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Authors
Clarke, John
Tennant, William E.
Woody, D.

Publication Date
1970-12-01

University of California

Ernest O. Lawrence Radiation Laboratory

Berkeley, California

Submitted to Journal of Applied Physics

UCRL-20360
Preprint

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

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John Clarke, William E. Tennant, and D. Woody

December 1970

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John Clarke, William E. Tennant, and D. Woody

Department of Physics, University of California
and
Inorganic Materials Research Division,
Lawrence Radiation Laboratory,
Berkeley, California 94720

ABSTRACT

The concept of an effective noise temperature is developed and applied to superconducting galvanometers. It is shown that even when the device is being used at a temperature above its noise temperature, Johnson noise in a source resistance may be observed only if this resistance is below a certain value, defined as the critical resistance. A detailed design is given for a superconducting transformer which matches the galvanometer to higher resistances and thereby appreciably increases the critical resistance. The transformer may be used at zero frequency. A practical transformer is described which has been used with the Josephson device known as the Slug. The transformer increased the critical resistance from $7\times10^{-8}\ \Omega$ to $2\times10^{-2}\ \Omega$ and could be readily adapted to match to resistances up to $1\ \Omega$.

I. INTRODUCTION

In recent years, a number of Josephson-effect[1] devices have been developed for measuring small voltages at liquid helium temperatures. These have included double-junction interferometers, such as the Squid[2] and the Slug,[3,4] and devices using single-junctions in a finite voltage mode, such as those developed by Mercereau,[5] Nisenoff,[6] and Goodkind and Stolfa.[7] All of these devices are capable of observing Johnson noise at He^4 temperatures in resistances on the order of 10^{-5} - 10^{-8} Ω with a time-constant of 1 second, and in lower resistances with a longer time-constant. However, if one uses circuit impedances of the order of 10^{-5} Ω or higher, these devices are not usually limited by Johnson noise. The resolution appears to be set instead either by inherent noise in the junction(s), by external noise pickup, or by noise at the input of the room temperature amplifier connected to the junction. The limited use of the Josephson devices in circuits of higher impedance is a consequence of their poor current sensitivity, usually in the range 10^{-7} - 10^{-8}A.

The best conventional voltage amplifiers are limited by Johnson noise in source resistances of perhaps[8] 10^4 Ω or higher at 4K. Matching transformers can improve the low resistance performance to enable Johnson noise to be observed in somewhat lower resistances. However, there remain several decades of resistance over which Johnson noise at 4K cannot be observed.

This paper describes a d.c. superconducting transformer which may be used with the superconducting devices to improve their current resolution.[9] The transformer was used in conjunction with a Slug but the general design principles are applicable to other types of Josephson device and also to the cryotron amplifier.[10] We have increased the resistance range over which the Slug was limited by Johnson noise from about $7 \times 10^{-8}\ \Omega$ to $2 \times 10^{-2}\ \Omega$ with a circuit time-constant of about 0.3 second. The same design could be used to extend the resistance range to the order of 1 Ω.

In Section II, we briefly review the properties of the Slug and describe a new configuration which achieves a lower circuit inductance. Section III is concerned with the criterion for an "ideal" voltmeter, that is, a voltmeter which is limited by the Johnson noise developed in the circuit resistance. The concept of equivalent noise temperature is developed. Section IV describes a detailed theory of the superconducting transformer and Section V gives the experimental details and results. In Section VI, we discuss the implications of our results.

II. THE SLUG

The Slug consists of a bead of tin-lead solder a few mm. long frozen on to a few cm. of niobium wire.[3,4] The niobium wire may be insulated, in which case the insulation is removed over a short length, or it may be bare. The solder forms a mechanical contact with the oxide layer of the niobium and at He^4 temperatures, Josephson tunneling occurs between wire and solder.[11] The maximum supercurrent (critical current) of the junction is typically 1 mA and the normal state resistance typically 0.5 Ω. An additional current, I, passed along the niobium wire causes the critical current to oscillate, the period in I being typically 100 μA to 1 mA. By resolving a change in critical current corresponding to a small fraction of one oscillation, one may detect a change in I as low[12] as 5×10^{-8} A when the Slug is biased on the steep portion of an oscillation. The device is a galvanometer having a zero resistance, a current sensitivity of 5×10^{-8} A and an inductance which is just that of the niobium wire, of perhaps 10^{-8} H. However, we have recently used "hairpin" Slugs in which the niobium wire is bent back upon itself within the solder (Fig. 1). The two niobium leads emerge close together and may be wired into a circuit with less inductance than is the case with conventional Slugs. The characteristics of the Slug were not significantly modified. We connected a known resistance in series with a hairpin Slug taking care to reduce the length of the niobium leads to a minimum, in practice 1-2 mm. The total circuit inductance, estimated from time-constant measurements, was 8×10^{-9} H.

Several techniques have been used to determine the critical current of the Slug. These include both ac[3,4,13] and dc[14] techniques. All methods appear to be comparable in sensitivity, at least in the open laboratory where the detection limit is usually set by noise pickup from other laboratory equipment and fm stations.

III. THE VOLTAGE RESOLUTION OF IDEAL VOLTMETERS

A circuit in which a steady voltage is maintained by a current flow must contain a finite resistive element. In this discussion, a voltmeter is said to be "ideal" if its resolution is limited by the Johnson noise developed in this resistance. We develop a criterion which describes the performance of a galvanometer used as a voltmeter and which is applicable to both conventional and superconducting galvanometers.

