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Holographic Accelerated Heavy Quark-Anti-Quark Pair

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The problem of a heavy quark-anti-quark pair which have constant eternal acceleration in opposite directions in the vacuum of deconfined maximally supersymmetric Yang-Mills theory is studied both in perturbation theory and at strong coupling using AdS/CFT. Perturbation theory is summed to obtain what is conjectured to be an exact result. It is shown to agree with a particular prescription for computing the disc amplitude in the string theory dual and it yields a value $s = \sqrt{\lambda}$ for the entanglement entropy of the quark and anti-quark.

The system of a quark and an anti-quark which accelerate eternally on mirror-symmetric trajectories has recently received some attention [1]-[8]. The quark and anti-quark are never in causal contact. However, together, they form a colour singlet. Their quantum states are therefore highly entangled. Moreover, they can interact with each other by exchanging space-like gluons and other quanta of the field theory that they are embedded in. How their properties would be modified by such interactions is an interesting question. For some quantum field theories, such as $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, this question can be addressed in both the weak coupling limit using perturbation theory and in the strong coupling limit by studying the string theory dual of the quark-anti-quark pair, an open string traveling on the $AdS_5 \times S^5$ background. In this paper, we will study an accelerating quark-anti-quark pair in $\mathcal{N} = 4$ Yang-Mills theory. To be concrete, we assume that the acceleration is generated by a constant electric field E . The quark and anti-quark are scalar components of the massive W-boson supermultiplet which is created when, in $\mathcal{N} = 4$ supersymmetric Yang Mills theory, $U(N+1)$ gauge symmetry is spontaneously broken to $U(N) \times U(1)$. The electric field is in the unbroken $U(1)$. We will consider the quantum amplitude for a process where we inject the quark and anti-quark into the system at early

times and at very high velocities on a collision course. The electric field decelerates both the quark and anti-quark. They stop and turn around before they collide and then accelerate back, in opposite directions to spatial infinity. We will take the heavy quark limit where this semi-classical description of quark propagation is valid. In particular, this limit should suppress competing processes such as Schwinger pair production by the electric field and the Drell-Yan process where the quark and anti-quark annihilate to form a space-like gluon which then decays to a quark-anti-quark pair. We will eventually take the large N limit where internal loops of the W-boson and emission of bremsstrahlung are suppressed.

The trajectories which solve the classical equation of motion of the quark and the anti-quark are

$$x^\mu(\tau) = \frac{M}{E} \left(\sinh \frac{E}{M} \tau, \cosh \frac{E}{M} \tau, 0, 0 \right), \quad (1)$$

$$\tilde{x}^\mu(\tilde{\tau}) = \frac{M}{E} \left(\sinh \frac{E}{M} \tilde{\tau}, -\cosh \frac{E}{M} \tilde{\tau}, 0, 0 \right), \quad (2)$$

respectively and are depicted in figure 1. Here, $\tau, \tilde{\tau}$ are the proper times and M is the quark mass. These trajectories are tuned so that the distance of closest approach is given by $2\frac{M}{E}$. The amplitude can be extracted from the 4-point function

$$\begin{aligned} g(x_f, \tilde{x}_f; x_i, \tilde{x}_i) &= \\ & \int_0^\infty \frac{dT}{T} \int [dx^\mu] e^{iS[x, T]} \int_0^\infty \frac{d\tilde{T}}{\tilde{T}} \int [d\tilde{x}^\mu] e^{i\tilde{S}[\tilde{x}, \tilde{T}]} W[x, \tilde{x}] \\ & \approx \exp(-i\mathcal{E}\tau_P) \end{aligned} \quad (3)$$

where we use the world-line path integral representation of the quark and anti-quark propagators, with actions

$$S = \int_0^1 d\sigma \left[\frac{(\dot{x}^\mu(\sigma))^2}{4T} - M^2 T + \frac{E}{2} (x^1 \dot{x}^0 - x^0 \dot{x}^1) \right] \quad (4)$$

$$\tilde{S} = \int_0^1 d\sigma \left[\frac{(\dot{\tilde{x}}^\mu(\sigma))^2}{4\tilde{T}} - M^2 \tilde{T} - \frac{E}{2} (\tilde{x}^1 \dot{\tilde{x}}^0 - \tilde{x}^0 \dot{\tilde{x}}^1) \right] \quad (5)$$

and boundary conditions $x^\mu(\sigma=1) = x_f^\mu$ and $x^\mu(\sigma=0) = x_i^\mu$. Interactions with the remaining massless $U(N)$ adjoint representation fields of $\mathcal{N} = 4$ theory are taken into

