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Heterogeneity Impacts and Implications in Allocation and Location Processes

A dissertation submitted in partial satisfaction of the

requirements for the degree Doctor of Philosophy

in Geography

by

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September 2019

The dissertation of Xin Feng is approved.

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Heterogeneity Impacts and Implications in Allocation and Location Processes

To Yuanpei, Qian, and Ningmeng

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ABSTRACT

Heterogeneity Impacts and Implications in Allocation and Location Processes

by

Xin Feng

Location-allocation decisions are extremely important and directly influence the efficiency of the investment and operation of a given service. The efficiency of the service system results from the geographical arrangement of a given set of facilities, the manner in which their services are provided, and the spatial distribution of demand. However, there are often unrealistic assumptions of spatial and temporal homogeneity in associated location and allocation processes. For example, one assumption is that service assignment cost is fixed over space and time, not impacted by instantaneous travel movement changes caused by topography, time, direction, slope, weather, etc. Even though heterogeneity has been formalized in assignment processes, previous studies assume a pre-specified road network. Without the restriction of a network, how to structure and solve an allocation process is particularly challenging when heterogeneity must be taken into account across continuous space over time. Both raster and vector base methods are developed in this dissertation to construct service areas in order to minimize assignment cost. Generalized location-allocation models are proposed to improve planning and decision-making processes with appropriate description of travel accessibility and distributed demand. Emergency medical service delivery is utilized to demonstrate the feasibility, usefulness and significance of incorporating spatial and temporal heterogeneity in location and allocation processes across a continuous terrain. A primary question to be answered for this specific case study is how to locate medical drone base stations and allocate service in order to optimize overall response, especially given the spatiotemporal heterogeneity in distributed demand and varying service response times/costs. Results show that response potential is over- and under-estimated when heterogeneity and travel obstacles are disregarded. More importantly, travel times to patients across a region can be significantly reduced through better location and allocation decision making.

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Chapter 1 Introduction

This chapter will provide an overview of the problems to be addressed in this dissertation and why they are important. It is divided into the following sections: Motivation, Key Problems, Research Objectives, Significances and Organization of the Research. Among the five sections, the section of Key Problems reviews the major research concerning allocation and location modeling to the present day. The section of Significances seeks to provide a broad picture perspective on the contributions of my work. All model formulations will be given in a format consistent with mathematical programming.

1.1 Motivation

Spatial heterogeneity is widely accepted as an important feature of variability in geography and geographic information science. Spatial heterogeneity implies that geographic attribute variables exhibit uncontrolled variance (Goodchild, 2004). That is, the result of any analysis is unique to a study area. Further, there is a lack of spatial uniformity associated with spatial dependence and/or relationships between variables under study (Anselin, 1988). In addition to spatial heterogeneity, temporal heterogeneity is also fundamental for both human activities and physical processes. Hägerstrand notes the significance of temporal variability, effectively establishing time geography. This field introduces the time-space model to account for spatial location as well as the time dimension. In an era of data explosion, the shear amount of spatial and aspatial information has increased rapidly (Miller & Goodchild, 2015). Current data sources include remote sensing images, Google Street View, location-based data, LiDAR, GPS movements, consumer activity, etc. These detailed data enhance the possibility for representing and exploring the spatial and temporal heterogeneity in both geographic attribute variables and their relationships. Important questions, however, arise regarding the details and nuances that emerge. The ability to address these is a major challenge.

Heterogeneity is an important issue and has been considered in spatial analytics. Tong & Murray (2009) point out that the abstraction of geographic space, reducing the complexities of the real world to something more manageable, has introduced unintended measurement and interpretation errors. Spatial statistics, such as local indicators of spatial association (LISA) statistics (Anselin 1995), the G statistics (Getis & Ord, 1992) and geographically weighted regression (Brunsdon et al., 1996), attempt to account for, quantify and/or capture spatial heterogeneity. Moreover, other spatial analysis models, such as space-time kriging (Janis & Robeson, 2004) and geographically and temporally weighted regression (Huang et al., 2010; Fotheringham et al., 2015), capture both spatial and temporal heterogeneity simultaneously. However, spatial and temporal heterogeneity have not been taken into account comprehensively in many location modeling contexts. In fact, many spatial optimization problems, including facility coverage/service area, allocation, location, routing, etc., are all impacted by aspects of heterogeneity. Location-allocation models are key to ensuring efficient investment and operation, essential in many services systems. Major components of location-allocation models include identifying the best geographical arrangement of facilities and providing the best service allocation, but are highly inter-dependent.

Allocation can be regarded as a partition of space -- all demand in a sub-region is assigned to the same service facility. There are multiple partitioning methods in spatial analysis, including Voronoi diagram, k-means and k-medoids, Expectation Maximization, etc. One similarity of these methods is that space is partitioned based on closeness in distance or similarity of attributes. As a result, Voronoi diagrams have been widely used and have proven effective in service allocation. Given a dataset of *n* generators, *n* partitions are constructed in which each customer demand is associated with its closest member of the generator set. Existing Voronoi diagram approaches, however, only focus on characteristics of generators, without considering the possible heterogeneous costs or attributes between demand and generators. This may directly influence the attractiveness of generators for demand, but also appropriate ideal allocations. Research exploring an extension of the Voronoi diagram has considerable merit. Associated allocation processes that account for spatiotemporal heterogeneity are critically needed.

In addition to assigning demand to be served in an optimal manner, a location-allocation problem involves identifying the best sites for service facilities (Church & Murray, 2009). In order to determine where to site facilities within a service system, we need to know both the distribution of demand and service cost for demand assignment, all of which confounds processes of allocation. As mentioned previously, allocation is complicated because of the additional consideration of heterogeneity. Therefore, dealing with allocation simultaneously makes the location component complex as well.

In what ways does the heterogeneity of location and allocation matter? Medical emergency service delivery provides an interesting and challenging example of heterogeneity complexities. Unmanned Aerial Vehicles, often called drones, have rapidly emerged for commercial and personal use in recent years. Drones are a promising and effective transportation mode for emergency medical service delivery because they can travel faster than traditional ground-based vehicles, particularly when obstacles limit quick response or in cases of congestion. While medical aid can be found at hospitals, clinics, fire stations, and the like, drones can be used to extend special services, like drug and equipment delivery, to almost anywhere, providing a quick response without the use of more expensive (and larger) vehicles.