All low-frequency measurements with Josephson junction detectors ultimately depend on the modification of the critical current of the device by means of a magnetic field. If the field is generated by a current flowing in a superconducting wire, the detector may be used as a resistanceless galvanometer, as indicated in Fig. 2. The critical current is determined by room-temperature electronic instrumentation and we assume that in an integration time τ_G we can detect a current I_G in the galvanometer. In the case of the open-circuit galvanometer, this resolution has a fundamental limitation set by the inherent noise in the Josephson junction itself. In practice, however, this limit appears to be set at a much higher level by either external noise pickup or noise at the input of the room-temperature amplifier connected to the junction. In an unshielded room, the pickup noise usually dominates whereas in a shielded room the amplifier noise is more important.

The galvanometer circuit (Fig. 2) contains a voltage source, a resistance, R, representing the total resistance in the circuit, and a stray inductance, L_S, which includes the inductance of the galvanometer. The time-constant of the circuit is $\tau = L_S/R$. All voltmeters which draw current in their operation may be characterized by the same parameters. For non-superconducting instruments, the resistance of the voltmeter must be included in R.

Let us assume first that $\tau \gg \tau_G$ so that the total measurement time-constant is effectively $\tau = L_S/R$ and the corresponding noise bandwidth $(4\tau)^{-1}$. The Johnson noise voltage V_N across the resistance R at a temperature T is given by

$$\overline{V_N^2} = \frac{kTR}{\tau} = \frac{kTL_S}{\tau^2}. \tag{1}$$

The product $\overline{V_N^2}\,\tau^2$ is independent of R. For a superconducting galvanometer, R may be made arbitrarily small. Consequently, if the voltage measurement is limited by Johnson noise for one value of R in a noise bandwidth $R(4L_S)^{-1}$, then the voltmeter is ideal for all values of R in the corresponding bandwidth, provided that $\tau \gtrsim \tau_G$.

We can rewrite Eq. (1) in terms of a noise current, I_N, as

$$\overline{I_N^2} = \frac{kT}{L_S}. \quad (\tau >> \tau_G) \tag{2}$$

The criterion for an ideal voltmeter is $I_G^2 \leqslant I_N^2$. We may therefore form the product $L_S I_G^2$ and write the condition for an ideal voltmeter as $L_S I_G^2 \leqslant kT$. The equivalent noise temperature of the voltmeter is then

$$T_G = \frac{L_S I_G^2}{k}. \tag{3}$$

In practice, the time-constant τ_G of the Slug is usually a substantial fraction of a second and the circuit time-constant, τ, may sometimes be of the same order. Under these circumstances, the effective time constant for the system becomes $\tau + \tau_G$.[15] The Johnson noise current is then

$$\overline{I_N^2} = \frac{kT}{L_S + \tau_G R}. \tag{4}$$

The time-constant now has a lower limit of τ_G and the noise bandwidth an upper limit of $(4\tau_G)^{-1}$. As R is increased, there will be a value for which the Johnson noise current falls below I_G so that the voltmeter is no longer ideal. If we set $I_G^2 = \overline{I_N^2}$ in Eq. (4), we can establish a critical value of resistance, $R_c(T)$, below which the voltmeter is ideal and above which it is not:

$$R_c(T) = \frac{kT - LI_G^2}{I_G^2\tau_G} = \frac{k}{I_G^2\tau_G}(T - T_G). \tag{5}$$

$R_c(T)$ falls as the working temperature T is lowered and becomes zero at $T = T_G$.

We may summarize these results as follows. Suppose that we start with a low value of R, so that $\tau >> \tau_G$, and that the voltmeter is ideal. As we increase R, the Johnson noise current squared per unit bandwidth varies as R^{-1}. However, the bandwidth increases as R so that the total noise current is constant, and the voltmeter remains ideal. However, near $R = R_c$, a further increase in R begins to decrease the noise current per unit bandwidth but the bandwidth reaches a limiting value of $(4\tau_G)^{-1}$. The voltmeter is no longer ideal.

A voltmeter may therefore be characterized by two parameters, an equivalent noise temperature, T_G, and a critical resistance, $R_c(T)$. For working temperatures below T_G, the voltmeter is never ideal and for temperatures above, it is ideal for resistances $\leqslant R_c(T)$.

As an example, if we take as typical values for the Slug $I_G = 5 \times 10^{-8}$ A, $\tau_G = 0.2$ sec., and $L_S = 10^{-8}$ H, we find the product, $L_S I_G^2$, to be 2.5×10^{-23}J, and the equivalent noise temperature,

T_G, to be 1.8K. At 4.2K, the critical resistance, R_c is $7 \times 10^{-8}\ \Omega$, and the time-constant roughly 0.4 sec.