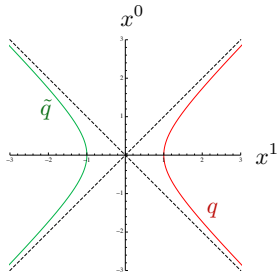


FIG. 1: The quark and anti-quark follow eternally accelerating trajectories on the right and left of the figure, respectively.

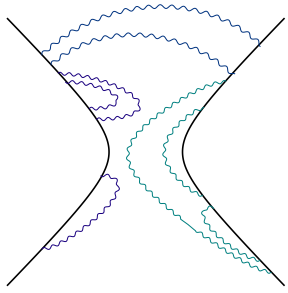


FIG. 2: The four point function is dominated by the amplitude where the quark and anti-quark follow the solid lines and exchange massless $\mathcal{N} = 4$ quanta along the wiggly lines. Only planar ladder diagrams are depicted here.

account by inserting the Wilson loop for the quark and anti-quark,

$$W[x, \tilde{x}] = \langle 0 | \text{Tr} U[x] \tilde{U}[\tilde{x}] | 0 \rangle \quad (6)$$

where

$$U[x] = \mathcal{T} e^{i \int_{-\infty}^{\infty} d\tau [A_{\mu}(x(\tau)) \dot{x}^{\mu}(\tau) + \sqrt{-\dot{x}(\tau)^2} \phi^1(x(\tau))]} \quad (7)$$

$$\tilde{U}[\tilde{x}] = \left(\mathcal{T} e^{i \int_{-\infty}^{\infty} d\tau [A_{\mu}(\tilde{x}(\tau)) \dot{\tilde{x}}^{\mu}(\tau) - \sqrt{-\dot{\tilde{x}}(\tau)^2} \phi^1(\tilde{x}(\tau))] \right)^{\dagger} \quad (8)$$

Our aim is to compute the phase and extract \mathcal{E} defined in (3) in the heavy quark limit, the large N planar limit and the large $M\tau_P$ limit.

Consider propagator for the free quark or antiquark, $\int_0^{\infty} \frac{dT}{T} \int [dx^{\mu}] \exp(iS[x, T])$ in the semi-classical large M limit. The classical equations of motion for the action (4) or (5) are solved by

$$x_0^{\mu}(\tau_P(\sigma - 1/2)), \tilde{x}_0^{\mu}(\tau_P(\sigma - 1/2)), T_0 = \frac{\tau_P}{2M} = \tilde{T}_0 \quad (9)$$

where $(x_0(\tau), \tilde{x}_0(\tilde{\tau}))$ are the trajectories (1,2). The classical actions evaluated on the solution are $S[x_0, T_0] = \tilde{S}[\tilde{x}_0, \tilde{T}_0] = [M - \frac{M}{2}] \tau_P$. The first term, $M\tau_P$, is the rest mass times the proper time. The second term, $\frac{M}{2}\tau_P$, is from the interaction with the electric field. The quark propagator is then $\int_0^{\infty} \frac{dT}{T} \int [dx^{\mu}] \exp(iS[x, T]) \sim \exp(-i[M - \frac{M}{2}] \tau_P)$. We are only interested in the asymptotic behaviour of the phase at large τ_P . Corrections to the coefficient of τ_P in the exponent are suppressed by powers of $\frac{E}{M^2}$ which we assume to be small.

