approximations of a large-scale DSGE model, especially in a multi-country analysis with numerous variables and equations. To mitigate the challenge, I embed a Ricardian trade model developed by [Eaton and Kortum \(2002\)](#) in a dynamic open economy macro framework. The trade model characterizes intra-temporal allocation across many economies

analytically, which allows me to solve for inter-temporal allocation with the linearization method for DSGE models efficiently. This novel approach that combines macro and trade methods greatly simplifies the portfolio choice analysis in a multi-country model, and has the potential for wide applications to other topics in open economy macro.

The model structure incorporates portfolio analysis in a multi-country real business cycle model. Countries trade intermediate goods with each other subject to bilateral trade cost. Moreover, they issue equities as claims to capital income net of investment expenditure. There is a representative household in each country which constructs a portfolio of different countries' equities to maximize expected lifetime utility. To reduce the impact of country-specific productivity shocks on consumption, the household's portfolio is shaped by the covariance between its consumption and asset returns. Besides, it considers bilateral financial friction when repatriating returns from another country. After providing qualitative analysis of mechanisms for portfolio choice, I calibrate the model to a world economy with 43 countries and the rest of the world for quantitative exercises. In particular, I conduct comparative statics analysis where I turn off mechanisms sequentially. The model-predicted changes to portfolios allow me to disentangle the relevance of these mechanisms for countries' observed asset positions. Moreover, I need only a few sufficient statistics including bilateral trade shares and portfolio weights to examine comparative statics. These statistics already embed the impacts of unobserved bilateral trade and financial costs, so that we do not need to calibrate these frictions when disentangling contributors to observed financial allocations across countries.

I start portfolio analysis with risk hedging mechanism, which has been proposed as a solution to the home bias puzzle. Using two-country models, economists contend that households may favor domestic assets which provide hedging benefits against the fluctuations in domestic labor income and in real exchange rate (RER).¹ However, my numerical exercises suggest that the importance of risk hedging is diminished in a general multi-country incomplete-market setting. The intuition for this result is that, when households construct a portfolio with 43 instead of 2 assets with non-perfectly correlated returns, any country-specific risk can be easily hedged away by this diversified portfolio. The covariance of macro fundamentals (including labor income and RER) with domestic asset returns is quantitatively similar to that with any foreign asset returns. Consequently, if risk hedging needs arise, instead of only domestic assets, households can adjust all the

¹For example, see [Obstfeld and Rogoff \(2000\)](#) and [Coeurdacier and Gourinchas \(2016\)](#). Nonetheless, theoretical predictions and empirical findings are both mixed to rationalize home bias by risk hedging in the literature, as summarized by [Coeurdacier and Rey \(2013\)](#).

assets simultaneously which offer similar hedging benefits in this multi-country setting, where the importance of risk hedging for asset home bias declines. Therefore, financial friction, which causes market incompleteness to make asset covariances relevant for portfolio choice, is a more viable explanation than risk hedging for home bias.

I proceed to examine mechanisms of bilateral asset positions beyond home bias, and find that financial friction and diversification motive are important determinants of cross-country financial allocation. The international macro literature has proposed risk sharing as a driver for portfolio choice.² Nonetheless, risk sharing is potentially impaired by barriers to global financial allocation. To measure market incompleteness caused by the existence of these frictions, I solve for the world portfolio absent financial friction and find this portfolio to deviate remarkably from data. Therefore, markets are not complete to ensure efficient allocation of financial resources, which induces households to consider the asset covariance structure for risk diversification. The model suggests that portfolios are heavily influenced by the heterogeneity in bilateral financial frictions and asset covariances across countries. In counterfactual exercises where I set each holder country's financial frictions and return covariances with others to be the same as their median level across investment destinations, portfolios change substantially from their observed composition: the weight of assets from countries with higher frictions and covariances rises significantly in portfolios. The large departure of model-predicted from observed portfolios implies that financial friction and risk diversification motive play a crucial role in drawing the landscape of global financial allocation.

These mechanisms also explain the strong connection between bilateral financial and trade linkages. In the data I document an empirical regularity that country pairs with stronger trade ties also invest more in each other's assets. If we measure bilateral linkages as the share of goods or assets in each other's total trade volume or portfolio, the correlation between bilateral trade and financial linkages across country pairs is as high as 0.83 in the data. To explain this fact, I conduct counterfactual portfolio analysis and find the correlation to drop to 0.21 if markets were complete and to almost zero when heterogeneity in financial frictions is turned off. Meanwhile, the correlation does not decline sharply when the hedging of RER risk or risk sharing through RER adjustment is shut down. Based on these results, the high correlation of trade costs and financial frictions is the main driver for synchronous bilateral trade and bilateral financial linkages.³

²For example, [Obstfeld and Rogoff \(2000\)](#) and [Heathcote and Perri \(2013\)](#).

³The correlation of the two frictions across the two channels can be understood from the fact that bilateral trade costs and financial frictions are shaped by common factors such as geographic distance, cultural similarity, information accessibility, and regional economic integration.

This paper contributes to the international macro literature on portfolio choice by disentangling the mechanisms of financial allocation across 43 countries with frictional trade and financial linkages. As surveyed by [Coeurdacier and Rey \(2013\)](#), economists have examined risk-hedging motives to explain the skewness of portfolios towards domestic assets (for example, [Obstfeld and Rogoff \(2000\)](#) and [Coeurdacier and Gourinchas \(2016\)](#)). Others have proposed financial frictions such as transaction barriers and information frictions as determinants for investors' portfolio choice (for example, [Portes and Rey \(2005\)](#) and [Okawa and van Wincoop \(2012\)](#)). I incorporate both mechanisms in a general equilibrium model and disentangle their impacts on global financial allocation. This paper is particularly related to the works that focus on the implications of trade pattern for financial allocation, including [Coeurdacier \(2009\)](#), [Heathcote and Perri \(2013\)](#), [Steinberg \(2018\)](#), [Hu \(2020\)](#), and [Chau \(2022\)](#). Many of these studies solve for portfolios in complete markets, while this paper generalizes the analysis to a setting with financial frictions. Quantitative predictions from the model suggest that existence and variation of bilateral financial frictions are important drivers for cross-country portfolio allocations.

I solve for portfolios with the method developed by [Devereux and Sutherland \(2011\)](#) and [Tille and van Wincoop \(2010\)](#), who use a second-order approximation of the Euler equations to overcome the certainty equivalence of different assets in a first-order approximation. Compared to alternative methods for portfolios, this macro approach does not require separate utility assumptions for agents' intratemporal financial allocation, which is instead determined by endogenous second-moment variables in general equilibrium.⁴ Moreover, this portfolio technique offers wide macro applications given its compatibility with DSGE frameworks, which facilitates the understanding of macro implications of global financial allocation. Many works in open economy macro build upon models with a small open economy or two symmetric countries to deliver important and elegant mechanisms. This paper provides an alternative approach by simplifying the quantitative analysis with many countries of uneven sizes linked through multilateral financial and trade relations subject to bilateral frictions. This multi-country approach is valuable for macro economists to examine the patterns and determinants of globalization.

⁴Alternative methods include the asset demand system employed by [Kleinman et al. \(2024\)](#) and the rational inattention logit demand system adopted by [Pellegrino et al. \(2024\)](#), both of whom provide tractable and elegant analytical expressions for intra-temporal financial allocation. The macro approach followed by this paper focuses more on agents' inter- and intra-temporal investment decisions shaped by endogenous second-moment variables under stochastic shocks, which makes it possible to examine mechanisms like risk sharing, risk hedging, and risk diversification in general equilibrium.

2 Model

This section consists of two parts. Section 2.1 develops a multi-country model with frictional trade and financial channels. Section 2.2 describes the computation and calibration strategies to solve the model.

2.1 Setup

The world comprises countries of uneven sizes indexed by $i = 1, 2, \dots, I$. There is a representative household in each country whose objective is to maximize lifetime utility

$$\max E_0 \sum_{t=0}^{\infty} \nu_t \left(\frac{C_{i,t}^{1-\gamma}}{1-\gamma} - \kappa_1 \frac{L_{i,t}^{1+\kappa_2}}{1+\kappa_2} \right), \quad (1)$$

where $C_{i,t}$ and $L_{i,t}$ are consumption and labor supply respectively, γ is the coefficient of relative risk aversion, κ_1 is a multiplier for labor supply, and κ_2 is the inverse of the Frisch elasticity. Moreover, I follow [Devereux and Sutherland \(2009\)](#) to introduce an endogenous discount factor ν_t which satisfies

$$\nu_0 = 1, \quad \nu_{t+1} = \nu_t \beta(C_{i,t}) \quad \text{with} \quad \beta(C_{i,t}) = \omega_i C_{i,t}^{-\psi}, \quad (2)$$

where $0 \leq \psi < \gamma$ and ω_i is a country-specific multiplier. This endogenous discount factor, whose steady state level is denoted as $\bar{\beta}$, is introduced to ensure a stationary wealth distribution in incomplete markets, otherwise even transitory shocks may have permanent impacts on wealth such that the steady state is indeterminate in a model solved with linear approximation.⁵

Each country produces a final good consisting of a continuum of intermediate goods u traded across countries

$$Q_{i,t} = \int_0^1 [q_{iu,t}(u)]^{\frac{\epsilon-1}{\epsilon}} du]^{\frac{\epsilon}{\epsilon-1}}, \quad (3)$$

where ϵ is the elasticity of substitution in the CES aggregator. The final good can be spent on consumption $C_{i,t}$, capital investment $IV_{i,t}$, or for the production of intermediate goods whose nominal output is $Y_{i,t}$. The optimal consumption-investment decision is

⁵See a detailed discussion by [Schmitt-Grohé and Uribe \(2003\)](#). An alternative assumption to endogenous discount factors to obtain a stationary wealth distribution is the life-cycle elements introduced by [Tille and van Wincoop \(2010\)](#) who assume that agents liquidate wealth with a probability every period.

characterized by the Euler equation

$$C_{i,t}^{-\gamma} = \beta(C_{i,t})E_t\left[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}}\left((1-\delta)P_{i,t+1} + \frac{\partial Y_{i,t+1}}{\partial K_{i,t+1}}\right)\right], \quad (4)$$

where $P_{i,t}$ is the price of the final good and δ is the depreciation rate of capital $K_{i,t}$.⁶ The law of motion for capital will be given by

$$K_{i,t+1} = (1-\delta)K_{i,t} + IV_{i,t}. \quad (5)$$

Country i 's technology of producing u , denoted as $Z_{i,t}(u)$, is drawn from a Fréchet distribution as in the [Eaton and Kortum \(2002\)](#) model⁷

$$\Pr(Z_{i,t} \leq z) = \exp(-T_{i,t}z^{-\theta}), \quad (6)$$

where $T_{i,t}$ captures the central tendency of the distribution which determines the average productivity in country i at t and θ governs the dispersion of the distribution.

To characterize the risk of the world economy for portfolio analysis, I follow the international business cycle literature to assume that country-level productivity is subject to stochastic shocks.⁸ Specifically, $T_{i,t}$ follows an AR(1) process over time

$$T_{i,t} = \rho T_{i,t-1} + (1-\rho)\bar{T}_i + \epsilon_{i,t}, \quad (7)$$

where \bar{T}_i is the productivity level in the steady state of the economy, shocks $\epsilon_{i,t}$ are drawn from a joint normal distribution with a zero mean and cross-country covariance Σ_T .