There are applications in which one would like to use an ideal voltmeter with low-temperature resistances appreciably above $7 \times 10^{-8}\ \Omega$, but below the values where a conventional (non-superconducting) voltmeter is applicable. One possible way of achieving this end is to increase the galvanometer sensitivity, I_G. There are certainly Josephson devices[5-7] which are more sensitive than the Slug, with resolutions of 10^{-8} A or better. However, a method of improving the current sensitivity of any given superconducting galvanometer is to use an impedance-matching superconducting transformer, which may be used at zero frequency. The galvanometer is mounted in a superconducting loop so that a magnetic field applied to the loop generates a circulating current which is measured by the galvanometer. This loop forms the secondary of the transformer. If the field is generated by an ideally coupled primary coil of N turns, the current gain of the transformer will be N. In practice, the coupling will be less than perfect and the current gain less than N. A further effect of the coupling is to reduce the effective primary inductance. Elementary transformer theory shows that coupling a resistanceless secondary to a primary inductance L_P lowers the effective inductance of the primary to $L_P(1-k^2)$, where k is the coupling constant. This decrease in inductance arises from the flux cancellation of the secondary screening currents. A coil has self-inductance because it is linked by magnetic flux proportional to the current flowing through it. A superconducting secondary loop placed in this flux generates a supercurrent which excludes flux from its interior. The field of this screening current cancels a fraction k^2 of the flux linking the primary and thereby reduces its self-inductance by an amount L_Pk^2.

In the case of an ideal transformer (k = 1), the current sensitivity referred to the primary will be $I_P = I_G/N$ and there will be no contributions to the inductance of the primary circuit from the primary coil. If we assume the galvanometer inductance to be L_S, an inductance $L = N^2L_S$ will be reflected into the primary. However, the noise temperature $k^{-1}LI_P{}^2=k^{-1}L_SI_G{}^2$ is the same as for the galvanometer alone. If we replace I_G by I_G/N in Eq. (5), we see that the critical resistance in the primary circuit of the transformer-galvanometer combination increases as N^2. In principle then, the resistance range of a superconducting galvanometer may be increased indefinitely by means of an ideally coupled transformer. In practice, of course, the coupling is not ideal and the range may be extended only by a finite amount. The following sections deal with the theory, design, and construction of a superconducting transformer.

IV. THEORY OF THE TRANSFORMER

The main objective of our transformer design is to make the coupling between primary and secondary as high as possible in order to maximize the current gain and at the same time minimize the primary inductance. We now explore this problem in some detail.

In practice, the greatest problem in obtaining ideal coupling is the presence of the irregularly-shaped Slug in the secondary. We assume that the stray inductance, L_U, associated with the Slug is completely uncoupled from the primary. The remaining inductance, L_C, in the secondary loop may be at least partially coupled to the primary. We assume that the transformer contains a mu-metal core to increase this coupling. We introduce the following parameters for the primary and secondary (see Fig. 2).

$N \equiv$ number of primary turns.

$I_P \equiv$ primary current.

$L_P \equiv$ primary inductance.

$\Phi_P \equiv L_P I_P/N$ = flux linking primary in the ideal situation where all the flux generated by I_P passes through all N turns.

$I_S \equiv$ secondary current.

$L_C \equiv$ portion of secondary inductance which may be partially coupled to primary.

$L_U \equiv$ uncoupled portion of secondary inductance (described above).

$\phi_C \equiv L_C I_S$.

The following parameters depend on the geometry of the primary and secondary taken together:

$M \equiv$ mutual inductance of L_P and L_C.

$\phi_{PC} \equiv M\, I_P \equiv k_{PC}\, \phi_C$ = portion of ϕ_C linking L_P in the absence of screening currents in the primary ($k_{CP} \leqslant 1$).

$\mu \equiv$ factor by which flux linking L_C is amplified by the mu-metal core.

We note that k_{PC} and k_{CP} need not be equal if the geometries of the primary and secondary are different. For example, if a single turn secondary of fixed radius were coaxial at the mid-point of a long solenoid of N turns and the same radius, virtually all of the flux generated by a current in the solenoid would pass through the secondary ($k_{PC} \simeq 1$). On the other hand, most of the flux generated by a current in the secondary would link only a few turns of the solenoid[16] ($k_{CP} \ll 1$). Also, notice that a relationship exists between k_{PC} and k_{CP}, namely

$$k_{PC} L_P / N = M = k_{CP} \, N \, L_C . \tag{6}$$

The basic quantities of interest in transformer design are the amplification

$$A \equiv |I_S| / |I_P| , \tag{7}$$

and the effective primary inductance, L_E. The effective primary inductance is proportional to the total flux ϕ_T which links the primary. There are two contributions to ϕ_T. The first is the flux ϕ_P generated by I_P and the second is the flux generated by the response of the superconducting secondary and the core. Let us consider first the situation in which $L_U = 0$. In this case, screening currents in L_C prohibit flux from linking the secondary or core. The effect of this screening current is to contribute to ϕ_T an amount $-k_{CP}\phi_{PC} = -k_{CP}k_{PC}\phi_P$.

The effective inductance is therefore $L_P(1-k_{CP}k_{PC})$. On the other hand, if L_U is non-zero, some flux is able to link L_C and the core. However, the total flux in the secondary remains zero because the flux linking L_C is exactly cancelled by that linking L_U.