Now, consider the coupling to the remaining massless fields of the $\mathcal{N} = 4$ theory. To analyze the integrals in (3) semi-classically we should take into account the contributions of the Wilson loop which would contribute a force, $-i\delta \ln W[x, \tilde{x}] / \delta x^{\mu}(\tau)$, to the equation of motion for the quark and similar for the anti-quark. However, since, by symmetry,

$$\left. \frac{\delta \ln W[x, \tilde{x}]}{\delta x^{\mu}(\tau)} \right|_{x, \tilde{x}=x_0, \tilde{x}_0} = 0, \quad \left. \frac{\delta \ln W[x, \tilde{x}]}{\delta \tilde{x}^{\mu}(\tau)} \right|_{x, \tilde{x}=x_0, \tilde{x}_0} = 0,$$

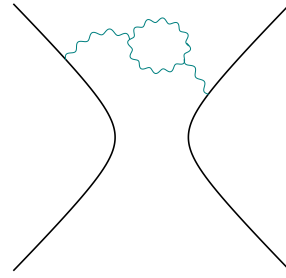


FIG. 3: Leading contribution with internal vertices: Feynman diagram with a self-energy correction.

the classical equations of motion which govern the semi-classical limit of the four-point function are still solved by the same classical trajectories (9) as in the absence of the Wilson loop. The semiclassical limit is then given by

$$g(x_f, \tilde{x}_f; x_i, \tilde{x}_i) \approx e^{-2i[M - \frac{M}{2}] \tau_P} W[x_0, \tilde{x}_0] \quad (10)$$

and, estimating the leading order reduces to evaluating the Wilson loop on the trajectories (1,2). To compute the Wilson loop, we expand the time ordered products in powers of the exponents in (7) and (8) and evaluate the resulting correlation functions using the standard Feynman-Dyson technique. For details of conventions and notation, we refer the reader to reference [11] where a similar computation is done for the Euclidean circle Wilson loop. The free scalar field and the free vector field propagators in the Feynman gauge are $\langle \mathcal{T} \phi^{ab}(x) \phi^{cd}(y) \rangle_0 = \frac{g^2 \delta^{ad} \delta^{bc}}{2} g(x - y)$ and $\langle \mathcal{T} A_{\mu}^{ab}(x) A_{\nu}^{cd}(y) \rangle_0 = \frac{g^2 \delta^{ad} \delta^{bc} g_{\mu\nu}}{2} g(x - y)$ with $g(x) = 1/[4\pi^2 x^2 + i\epsilon]$. The essential observation is that, if we sum the contribution of vector and scalar propagators between any two points on the trajectories, the effective propagator, for example, $\frac{\sqrt{-\dot{x}(\tau_1)^2} \sqrt{-\dot{x}(\tau_2)^2} \dot{x}_{\mu}(\tau_1) \dot{x}^{\mu}(\tau_2)}{8\pi^2 [x(\tau_1) - x(\tau_2)]^2 + i\epsilon} = \frac{E^2}{16\pi^2 M^2}$, is a constant. Then, since the effective propagators are constants, one can sum rainbow ladder diagrams of the type depicted in figure 2 by simply solving the algebraic problem of contracting Lie algebra indices. As for the case of the Euclidean circle studied in reference [11], this problem is summarized by a Gaussian matrix model. Moreover, the matrix model integral can be solved exactly, for any value of N and λ . The result for the Euclidean circle at large N was given in reference [11] and for any N in [12] and can be modified for the Lorentzian case to get

$$W[x_0, \tilde{x}_0] = N L_{N-1}^1 \left[\frac{\varepsilon^2}{N} (M\tau_P)^2 \right] e^{-\frac{\varepsilon^2}{2N} (M\tau_P)^2} \quad (11)$$

where $L_n^m(x) = \frac{1}{n!} e^x x^{-m} \left(\frac{d}{dx}\right)^n (x^{m+n} e^{-x})$ is the Laguerre polynomial and

$$\varepsilon = \frac{\sqrt{\lambda}}{2\pi} \frac{E}{M^2} \quad (12)$$

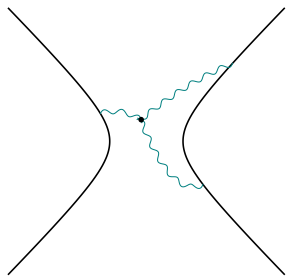


FIG. 4: Leading contribution with internal vertices: Feynman diagram with one internal vertex.

This sum over ladder diagrams has left out diagrams with internal vertices. Their leading orders are depicted in figures 3 and 4. In the case of the circle Wilson loop discussed in [11], the contributions of these diagrams canceled exactly, leading to the conjecture, later proved using localization [13], that all diagrams with internal vertices cancel and that the sum of ladders is exact. The Wilson loop (6) with trajectories (1,2) is supersymmetric in that it commutes with half of the supercharges. It is straightforward to see by explicit calculation that, as $\tau_P \rightarrow \infty$, the leading corrections cancel in the present Lorentzian case also. This leads us to the conjecture that, for large τ_P , (11) approaches an exact formula. We will proceed with the assumption that it is exact, with emphasis that, at this point, it is an assumption.