Travel time is essential for emergency medical service response as any reduction may increase the chances of patient survival. An important consideration for drones is that travel times are impacted in various ways by real-time local conditions, including weather and terrain. Previously it has been assumed that drone flight speed is fixed over space and time. Because of this assumption, the cost of service for demand from/to a facility in continuous space is usually derived solely according to distance or a fixed cost over time. However, in many practical cases, speed is not homogeneous, and distance may not accurately represent cost. Wind magnitude and speed as well as "no drone zones" may vary travel potential over space and time. This variation should be accounted for when determining allocation and location so as to enhance the accuracy of response time, improving the efficiency of the entire service system. Failure to consider heterogeneity highlights limitations of existing location-allocation approaches. Further, this puts lives at risk in the case of emergency medical drone delivery as any delay in service response increases the likelihood of death.

1.2 Key Problems

The discussion of spatial and temporal heterogeneity will benefit from a more detailed look at problem formulations. Key location-allocation models (shown in Figure 1.1) include median problems (e.g., Vector Assignment P-Median, P-Median, P-Median Scheduling and Location Problem, Stochastic Median Problem, and Progressive P-Median Problem, among others) and coverage problems (e.g., Backup, Maximum Expected, Maximal Covering, and Location Set Covering, among others). Shown in Figure 1.1 is an interesting relationship between these core modeling approaches, where some are characterized by Single Facility

Service and others as Multiple Facility Service. Beyond this, some models can be structured as special cases of others. Such relationships were originally noted in Church and ReVelle (1976) and Church and Weaver (1986) as well as more recently in Lei and Church (2011, 2014) and Lei et al. (2016).

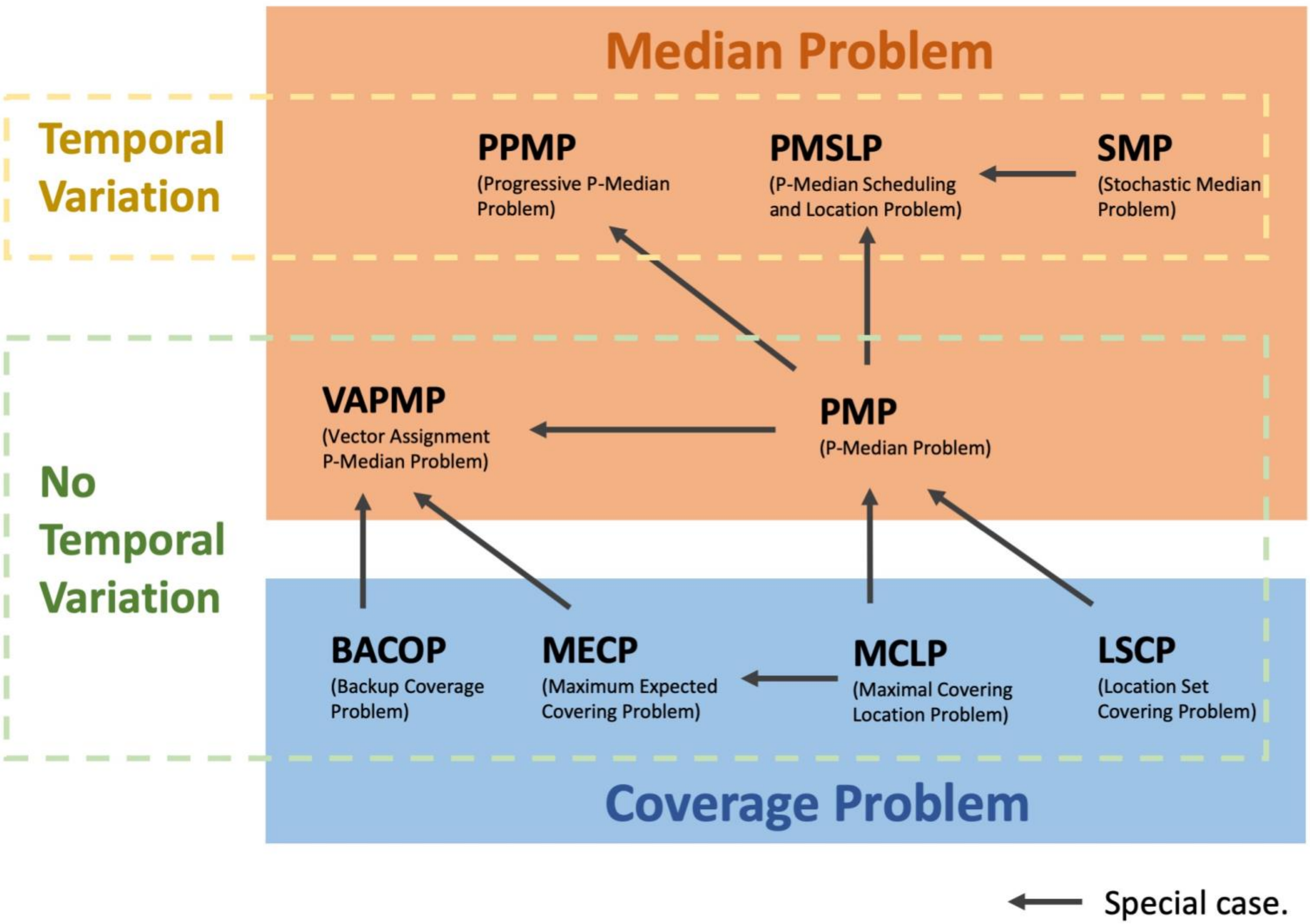


Figure 1.1 Selected median and covering problems and their relationships

The Vector Assignment P-Median Problem (VAPMP) provides a good basis for a review of allocation-location problems underpinning the research carried out in this dissertation since it is a generalized P-Median Problem. The goal of the VAPMP is to locate p-facilities in a way that total weighted distance is minimized. Weaver and Church (1985) formulated the VAPMP as follows:

j = index of demand areas/nodes (1, 2, …, n)

g = index of potential facility sites (1, 2, …, m)

$p =$ the number of stations to be located

a_j = amount of demand in area j

d_{jg} = shortest distance from demand area j to potential facility site g

b_{jk} = the fraction of demand j serviced by the k^{th} closest facility

$$Z_g = \begin{cases} 1 & \text{if facility at site } g \text{ is located} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{jg}^k = \begin{cases} 1 & \text{if demand } j \text{ assigns to facility } g \textit{ as the } k^{th} \textit{ closest} \\ 0 & \text{otherwise} \end{cases}$$