The quantity of flux ϕ which now links L_C and therefore partially links the primary may be found by solving the equations:

$$\phi = \mu\ (\phi_{PC} + L_C I_S) \tag{8}$$

and

$$0 = L_U I_S + \phi, \tag{9}$$

to obtain

$$\phi = \mu\phi_{PC}\frac{L_U}{\mu L_C + L_U}\ . \tag{10}$$

The effective inductance, L_E, is given by

$$L_E = \frac{N}{I_P}\ (\phi_P(1 - k_{PC}k_{CP}) + k_{CP}\phi)$$

$$= L_P\ [(1 - k_{PC}k_{CP}) + k_{PC}k_{CP}\frac{\mu L_U}{\mu L_C + L_U}]. \tag{11}$$

We determine the amplification A from Eqs. (6) - (9):

$$A \equiv \left|\frac{I_S}{I_P}\right| = \frac{\mu k_{PC}L_P}{N(\mu L_C + L_U)} = \frac{\mu L_C k_{CP}\ N}{\mu L_C + L_U}. \tag{12}$$

L_E may be written in terms of the independent parameters A, L_C, L_U, μ, k_{CP}, and k_{PC} in the form:

$$L_E = L_C A^2 \left(1 + \frac{L_U}{\mu L_C}\right)^2 \left(\frac{1}{k_{CP}k_{PC}} - 1\right) + A^2 L_U \left(1 + \frac{L_U}{\mu L_C}\right). \tag{13}$$

Eq. (13) expresses the result that for a given value of A, the lowest value of L_E is achieved by making L_U as small as possible and the coupling coefficients as near to unity as possible.

In the limit $L_U << \mu L_C$, the current amplification becomes

$$A = Nk_{CP}. \tag{14}$$

In this limit the effective inductance may be written from Eqs. (6), (11), and (14) as

$$L_E = L_P(1 - k_{PC}k_{CP}) + A^2 L_U. \tag{15}$$

If $L_C >> L_U$, this result follows even for $\mu = 1$ and there is little to be gained by using a high permeability core. However, if L_C and L_U are comparable, there is a distinct advantage to using a core.

We may summarize our conclusions by saying that for a given value of gain, the transformer should be designed to have the lowest value of primary inductance, L_P, the lowest value of stray inductance, L_U, from the galvanometer, and the best possible coupling between primary and secondary. If the inductances L_U and L_C are comparable, there is a considerable advantage to using a high permeability core; if $L_C >> L_U$, there is little advantage.

V. PRACTICAL TRANSFORMERS

A. Transformer Design and Construction

Our basic design philosophy was to wind the primary as a long, single-layer solenoid which, for given radius and number of turns, has a much lower inductance (L_P) than a short multilayer coil. In order to achieve good coupling to the primary, the secondary must be correspondingly long. A typical transformer design is illustrated in Fig. 4. The primary coil was typically several hundred turns of 0.005 cm. diameter insulated niobium wire. It was wound on a teflon rod 1 cm. in diameter and 11 cm. long, and protected with a layer of varnish. The secondary consisted of a rectangular sheet of lead of 0.015 cm. thickness wrapped around the primary with about 1 cm. overlap. A strip of masking tape between the overlapping edges of the lead cylinder prevented their shorting together. Two tabs 6 mm. wide were raised from the lead sheet, separated from each other by a thin layer of tape. The two niobium leads of a "hairpin" Slug were soldered one to each tab with a low melting-point solder, $BiIn_2$. These connections were not true solder joints but rather mechanical joints, so that solder and niobium were separated by a superconducting weak link. In practice, these joints were capable of carrying supercurrents of at least several tens of mA. The niobium wires of the Slug were made as short as possible, to minimize the stray inductance, L_U. The uncovered length of the niobium wires between the tabs and the Slug was 1 to 2 mm. and their separation on the same order.

In a slightly different design, the primary was wound over the secondary. The performance of the two types was similar. In a further configuration, the primary was wound as a multilayer coil about 4 mm. long. Lastly, a transformer was constructed in which the edges of the lead cylinder were not overlapped but separated by a gap of about 1 mm.

A permeable core consisting of 0.1 mm. mu-metal rolled into a rod could be inserted into a 7.5 mm. diameter hole in the teflon former. The effective premeability of this core, that is the factor by which the inductance of the primary (in the absence of the secondary) was increased, was about 15.

B. Measurement of Transformer Characteristics

The two important parameters of the transformer are its current gain and the effective inductance of the primary.

In principle, the measurement of the current gain is straightforward. A small current (I_P) is introduced into the primary and a corresponding current (I_S) fed into the niobium wire of the Slug so as to exactly cancel the effect of I_P. The ratio I_S/I_P represents the amplification. In practice, the current I_S must be introduced at the tabs of the secondary so that the applied current divides, a fraction $\mu L_C/(\mu L_C + L_U)$ actually flowing through the Slug. The measured current amplification is therefore higher than the true amplification. This difficulty was overcome by replacing the lead tabs on the lead cylinder with indium tabs.[17] The true period of the Slug was determined with the tabs in the normal state. The temperature was lowered so that the tabs became superconducting and the apparent period determined. This technique enabled the true gain of the transformer to be determined. In addition, the uncoupled inductance (L_U) could be estimated from the relation

$$\frac{P_N}{P_S} = \frac{1}{1 + L_U/(\mu L_C)} , \tag{16}$$

where P_N and P_S are the measured periods of the Slug with the indium tabs in the normal and superconducting states respectively.

The effective inductance of the primary was measured by connecting it in parallel with a known resistance (R), chosen so that the primary-time constant, $\tau = L_E/R$, was several seconds. A current was switched into the resistance. As this current flowed into the primary, the critical current of the Slug oscillated with a frequency which decayed exponentially with time. This exponential decay enabled us to estimate τ and hence L_E.