Production of intermediate goods in country i combines labor L_i , capital K_i , and i 's final good. The prices of these inputs are wage w_i , capital rental fee r_i , and price of the

⁶The Euler equation implies that firms' future cash flows are discounted using the discount factor of country i , which may not be equal across countries in incomplete markets.

⁷There are three major benefits of using the [Eaton and Kortum \(2002\)](#) (EK) framework. First, the model characterizes the world trade pattern with more theoretical underpinnings — including Ricardian productivity, trade cost, market size, factor intensity, and input-output linkages — than standard macro models. Second, countries' productivity consistent with their trade patterns and relevant for their risk characteristics can be calibrated from bilateral trade data based on the EK model. Third, intra-temporal allocation across many countries is determined by the gravity trade system in a closed form, which makes this large-scale DSGE framework significantly easier to solve. See more details in the computation section.

⁸For example, canonical frameworks by [Mendoza \(1991\)](#) and [Backus et al. \(1992\)](#) assume an AR(1) process for country-level productivity. Besides productivity shocks, this model can be adapted to accommodate other risks which drive countries' output fluctuations that induce households to construct portfolios for risk sharing. Future extensions can also examine shocks originated from the financial sector, which will further enrich the mechanisms of comovement between trade and financial channels.

final good P_i respectively. Let τ_{ij} be the iceberg trade cost for exports to j , $\frac{\mu}{1-\mu}$ be the capital-to-labor ratio and $1-\eta$ be the share of the final good in production,⁹ then shares of i 's goods in j 's expenditure under the Fréchet distribution will be

$$\pi_{ij,t} = \frac{T_{i,t}[\tau_{ij}(r_{i,t}^\mu w_{i,t}^{1-\mu})^\eta P_{i,t}^{1-\eta}]^{-\theta}}{\Phi_{j,t}}, \quad \text{where} \quad \Phi_{j,t} = \sum_{k=1}^I T_{k,t}[\tau_{kj}(r_{k,t}^\mu w_{k,t}^{1-\mu})^\eta P_{k,t}^{1-\eta}]^{-\theta}, \quad (8)$$

while $\Phi_{j,t}$ is linked to price in country j through a Gamma function: $P_{j,t} = \Gamma\Phi_{j,t}^{-\frac{1}{\theta}}$. If $X_{j,t}$ denotes country j 's expenditure

$$X_{j,t} = (1-\eta)Y_{j,t} + P_{j,t}(C_{j,t} + IV_{j,t}) \quad (9)$$

and $Y_{i,t}$ is country i 's nominal output, then i 's goods market clearing condition follows

$$Y_{i,t} = \sum_{j=1}^I \pi_{ij,t} X_{j,t}. \quad (10)$$

Households have access to international financial markets where assets from different countries are traded. I follow the asset home bias literature including [Coeurdacier and Rey \(2013\)](#) and [Heathcote and Perri \(2013\)](#) to assume that countries issue equities whose dividends are claims to capital income net of investment expenditure¹⁰

$$d_{i,t} = \eta\mu Y_{i,t} - P_{i,t}IV_{i,t}, \quad (11)$$

⁹Including final goods used as intermediate inputs in production is a standard modeling strategy in the quantitative trade literature to capture input-output linkages of global trade patterns. Capital-to-labor ratios are assumed to be same across countries here for expositional simplicity but are allowed to be different in quantitative exercises. Country-specific capital share (μ_i) calibrated to the data will capture the cross-country heterogeneity of industrial profiles shaped by factor endowments, which potentially influence both trade and financial allocations.

¹⁰Equity is the only type of asset in this paper modeled as all claims to capital income, which should in theory represent other forms of assets including bonds, bank loans, derivatives, reserves, and FDI. High-quality data that cover bilateral investment positions for all these assets are nonexistent. This paper focuses on equities for which I have data for the calibration of portfolio weights. If there are bond data merge-able with equity data with consistent measure and coverage, it will be interesting to introduce both assets in the model and compare their risk sharing and hedging benefits. In particular, [Coeurdacier and Gourinchas \(2016\)](#) find bonds to be the main hedging instrument for the real exchange rate risk, which explains equity home bias driven by equities' conditional hedging positions. Risk hedging plays a much lesser role in this multi-country framework, given the covariance between macro fundamentals and asset returns, and the asset covariance structure of a diversified portfolio with many assets (see section 3). Therefore, abstracting from bonds, whose difference from equities lies in hedging positions according to the aforementioned literature, does not substantially alter model predictions.

which together with equity prices $q_{i,t}$ define financial returns

$$R_{i,t+1} = \frac{d_{i,t+1} + q_{i,t+1}}{q_{i,t}}. \quad (12)$$

Markets are incomplete due to barriers to global financial investment. In particular, financial frictions potentially vary across country pairs, which justifies the gravity model of capital flows documented by [Portes and Rey \(2005\)](#). I follow [Heathcote and Perri \(2004\)](#) and [Aviat and Coeurdacier \(2007\)](#) by introducing financial friction as an iceberg transaction cost f_{ij} , such that the household in country i expects to collect $e^{-f_{ij}} R_{j,t+1}$ when repatriating asset returns from country j . Besides, these frictions are second-order in magnitude (proportional to the variance of shocks) to be consistent with the solution method for portfolio choice in a DSGE framework developed by [Devereux and Sutherland \(2011\)](#) and [Tille and van Wincoop \(2010\)](#) (DSTW).¹¹ Acknowledging that assets are distinguishable by their risk characteristics, these authors combine a second-order approximation of the Euler equation

$$\frac{C_{i,t}^{-\gamma}}{P_{i,t}} = \beta(C_{i,t}) E_t \left[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} R_{i,t+1} \right] = \beta(C_{i,t}) E_t \left[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} e^{-f_{ij}} R_{j,t+1} \right], \quad \forall i, j \in \{1, \dots, I\}. \quad (13)$$

with a first-order approximation of other equations in the model in order to determine a zero-order (i.e. steady-state) portfolio.

Let $\lambda_{ik,t}$ be i 's purchase of k 's assets at the end of period t , i 's budget constraint is

$$P_{i,t} C_{i,t} + \sum_{k=1}^I \lambda_{ik,t} q_{k,t} = w_{i,t} L_{i,t} + \sum_{k=1}^I \lambda_{ik,t-1} (q_{k,t} + d_{k,t}). \quad (14)$$

¹¹ Financial frictions are modeled as transaction costs on foreign returns following the tradition of the international macro literature and will be quantified as frictions that cause market incompleteness in the quantitative analyses of the model. I do not take a strong stand on the underlying structure of these barriers to financial investment, which may take many forms in the real world. For example, f_{ij} in this model can reflect a mix of worldwide factors including global financial liquidity, country-specific factors including capital account openness, and pair-specific factors including geographic distance and bilateral financial agreements. It can take alternative forms such as informational frictions, as [Okawa and van Wincoop \(2012\)](#) find that these types of frictions yield comparable implications for the gravity model of financial flows. The frictions are second-order in magnitude to retain the certainty equivalence of assets to the first order approximation of the model, so that the solution method for portfolio choice developed by DSTW can be applied. Quantitative exercises in the next section show that these frictions, despite their small magnitudes, generate sizable impacts on portfolios.

Meanwhile, the supply of assets issued by any country k is normalized at unity

$$\sum_{i=1}^I \lambda_{ik,t} = 1. \quad (15)$$

If we introduce net holdings defined as

$$\alpha_{ii,t} = q_{i,t}(\lambda_{ii,t} - 1), \quad \alpha_{ij,t} = q_{j,t}\lambda_{ij,t}, \quad \forall j \neq i, \quad (16)$$

we can re-write the budget constraint as

$$P_{i,t}C_{i,t} + \sum_{k=1}^I \alpha_{ik,t} = \underbrace{\eta(1-\mu)Y_{i,t}}_{\text{Labor Income}} + \underbrace{\eta\mu Y_{i,t} - P_{i,t}IV_{i,t}}_{\text{Domestic Dividend Income}} + \sum_{k=1}^I \alpha_{ik,t-1} e^{-f_{ik}} \underbrace{\frac{q_{k,t} + d_{k,t}}{q_{k,t-1}}}_{R_{k,t}} \quad (17)$$

and the asset market clearing condition as

$$\sum_{i=1}^I \alpha_{ik,t} = 0. \quad (18)$$

It follows from the budget constraint that country i 's evolution of wealth is

$$D_{i,t} = D_{i,t-1} e^{-f_{iI}} R_{I,t} + \sum_{k=1}^{I-1} \alpha_{ik,t-1} (e^{-f_{ik}} R_{k,t} - e^{-f_{iI}} R_{I,t}) + \eta Y_{i,t} - P_{i,t}(C_{i,t} + IV_{i,t}), \quad (19)$$

where the net wealth position $D_{i,t}$ is the sum of bilateral holdings

$$D_{i,t} = \sum_{k=1}^I \alpha_{ik,t}. \quad (20)$$

To conclude the description of the model setup, the general equilibrium of the model consists of a set of prices and quantities such that 1) households choose consumption and investment to maximize expected lifetime utility, 2) firms set output and price to maximize profit, and 3) factor, goods, and asset markets all clear.¹² The steady state of the economy is obtained by dropping stochastic shocks and time subscripts in all the equations, which characterizes the stationary equilibrium of this DSGE model.

¹²The goods and asset market clearing conditions are given by Equations 10 and 18 respectively. The factor market clearing condition ensures that households' labor supply and capital accumulation decided by utility maximization equal firms' demand for labor and capital driven by profit maximization.

2.2 Computation

This section describes two strategies employed in this paper to facilitate portfolio choice analysis in a large-scale DSGE model. The first is system reduction to make the computation more efficient, and the second is comparative statics with sufficient statistics.

Solving for equilibrium portfolios in this DSGE model requires loglinearizing the model around a steady state to predict the dynamic responses of endogenous variables to stochastic shocks (see Online Appendix C.1 for details). This is computationally challenging to implement in a framework with many countries of uneven sizes.¹³ To mitigate this challenge, I reduce the linear system to include state variables and forward-looking control variables only when performing eigen-decomposition of the dynamic model. This idea of system reduction, introduced by the macro literature including King and Watson (2002) and Hernandez (2013), yields substantial gains in efficiency. I adapt their idea to the international context by embedding a gravity trade model in an open economy macro framework. The gravity trade model efficiently characterizes intra-temporal allocation across many economies, which makes the system reduction method easier to implement. Online Appendix C.2 provides more implementation details with Uhlig (1995)'s toolkit.

The second challenge lies in the difficulty of calibrating bilateral trade and financial frictions across all the country pairs, which would often require additional assumptions and take much effort. To alleviate this problem, I follow the trade literature to study comparative statics with sufficient statistics when examining countries' observed financial allocations. In particular, I start with countries' bilateral trade and portfolio shares in the data, which are sufficient statistics that already embed unobserved bilateral trade and financial frictions, and conduct comparative statics analysis by shutting down potential determinants for portfolio choice sequentially. The model-predicted changes to portfolios, when relevant second-moment variables are excluded from portfolio equations, allow me to disentangle different mechanisms for observed financial allocation without having to calibrate trade or financial frictions across all the countries.

The appendix provides more computation details: B introduces data sources and calibration strategies. C applies DSTW's solution method to a multi-country framework for both qualitative and quantitative portfolio analyses.