Now, we observe that (11) has Gaussian damping with proper time and its contribution to the amplitude at large τ_P vanishes. This can be attributed to the fact that accelerating quarks emit bremsstrahlung and the amplitude for finding only a quark-anti-quark pair in the final state is vanishingly small. It is also easy to see that emission of bremsstrahlung requires nonplanar Feynman diagrams which are suppressed in the large N limit. This suggests that the proper asymptotic behaviour should be restored if we first project onto planar diagrams and then take τ_P large. Indeed, by looking at the large N limit of (11), we see that the Gaussian damping goes away,

$$\lim_{N \rightarrow \infty} W[x_0, \tilde{x}_0] = \frac{N}{\varepsilon M \tau_P} J_1(2\varepsilon M \tau_P) \quad (13)$$

$$\lim_{N, \tau_P \rightarrow \infty} W[x_0, \tilde{x}_0] = N \frac{-i}{\sqrt{4\pi} (\varepsilon M \tau_P)^{\frac{3}{2}}} e^{i 2\varepsilon M \tau_P} \quad (14)$$

Here, $J_\alpha(x)$ are Bessel functions of the first kind and $J_\alpha(x) \sim \sqrt{\frac{2}{\pi x}} \cos(x - \frac{\pi}{2}\alpha)$ at large argument. The cosine is a sum of two exponentials. The large $M\tau_P$ limit must be defined with an $i\varepsilon$ prescription that picks out the appropriate exponential. The resulting phase, when combined with (10), gives the result for the energy of the quark-anti-quark pair,

$$\mathcal{E} \approx 2 \left[M - \varepsilon M - \frac{M}{2} \right] \quad (15)$$

The pre-factor in (14) is also interesting as it summarizes the effect of fluctuations corresponding to those of the disc geometry in the string theory dual. However, we have not computed the contribution of fluctuations of the quark and anti-quark paths, which would contribute at the same order, and are dual to fluctuations of the disc boundary in the string theory. We leave this as a project for future work.

When $\sqrt{\lambda}$ is large so that $\varepsilon \gg E/M^2$, the correction in (15) would be larger than the order E/M^2 terms which would arise from taking into account fluctuations of the trajectory about the classical limit and which we have ignored. However, we cannot let ε be of order one, as the amplitude for Schwinger pair production, which also occurs in a constant external electric field, is modified at strong coupling [10]. There is a critical electric field where the tunnelling barrier to pair production vanishes and it loses its usual exponential suppression. That critical electric field is at $\varepsilon = 1$. If we want to avoid this regime, we must also take $\varepsilon \ll 1$. This restricts the region of validity of our computation and, simultaneously, the regime where the correction from the Wilson loop is significant, to

$$\frac{E}{M^2} \ll \varepsilon = \frac{\sqrt{\lambda}}{2\pi} \frac{E}{M^2} \ll 1 \quad (16)$$

This is the strong coupling regime of Yang-Mills theory.

The AdS/CFT dual of $\mathcal{N} = 4$ Yang-Mills theory is IIB string theory on the $AdS_5 \times S^5$ background,

$$ds^2 = \frac{\sqrt{\lambda} \alpha'}{u^2} (\eta_{\mu\nu} dx^\mu dx^\nu + du^2) \quad (17)$$

The AdS boundary is located at $u = 0$ and the Poincare horizon at $u = \infty$. The quark and anti-quark are oppositely oriented open strings which stretch from the Poincare horizon of $AdS_5 \times S^5$ to a probe D3 brane that is suspended parallel to the horizon at radial coordinate $u = u_M$. The quark mass M is the energy of a static string suspended between the probe and the Poincare horizon. This determines the radius at which the probe is suspended, $u_M = \frac{\sqrt{\lambda}}{2\pi M}$. The probe brane has an internal constant U(1) electric field E , where E is assumed to be small enough that it does not back-react on the probe brane embedding. (This is the condition $\varepsilon \ll 1$.) We need to compute the disc amplitude, where the boundary of the disc lies on the probe brane and where the asymptotic states are the in-coming and out-going quark-anti-quark pair. In the limit of large λ , the open string sigma model is semiclassical and it suffices to study classical solutions of the Nambu action,

$$S_N = \frac{-1}{2\pi\alpha'} \int d^2\sigma \sqrt{-\det g} + \frac{E}{2} \oint d\tau (x^1 \dot{x}^0 - x^0 \dot{x}^1) \quad (18)$$

with g the world-sheet metric. The last term in (18) is the line integral on the world-sheet boundary of the