Finally, we measured $k_{PC}k_{CP}$ by winding a 500 turn superconducting coil on a hollow lead cylinder whose overlapping edges were soldered together. From the measured effective inductance of the primary, we deduced a value for $k_{PC}k_{CP}$ of 0.94.

C. Performance of Transformers

The specification and performance of four transformers are summarized in Table I. The values of the primary inductance, L_P, were calculated rather than measured. The noise temperature for the transformer-Slug combination is $k^{-1}L_E(I_G/A)^2$, where L_E is the effective primary inductance and I_G/A the current resolution in the primary circuit.

It is clear that transformer 1 had the highest performance. Removal of the mu-metal core, as in transformer 2, increased the noise temperature by about 4. Failure to overlap the edges of the lead cylinder, as in transformer 3, increased the noise temperature by a further factor of 4. Transformer 4, with a short multilayer primary and no core, had a noise temperature about 50% higher than transformer 2.

It is apparent that only transformer 1 had an ideal performance in the He^4 range. However, it is important to realize that the use of a more sensitive Josephson device would improve the noise temperatures appreciably. For example, if we substituted a Slug with a current resolution of 10^{-8} A, the noise temperatures would be reduced by a factor of 25 and they would all be comfortably in the He^4 range.

VI. DISCUSSION

We now examine the parameters of transformer 1 in relation to the theory developed in Section IV. We first estimate the stray inductance, L_U, associated with the Slug and its wiring to the transformer. We assume the following values for the various parameters: $N = 1000$, $L_C = 8 \times 10^{-10}$ H (calculated), $\mu = 15$ (measured), $L_P = 0.8$ mH (calculated), $L_E = 4$ mH (measured), $k_{PC}k_{CP} = 0.94$ (measured), and $A = 565$ (measured). We further assume $k_{PC} = k_{CP}$. From Eq. (12), we find $L_U \simeq 8$ nH, from Eq. (13), 8 nH, and from Eq. (16), 9 nH. These values are consistent with each other and with the inductance of a Slug and its wiring measured independently (Section II). We conclude that there is little flux leakage from the split cylinder forming the secondary. Flux leakage along the slit would have lowered the values of k_{PC} and k_{CP} and led to a high apparent value of L_U as estimated using $k_{PC}k_{CP} = 0.94$.

It appears that $\mu L_C \simeq 1.5\ L_U$ for transformer 1. Consequently, a core of higher permeability could have been used with advantage. Eq. (15) is not a very good approximation for the effective primary inductance. With a higher permeability core, the second term on the right of Eq. (15) would have dominated the first and the equation reduced to $L_E \approx A^2 L_U$. Under these circumstance, the noise temperature of the transformer-Slug combination would have been the same as that of the Slug alone; in fact it was about 25% higher.

We turn now to the performance of our transformer as a voltmeter. If we assume a Slug current resolution of 5×10^{-8} A, the sensitivity of the transformer-Slug combination is about 9×10^{-11} A. The equivalent noise temperature is 2.3K and the critical resistance at 4.2K is, from Eq. (5), about 1.6×10^{-2} Ω. The circuit time-constant is thus 2.5×10^{-1} sec. and the overall time-constant roughly 0.5 sec. The corresponding Johnson noise at 4.2K with the critical resistance is 1.4×10^{-12} V.

One may speculate on possible extensions of our design. It seems reasonable, from a comparison of transformers 2 and 4, that one could wind a multilayer primary without seriously affecting the values of the coupling constants. For example, a six-layer primary of 6000 turns could be expected to have an amplification of perhaps 3000 and an effective inductance of perhaps 150 mH. This latter estimate assumes that the reflected Slug inductance, $A^2 L_U$, still makes the major contribution to L_E. This transformer would have a critical resistance of about 1 Ω at 4.2K.

ACKNOWLEDGEMENTS

We are indebted to Professor P. L. Richards and to Dr. T. L. Thorp for many helpful discussions and to Professor Richards for a critical reading of the manuscript. This work was performed under the auspices of the U. S. Atomic Energy Commission.

Table I. Specification and performance of Superconducting Transformers. The values of the noise temperature, $T_G = k^{-1}L_E(I_G/A)^2$, are based on a value of $I_G = 5 \times 10^{-8}$ A.

	Type	number of turns	Gain (A)	L_E (mH)	L_P (mH)	T_G (K)
1	Monolayer primary, about 10 cm. long, and 1 cm diameter, secondary on top of primary. Core with effective $\mu \approx 15$.	1000	565	4	0.8	2.3
2	As 1, but with no core.	1000	100	0.5	0.8	9.1
3	Monolayer primary, about 5 cm. long, 1.4 cm diameter, primary on top of secondary. Edges of lead cylinder not overlapped. No core.	500	55	0.6	0.6	36
4	Short multilayer coil, about 4 mm long, wound on top of secondary on 2 cm. diameter former. No core.	1000	385	11	30	13.4

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12. The best Slugs we have made could resolve better than 10^{-8} A. We have taken 5×10^{-8} A as a more typical figure.
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15. The response of the combined system (L_S/R circuit and room temperature amplifier) to a step-function input is the sum of two exponentials, with time-constants L_S/R and τ_G. However, the current noise per unit bandwidth referred to the input of the galvanometer integrated over all frequencies yields the result given in the text.