A few items are worth expanded discussion. The idea behind the VAPMP is that service is likely to come from different facilities, depending on the state of the system at a given time. If the closest facility is busy, as an example, then the next closest facility may be utilized. This reflects service dispatching strategy, but also consumer behavior. Accordingly, the requirement would be that demand in area j would be entirely served through some combination of closest facilities, thus $\sum_k b_{jk} = 1$. This means that the total fractional assignment would sum to one for a demand. The associated VAPMP formulation is as follows:

$$Minimize \sum_j \sum_g \sum_k a_j d_{jg}\, b_{jk} X_{jg}^k \tag{1.1}$$

Subject to:

$$\sum_g X_{jg}^k = 1 \qquad \forall j, k \tag{1.2}$$

$$\sum_k X_{jg}^k \leq Z_g \qquad \forall j, g \tag{1.3}$$

$$\sum_g Z_g = p \tag{1.4}$$

$$X_{jg}^k = \{0, 1\} \quad \forall j, k, g \qquad Z_g = \{0, 1\} \quad \forall g \tag{1.5}$$

The objective (1.1) of the VAPMP seeks to minimize the total weighted assignment of demand to service facilities. Constraints (1.2) require that assignment be made for each demand and closeness level. Constraints (1.3) ensure that assignment cannot occur unless the facility is sited. Budgetary conditions are imposed in constraints (1.4). Finally, binary requirements are stipulated in constraints (1.5).

As noted above, the VAPMP relies on a vector of utilization for each demand, where the k^{th} component is the fraction utilized by (or assigned to) the k^{th} closest facility. When the number of options for k is equal to 1, the VAPMP simplifies to the classical median problem, the *p*-Median Problem (PMP). The PMP, therefore, is a special case of VAPMP, and assumes that each demand is always served by the closest facility.

The second type of model worth reviewing involves covering. Toregas et al. (1971) are usually regarded as the first to propose a location problem to cover demands. They proposed a model, the Location Set Covering Problem (LSCP), to find the fewest number of facilities in order to cover all demands. A demand is covered if a facility is located within a known service distance or time standard of that demand. The problem is formulated using the following additional notation:

S = maximum distance/service time

$N_j = \{g \in G \mid d_{jg} \leq S\}$, the set of facilities within distance/time that can provide service coverage for demand j

$$Z_g = \begin{cases} 1 & \text{if facility at site } g \text{ is located} \\ 0 & \text{otherwise} \end{cases}$$

The formulation of the LSCP follows:

$$Minimize \quad \sum_g Z_g \tag{1.6}$$

Subject to:

$$\sum_{g \in N_j} Z_g \geq 1, \quad \forall j \in J \tag{1.7}$$

$$Z_g = \{0, 1\} \qquad \forall g \in G \tag{1.8}$$

The objective of the LSCP is to locate the minimum number of facilities. Constraints (1.7) ensure that each demand is within the service coverage of at least one sited facility. Binary requirements for decision variables are given in constraints (1.8).

The LSCP is one of the first models proposed to cover demand. The LSCP has been widely applied to locate various public resources. However, the LSCP is not appropriate when the cost of complete coverage is not affordable or is unnecessary. To address this limitation, Church and ReVelle (1974) proposed the Maximal Covering Location Problem (MCLP), which aims to maximize coverage given a limit on the number of facilities to be sited. Rather than requiring complete coverage, the MCLP represents a relaxation of sorts. The model structure therefore facilitates a tradeoff between the number of facilities sited (total investment) and coverage provided. Consider the following additional notation:

$$Y_j = \begin{cases} 1 & \text{if demand } j \text{ is covered} \\ 0 & \text{otherwise} \end{cases}$$

The MCLP formulation is as follows:

$$Maximize \sum_j a_j Y_j \tag{1.9}$$

Subject to:

$$\sum_{g \in N_j} Z_g \geq Y_j \qquad \forall j \tag{1.10}$$

$$\sum_g Z_g = p \tag{1.11}$$

$$Z_g = \{0, 1\} \qquad \forall g \in G \qquad\qquad Y_j = \{0, 1\} \qquad \forall j \in J \tag{1.12}$$

The objective function of the MCLP, (1.9), is to maximize the total demand covered by p facilities. Constraints (1.10) ensure that demand j can only be covered if at least one facility has been located within distance/time that can provide service coverage. The total number of facilities to be sited is stipulated in constraint (1.11). Binary conditions on decision variables are detailed in constraints (1.12).

Suggested in Figure 1.1 is that both the LSCP and MCLP could be solved (directly or indirectly) as a modified PMP (see also Church and Murray, 2018). Beyond this, there is a relationship between the VAPMP and the PMP, and as a result other model forms. The MCLP is a special case of the Maximum Expected Covering Problem (MECP) (Daskin, 1983). The MCLP assumes facilities are always available for service while the MECP allows facilities to have a probability of unavailability. Hogan and ReVelle (1986) extended the work of Daskin (1983) by proposing the Backup Coverage Problem (BACOP). The goal of the BACOP is to maximize backup coverage subject to the constraint that all demand is covered at least once. The BACOP has been formulated in the following way:

$$Y_j' = \begin{cases} 1 & \text{if demand } j \text{ is covered at least twice} \\ 0 & \text{otherwise} \end{cases}$$

$$Maximize \sum_j a_j Y_j' \tag{1.13}$$

Subject to:

$$\sum_{g \in N_j} Z_g - Y_j' \geq 1 \qquad \forall j \tag{1.14}$$

$$\sum_g Z_g = p \tag{1.15}$$

$$Z_g = \{0,1\} \quad \forall g \in G. \qquad Y_j' = \{0,1\} \quad \forall j \in J \tag{1.16}$$

The objective (1.13) of the BACOP is to maximize coverage of demand with two or more facilities. Constraints (1.14) account for coverage of two or more facilities. Constraint (1.15) stipulates that p facilities are to be sited. Binary conditions are imposed in constraints (1.16).

It has been well documented that there are theoretical linkages between covering problems and median problems. The LSCP, MCLP and other covering models have been shown to be special cases of the PMP (e.g., Church & ReVelle, 1976). With appropriate distance and population transformation functions, these covering models can be solved as specially defined median problems (Church & Weaver, 1986). The LSCP, MCLP and other covering models share the assumption that a demand is being served by one facility. However, multiple cover models, like the MECP and BACOP, allow for multiple facility coverage (service) properties. They are special cases of the more general VAPMP as well. In this dissertation, I focus on location-allocation models with one facility service property.

