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In a recent letter, Hoh[1] has given a theory attempting to explain the marked change in the behavior of the positive column of a glow discharge in a magnetic field as the magnetic induction is increased beyond a critical value, B_c. In particular, Hoh bases his calculation of B_c on a criterion due to Bohm[2] regarding a requirement for the formation of a wall sheath. It is the author's opinion that the use of this criterion in the manner of Hoh is not justified, a conclusion which appears to be supported at least in part by the experiments of Allen, Paulikas, and Pyle.[3] As the confusion regarding the use of Bohm's criterion has also appeared in other circumstances, it seems worth while to re-examine this matter here.

We may first quickly review the Bohm derivation of the potential distribution within a sheath.[2] Assuming the rectilinear geometry of Fig. 1, we have Poisson's equation

$$d^2V/dx^2 = 4\pi e(n_i - n_e) , \qquad (1)$$

where V is the negative of the electrostatic potential relative to that deep inside the plasma, e is the electronic charge, and n_i and n_e are, respectively, the number densities of (singly charged) ions and electrons. If the wall is at a negative potential $V = V_w$, where eV_w is large compared to the electron thermal energy in the plasma, the electrons can be considered to be in thermal

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equilibrium, and their density variation is given by the Boltzmann factor, a circumstance which holds regardless of the presence or absence of a magnetic field. Thus we have

$$n_e = n_{e0} \exp\left[-e(V - V_0)/kT_e\right], \tag{2}$$

where T_e is the electron temperature and k is Boltzmann's constant, and where n_{e0} and V_0 are the values of the electron density and (the negative of) the electrostatic potential, respectively, at the position x_0, the sheath edge.

The ion density is given by the equation of continuity, so that if v_i is the ionic drift velocity in the direction of the wall, we have

$$n_i v_i = \text{const.} \tag{3}$$

If the electron temperature is large compared with that of the ions, eV_0/k is of the order of magnitude of the former,[4,5] and the ionic drift velocity at a distance $x \leq x_0$ is very nearly that which is acquired in falling through the potential $V(x)$. Hence Eq. (3) becomes

$$n_i = n_{i0}(V_0/V)^{1/2}, \tag{4}$$

where n_{i0} is the ion density at x_0. Equation (4) holds as long as the ion mean free path and the ion cyclotron radius are both large compared with the sheath thickness x_0.

Equations (1), (2), and (4) can be combined, multiplied by dV/dx, and integrated once. The result is

$$\frac{1}{2}(dV/dx)^2 = 4\pi\left\{2n_{i0}e(VV_0)^{1/2} + n_{e0}kT_e \exp\left[-e(V - V_0)/kT_2\right]\right\} + C \tag{5}$$

At this point, Bohm introduces the <u>approximate</u> boundary condition in which he assumes that throughout most of the <u>sheath,</u> the electric field is large compared with that at the sheath edge, and further that the plasma is very nearly neutral at x_0. To the extent that this is allowable (i.e., to the extent, really, that the concept of a sheath is valid at all), one is able to write

$$n_{i0} \cong n_{e0} = n_0 \,, \tag{6}$$

$$\left(\frac{dV}{dx}\right)^2 \approx \left(\frac{dV}{dx}\right)^2 - \left(\frac{dV}{dx}\bigg|_{x0}\right)^2$$

$$\approx 8\pi n_0 \left\{ 2eV_0\left[\left(\frac{V}{V_0}\right)^{1/2} - 1\right] + kT_e \left[\exp \frac{-e(V - V_0)}{kT_e} - 1\right]\right\}. \tag{7}$$

Bohm next notes that in order that the right-hand side of Eq. (7) remain always positive for $x \ll x_0$, as is required in this approximation since the left-hand side is necessarily so, one must have

$$V_0 \geq kT_e / 2e \,, \tag{8}$$

or, what is equivalent, the ion drift velocity at the sheath edge must satisfy

$$v_{i0} \geq (kT_e / m_i)^{1/2} \,, \tag{9}$$

where m_i is the ionic mass.

The inequality (8) is frequently called the Bohm criterion for the stability of a sheath, and it has seen wide use since the initial appearance of Bohm's work. Nevertheless, the reference to "sheath stability" in this nomenclature seems somewhat unfortunate. The important point, which it appears is sometimes missed, is that the evaluation of the constant of integration C as above is an approximation only, and that the result of the approximations can in any case be expected to give only a very poor representation of the potential distribution in the vicinity of x_0. The situation is somewhat as diagrammed in Fig. 1, the error at positions just less than x_0 being chiefly due to the neglect of $-[V'(x_0)]^2$ in Eq. (7).