16. It is also interesting to notice that for a primary and a secondary to be perfectly coupled ($k_{PC} = k_{CP} = 1$, $L_U = 0$), in the absence of screening currents all flux which links one circuit must link the other. Therefore the geometries of the primary and secondary must be exactly superposed--the path taken by the secondary current must be identical to that taken by the primary current. In the case where the paths are not identical (all real situations), there are always fine flux lines linking one circuit which do not link the other.

17. 1% Bi was added to the In to increase the normal-state resistance of the tabs. The time-constant of the secondary circuit was then conveniently short.

FIGURE CAPTIONS

Fig. 1 (a). A conventional Slug, consisting of a bead of tin-lead solder on a length of niobium wire from which the insulation has been scraped over a small section. (b) A "hairpin Slug", in which the niobium wire is folded back on itself. This configuration has a lower inductance than that of (a).

Fig. 2. A voltmeter circuit containing a resistanceless galvanometer in series with a voltage source (V), a resistance R and an inductance L_S which represents the total circuit inductance. The galvanometer response is detected by external circuitry which requires a time τ_G to detect a current I_G in the circuit.

Fig. 3. Schematic of superconducting transformer. The primary contains N turns of inductance L_P. The secondary has an inductance L_C partly coupled to the primary and an inductance L_U completely uncoupled from the primary. M is the mutual inductance between L_P and L_C and μ the permeability of the core. The critical current of the Slug is measured by means of the leads (i) and (v). A current, I_B, biases the Slug on the steepest part of an oscillation.

Fig. 4. Sketch of practical superconducting transformer and Slug.

1 - mu-metal core

2 - teflon former

3 - masking tape insulation between primary and secondary

4 - lead tape secondary

5 - masking tape insulation between overlapped portions of secondary

6 - primary windings

7 - masking tape insulation between tabs

8 - lead tab cutout of secondary

9 - solder joining lead and indium tabs

10 - indium tab

11 - solder joining slug lead to tab

12 - superconducting slug leads (niobium)