The models detailed so far account for spatial heterogeneity though the use of a_j, but assume temporal homogeneity in demand for service. There are some models focusing on the temporal aspects of demand and/or travel cost associated with location-allocation decisions (e.g. Bloxham & Church, 1991; Dao et al. 2012). Bloxham & Church (1991) proposed the P-Median Scheduling and Location Problem (PMSLP), which aims to locate facilities and simultaneously exploit the hours of operations. This is done so as to minimize the total travel cost for all users. The PMSLP assumes that demand could be different over time, and as such facilities should be located based on time of operation. The model is formulated with the following additional notation:

a_j^t = amount of demand at node j during time period t

d_{jg}^t= shortest distance or time between node j and potential facility site g during period t

$\bar{p}$ = the total number of open time periods distributed among the p facilities.

$$X_{jg}^{t} = \begin{cases} 1 & \text{if demand } j \text{ assigns to facility } g \text{ in time period } t \\ 0 & \text{otherwise} \end{cases}$$

$$X_{g}^{t} = \begin{cases} 1 & \text{if a facility } g \text{ is open during period } t \\ 0 & \text{otherwise} \end{cases}$$

Given this notation, the formulation of the PMSLP follows:

$$Minimize \sum_{j} \sum_{g} \sum_{t} a_j^t d_{jg}^t X_{jg}^t \tag{1.17}$$

Subject to:

$$\sum_{g} X_{jg}^{t} = 1 \qquad \forall j, t \tag{1.18}$$

$$X_{jg}^{t} \leq X_{g}^{t} \qquad \forall j, g, t \tag{1.19}$$

$$X_{g}^{t} \leq Z_{g} \qquad \forall g, t \tag{1.20}$$

$$\sum_{g} Z_{g} = p \tag{1.21}$$

$$\sum_{g} \sum_{t} X_{g}^{t} = \bar{p} \tag{1.22}$$

$$X_{jg}^{t} = \{0, 1\} \quad \forall j, g, t \qquad X_{g}^{t} = \{0, 1\} \quad \forall g, t \qquad Z_{g} = \{0, 1\} \quad \forall g \tag{1.23}$$

The objective function of the PMSLP, (1.17), seeks to minimize the total requisite travel to facilities for all users over all time periods. Constraints (1.18) ensure that demand j assign to a facility g in time period t and that this hold true for all demands and all time periods. Constraints (1.19) require that demands are allocated to only open facilities. Constraints (1.20) prevent a potential facility site from being scheduled to operate unless it has been selected. The total number of facilities to be sited is stipulated in constraint (1.21). Constraint (1.22) specifies the extent of system operation. Binary conditions on decision variables are detailed in constraints (1.23).

In addition to the PMSLP, there are a few extensions of the P-Median problem that also consider temporal variation. For example, Mirchandani (1980) captured the effect of temporal variation in demand for services and network travel states, calling this the Stochastic Median Problem. Drezner (1995) proposed the progressive P-Median model in which demand has a functional relationship with time and is time-dependent. Chukwusa (2014) proposed a trend-weighted location allocation model in order to account for demand changing over time. The impact of temporal variations in demand was examined on emergency medical service location-allocation decisions.

1.3 Research Objectives

There are five major components of the proposed research related to spatial and temporal heterogeneity and associated allocation and location problems:

1. Solve an allocation problem in spatially heterogeneous space using a new extension of the Voronoi diagram – heterogeneous Voronoi diagram, using a raster-based approach.
2. Expand the allocation problem in time-varying heterogeneous space and solve it with a vector-based approach.
3. Develop a location-allocation model with the consideration of spatiotemporal heterogeneity in distributed demand and varying service response costs.
4. Apply the proposed model to emergency response: locating medical drone base stations and allocating service in order to optimize overall response.

5. Ensure reproducibility and replicability in developed methods

1.4 Significance

The location-allocation problems reviewed in section 1.2 have been applied in various private and public settings. However, they rely on a common assumption of spatiotemporal homogeneity. In reality there is spatiotemporal heterogeneity in allocation, and this can have a major influence on location selection. Without loss of generality, Figure 1.2 summarizes the objective functions and allocation variables for the PMP and MCLP (as examples of median problems and covering problems in Figure 1.1) in the cases of homogeneous and heterogeneous travel. For most existing location problems, d_{jg}, the shortest distance or travel time from demand j to facility g, is fixed and known in advance. There has been limited work specifically focused on allocation because d_{jg} is easily calculated when spatial and temporal homogeneity is assumed. However, the assumption may be problematic in many application contexts. Assignment cost (denoted as $c_g(\vec{j}, t)$ in Figure 1.2) could vary over space and time, making it complex and difficult to quantify in advance. If travel time / cost is dependent on location, direction of travel (inertia) and time of day, then traditional assumptions of service by closest facility will not hold. Variation in assignment cost / time factors will lead to a change in allocation decisions, e.g., $X_g(\vec{j}, t)$ in Figure 1.2.

The interdependent relationship between allocation and location makes the issue of spatiotemporal heterogeneity a challenge indeed, as allocation decisions influence location selection (e.g., $Y(\vec{j}, t)$ in Figure 1.2) and location decisions influence allocation choices. The first two research objectives in this dissertation are focused on addressing specifically the allocation problem through the use of heterogeneous assignment costs over space and time.

The third research objective then proceeds to the development of a location-allocation model that takes into consideration spatiotemporal heterogeneity, both with respect to distributed demand and varying service response costs.