¹³Computation is challenging because the coefficient matrices that cover the world economy are very large with numerous equations and variables. Besides, the matrices are badly scaled with countries' uneven sizes and sparse bilateral linkages. Inverting such a large matrix to compute the eigenvalues of the linear system requires high computing power and may sometimes give inaccurate numerical results.

3 Portfolio Analysis

This section performs portfolio analysis with the model to identify determinants of financial allocation in a multi-country framework with incomplete markets.

3.1 Mechanisms for Portfolio Choice

To examine drivers of financial allocation in this model, I evaluate a general portfolio equation:¹⁴

$$\frac{\gamma}{\zeta} E_t[\eta \tilde{Y} \tilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s) \tilde{P} \tilde{R}' - s \tilde{I} \tilde{R}' - \tilde{D} \tilde{R}' + \tilde{Z} \tilde{R}' + \check{\alpha} \tilde{R} \tilde{R}'] = \frac{1}{2} F. \quad (21)$$

From this equation, cross-country covariance matrices relevant for portfolio choice are the covariances between excess asset returns and countries' relative domestic macro variables, including with income ($\tilde{Y} \tilde{R}'$), price ($\tilde{P} \tilde{R}'$), investment ($\tilde{I} \tilde{R}'$), wealth ($\tilde{D} \tilde{R}'$), and the covariance-variance structure of asset returns ($\tilde{R} \tilde{R}'$). \tilde{Z} represents the numeraire country's excess consumption, influenced by ζ and s which are countries' equilibrium consumption-to-output and investment-to-output ratios respectively. Besides these covariance terms (second-moment variables), the portfolio matrix $\check{\alpha}$ is also influenced by the matrix of financial frictions F .

The coefficients of second-moment variables in Equation 21 suggest the direction of their influences on portfolio choice $\check{\alpha}$:

$$(i) \frac{\partial \check{\alpha}}{\partial \tilde{Y} \tilde{R}'} < 0, \quad (ii) \frac{\partial \check{\alpha}}{\partial \tilde{P} \tilde{R}'} > 0, \quad (iii) \frac{\partial \check{\alpha}}{\partial \tilde{I} \tilde{R}'} > 0, \quad (iv) \frac{\partial \check{\alpha}}{\partial \tilde{D} \tilde{R}'} > 0, \quad (v) \frac{\partial \check{\alpha}}{\partial \tilde{R} \tilde{R}'} < 0. \quad (22)$$

To provide intuition for (i) which predicts households would favor assets whose returns decrease with their country's domestic income, I decompose aggregate income Y into labor income wL and capital income rK which decides domestic return R .¹⁵ The former is relevant for risk hedging, since households would prefer assets whose returns decrease with domestic labor income to hedge against labor income risk (see [Baxter and Jermann \(1997\)](#) for a discussion). The latter is relevant for risk diversification, since households would choose assets whose returns are less correlated with theirs to diversify risks. Hence, both

¹⁴See Online Appendix B.2 for its derivation. The portfolios in matrix $\check{\alpha}$ solved from this equation are countries' equilibrium asset positions adjusted for their output and discount factor: $\check{\alpha}_{ij} = \frac{\check{\alpha}_{ij}}{\beta Y_i}$.

¹⁵This is because dividends are claims to capital income net of investment as defined in Equation 11. Dividends together with capital gains define asset returns (Equation 12).

risk hedging and risk diversification motives account for prediction (i). Meanwhile, (ii) can be explained by the real exchange rate (RER) risk hedging of risk-averse households, who would prefer assets with returns increasing with the domestic price level. Such assets provide hedging benefits for households to sustain purchasing power when local goods are relatively more expensive (see [Kollmann \(2006\)](#) and [Coeurdacier \(2009\)](#) among others). (iii) states that households should choose assets whose returns increase with domestic investment expenditure. As argued by [Heathcote and Perri \(2013\)](#), this cross-country investment financing facilitates risk sharing. Moreover, capital accumulation, by changing the comovement between labor and financial income, also shifts the optimal hedging position against labor income risk. (iv) predicts that households would prefer assets whose returns increase with relative wealth fluctuations. Since wealth fluctuates over time due to time-variation in portfolio returns, this term reflects a hedge against changes in future expected portfolio returns as discussed by [Tille and van Wincoop \(2010\)](#). (v) considers the asset covariance-variance structure and predicts that households assign more portfolio weights to the assets whose returns are less correlated with others' for risk diversification. This term is different from (i) which only considers the covariance with domestic asset return for diversification. $\tilde{R}\tilde{R}'$ covers the covariance across all assets and hence captures multilateral resistance beyond bilateral correlation in the gravity model of international financial flows, consistent with the analysis by [Okawa and van Wincoop \(2012\)](#) and [Bergin and Pyun \(2016\)](#). This covariance structure will determine the portfolio adjustment across assets if any risk hedging or sharing needs arise. After this qualitative analysis, I conduct quantitative exercises to evaluate the impacts of these potential determinants on portfolio choice.

3.2 Asset Home Bias in a Multi-Country Setting

I start quantitative analysis with mechanisms from the asset home bias literature. Many home bias papers assume two symmetric countries in a world with perfect risk sharing.¹⁶ Under the assumption, [Coeurdacier and Rey \(2013\)](#) derive a general expression

¹⁶Risk sharing is achievable under various assumptions. For example, [Coeurdacier and Rey \(2013\)](#) follow the [Backus and Smith \(1993\)](#) condition implied by complete markets when deriving portfolios. [Coeurdacier and Gourinchas \(2016\)](#) show markets are locally complete to support risk sharing if the set of asset returns spans the space of shocks.

for the share of domestic asset in a country's portfolio

$$\hat{\alpha}_{ii} = \underbrace{\frac{1}{2}}_{\text{Risk sharing (diversification)}} - \underbrace{\frac{1}{2} \frac{1-\mu}{\mu} \frac{\text{cov}(\tilde{w}L_{i/j}, \tilde{R}_{i/j})}{\text{var}(\tilde{R}_{i/j})}}_{\text{Hedging labor income risk}} + \underbrace{\frac{1}{2} \frac{1-1/\gamma}{\mu} \frac{\text{cov}(\tilde{P}_{i/j}, \tilde{R}_{i/j})}{\text{var}(\tilde{R}_{i/j})}}_{\text{Hedging RER risk}}, \quad (23)$$

where cov and var represent covariance and variance respectively, and $\tilde{A}_{i/j}$ denotes the log-deviation of any variable A 's cross-country ratio from its steady-state value

$$\tilde{A}_{i,t} = \ln\left(\frac{A_{i,t} - \bar{A}_i}{\bar{A}_i}\right), \quad \tilde{A}_{i/j,t} = \tilde{A}_{i,t} - \tilde{A}_{j,t} \quad \text{for} \quad A_{i/j,t} = A_{i,t}/A_{j,t} \quad \forall i, j \in \{1, \dots, I\}. \quad (24)$$

In Equation 23, the first term reflects households' incentive to diversify portfolios across different countries' assets to reduce the impact of country-specific risks on consumption (see Lucas (1982)). The second and third terms capture the hedging of labor income and RER risks respectively ((i) and (ii) in 22). This model departs from these home bias papers along two dimensions. First, I acknowledge the existence of financial frictions when solving for portfolios. International risk sharing is hence not perfect and the first term in 23 is no longer a constant that reflects a country's relative size in the world economy. Instead, households need to consider asset covariances and financial frictions when constructing the optimal portfolio for risk diversification. Second, in this framework with I economies, there are $I-1$ foreign countries instead of 1. The correlations between the relative return of individual assets and macro fundamentals are lower in this setting while the diversification benefits offered by a large group of assets with non-perfectly correlated returns are higher. Therefore, risk sharing (diversification) becomes relatively more important than risk hedging for portfolios in this multi-country framework with incomplete markets than in a two-country framework with complete markets.

To highlight these differences of predictions, I compare numerical results in 2- and 43-country frameworks.¹⁷ The proposed risk-hedging mechanism in 23 is affected by the covariances of variables which appear as second moments in 21. To compute these second moments around the calibrated steady state, I solve for the first-order dynamics of endogenous variables with the linearization method for DSGE frameworks and calculate the product of variables, which will appear as second moments in portfolio equations.¹⁸

¹⁷See calibration strategies of both settings in Appendix B.

¹⁸See Online Appendix C.1 for details. In particular, Equations C.4 through C.11 describe Devereux and Sutherland (2011)'s method to compute the product of consumption differential and asset returns. This method applies to the calculation of other second moments for portfolio analysis.

Comparative statics analysis, with second moments relevant for different mechanisms excluded from portfolio equations, allows me to identify the parts of countries' observed asset positions attributable to these mechanisms and hence disentangle their potential contribution to asset home bias in the data. I start with a general portfolio equation:

$$\frac{\gamma}{\zeta} E_t[\eta \tilde{Y} \tilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s) \tilde{P} \tilde{R}' - s \tilde{I} \tilde{R}' - \tilde{D} \tilde{R}' + \tilde{Z} \tilde{R}' + \check{\alpha}^0 \tilde{R} \tilde{R}'] = \frac{1}{2} F \quad (25)$$

where α^0 is observed portfolios, and predict portfolio changes from alternative portfolio equations where coefficients of relevant second moments are modified.

First, to disentangle the hedging against labor income risk, I exclude labor income from households' wealth constraint and rewrite the portfolio equation as

$$\frac{\gamma}{\zeta} E_t[\mu \eta \tilde{Y} \tilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s) \tilde{P} \tilde{R}' - s \tilde{I} \tilde{R}' - \tilde{D} \tilde{R}' + \tilde{Z} \tilde{R}' + \check{\alpha}^1 \tilde{R} \tilde{R}'] = \frac{1}{2} F, \quad (26)$$

where the coefficient of $\tilde{Y} \tilde{R}'$ changes from η in 25 to $\mu \eta$ in 26, whose difference $(1 - \mu) \eta$ reflects labor share in output ($\frac{\bar{w} \bar{L}}{\bar{Y}}$). Subtracting this labor income term isolates the potential contribution of the covariance between labor and financial income to observed portfolios. When predicting the change of $\check{\alpha}^1$ from $\check{\alpha}^0$ driven by risk hedging, I take the difference between 25-26, on the right of which are F matrices consisting of existing financial frictions to be cancelled out, so that we do not need to calibrate these frictions.¹⁹

Second, to disentangle the hedging of RER risk, I assume $\gamma = 1$ for the coefficient of $\tilde{P} \tilde{R}'$ under which assumption the portfolio equation turns into²⁰

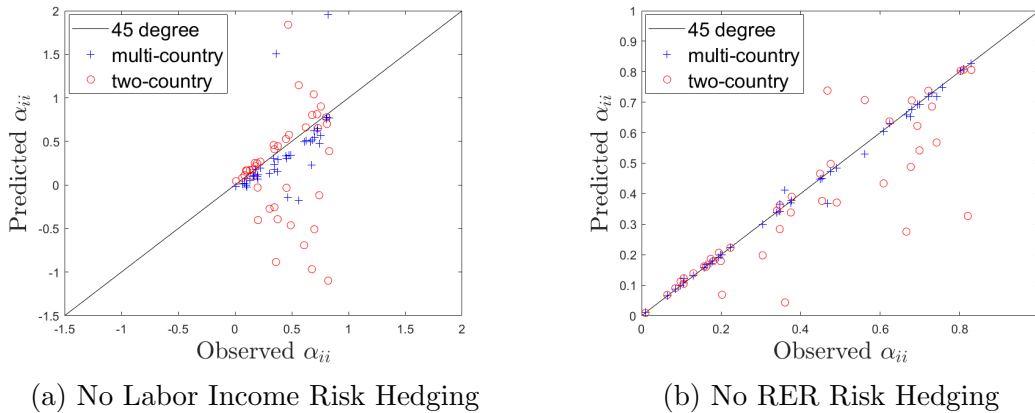
$$\frac{\gamma}{\zeta} E_t[\eta \tilde{Y} \tilde{R}' - s \tilde{P} \tilde{R}' - s \tilde{I} \tilde{R}' - \tilde{D} \tilde{R}' + \tilde{Z} \tilde{R}' + \check{\alpha}^2 \tilde{R} \tilde{R}'] = \frac{1}{2} F, \quad (27)$$

where the coefficient of $\tilde{P} \tilde{R}'$ in Equation 25 is $\frac{\zeta}{\gamma} - \zeta - s = -s$ under log-utility ($\gamma = 1$). γ as the coefficient of risk aversion decides whether households would like to increase income to sustain purchasing power or to reduce consumption when local goods are more expensive. For households with log-utility, the two forces offset each other and leave a portfolio, which generates financial income, independent of fluctuations in RER.