13 - slug galvanometer

14 - constantan leads to slug

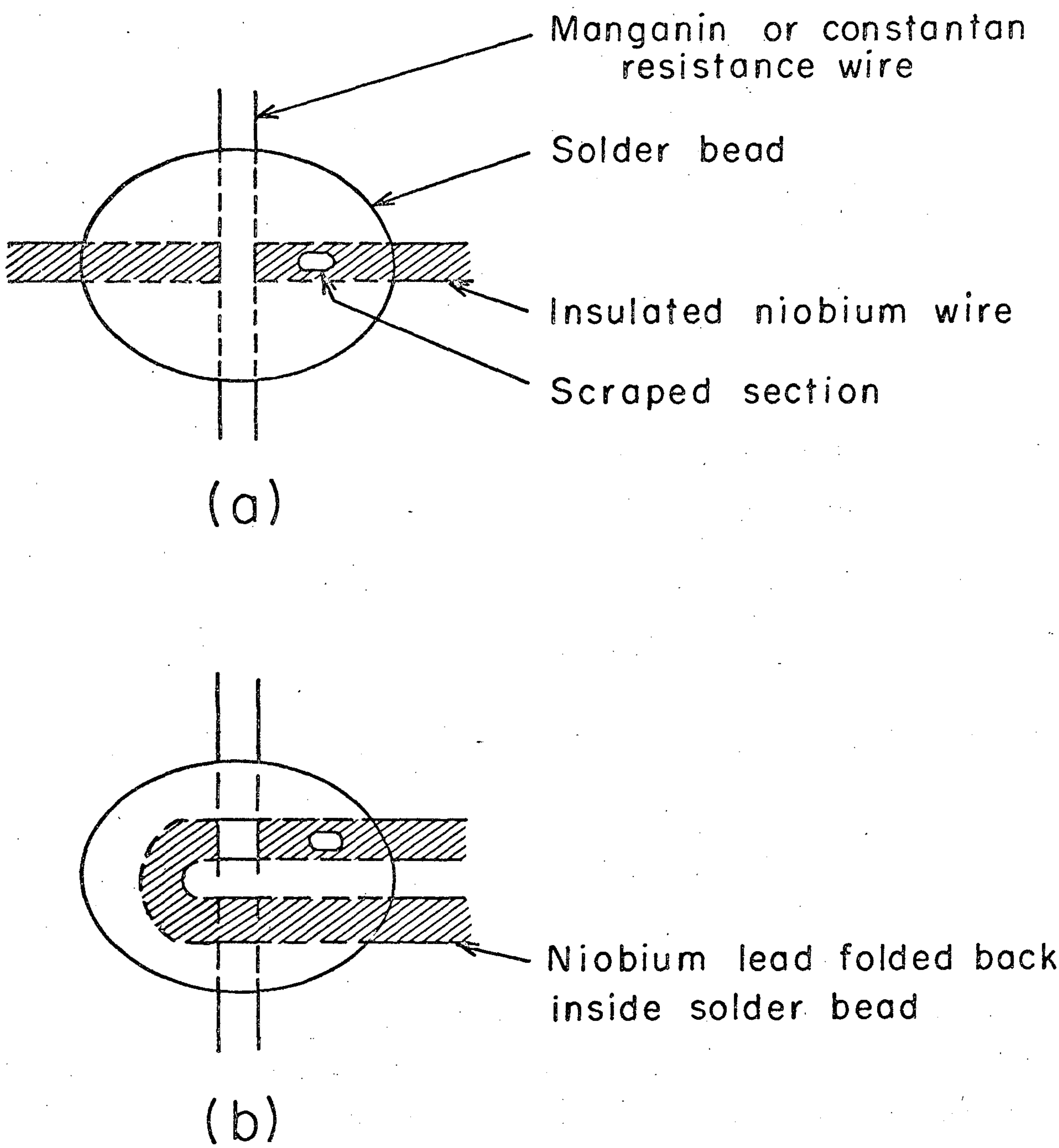

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Fig. 1

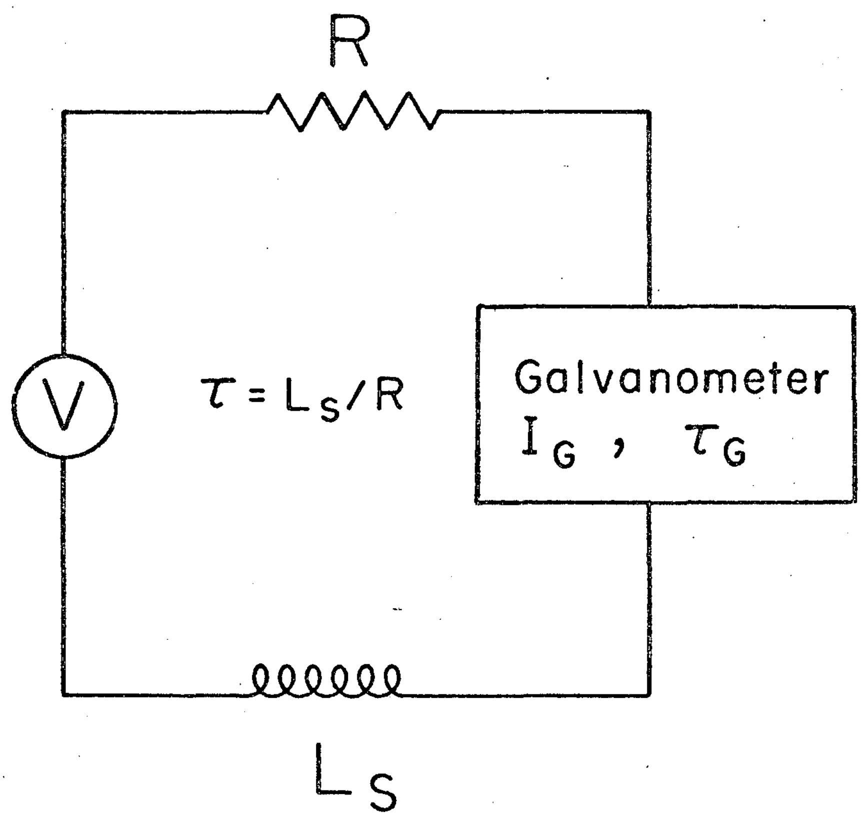

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Fig. 2