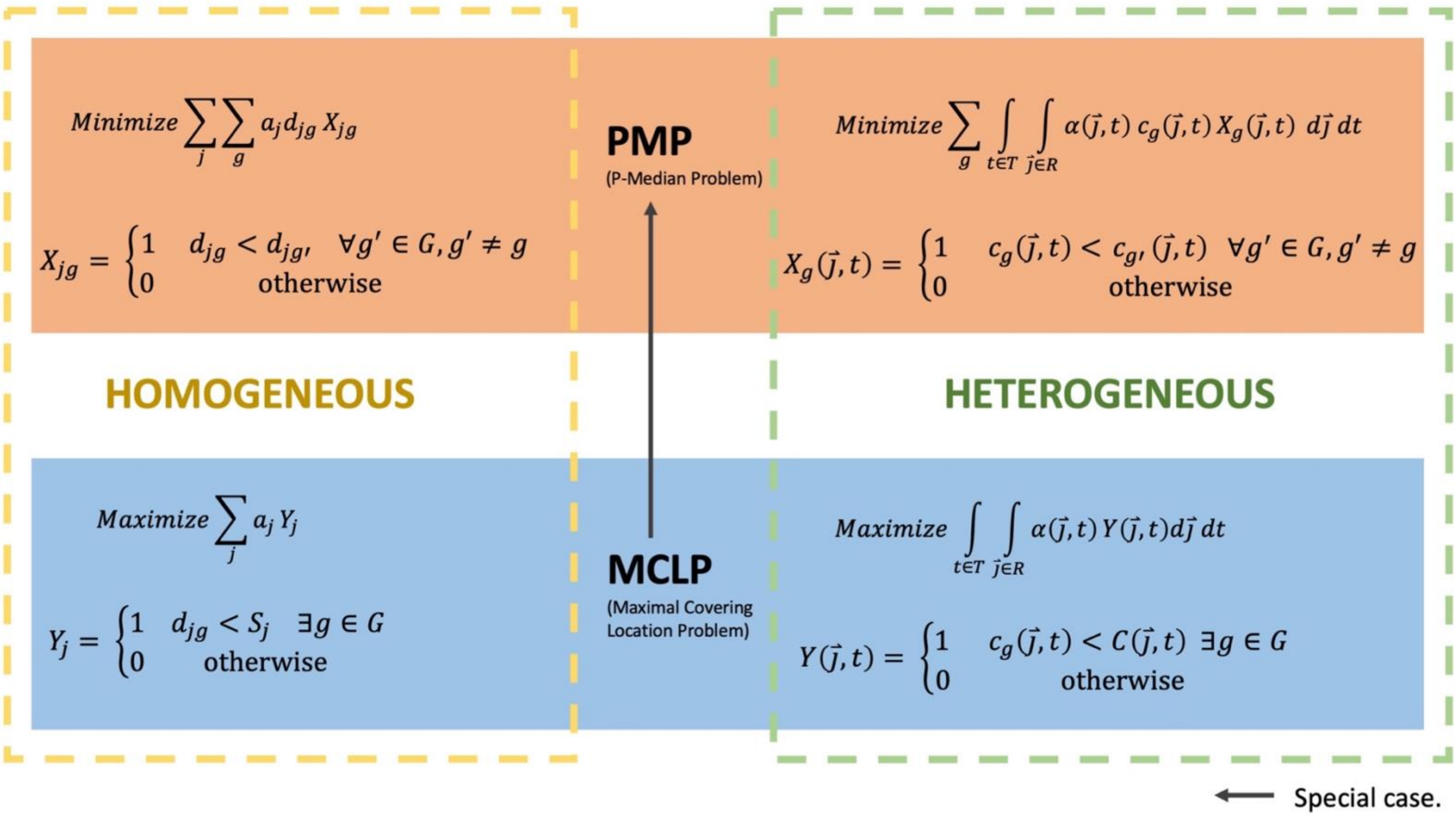


Figure 1.2 Homogeneous and heterogeneous version of p-median problem and maximal covering location problem.

Emergency medical response, especially locating medical drone base stations and allocating service, is an ideal application for an allocation-location model (research objective 4). Most existing research supporting emergency medical drone delivery has applied covering models like the LSCP, MCLP, BACOP, and their extensions (Pulver & Wei, 2016, 2018). This is understandable because drones are limited in travel time, and this is intuitively conceived in terms of a certain radius of service coverage. However, emergency response is about not only serving as many potential patients as possible but also minimizing response time. Minimizing total weighted assignment cost is precisely the objective of the PMP.

As mentioned above, covering models, like the LSCP and MCLP, can be considered special cases of the PMP with distance and population transformation. An extension of the PMP is therefore a good option for dealing with allocation and location of medical drone base stations, ensuring service quality, improving operational efficiency, and considering the bounds of service area as well. A mathematical formulation and implementation are detailed in Chapter 4.

A final component of this dissertation recognizes the importance of ensuring reproducibility and replicability of the work (research objective 5). This is addressed in a number of ways, including model formulation, specification, implementation and explicit characterization of assumptions. Supporting this is pseudo-code at the end of each chapter. Rey (2009, 2018) claims that the true value of open source is its potential to revolutionize and fundamentally enhance geospatial education and research. Code should be regarded as text and a part of research. Further, it provides a pathway to enhance geospatial education and research (Rey, 2009; 2018). The added pseudo-code enables the reader to know more details about the implementation of the proposed models and provides convenience for reproducing and replicating the work in this dissertation.

1.5 Organization of Research

The aim of this research is to explore the impacts and implications of heterogeneity, spatial and/or temporal, in allocation and location processes. The dissertation is structured as follows.

Chapter 2 starts with an introduction of allocation, and then discusses the Voronoi diagram. The Voronoi diagram has been widely and efficiently used in allocation. After reviewing existing Voronoi diagram approaches, it is demonstrated that an assumption of homogeneous space is central to existing approaches. A new Voronoi diagram is then defined – the heterogenous Voronoi diagram. Next, a geographic information system based method is developed to account for spatial heterogeneity by describing continuous space using a discrete approximation. This is followed by an application of the developed approach to demonstrate feasibility, usefulness and significance in incorporating geographic heterogeneity in the allocation process.

Following the discussion of heterogeneity, Chapter 3 adds the consideration of temporal heterogeneity. The mathematical formulation of the allocation problem is formalized to account for spatiotemporal heterogeneity in accessibility by introducing ordinary differential equations. A vector approach – marker particle based front tracking method – has been used to address spatiotemporal heterogeneity in the allocation problem. The implications of spatiotemporal heterogeneity in allocation problems are then examined through an application case study.

Chapter 4 introduces the concept of heterogeneity in allocation along with the process of siting decisions, or location. After reviewing a classic location-allocation approach, the p-median problem, a generic location-allocation model is proposed that considers heterogeneity as instantaneous measures over continuous space and time. Following the model, a spatial optimization solution approach is detailed, which simultaneously considers demand allocation (Chapters 2 and 3) and facility location. Next, the siting of drone-equipped stations for medical emergency delivery is carried out.

Chapter 5 summarizes the contribution to theories and methods of spatial analytics. It also serves to provide concluding comments on research findings along with implications. Finally, directions for future research are discussed.

Chapter 2 Allocation Using a Heterogeneous Space Voronoi Diagram[1]

2.1 Introduction

Allocation is the process of best determining who is served by which facility given both simple and complex requirements (Church & Murray, 2009). Compared with locating facilities, allocating services has received less interest and emphasis since demands are generally assumed to be assigned based upon closest facility criteria. The property of closest assignment is a reasonable assumption/requirement in many contexts (Gerrard & Church, 1996). However, most existing research with respect to closest assignment assumes that accessibility is homogeneous; that is, distance (measured in some unit of length) accurately reflects the cost of traveling through a region (Gerrard & Church, 1996). In reality, spatial homogeneity simply does not exist in many situations; different kinds of travel costs (or service costs in allocation), such as economic, time, and energy, are generally not evenly distributed. The best accessibility or travel time have not been reflected in allocation in such contexts with the assumption of spatial homogeneity. Additionally, without a pre-specified network linking demands and facilities, the measure of closeness becomes far more complicated.