¹⁹As mentioned earlier in Footnote 18, the remaining second moments from these equations, including $\tilde{Y} \tilde{R}'$ and $\tilde{P} \tilde{R}'$, are solved with the linearization method around the calibrated steady state with observed trade shares and financial allocations. The objective of these exercises is to quantify the proportion of observed portfolios attributable to risk hedging shaped by these second moments. Hence, these second moments around the calibrated steady state derived under observed portfolios are fixed in these exercises.

²⁰It is worth noting that we cannot assume log-utility ($\gamma = 1$) for the whole equation, since that will affect other terms including the hedging against expected portfolio returns (iv in 22).

Figure 1: Domestic Asset Holding and Risk Hedging



This figure presents the share of domestic assets in countries' portfolios when risk hedging is turned off. Domestic shares observed in the data are on the horizontal axis and model-predicted shares absent labor income or RER risk hedging are on the vertical axis. Red circles and blue stars represent predictions from settings with 2 and 43 countries respectively. See calibration strategies of both settings in Appendix B, cross-country median values and standard deviations summarized in Table 1, and complete results by country in Table A.2.

Figure 1 and Table 1 present the numerical results for the impacts of risk hedging on observed home bias. The weight of domestic assets in portfolios from the Factset/Lionshare data has a median value of 0.378 across sample countries. When portfolio choice abstracts from labor income and RER risk considerations, this median value decreases to 0.171 and 0.234 respectively in a two-country case. Based on these median values, labor income and RER risks account for 55% and 38% observed asset home bias. This finding that the two risks would tilt portfolios towards domestic assets on average is consistent with the predictions from many existing papers in the home bias literature. However, the median value from this analysis is not informative about variations in the direction or magnitude of predictions across sample countries. About half of the 43 countries exhibit weaker home bias when either risk hedging is turned off. While the majority of countries do not adjust portfolios from observed ones by a great magnitude, several countries' changes in portfolios are large, which suggests that risk hedging is important for their observed home bias.

To diagnose this variation, I examine the underlying mechanism for portfolio choice driven by second moments in the model. When risk hedging is turned off, the shift of an

asset position from its original level is determined by the covariance-variance terms in 23:

$$\frac{\text{cov}(\widetilde{wL}_{i/I}, \widetilde{R}_{i/I})}{\text{var}(\widetilde{R}_{i/I})}, \quad \frac{\text{cov}(\widetilde{P}_{i/I}, \widetilde{R}_{i/I})}{\text{var}(\widetilde{R}_{i/I})}. \quad (28)$$

These two terms represent the ‘hedge ratio’ against labor income and RER risks respectively. The greater their absolute value is, the larger is the portfolio change when risk hedging is shut down. The sign and magnitude of these hedge ratios vary across countries as summarized in Table 1 and reported in Table A.2, which explains the large cross-country variation in portfolio adjustment. The second moments in these ratios are shaped by many factors not incorporated in a symmetric two-country complete-market model, including uneven country sizes, asymmetric trade shares with foreign countries, covariance structure of productivity shocks, and magnitude of financial frictions relative to second moments. Cross-country differences of these factors contribute to the large variation in the importance of risk hedging mechanism for asset home bias. Therefore, the departure of predicted from observed asset home bias is more substantial for some countries than others in a two-country model.

Predictions from a multi-country model look different: the majority of countries alter their domestic asset positions only by a small margin when risk hedging is turned off in Figure 1. This finding suggests that risk hedging becomes less relevant in a multi-country scenario than in a two-country scenario for asset home bias. The main explanation for this contrast between the two scenarios is that, households construct portfolios with 43 instead of 2 assets with non-perfectly correlated returns. If markets are not complete, this large set of assets offers great risk sharing benefits, which weaken the covariance between macro fundamentals and an individual asset’s return relative to other assets’. Moreover, when risk hedging needs arise, households adjust 43 assets simultaneously instead of only the domestic asset in their portfolios.

If we compare the hedge ratio for country i ’s domestic asset

$$\frac{\text{cov}(\widetilde{wL}_{i/I}, \widetilde{R}_{i/I})}{\sum_{k=1}^{I-1} \text{cov}(\widetilde{R}_{i/I}, \widetilde{R}_{k/I})}, \quad \frac{\text{cov}(\widetilde{P}_{i/I}, \widetilde{R}_{i/I})}{\sum_{k=1}^{I-1} \text{cov}(\widetilde{R}_{i/I}, \widetilde{R}_{k/I})}, \quad (29)$$

to the hedge ratio in the two-country case (28), the denominator embeds the covariance structure of domestic and all the foreign assets in a multi-country case. Table 1 reports

that the median hedge ratios of domestic and foreign assets are quantitatively similar:

$$\begin{aligned} \frac{\text{cov}(\widetilde{wL}_{i/I}, \widetilde{R}_{i/I})}{\sum_{k=1}^{I-1} \text{cov}(\widetilde{R}_{i/I}, \widetilde{R}_{k/I})} &\approx \frac{\text{cov}(\widetilde{wL}_{i/I}, \widetilde{R}_{j/I})}{\sum_{k=1}^{I-1} \text{cov}(\widetilde{R}_{j/I}, \widetilde{R}_{k/I})}, \\ \frac{\text{cov}(\widetilde{P}_{i/I}, \widetilde{R}_{i/I})}{\sum_{k=1}^{I-1} \text{cov}(\widetilde{R}_{i/I}, \widetilde{R}_{k/I})} &\approx \frac{\text{cov}(\widetilde{P}_{i/I}, \widetilde{R}_{j/I})}{\sum_{k=1}^{I-1} \text{cov}(\widetilde{R}_{j/I}, \widetilde{R}_{k/I})}, \quad \forall j \neq i. \end{aligned} \tag{30}$$

This result is in contrast to the two-country case where the hedge ratios of domestic and foreign assets sum up to zero. Now that all the assets offer similar hedging benefits, any portfolio adjustment driven by risk hedging motives is spread across domestic and $I-1$ foreign assets. Therefore, the influence of risk hedging on asset home bias declines in a multi-country model. Between the two risks, RER risk hedging generates weaker adjustment in domestic asset holdings, supported by its smaller absolute value of hedge ratios reported in Table 1. This happens as goods prices are jointly determined by all the countries in the trade structure where the comovement between RER fluctuations and asset returns becomes significantly weaker. Hence, shutting down RER risk barely affects domestic asset positions.

The prediction from this multi-country model that risk hedging is not essential for observed asset home bias is supported by the empirical evidence from [Massa and Simonov \(2006\)](#) and [van Wincoop and Warnock \(2010\)](#). They find the correlation between asset returns and labor income or real exchange rate to be too low to rationalize home bias by risk-hedging incentives. On the theoretical front, economists also disagree on the implications of risk hedging for portfolio choice. [Coourdacier and Rey \(2013\)](#) survey different modeling strategies and find their predictions to be sensitive to parametric assumptions, especially in complete markets with two symmetric countries. Therefore, this paper complements the home bias literature by evaluating the importance of risk hedging mechanism in a more general multi-country framework with incomplete markets.

3.3 Determinants of Bilateral Asset Positions

Another major contribution of this multi-country framework to the international finance literature lies in its ability to examine drivers of bilateral asset positions beyond asset home bias. The qualitative analysis provided for 22 still applies here to explain the potential impacts of second-moment variables, which reflect bilateral and multilateral covariances across countries, on observed portfolios. Besides these second moments, financial frictions in international markets may also affect global financial allocation.

Table 1: Summary of Asset Home Bias and Risk Hedging

(I) Domestic Asset Share		Data	No Labor Income Risk	No RER Risk	
Two-country	Median	0.378	0.171	0.234	
	Std Dev	0.256	0.745	0.401	
Multi-country	Median	0.378	0.345	0.377	
	Std Dev	0.256	0.267	0.267	
(II) Hedge Ratio		Labor Income Risk		RER Risk	
		Domestic Asset	Foreign Asset	Domestic Asset	Foreign Asset
Two-country	Median	-0.235	0.235	0.193	-0.193
Multi-country	Median	0.403	0.318	-0.030	-0.008

Panel (I) presents the share of domestic assets in countries' portfolios in the data and in the model where labor income and RER risks are turned off. Panel (II) presents the model-predicted hedge ratio of domestic assets and the median hedge ratio of all the foreign assets from each holder country's perspective against labor income and RER risks (30). Results are reported for 1) a multi-country case where there are 43 countries with bilateral trade and financial linkages, and 2) a two-country case where each of the countries in the sample treats itself as the domestic economy and all the other countries in the world as the aggregate foreign economy. See calibration strategies of both settings in Appendix B, cross-country plots in Figure 1, and complete results by country in Table A.2.

To examine how portfolios are decided by second moments and financial frictions, I use this model to explain the strong correlation between bilateral trade and financial linkages across countries in the data. When calculating trade and financial linkages measured as the mean value of bidirectional shares of goods or assets in each other's total trade volume or portfolio averaged over time for a country pair (denoted as $\bar{\pi}$ and $\bar{\alpha}$ ²¹):

$$\pi_{ij} = \frac{\bar{\pi}_{ij} + \bar{\pi}_{ji}}{2}, \quad \alpha_{ij} = \frac{\bar{\alpha}_{ij} + \bar{\alpha}_{ji}}{2}, \quad \forall i, j \in \{1, 2, \dots, I\}, \quad (31)$$

I find the correlation between trade (π_{ij}) and financial (α_{ij}) linkages to be as high as 0.825 in the sample of country pairs. This finding that countries which trade more with each other in the goods market also hold more of each others' assets in the financial market verifies the strong connection between the two channels of globalization.