In view of the above, the nonsatisfaction of Bohm's criterion should be considered more as a warning against a breakdown of the theoretical model than as a condition on the system itself. It is true that when this requirement is not met, no sheath in the ordinary sense is set up. However, it is also true that in such a case the forces which try to establish the sheath are also absent. To illustrate in another way, we note that when the wall potential V_w is large, there are always positions at which V takes on a value greater than $kT_e/2e$, and in this case Bohm's treatment merely says that x_0 must be one of these points. On the other hand, in a circumstance in which the wall potential can never be more than of the order of $kT_e/2e$, as in the case considered by Hoh, no sheath appears at all and the theoretical analysis which assumes its presence is in any event inapplicable. Not only does the approximate boundary condition fail, but the wall begins to draw an appreciable electron current and Eq. (2) breaks down as well.

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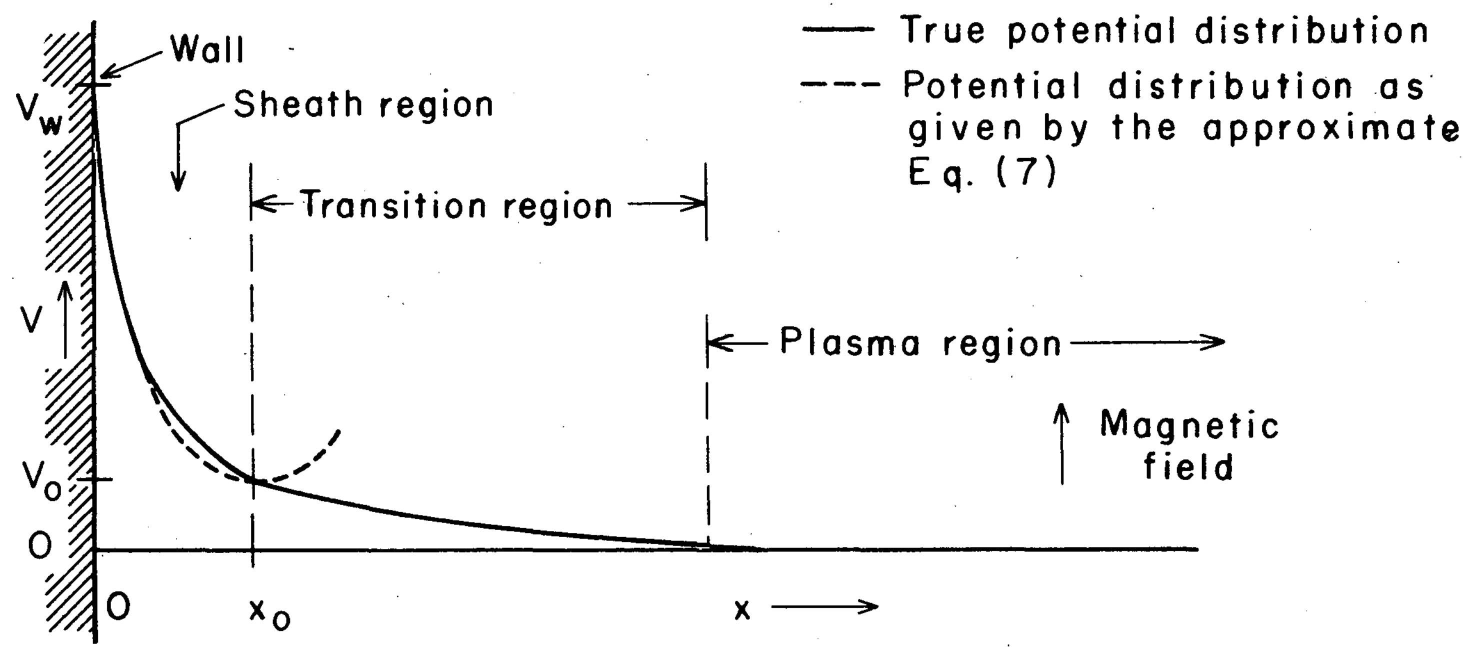
3. T.K. Allen, G.A. Paulikas, and R.V. Pyle, Phys. Rev. Letters (to be published).
4. L. Tonks and I. Langmuir, Phys. Rev. 34, 876 (1929). See especially p. 881 and part IV, p. 901 ff.
5. D. Bohm, loc. cit., p. 40.

Figure Caption

Fig. 1. Potential variation in the vicinity of the sheath.



Wall
Sheath region
True potential distribution
Potential distribution as given by the approximate Eq. (7)
Transition region
Plasma region
Magnetic field
V_w
V
V_0
0
0
x_0
x

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