The Voronoi diagram has been widely used to delineate continuous space in allocation problems (Okabe et al., 1992). The essence of the method is that, given a finite set of distinct

[1] This chapter represents a revised version of a paper published in *Journal of Geographical Systems*, co-authored with Dr. Alan Murray.

points (also called generators), each location is associated with its closest member of the point generator set, where Euclidean distance is assumed. The association process produces a tessellation of the plane, called the ordinary Voronoi diagram (Okabe et al., 1992). Each tessellation then reflects that the demand in each Voronoi polygon is assigned to its closest facility (generator). Since the Voronoi diagram has proven useful and efficient in the allocation and partitioning of space, it has been widely adopted and relied upon for representing access, service assignments and trade areas. Okabe et al. (1992) list the many fields in which the Voronoi diagram has been applied, including astronomy, geography, zoology, and others. In geography, market and trade area analysis is a classic application domain. Voronoi diagrams have received continued and sustained attention, serving as the foundation of major contributions in many economic and urban analysis contexts. Beyond this, Voronoi diagrams have also proven effective for solving a class of continuous location optimization models, such as median, center and other location-allocation models (Okabe & Suzuki, 1997; Wei & Murray, 2006; Murray et al., 2008).

Although Voronoi diagrams have been the focus of much academic research and their utility is far ranging, their capability for dealing with spatial variability is limited. Market area analysis has been an application field of the Voronoi diagram, for which it is used to describe the allocation of customer demand for two or more competing centers. Various characteristics of centers could be represented as weighted generator points in the weighted Voronoi diagram (Shieh, 1985). However, heterogeneous costs or attributes away from generator points are not considered, yet may directly influence the attractiveness of centers to customers. For example, Figure 2.1 shows Walmart stores in a portion of the Seattle metropolitan area. Based on closest assignment, the trade areas for each store are derived using the Voronoi diagram, a commonly assumed and applied retail business analysis approach (see Bogue, 1949; Snyder, 1962; Dacey, 1965; Boots & South, 1997; Mendes & Themido, 2004; Cachon, 2014). This approach necessarily assumes that travel costs/distance are homogeneous. A problematic byproduct of such an assumption may be observed in Figure 2.1, as the indicated service area (red circle) is allocated to a store on the opposite side of Lake Washington, yet this would require travel over water or across a toll bridge. In reality, this demand area is actually better served by another store. So closest assignment assumed in the Voronoi diagram is not appropriate in this case. Accessibility for different stores, therefore, varies due to heterogeneity in neighboring land cover type, economy, time, and energy cost.

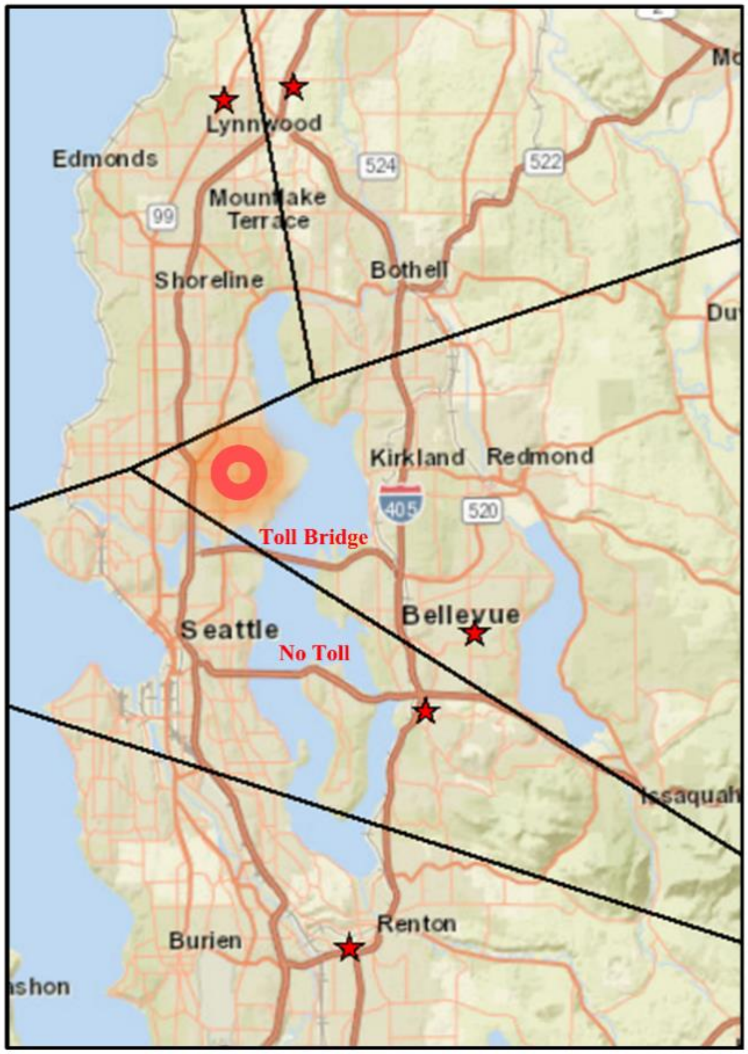


Figure 2.1 Voronoi diagram showing derived service areas of indicated stores

In many locational scenarios, closest assignment is very useful and necessary. Besides market area analysis, other location optimization problems have utilized closest assignment through Voronoi diagrams (Okabe et al., 1992; Novaes et al., 2009). The coverage/service areas of different kinds of facilities, like a fire station, EMS vehicle, hospital, transit station, post office, etc., have often been derived in terms of the closest distance. However, because of associated attributes and conditions, including direction, slope, winding, speed limit, road volume, lane width, tolls, etc., travel time and/or costs may vary (see Toregas et al., 1971; Singh et al., 1998). Distinctions along these lines, unfortunately, are not reflective of closest assignment, which is assumed in the Voronoi diagram.

Work in continuous space movement and behavior too often assumes spatial homogeneity. For example, homogeneity in allocation is imbedded in service/mission tasking in a marine environment using Voronoi diagram. Associated marine applications include environmental monitoring, surveillance, optimal pursuit of multiple targets, etc. (Gold & Condal, 1995; Bakolas & Tsiotras, 2010). Previous studies have demonstrated that ocean currents play an important role in time and energy associated with the movements of autonomous agents (Witt & Dunbabin, 2008; Dahl et al., 2011). Therefore, the spatiotemporal variability in ocean current magnitude and direction must be considered when deriving allocation schemes for autonomous agents, something traditionally done using a Voronoi diagram.