There are two potential explanations for this empirical regularity based on the model. First, trade linkages influence cross-country covariances of macro and financial variables relevant for risk hedging and risk sharing, to the extent that bilateral trade linkages predict financial linkages. Second, cross-country financial and trade frictions are shaped by common factors such as geographic distance, cultural similarity, information accessibility, and regional economic integration, all of which are embedded in the friction matrix

²¹The relationship between bilateral portfolio weights denoted as $\hat{\alpha}_{ij}$ and its theoretical counterpart solved from the model $\check{\alpha}_{ij}$ is described by Equation B.2. In quantitative exercises, I solve for $\check{\alpha}_{ij}^{ctf}$ from the model and then use this equation to convert it to $\hat{\alpha}_{ij}$ with which I compute α_{ij} in 31.

F .²² Hence, the high correlation of the two frictions potentially generates synchronous linkages in the two channels. I use this rich structural model to distinguish between these two hypotheses, driven by either second moments or financial frictions, to explain the connection between bilateral asset positions and bilateral trade ties.

I will focus on the design of exercises for financial friction and risk sharing. Together with the risk hedging analysis provided earlier, these exercises disentangle the contribution of second moments to observed bilateral asset positions. To examine the degree of risk sharing affected by the existence of financial frictions, I rewrite the general portfolio equation (25) with the matrix of frictions equal to zero:

$$\frac{\gamma}{\zeta} E_t[\eta \tilde{Y} \tilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s) \tilde{P} \tilde{R}' - s \tilde{I} \tilde{R}' - \tilde{D} \tilde{R}' + \tilde{Z} \tilde{R}' + \tilde{\alpha}^3 \tilde{R} \tilde{R}'] = 0. \quad (32)$$

Portfolios predicted from this equation absent financial friction will support perfect risk sharing, since the number of nonperfectly correlated assets equals the number of shocks in this model so that markets are locally complete around the steady state (Coeurdacier and Gourinchas (2016)). Therefore, the deviation of predicted from observed portfolios measures market incompleteness. To solve for portfolios in locally complete markets from 32, I use the analytical solution from Devereux and Sutherland (2011) in such a case without financial friction (Equation C.8 in Appendix C). Meanwhile, observed portfolios $\tilde{\alpha}^0$ allow me to recover the matrix of cross-country financial frictions F from Equation 25 (equivalent to Equation C.9 in Appendix C). Since these frictions are estimated from the portfolio equation as wedges that generate the deviation of consumption allocation from the prediction under complete markets, they can be interpreted as all barriers to financial arrangements that account for market incompleteness.²³

If friction exists to cause asset market incompleteness, international trade facilitates risk sharing through price adjustment, a classic mechanism that has been studied in the international macro literature.²⁴ To test whether this mechanism may contribute to

²²As mentioned in Footnote 11, barriers to financial allocation in this model may take various forms including these factors, which influence countries' portfolio choice by shaping financial frictions.

²³This estimation strategy for financial frictions hence allows us not to take a strong stand on the exact form these frictions take in the real world. Following this strategy, I find elements in the F matrix are estimated to be very small with most of which at e^{-4} levels, whose median value is $-6.4e^{-5}$. This small magnitude is consistent with the theoretical assumption for these frictions that they are second-order in magnitude to retain the certainty equivalence of assets to the first-order approximation of the model, which is a necessary condition for the DSTW portfolio solution method to be valid. Despite their small magnitude, these frictions can generate low bilateral portfolio weights consistent with the data.

²⁴For example, Obstfeld and Rogoff (2000), Corsetti et al. (2008), and Fitzgerald (2012) suggest that trade affects consumption risk sharing through terms-of-trade (TOT) adjustment in response to supply

countries' observed financial allocation, I shut down the influence of RER variability on cross-country consumption differential and derive portfolio from²⁵

$$\frac{\gamma}{\zeta} E_t[\eta \tilde{Y} \tilde{R}' + (-\zeta - s) \tilde{P} \tilde{R}' - s \tilde{I} \tilde{R}' - \tilde{D} \tilde{R}' + \tilde{Z} \tilde{R}' + \tilde{\alpha}^4 \tilde{R} \tilde{R}'] = \frac{1}{2} F. \quad (33)$$

Differences in bilateral financial frictions also potentially drive cross-country asset positions. To turn off heterogeneity of relative frictions which influence country i 's choice among different investment destinations, I set all of its bilateral financial frictions with other countries to be the same as its median level across foreign destinations solved from Equation 25 denoted as \bar{F}_i . Therefore, elements in this matrix of financial frictions (denoted as F^*) become

$$F^*(i, j) = \bar{F}_i, \quad \forall j \neq i \in \{1, 2, \dots, I-1\} \quad (34)$$

in the portfolio equation under homogenous friction for each holder country²⁶

$$\frac{\gamma}{\zeta} E_t[\eta \tilde{Y} \tilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s) \tilde{P} \tilde{R}' - s \tilde{I} \tilde{R}' - \tilde{D} \tilde{R}' + \tilde{Z} \tilde{R}' + \tilde{\alpha}^5 \tilde{R} \tilde{R}'] = \frac{1}{2} F^*. \quad (35)$$

Another important implication of market incompleteness is that, countries need to consider the asset covariance structure when constructing optimal portfolios for risk diversification. As discussed earlier, risk diversification affects portfolio choice in two aspects ((i) and (v) in 22). First, the incentive of risk diversification is driven by households' need to diversify the risk of domestic assets. Second, the pattern of risk diversification is shaped by the asset covariance-variance structure based on which portfolio weights are determined. Therefore, to turn off the influences of risk diversification on observed financial allocation, first I exclude a country's domestic asset return denoted as R^H from its wealth constraint to shut down the need for risk diversification. Second I shut down the heterogeneity in bilateral asset covariances when solving for bilateral portfolio weights.

shocks. This paper contributes to this macro literature by embedding the global trade structure in a multi-country DSGE framework, whose prediction for countries' TOT and RER variability differs from that in a standard two-country setup.

²⁵This equation for RER risk sharing is different from Equation 27 for RER risk hedging. Specifically, here I turn off the relative price change across countries which reflects RER adjustment in price-adjusted consumption differential: $\tilde{P}_{i,t+1} - \tilde{P}_{I,t+1} = 0, \forall i \in \{1, 2, \dots, I-1\}$ in Equation C.2 used for C.1.

²⁶To predict portfolios under counterfactual financial frictions or asset covariances, I first form an initial guess of portfolios $\tilde{\alpha}$, under which I compute new second moments from the model as functions of $\tilde{\alpha}$, after that I update $\tilde{\alpha}$ with the second moments, until the portfolio determination equation is satisfied under specific $\tilde{\alpha}$ and its corresponding second moments. This procedure enables second moments to adjust endogenously with portfolios in alternative scenarios.

Specifically, a holder country i 's asset covariances with others is set to be the same as their median level across foreign investment destinations denoted as $\bar{R}R_i$, which implies that elements in the asset covariance matrix (denoted as RR'^*) become²⁷

$$\tilde{R}\tilde{R}'^*(i, j) = \bar{R}R_i, \quad \forall j \neq i \in \{1, 2, \dots, I-1\}. \quad (36)$$

Hence, the portfolio equation absent risk diversification is

$$\frac{\gamma}{\zeta} E_t[\eta \tilde{Y} \tilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s) \tilde{P} \tilde{R}' - s \tilde{I} \tilde{R}' - \tilde{D} \tilde{R}' + \tilde{Z} \tilde{R}' - \tilde{R}^H \tilde{R}' + \alpha^6 \tilde{R} \tilde{R}'^*] = \frac{1}{2} F. \quad (37)$$

Equation 37 can be further split into two equations, whose difference from the original portfolio equation 25 identifies part of portfolios influenced separately by the incentive ($\tilde{R}^H \tilde{R}'$) and the pattern ($\tilde{R} \tilde{R}'^*$) of diversification. Characterized by the portfolio equations above, the departure of model-predicted from observed portfolios disentangles the impacts of risk hedging, risk sharing, financial friction, and risk diversification on countries' observed bilateral asset positions.

Table 2: Bilateral Financial and Trade Linkages

	Data (1). Eqn 25	No labor income risk hedging (2). Eqn 26	No RER risk hedging (3). Eqn 27
Corr (α_{ij}, π_{ij})	0.825	0.527	0.816
Std Dev (α_{ij})	0.076	0.079	0.078
	Complete markets (no friction) (4). Eqn 32	No RER risk sharing (5). Eqn 33	Homogeneous financial friction (6). Eqn 35
Corr (α_{ij}, π_{ij})	0.207	0.825	0.027
Std Dev (α_{ij})	3.561	0.075	6.858

This table presents the correlation coefficient between bilateral trade and financial linkages (defined in 31) and the standard deviation of bilateral financial linkages in scenarios where risk hedging is turned off, markets are complete, RER does not influence consumption risk sharing, and each country faces the same financial friction and asset covariance across investment destinations. The corresponding portfolio equations for different scenarios are marked in the table.

Table 2 summarizes model-predicted bilateral financial linkages (α_{ij} defined in 31) in different scenarios. Since the variable's median values are always close to zero,²⁸ I focus on second moments including its dispersion measured by standard deviation and

²⁷This exercise focuses on the variation of bilateral covariance between a holder country's domestic asset with all the foreign assets, taking the covariance among foreign assets (multilateral resistance) constant for comparative statics analysis.

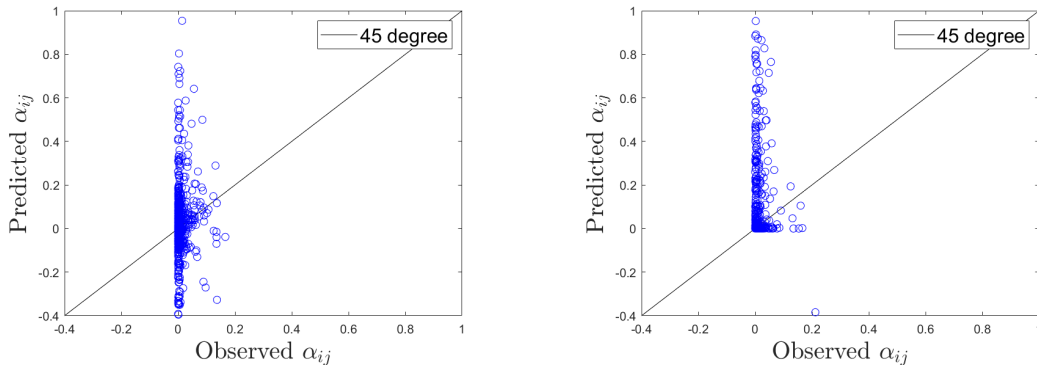
²⁸For example, the median value of α_{ij} across country pairs is $2e^{-3}$ in the data, $2e^{-3}$ under homogeneous financial frictions, and $1e^{-3}$ under homogeneous asset covariances across investment destinations.

its correlation with bilateral trade linkages π_{ij} . Both moments suggest that asset market incompleteness and heterogeneous financial frictions are major determinants of cross-country financial linkages (columns (4) and (6) in the table). Compared to them, risk hedging is not as important for observed bilateral asset positions (columns (2) and (3)). For example, the correlation between bilateral financial and trade linkages drops from 0.825 in the data to 0.527 and 0.816 when either labor income or RER risk hedging is turned off. The same reasoning for the home bias analysis applies here to explain the negligible effect of RER risk hedging: the covariance between RER fluctuations and individual asset returns is low since price is jointly determined by all the countries in the trade structure. This also explains the result in column (5) where shutting down risk sharing through RER adjustment in the trade channel barely changes the relation between bilateral financial and bilateral trade linkages.