U(1) gauge field corresponding to a constant electric field. Then, a classical solution is a world sheet which is itself a part of AdS_2 , the locus of

$$u^2 + (x^1)^2 - (x^0)^2 = \frac{M^2}{E^2} \quad (19)$$

This world-sheet intersects the probe brane at the quark and anti-quark trajectories (1,2). There are two world-sheet event horizons which are located at $u_E = \frac{M}{E}$. The metric near the quark trajectory is

$$ds_g^2 = \sqrt{\lambda} \alpha' \left[- \left(\frac{1}{u^2} - \frac{E^2}{M^2} \right) d\tau^2 - \frac{2}{u^2} d\tau du \right]$$

One can check that this agrees with the general solution for the extremum of the Nambu action with arbitrary curve on the AdS_5 boundary which was found by Mikhailov [9]. As Mikhailov observed, the determinant of the world-sheet metric does not depend on the trajectory, and in this particular case it does not depend on the electric field. An integration to find the proper area of the full AdS_2 world sheet yields an on-shell action which contains only the quark mass (plus interaction with electric field), $S_N = -2 \left[M - \frac{M}{2} \right] \tau_P$, which disagrees with the Yang-Mills computation since the energy shift of the quark is absent. (The factor of 2 is from integration over two identical regions.) What the Yang-Mills computation suggests is, rather, that we should use the proper area between the probe brane and the event horizons

$$\begin{aligned} S_N &= 2 \left[-\frac{\sqrt{\lambda}}{2\pi} \int_{-\frac{\tau_P}{2}}^{\frac{\tau_P}{2}} d\tau \int_{u_M}^{u_E} \frac{du}{u^2} + \frac{M}{2} \tau_P \right] \\ &= -2 \left[M - \varepsilon M - \frac{M}{2} \right] \tau_P \end{aligned} \quad (20)$$

We see that, in this case, the semiclassical disc amplitude, $\exp(iS_N) = \exp(-2i \left[M - \varepsilon M - \frac{M}{2} \right] \tau_P)$, matches the Yang-Mills result (15) exactly. Although this certainly deserves further study, on the face of it, the Yang-Mills

computation favours a picture of the world-sheet advocated in reference [9] and elaborated in [4] where the world sheet, rather than being the maximally extended AdS_2 wormhole which is the full locus of (19), is identical to that surface only between the event horizons and the probe brane. Between the event and Poincare horizons, it is replaced by the null surface which has vanishing proper area. The reasoning is that, with causal propagation of influences on the world-sheet, the quark acceleration has no effect on the world-sheet beyond the horizon.

We note that the Unruh temperature of the accelerated reference frames is $T_U = \frac{E}{2\pi M}$ and $\frac{1}{2}\mathcal{E} = M - \frac{M}{2} - \sqrt{\lambda} T_U$ can be interpreted as the free energy of the quark in its rest frame. In the limit, $T_U \ll M$, which we have considered, most of the thermal effects due to the accelerated reference frame are ignored. The small correction that we see is due to interactions and it is significant only at strong coupling. The entropy which we deduce from this free energy is $s = \sqrt{\lambda}$. This entropy must be entirely due to quantum entanglement of the accelerated quark with degrees of freedom behind its horizon. Why this value differs from other estimates of this entanglement entropy [5]-[8] is an interesting question.

Finally, there is an obvious question: what about a single quark, without the anti-quark? In the Yang-Mills calculation, there is a big difference with the quark-anti-quark pair, the cancellation of contributions from the diagrams in figures 3 and 4 does not occur for a single quark and we are not able to do a Yang-Mills calculation of its energy shift beyond order λ^2 . It would be interesting to see if this difference is reflected in the string theory dual. We will revisit this issue in a companion paper[14].

Acknowledgments

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