This chapter addresses allocation when heterogeneity is an issue. A new Voronoi diagram, the heterogeneous Voronoi diagram, is developed to solve allocation problems in continuous heterogeneous space. In what follows, I review relevant literature. Then, a mathematical modeling structure for constructing the heterogeneous Voronoi diagram is introduced. Following this, a case study concerning allocation in emergency drone delivery is detailed. Application results highlight the significance and utility of a heterogeneous Voronoi diagram, as well as demonstrate the computational feasibility of the proposed approach to support planning and decision making.

2.2 Background

Allocation optimization has not necessarily been of primary interest in location science. One reason for this is that demands are generally assumed to be assigned to their closest facilities. Closest assignment has been achieved in two ways in location planning approaches. In some cases, models have an implicit nearest allocation property. For example, demands are always assigned to the closest facilities in the p-median problem (see Hakimi, 1964; 1965; ReVelle & Swain, 1970) as the objective is to minimize the total weighted distance in the allocation of demand and facilities. Even though the model does not necessary impose that each demand to be assigned to its closest facility, this is the optimal strategy given the orientation of the objective. In this sense, properties have been established that exploit closest assignment, and accordingly this has led to efficient solution approaches. In other cases, however, closest assignment is embedded through some explicit construction in an optimization model, where closest assignment is required but does not automatically occur (Gerrard & Church, 1996). This means that special constraints must be added to a model to ensure closest assignment as approached in budget constrained median problem of Rojeski and Revelle (1970) and the weighted benefit maximal covering location problem Gerrard and Church (1996).

When attributes are homogeneous or implicit closest assignment works, allocation of demand is often an inconsequential part of the locational modeling process. With a pre-specified network, the characteristics of the linkages between demand and facility, such as distance, time costs, etc., can be easily obtained. Assignment based on closest criteria is achieved by comparing these characteristics of demands to each facility (Rojeski & ReVelle, 1970; Wagner & Falkson, 1975; Church & Cohon, 1976; Hanjoul & Peeters, 1987; Gerrard

& Church, 1996). Many additional issues in allocation problems, however, need to be addressed when there is no pre-specified network. A key question, therefore, is how to identify the optimal path between demand and a facility? A more generalized definition of "closeness" is necessary based on heterogeneous attributes of continuous space. With spatial variability in mind, a new method is needed to address more generalized allocation processes.

Dirichlet and Voronoi were among the first to suggest allocation of space based upon proximity to generator points. The initial concept involved sets of points regularly spaced in crystallography. The fields of meteorology (Thiessen, 1911) is also of interest. Voronoi regions were rediscovered at various times in physics, chemistry, and ecology, and other disciplines (Meijering, 1953; Wigner & Seitz, 1993). While the Voronoi diagram has been developed and applied extensively in the natural sciences, utilization in the social sciences has also been far ranging. Bogue (1949) used the Voronoi diagram to define market areas for US metropolitan centers. This work was extended to focus on individual retail stores (Snyder, 1962; Dacey, 1965). Geographers used Voronoi concepts in the analysis of 2-D point patterns (Boots, 1974) as well as different types of human territorial systems (Huff & Lutz, 1979). The Voronoi diagram and its extensions have played important roles in transportation (Novaes et al., 2009), path choice (Sharifzadeh & Shahabi, 2008), allocation of resources (Okabe et al., 2008), and regional analysis (Mu & Wang, 2006), among others.

The Voronoi diagram serves to allocate demand in space when the generator facilities are known. The objective of an allocation problem is to minimize the total distance from sited facilities to demands, often with the constraint that only one facility can be associated with each demand. Based on the definition of the Voronoi diagram, the generator would have the shortest distance to one demand point if the demand is within the corresponding Voronoi polygon. Each demand is allocated to its closest corresponding generator. The solution of the allocation problem is optimal when the distance from a sited facility to each demand is a global minimum.

Spatial heterogeneity is a significant and meaningful concept in the study of populations, communities, ecosystems, and landscapes (Shaver, 2005). It refers to the uneven distribution of an attribute within an area. Such attributes may be concentrations of plant or animal species (biological), terrain formations (geological), individuals (population), or environmental characteristics (e.g., rainfall, temperature, wind) and so on. In spatial econometrics, Anselin (1988) defined spatial heterogeneity as a lack of spatial uniformity associated with spatial dependence and/or relationships between variables under study (see also Anselin, 2013). With this definition, certain attributes, as well as the cause-and-effect relationship, are heterogeneous when there is a lack of spatial uniformity. An example is where the distribution of an ethnic population in most urban areas. Segregation in residential housing, population growth, and community dynamics, etc. are often observed (Pickett & Cadenasso, 1995). Spatial heterogeneity results because of the influence of self-selection and other dynamics on population distributions, with individuals opting to reside near friends, families and others of the same cultural background, values, mores, etc.

Spatial analysis is particularly dependent on the description of space. It has been well documented that how we represent spatial phenomena will influence analysis and findings. When a continuous surface is approximated, cumulative errors and uncertainty will inevitably be introduced into subsequent results (Goodchild, 1992; Yao & Murray, 2013; 2014). As errors/uncertainties are unavoidable, research devoted to error modeling and propagation continues (see Goodchild & Gopal, 1989; Heuvelink, 1998; Cressie & Wikle, 2015). Taking into account the errors/uncertainties inherent in spatial representation is essential, with methods structured accordingly (Yao & Murray, 2013; 2014).

There has been a limited capacity to account for spatial heterogeneity and error in the construction of a Voronoi diagram. Differing from the ordinary Voronoi diagram where generator objects importance is not differentiated, a weighted Voronoi diagram designates weights for generators to reflect varying significance (Boots, 1980). The varied properties of each generator could be the population of a neighborhood, the area of a shopping center, the storage capacity of a warehouse, the popularity of a restaurant, and so forth. The polygons associated with weighted Voronoi diagram are defined in terms of a distance modified by the generator weights. This extension has been meaningful in geographical terms and has provided a mechanism to influence allocation regions. However, a weighted Voronoi diagram only considers generator points. The so called "weighted distance" between any point on the plane and pre-specified generator is based on the varying significance of the generator instead of the space in between. Accordingly, the construction of Voronoi diagrams is based on an assumption of homogenous accessibility between generators in geographic contexts. Accounting for spatial heterogeneity in the derivation of a Voronoi diagram offers much potential to address more generalized allocation processes, but also provides opportunity to view and account for spatial error in important ways.