Compared to risk hedging, financial friction plays a more crucial role. Column (4) suggests that in complete markets where there is no friction, the correlation between trade and financial linkages drops by 75% to 0.207. Hence, portfolios would shift substantially from their observed allocations if markets were complete. The huge contrast between observed and predicted portfolios measures the degree of market incompleteness caused by the existence of cross-country financial frictions. The drastic decline of the correlation between trade and financial linkages is attributable to the fact that bilateral financial frictions are highly correlated with bilateral trade costs. For example, these two frictions covary with gravity variables such as geographic distance, common border, language, and legal system. Therefore, setting all the financial frictions to zero leaves bilateral asset holdings less correlated with bilateral trade ties, the latter of which is still affected by these gravity variables.

To further establish the importance of the variation beyond the existence of financial frictions for bilateral asset positions, I examine portfolios by assuming a country's bilateral frictions are the same as their median value across destinations (34). Column (6) of Table 2 reports that the correlation between financial and trade linkages declines even further to almost zero (0.027) in this scenario. The interpretation for this result is that, the assumption of homogeneous financial friction redistributes portfolio weights from investment destinations where financial friction is lower to where the friction is higher. If bilateral financial frictions and trade costs are positively correlated, country-pairs with lower trade costs now face higher financial frictions, and vice versa for country-pairs with higher trade costs which face lower financial frictions. This reasoning explains the further correlation decline between bilateral trade and financial linkages under the assumption.

Figure 2: Bilateral Financial Linkages and Financial Frictions



(a) In Complete Markets (Under No Friction) (b) Under Homogeneous Financial Friction

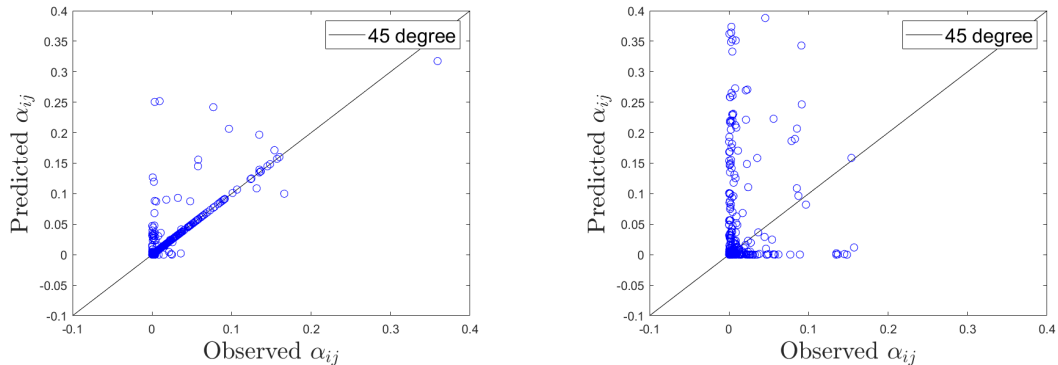
This figure presents bilateral financial linkages (defined in 31) in complete markets with no financial friction and where a holder country’s bilateral financial frictions across foreign investment destinations are the same. Observed financial linkages are on the horizontal axis and model-predicted linkages are on the vertical axis.

To illustrate this point, Figure 2 plots the comparison between the scenarios in complete markets (under no friction) and under homogeneous friction. Their main difference is the kink in 2b which represents the portfolio redistribution under homogeneous financial friction. This is different from the pattern in 2a where countries freely decide on portfolio allocations including taking short-sale positions absent financial friction.²⁹ Since model-predicted financial linkages deviate significantly from observed ones in both scatter plots which look different from each other, financial frictions influence global asset allocation through two distinct mechanisms. First, the variation in bilateral frictions alters the direction of global financial investment. Second, the existence of these frictions causes market incompleteness and impedes international risk sharing. These mechanisms can also explain the strong connection between bilateral financial and trade linkages, which is driven by the high correlation of financial frictions and trade costs.

These model predictions add new insights to a classic literature led by Portes and Rey (2005) and Lane and Milesi-Ferretti (2008), who document countries’ portfolio bias towards trading partners and geographically close economies. To explain the implications of trade linkages for international portfolios, theoretical papers including Coeurdacier (2009)

²⁹The model does not impose a short-sale constraint so that asset holdings can be negative. Having the constraint in the model will further complicate the portfolio choice problem by introducing nonlinearity into its computation. Since this paper focuses on comparative statics, relative changes of predicted from observed portfolios are more important than their absolute values and signs. From Figure 2, the impact of financial frictions on portfolios is sizable, even though they are assumed to be second-order in magnitude to satisfy the validity requirement for DSTW’s solution method.

Figure 3: Bilateral Financial Linkages and Risk Diversification



(a) No Diversification for Domestic Asset (b) Under Homogeneous Asset Covariance

This figure presents bilateral financial linkages (defined in 31) in the cases where there is no need to diversify the risk associated with domestic asset returns and where a holder country’s bilateral asset covariances with assets of different foreign investment destinations are the same. Financial linkages observed in the data are on the horizontal axis and model-predicted linkages are on the vertical axis.

and Heathcote and Perri (2013) have focused on risk sharing and hedging mechanisms in frictionless complete-market settings. This paper develops a more general multi-country model with incomplete markets to disentangle the impacts of risk sharing or hedging mechanisms shaped by second moments and the influences of potential financial frictions on observed cross-country asset allocations. The model confirms financial frictions to be an important driver for synchronous bilateral trade and financial linkages.

As risk sharing is not perfect in the presence of financial frictions, households need to consider the asset covariance structure to diversify country-specific risks. To investigate the effects of risk diversification on portfolio choice, I examine the incentive and the pattern of diversification separately in terms of their impacts on cross-country financial linkages (Equation 37). Figure 3a shows that if there is no need to diversify the risk of domestic assets, most countries adjust their portfolios only by a negligible magnitude. This can be understood from the fact that a holder country combines its domestic asset with many foreign assets with non-perfectly correlated returns in the portfolio, which makes domestic risk easily diversified away already so that portfolio adjustment is not necessary. However, the weight of each foreign asset in the portfolio is heavily influenced by that asset’s covariance with others. In Figure 3b where the heterogeneity in bilateral covariances is shut down, portfolio weights are redistributed such that the share of assets drops for those whose covariance with the domestic asset is low and vice versa. The substantial deviation of model-predicted from observed financial linkages suggests

that risk diversification motive is a major determinant of countries' observed bilateral asset positions: countries' choice of investment destination is strongly directed by the diversification benefits offered by different assets.

3.4 Empirical Evidence

To empirically validate the importance of risk diversification and financial friction for portfolios, I test whether their measures covary with observed bilateral asset positions. For risk diversification, I consider cross-country correlation of productivity shocks (see Appendix B for estimation) which are the drivers of risks in the model. For bilateral frictions, I use a country-pair's average Chinn-Ito index values to measure their capital account openness. Meanwhile, I obtain geographic distance and other gravity variables from the CEPII dataset. Besides, I include a country pair's bilateral RER as a control variable and get its measure from the ratio of the pair's CPI-based real effective exchange rates from the IMF divided by their ratio of nominal exchange rates.³⁰

Table 3 reports the empirical results for the variation of bilateral asset positions. Column (1) shows that bilateral holdings increase in the GDPs of the holder (origin) and asset (destination) countries and decrease in geographic distance, consistent with the gravity model of international finance documented by [Portes and Rey \(2005\)](#). Column (2), by adding origin-, destination-, time-fixed effects and gravity variables, accounts for most of the variability of asset positions in the data implied by the high R^2 value. Column (3) further improves the prediction and shows that capital account openness facilitates cross-country investment. Controlling for this institutional feature, stronger productivity comovement reduces bilateral asset positions, as suggested by the negative coefficient for the interaction term of productivity correlation and the Chinn-Ito index in column (4). This finding that portfolio choice is influenced by risk diversification echoes [Coeurdacier and Guibaud \(2011\)](#) and [Bergin and Pyun \(2016\)](#) who empirically and theoretically establish the importance of cross-country covariance structure for asset positions. Last, column (5) considers RER but does not find it to be an important determinant of bilateral holdings, consistent with the theoretical result that RER risk hedging is not as essential in this multi-country setting. Therefore, these findings provide empirical support for the predictions from the model about determinants of global financial allocation.

³⁰Adjusting variables for nominal exchange rate is a common empirical strategy in the home bias literature (for example [Coeurdacier and Gourinchas \(2016\)](#)). The purpose is to control for nominal exchange rate fluctuations in the data which cannot be explained by macro fundamentals in an open economy macro model (also known as the exchange rate disconnect puzzle).

Table 3: Sources of Variation for Bilateral Asset Positions

Dep. Var: log(Bilateral Holdings)	(1)	(2)	(3)	(4)	(5)
log(GDP _o)	1.245 *** (0.034)	1.108 *** (0.061)	1.085 *** (0.062)	1.111 *** (0.059)	1.215 *** (0.065)
log(GDP _d)	1.442 *** (0.032)	-0.012 (0.093)	0.042 (0.094)	0.112 (0.089)	0.196 * (0.101)
log(dist)	-0.709 *** (0.037)	-1.167 *** (0.021)	-1.202 *** (0.022)	-1.033 *** (0.022)	-1.076 *** (0.025)
Chinn-Ito			0.674 ** (0.298)	2.288 *** (0.293)	3.385 *** (0.918)
corr(T)				5.049 *** (0.253)	5.230 *** (0.268)
Chinn × corr(T)				-4.501 *** (0.269)	-4.747 *** (0.284)
RER					0.512 (0.689)
Chinn × RER					-0.975 (0.855)
Fixed Effects	N	Y	Y	Y	Y
Gravity Variables	N	Y	Y	Y	Y
Observations	22,448	22,448	20,807	20,807	17,105
Adjusted R ²	0.123	0.957	0.959	0.961	0.965

Robust standard errors in parentheses. ***significant at 1%, ** significant at 5%, and * significant at 10%. The dependent variable is bilateral asset holdings in logs from Factset/Lionshare. GDP_o and GDP_d are the GDPs of the investment origin (holder) and destination (asset) country. dist is the population-weighted distance between countries from CEPII. Chinn-Ito is a capital account openness measure here averaged over the country pair. corr(T) is estimated bilateral correlation of productivity shocks (see Online Appendix A.1 for estimation). Fixed Effects include origin-, destination-, and time-FE. Gravity variables include CEPII's dummy variables for contiguity, regional trade agreements, common official language, religion, and legal origin. RER is the ratio of origin to destination country's real exchange rate, computed as their IMF's REER divided by nominal exchange rate ratios.

4 Conclusion

This paper examines portfolio choice in a DSGE model with 43 countries linked through trade and financial channels. I provide comparative statics analyses with the structural model where I quantify the implications of potential mechanisms for asset allocation, including risk hedging, risk sharing, risk diversification, and financial friction. The model predicts that hedging of labor income or RER risk becomes less important in a multi-country setting for domestic asset holdings. Furthermore, the model suggests that variations in financial friction and asset covariance are major determinants of observed bilateral asset positions across countries. These general equilibrium analyses provide new insights into cross-country financial allocation.