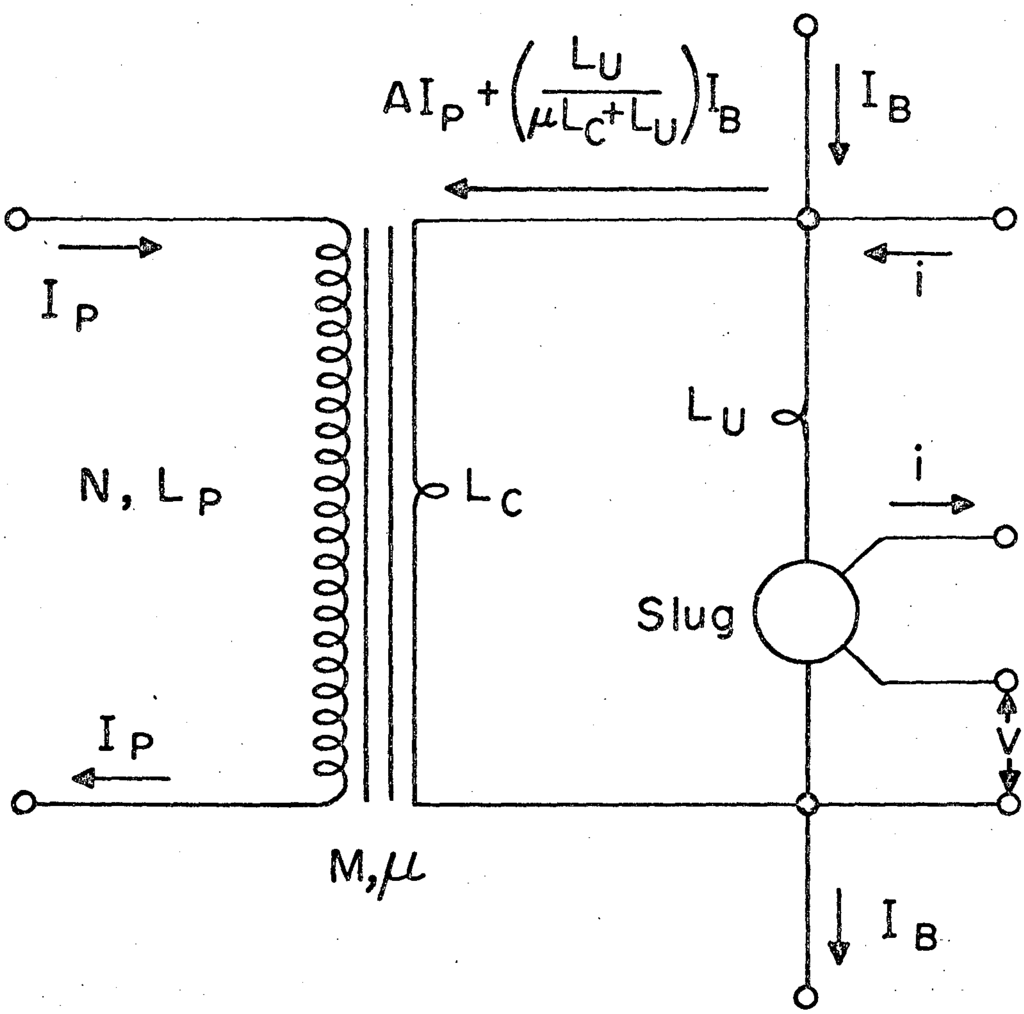

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Fig. 3

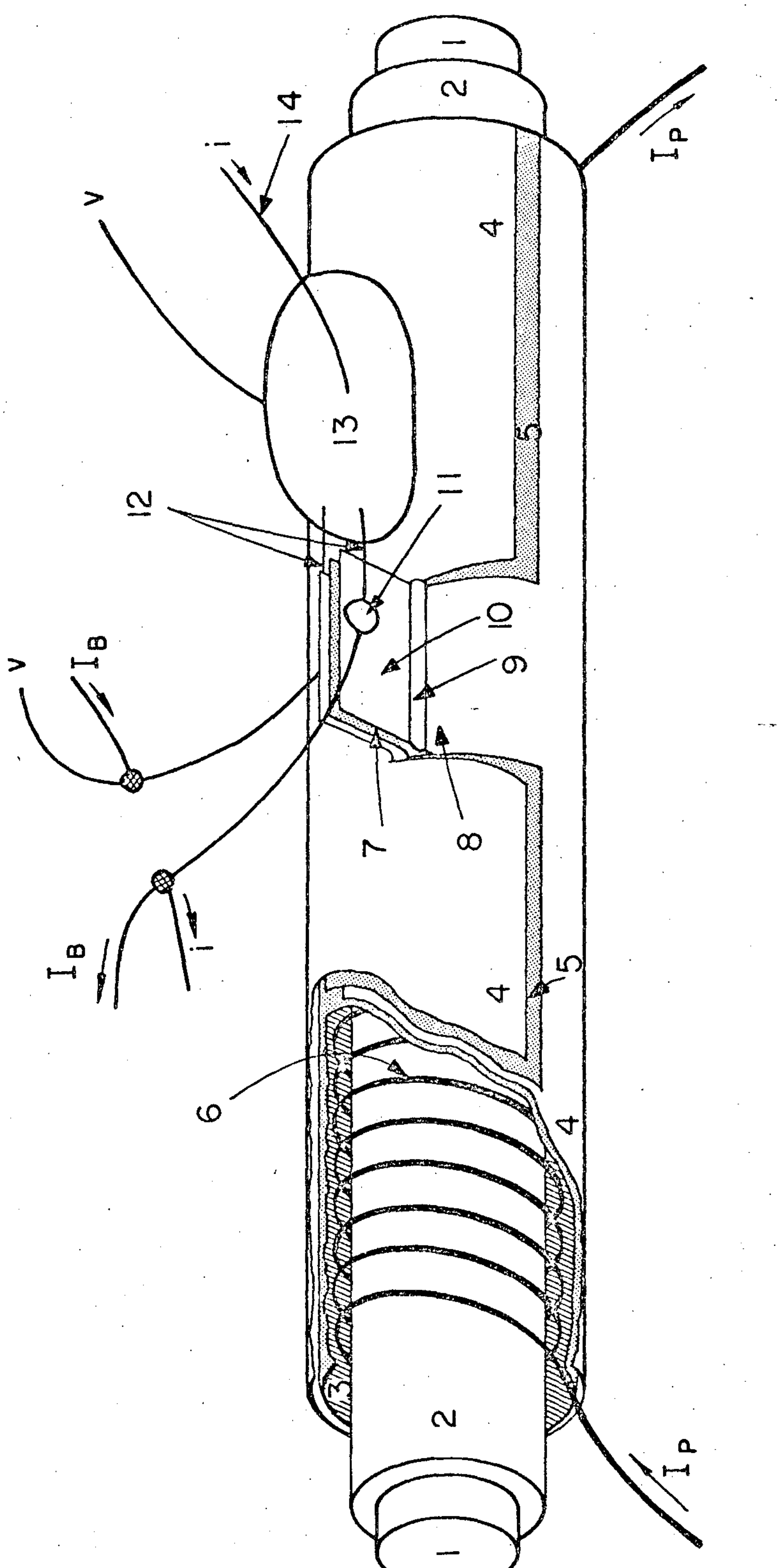

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Fig. 4