To obtain these numerical predictions from the model, I develop an approach that

simplifies the solution method for a multi-country macro model. In particular, I embed a Ricardian trade model in an open economy DSGE framework. The trade model efficiently characterizes intra-temporal allocation across many countries, which allows me to reduce the dynamic system when solving inter-temporal allocation with the linearization method for DSGE frameworks. Beyond portfolio choice, this approach which combines trade and macro methods has many other applications in open economy macro. We can follow the approach to develop international macro models with many countries linked through frictional financial and trade channels, to explore questions such as the transmission of monetary policy, optimal currency invoicing, and financial allocation during major events such as global liquidity crunch and trade wars. The predictions from this multi-country model, by capturing both bilateral and multilateral comovements across many economies, are richer than those from a small open economy or a two country model.

This paper uses the local linearization method developed by DSTW to derive portfolio choice around a deterministic steady state. Their method offers a powerful yet tractable toolkit widely applicable to DSGE models. The derived solution is close to exact around the point of approximation, but it is less accurate when there are large deviations from the steady state or when the problem exhibits strong non-linearity (see a discussion by [Coourdacier and Rey \(2013\)](#)). Therefore, if global solution methods (such as policy or value function iterations) for the portfolio choice problem with comparable applicability and tractability become available in the future, financial allocation can be endogenously determined in more general economic environments. Important questions including those related to sovereign defaults and disaster risks can be answered with the development of such solution techniques.

This paper focuses on comparative statics using observed bilateral trade shares and portfolio weights as sufficient statistics for unobserved trade and financial frictions across countries around the calibrated steady state to perform portfolio analysis. Nonetheless, this study does not predict counterfactual outcomes when economic conditions such as trade or financial policies shift the steady state of the real economy. In another paper ([Hu \(2024\)](#)), I use the ‘exact hat algebra’ technique from the trade literature for such policy analysis, where trade and financial re-allocations jointly determine a new steady state under policy shifts. These two papers, by characterizing the interaction between trade and financial channels, facilitate a comprehensive and deep understanding of globalization.

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Appendices

A Tables

Table A.1: List of Sample Countries

Name	Code	Name	Code	Name	Code	Name	Code
Australia	AUS	France	FRA	Luxembourg	LUX	Russia	RUS
Austria	AUT	Germany	DEU	Malaysia	MYS	Singapore	SGP
Bahrain	BHR	Greece	GRC	Mexico	MEX	Slovenia	SVN
Belgium	BEL	Hong Kong	HKG	Netherlands	NLD	Spain	ESP
Brazil	BRA	Hungary	HUN	New Zealand	NZL	Sweden	SWE
Canada	CAN	Ireland	IRL	Norway	NOR	Switzerland	CHE
Chile	CHL	Israel	ISR	Philippines	PHL	U.A.E.	ARE
China	CHN	Italy	ITA	Poland	POL	United Kingdom	GBR
Czech	CZE	Japan	JPN	Portugal	PRT	United States	USA
Denmark	DNK	Korea	KOR	Qatar	QAT	South Africa	ZAF
Finland	FIN	Kuwait	KWT	Romania	ROU		

B Calibration

The model is calibrated to a world economy with 43 countries (listed in Table A.1) plus the rest of the world (ROW). The time-averaged values of the following variables over the sample period 2001-2021 are used to calibrate the steady state of the economy.³¹

³¹This sample selection for both countries and time is largely decided by the availability of bilateral trade flows and financial holdings data.

Table A.2: Asset Home Bias and Risk Hedging

Country	(I) Domestic Asset Share					(II) Hedge Ratio							
	Data	No Labor Income Risk		No RER Risk		Labor Income Risk				RER Risk			
		multi-co	two-co	multi-co	two-co	Domestic Asset		Foreign Asset		Domestic Asset		Foreign Asset	
					multi-co	two-co	multi-co	two-co	multi-co	two-co	multi-co	two-co	
AUS	0.61	0.43	-0.69	0.61	0.50	-0.03	3.11	-1.23	-3.11	0.12	-1.10	0.58	1.10
AUT	0.10	0.11	0.17	0.10	0.00	0.47	-0.30	0.43	0.30	-0.04	0.22	-0.04	-0.22
BHR	0.81	0.81	0.70	0.81	0.78	0.43	1.19	-0.19	-1.19	0.21	0.89	-0.17	-0.89
BEL	0.11	0.12	0.17	0.12	-0.02	0.48	-0.23	0.44	0.23	0.14	0.21	0.04	-0.21
BRA	0.70	0.54	-0.51	0.69	0.54	0.36	3.45	-0.40	-3.45	-0.03	-1.26	0.45	1.26
CAN	0.45	0.38	-0.03	0.45	0.31	0.43	1.48	0.23	-1.48	-0.02	-0.68	0.03	0.68
CHL	0.72	0.74	0.81	0.72	0.62	0.41	-0.41	0.00	0.41	-0.05	0.27	0.10	-0.27
CHN	0.69	0.62	1.04	0.69	0.62	0.43	-1.55	-0.85	1.55	-0.14	-5.31	0.23	5.31
CZE	0.19	0.21	0.25	0.19	0.10	0.28	-0.25	0.25	0.25	-0.08	0.20	-0.06	-0.20
DNK	0.13	0.14	0.17	0.13	0.05	0.43	-0.23	0.42	0.23	0.00	0.22	0.00	-0.22
FIN	0.45	0.47	0.53	0.45	0.33	0.39	-0.31	0.36	0.31	-0.01	0.22	0.00	-0.22
FRA	0.35	0.28	-0.26	0.34	0.18	0.37	1.56	0.32	-1.56	-0.01	-0.44	-0.01	0.44
DEU	0.20	0.18	-0.03	0.20	0.12	0.38	1.27	0.37	-1.27	-0.01	-0.50	-0.01	0.50
GRC	0.36	0.04	-0.88	0.41	1.51	0.58	-0.53	6.00	0.53	-0.17	0.26	-3.33	-0.26
HKG	0.17	0.19	0.25	0.17	0.11	0.43	-0.42	1.11	0.42	-0.15	0.29	-0.39	-0.29
HUN	0.35	0.36	0.41	0.37	0.23	0.25	-0.25	0.27	0.25	-0.04	0.19	-0.03	-0.19
IRL	0.16	0.16	0.10	0.17	0.11	0.28	0.68	0.18	-0.68	-0.47	0.66	-0.04	-0.66
ISR	0.68	0.71	0.80	0.68	0.52	0.45	-0.38	0.22	0.38	-0.03	0.23	0.03	-0.23
ITA	0.20	0.07	-0.40	0.20	0.07	0.38	1.98	0.28	-1.98	-0.01	-1.30	-0.01	1.30
JPN	0.49	0.37	-0.46	0.48	0.34	0.44	2.80	0.40	-2.80	-0.05	-0.99	-0.04	0.99
KOR	0.73	0.69	0.65	0.73	0.65	-0.10	1.56	-0.30	-1.56	0.10	-0.67	0.21	0.67
KWT	0.22	0.22	0.27	0.22	0.19	0.18	-0.70	0.12	0.70	-0.04	0.03	-0.03	-0.03
LUX	0.01	0.01	0.04	0.01	-0.02	0.48	-0.58	0.40	0.58	-0.11	0.05	-0.12	-0.05
MYS	0.80	0.80	0.78	0.80	0.76	0.29	0.27	0.11	-0.27	0.04	0.44	0.01	-0.44
MEX	0.74	0.57	-0.12	0.72	0.47	0.48	1.73	1.44	-1.73	-0.17	-0.88	-0.56	0.88
NLD	0.09	0.09	0.11	0.09	0.02	0.41	-0.17	0.40	0.17	0.05	0.25	0.03	-0.25
NZL	0.48	0.50	0.58	0.47	0.34	0.38	-0.33	0.36	0.33	-0.02	0.21	-0.01	-0.21
NOR	0.06	0.07	0.08	0.06	0.01	0.39	-0.13	0.36	0.13	-0.02	0.30	-0.02	-0.30
PHL	0.47	0.74	1.84	0.37	-0.14	0.67	-1.34	26.00	1.34	-0.23	0.59	-9.00	-0.59
POL	0.76	1.12	0.90	0.75	0.56	0.28	-11.54	0.16	11.54	-0.06	4.62	0.00	-4.62
PRT	0.56	0.71	1.14	0.53	-0.18	0.39	-0.41	0.27	0.41	0.00	0.22	0.05	-0.22
QAT	0.18	0.18	0.21	0.18	0.18	0.15	-0.61	0.13	0.61	-0.05	0.07	-0.05	-0.07
ROU	0.82	0.33	-1.10	1.11	1.96	0.50	-0.48	0.57	0.48	-0.17	0.24	-0.22	-0.24
RUS	0.83	0.81	0.39	0.83	0.77	0.18	3.34	0.05	-3.34	-0.06	-1.20	0.00	1.20
SGP	0.11	0.10	0.17	0.11	0.07	0.48	-0.93	0.34	0.93	-0.15	0.09	-0.10	-0.09
SVN	0.62	0.64	0.66	0.63	0.51	0.23	-0.24	0.23	0.24	-0.01	0.19	-0.01	-0.19
ESP	0.30	0.20	-0.27	0.30	0.13	0.36	1.50	0.33	-1.50	0.01	-0.75	0.00	0.75
SWE	0.38	0.39	0.45	0.38	0.29	0.43	-0.35	0.41	0.35	-0.01	0.26	-0.01	-0.26
CHE	0.16	0.16	0.18	0.16	0.10	0.40	-0.16	0.36	0.16	0.03	0.26	0.02	-0.26
ARE	0.34	0.34	0.46	0.34	0.31	0.35	-1.46	0.07	1.46	0.16	-0.39	-0.02	0.39
GBR	0.38	0.34	-0.39	0.37	0.15	0.42	1.48	0.32	-1.48	-0.03	-0.18	-0.01	0.18
USA	0.68	0.49	-0.97	0.65	0.23	0.52	1.51	0.66	-1.51	-0.14	-0.38	-0.21	0.38
ZAF	0.67	0.28	-2.83	0.66	0.49	0.47	10.05	0.16	-10.05	-0.11	-3.15	0.05	3.15
Median	0.38	0.34	0.17	0.38	0.23	0.40	-0.23	0.32	0.23	-0.03	0.19	-0.01	-0.19
Std Dev	0.26	0.27	0.74	0.27	0.40	0.14	2.69	4.03	2.69	0.11	1.30	1.46	1.30

This table presents domestic asset positions and hedge ratios against labor income and real exchange rate (RER) risks. Section (I) reports the share of domestic assets in portfolios in the data and predicted by the model where risk hedging is turned off (derived from portfolio Equations 26 and 27). Section (II) reports the hedge ratios against the two risks (defined in 28) for domestic assets and for assets issued by all the foreign countries whose median values are computed for each holder country. Results are reported for 1) a multi-country case (labeled “multi-co”) where there are 43 countries with bilateral trade and financial linkages, and 2) a two-country case (labeled “two-co”) with each of the countries in the sample treating itself as the domestic economy and all the other countries in the world as the aggregate foreign economy. Table 1 summarizes the median and standard deviation of these variables and Figure 1 is the scatter plot for predicted versus observed domestic asset holding of sample countries.

Countries' GDP is from the Penn World Table and their equilibrium wealth $\check{D}_i = \frac{\bar{D}_i}{Y_i}$ is from the World Bank, which reports trade balance that corresponds to

$$TB_{i,t} = Y_{i,t} - X_{i,t} = \eta Y_{i,t} - P_{i,t}(C_{i,t} + IV_{i,t}). \quad (\text{B.1})$$

This trade balance as shares of GDP is relatively stable over time in the data whose time-averaged value is denoted as $tb_i = \frac{TB_i}{Y_i}$. We can plug it in countries' wealth constraint (19) in the steady state to recover equilibrium net wealth: $\check{D}_i = \frac{tb_i}{1-\frac{1}{\beta}}$.³²

Bilateral trade data are obtained from the Direction of Trade Statistics (DOTS) compiled by the IMF. I use the bilateral import (CIF) data to calculate a country's spending on goods imported from other countries. A country's spending on its own goods is computed as the difference between its gross expenditure and total imports, both available from the World Development Indicators (WDI) compiled by the World Bank.

Financial data are from Factset/Lionshare, a comprehensive dataset that provides information on institutional investors' holdings of equities across countries. I describe its details in Hu (2023) and its consistency in terms of portfolio composition with macro-level datasets such as IMF's International Financial Statistics. Factset/Lionshare compiles financial data by investors' origin and destination including for domestic assets, using which I calculate bilateral portfolio weights directly. Ideally, asset ownership should include all forms of capital, such as equity, debt, derivatives, and FDI. However, such comprehensive cross-country financial datasets are scarce, and it takes tremendous efforts to merge datasets covering different types of assets given the lack of universal identifiers for institutional holders. Another popular data source for the purpose is the Coordinated Portfolio Investment Survey (CPIS) and Coordinated Direct Investment Survey (CDIS). I look into these data and find their coverage to be much smaller than Factset/Lionshare's especially for non-OECD countries. Meanwhile, their methodology documentation states that data construction involves much imputation. Such procedure may have caused data anomaly such as negative assets which is difficult to interpret and treat properly, as excluding these values makes the matrix of bilateral portfolio weights even more sparse. For these reasons, I use Factset/Lionshare as the data source to calibrate bilateral portfolio weights, knowing it is not perfect either.

For country i 's holding of j 's asset, the relationship between its observed portfolio weight from the data denoted as $\hat{\alpha}_{ij}$ (with $\sum_j \hat{\alpha}_{ij} = 1$) and its theoretical counterpart

³²Devereux and Sutherland (2009) discuss the potential pitfalls of calibrating the steady state wealth to an exogenously determined level which may miss mechanisms such as precautionary saving and risk sharing influenced by second moments in the model, and hence solve for the stochastic (risky) steady state from the second-order approximation of the model instead. But since this model also embeds financial frictions which influence consumption, it will be difficult to disentangle these frictions and endogenous second moments following their approach. Therefore, I calibrate wealth to an empirical moment, and shut down different mechanisms by excluding relevant second moments to evaluate their quantitative importance for observed portfolios. Compared to volatile net foreign asset positions, trade balance as a relatively stable and easily observable empirical moment, with a consistent theoretical counterpart (B.1), is a feasible target to calibrate the steady state of the economy to.

solved from the model $\check{\alpha}_{ij}$ (with $\sum_j \check{\alpha}_{ij} = \frac{\check{D}_i}{\beta}$) is

$$\check{\alpha}_{ij} = \frac{\check{\alpha}_{ij}}{\beta} (\check{D}_i + \frac{\bar{q}_i}{\bar{Y}_i}) - \mathbb{1}(i=j) \frac{1}{\beta} \frac{\bar{q}_i}{\bar{Y}_i}, \quad \forall i, j \in \{1, 2, \dots, I\} \quad (\text{B.2})$$

where $\frac{\bar{q}_i}{\bar{Y}_i}$ is the equilibrium ratio of asset value to output based on the parameters in the model. This term is included since $\check{\alpha}_{ii}$ is defined as a net holding in 16, adding a unit share to which multiplied by the asset value yields the nominal value of domestic asset holding, consistent with the way portfolio weights are calculated in the data.

The risks of the world economy are driven by productivity fluctuations. Therefore, I estimate countries' dynamic productivity consistent with the Eaton and Kortum (2002) model following Levchenko and Zhang (2014) and compute its persistence ($\rho = 0.85$), mean, and covariance matrix (see Online Appendix A.1 for details).

Several quantitative exercises in Section 3 compare model predictions in settings with 2 and 43 countries. For the former, I collapse the multi-country to a two-country model where each country is treated as the domestic economy and the sum of all the other countries from this country's perspective as the foreign economy. Following this rule, I calculate the two-by-two matrices of financial and trade shares, and re-estimate the productivity of domestic and foreign economies based on country sizes and trade flows (see Online Appendix A.2 for details). Table B.1 below summarizes parametric assumptions of this model, most of which take standard values from the literature.

Table B.1: Parametrization

Parameter	Description	Value	Source
(I) Common Parameters			
θ	Trade Elasticity	4	Simonovska and Waugh (2014)
ψ	Elasticity of discount factor	0.01	Devereux and Sutherland (2009)
η	Share of intermediate input	0.312	Dekle et al. (2007)
γ	Coefficient of relative risk aversion	2	Macro literature
$\bar{\beta}$	Annual discount factor	0.9	Macro literature
κ	Inverse of the Frisch elasticity	2	Macro literature
δ	Capital Depreciation	0.1	Macro literature
ρ	Productivity Persistence	0.85	Estimated in Online Appendix A.1
(II) Country-Specific or Bilateral Variables			
\bar{Y}_i	Output		Penn World Table
$1-\mu_i$	Labor share		Penn World Table
\check{D}_i	Net Wealth		World Bank
$\check{\alpha}_{ij}$	Bilateral Portfolio Weights		Factset/Lionshare
$\bar{\pi}_{ij}$	Bilateral Trade Shares		Direction of Trade Statistics
\bar{T}_i	Productivity		Estimated in Online Appendix A.1

C Portfolio Analysis

This section characterizes the portfolio choice problem in a multi-country framework and summarizes the solution method to solve it (see Online Appendices B-C for more technical details). As shown in Online Appendix B.1, the system of portfolio determination equations can be summarized by

$$E_t(\tilde{C}_{x,t+1}^p \tilde{R}'_{x,t+1}) = \frac{1}{2}F + \mathcal{O}(\epsilon^3), \quad (\text{C.1})$$

where $\tilde{C}_{x,t+1}^p$ is the vector of price- and utility-adjusted consumption differential

$$\tilde{C}_{x,t+1}^p = [\gamma\tilde{C}_{1,t+1} + \tilde{P}_{1,t+1}, \gamma\tilde{C}_{2,t+1} + \tilde{P}_{2,t+1}, \dots, \gamma\tilde{C}_{I-1,t+1} + \tilde{P}_{I-1,t+1}] - \gamma\tilde{C}_{I,t+1} - \tilde{P}_{I,t+1}, \quad (\text{C.2})$$

and $\tilde{R}_{x,t+1}$ is the excess return relative to the numeraire asset

$$\tilde{R}'_{x,t+1} = [\tilde{R}_{1,t+1} - \tilde{R}_{I,t+1}, \tilde{R}_{2,t+1} - \tilde{R}_{I,t+1}, \dots, \tilde{R}_{I-1,t+1} - \tilde{R}_{I,t+1}], \quad (\text{C.3})$$

F is the matrix of relative financial frictions whose element in the i^{th} row j^{th} column is

$$F(i, j) = f_{iI} - f_{ij} - f_{II} + f_{Ij}. \quad (\text{C.4})$$

For qualitative analysis in Section 3.1, I rewrite Equation C.1 where consumption differentials are expressed as functions of relevant macro variables to determine the second moments important for households' portfolio choice (see Online Appendix B.2 for details). For quantitative portfolio analysis, I follow Devereux and Sutherland (2011)'s method (see Online Appendix C.1 for details) to rewrite Equation C.1 as

$$E_t(\tilde{C}_{x,t+1}^p \tilde{R}'_{x,t+1}) = (D_1 H + D_2) \Sigma_T (H' R'_1 + R'_2) = \frac{1}{2}F + \mathcal{O}(\epsilon^3), \quad (\text{C.5})$$

$$\text{where } D_1 = \frac{\partial C_{x,t+1}^p}{\partial \xi_{t+1}}, \quad D_2 = \frac{\partial C_{x,t+1}^p}{\partial \epsilon_{t+1}}, \quad R_1 = \frac{\partial R_{x,t+1}}{\partial \xi_{t+1}}, \quad R_2 = \frac{\partial R_{x,t+1}}{\partial \epsilon_{t+1}}, \quad H = \frac{\check{\alpha} R_2}{1 - \check{\alpha} R_1}. \quad (\text{C.6})$$

Equation C.5 is a general portfolio equation applicable to both complete and incomplete markets. If there is no financial friction which implies $F = 0$, C.5 becomes

$$(D_1 H^* + D_2) \Sigma_T (H^{*'} R'_1 + R'_2) = 0 + \mathcal{O}(\epsilon^3), \quad (\text{C.7})$$

where $H^* = \frac{\check{\alpha}^* R_2}{1 - \check{\alpha}^* R_1}$ is the excess portfolio return corresponding to the portfolio $\check{\alpha}^*$ that solves Equation C.7. The expression of $\check{\alpha}^*$ is derived by Devereux and Sutherland (2011):

$$\check{\alpha}^* = [R_2 \Sigma_T D'_2 R'_1 - D_1 R_2 \Sigma_T R'_2]^{-1} R_2 \Sigma_T D'_2. \quad (\text{C.8})$$

This portfolio $\check{\alpha}^*$ absent financial friction supports perfect risk sharing, since markets

are locally complete around the steady state.³³ In the expression (C.8), the coefficient matrices R_1, R_2, D_1, D_2 are extracted from the linearization of the model. Around the calibrated steady state, countries' observed moments including wealth \bar{D} , output \bar{Y} , and trade shares $\bar{\pi}$ are sufficient to determine these coefficient matrices, therefore R_1, R_2, D_1, D_2 and the resulting $\check{\alpha}^*$ are independent of observed portfolio $\check{\alpha}^0$.

If financial frictions exist in the real world to affect observed portfolio $\check{\alpha}^0$ and its corresponding $H^0 = \frac{\check{\alpha}^0 R_2}{1 - \check{\alpha}^0 R_1}$, $\check{\alpha}^0$ is the solution to a general portfolio equation written as

$$(D_1 H^0 + D_2) \Sigma_T (H^{0'} R_1' + R_2') = \frac{1}{2} F + \mathcal{O}(\epsilon^3). \quad (\text{C.9})$$

On the left of this equation are the elements we already collect, including R_1, R_2, D_1, D_2 , and $H^0 = \frac{\check{\alpha}^0 R_2}{1 - \check{\alpha}^0 R_1}$ calculated with observed $\check{\alpha}^0$. Using these elements to compute the left side of C.9 allows us to quantify the matrix of financial frictions F on the right side.

Online Appendix C.1 provides more technical details about the implementation of the portfolio solution method in this multi-country incomplete-market model. Deriving coefficient matrices R_1, R_2, D_1, D_2 typically requires linearizing the whole DSGE model to perform eigen-decomposition, which is computationally challenging to implement when there are many countries. Online Appendix C.2 describes a system reduction method combined with Uhlig (1995)'s toolkit to facilitate the computation.

³³Coeurdacier and Gourinchas (2016) show that in a setting where the number of nonperfectly correlated assets equals the number of shocks, both of which equal the number of countries in this model, spanning and rank conditions are satisfied such that markets are locally complete absent friction.