2.3 Methods

Consider a set of generators $g \in G$ in space S representing the service region. These may be any set of objects, such as points, lines, polygons, etc., possibly located in Euclidean space, $\mathbb{R}^2$. The distance, d_{pg}, between a point p and a generator g could be any metric, such as

Euclidean, rectilinear, etc. The Voronoi diagram is therefore defined as a set of polygons $\mathrm{V} = \{V_1, \ldots, V_{|G|}\}$, where polygon V_g is given by:

$$V_g = \{p \subseteq S \mid d_{pg} \leq d_{pg'}\ , \quad \forall g' \in G \ \&\ g \neq g'\} \tag{2.1}$$

A weighted Voronoi diagram expands the measurement of distance in different ways. One approach is through an additive weighted distance measure:

$$\hat{d}_{pg} = d_{pg} + \beta_g \tag{2.2}$$

A second is using a multiplicative weighted distance function:

$$\tilde{d}_{pg} = \alpha_g * d_{pg} \tag{2.3}$$

where α_g and β_g are weights associated with generator g. In the special case of $\beta_g = 0$ and $\alpha_g = 1$, the weighted Voronoi diagram is equivalent to the unweighted Voronoi diagram, (2.1).

An important issue is that weights in the weighted Voronoi diagram (α_g and β_g) are only related to generators. Addressing spatial heterogeneity of non-generator locations, therefore, requires extension of some sort. This is precisely the focus of this chapter. Specifically, the heterogeneous distance from an arbitrary point p to a generator g is as follows:

$$d_{pg}^H = \min_{c_{pg} \in \Omega_{pg}} \int_{c_{pg}} f(\overrightarrow{r_s})ds \tag{2.4}$$

The notation is as follows. Ω_{pg} is the set of all paths c_{pg} from g to p. A path c_{pg} is a specified piecewise continuous curve in the feasible domain U. $\overrightarrow{r_s}$ is a vector describing the instantaneous movement alongc_{pg}. $f(\ )$ accounts for attributes and movement, relating spatial accessibility and taking into account travel, traffic and other conditions. Accessibility is not only based on the current location along the curve, but also the direction of movement as one moves through a location. Accordingly, accessibility is better at some locations than other locations, but also accessibility is better in some directions than other directions. Although there are multiple continuous curves c_{pg} connecting p and g, the one that is minimal is consistent with Voronoi allocation. The heterogeneous Voronoi diagram is therefore defined as a set of polygons, $\mathrm{V}^H = \{\mathrm{V}_1^H, \ldots, \mathrm{V}_{|G|}^H\}$, where polygon V_g^H is given by:

$$V_g^H = \left\{p \subseteq U \mid d_{pg}^H \leq d_{pg'}^H, \quad \forall g' \in G \;\&\; g \neq g'\right\} \tag{2.5}$$

In computational geometry, much effort has focused on the development of techniques to derive a Voronoi diagram. Potential approaches are either vector-based or raster-based (Chen, 1999). Raster-based methods have been developed to be more efficient in forming a Voronoi diagram for spatial objects, especially line and area sets (Gold, 1992; Gold and Condal, 1995). Raster models are useful and efficient for storing and managing data that varies continuously in space while vector models are not.

In what follows, a raster-based specification of the heterogeneous Voronoi diagram is considered given clear computational advantages. Distance is a key concept in the generation of Voronoi diagrams (Li et al., 2004). Based on equation (2.4), heterogeneous distance can be defined in continuous space, but it remains to be shown how this may be accomplished in a raster context in order to derive V^H. Discretization of geographic space can be approached using the following notation:

i = index of spatial units (also g, k and j)

Φ_i= set of neighboring units where travel originating out of unit i is possible (arcs)

Ψ_i= set of neighboring units where travel going into unit i is possible (arcs)

δ_{ij}= heterogeneous cost to use directional arc from unit i to j

Spatial units, therefore, represent raster cells serving as an approximation of continuous space. There are various definitions of neighboring cells in a raster, including cells in the orthogonal directions, diagonal directions, directions two cells horizontally and one cell vertically, two cells vertically and one cell horizontally, etc. (Scaparra et al., 2014). δ_{ij} is a cost that may be calculated in advance, not only based on the attributes encountered in travel between neighboring units i and j but also taking into account the distance and direction from unit i to j. With this discrete representation of space, it is then possible to structure the heterogeneous allocation process. Consider the following allocation choice variable:

$$X_{kij} = \begin{cases} 1, & \text{if assignment of unit } k \text{ is based on use of arc from } i \text{ to } j \\ 0, & \text{otherwise} \end{cases}$$

The decision variable, X_{kij}, is to be determined whether arc from unit i to j is on the optimal trajectory to assign unit k. The discretized heterogeneous Voronoi diagram can formally be defined as the following optimization model:

$$Minimize \qquad \sum_{k} \sum_{i} \sum_{j \in \Phi_i} \delta_{ij} X_{kij} \tag{2.6}$$

$$\sum_{j \in \Phi_k} X_{kkj} = 1 \qquad \forall k \tag{2.7}$$

$$\sum_{g \in G} \sum_{i \in \Psi_g} X_{kig} = 1 \qquad \forall k \tag{2.8}$$

$$\sum_{j \in \Phi_i} X_{kij} - \sum_{j \in \psi_i} X_{kji} = 0 \qquad \forall k, i \ (i \neq k, i \notin G) \tag{2.9}$$

$$X_{kij} = \{0, 1\} \qquad \forall k, i, j \tag{2.10}$$

The objective, (2.6), seeks a minimum heterogeneous path from unit k to a generator g. Constraints (2.7) specify that a path is to originate out of each unit k. Constraints (2.8) indicate that a path originating at k must end at one generator. Constraints (2.9) are conservation of flow conditions, meaning that the amount of flow in and out of each unit should be the same. If an arc into a unit is used for assignment of unit k, then there must be another arc used coming out of the unit. Finally, constraint (2.10) indicate binary integer restrictions on decision variables.

Solution of this model could be accomplished in a number of ways. One may consider solving it using a commercial optimization package, such as Xpress, Cplex, or Gurobi. This is conceptually reasonable, but likely inefficient and time-consuming. A second approach is that the model can be divided into a set of sub-problems, focusing on finding the optimal allocation of each individual unit to all possible generators. Viewed as a sub-problem in this way, each can be solved separately and then combine to get a final solution or approximation of V^H. A third approach is to focus on each sub-problem and the associated heterogeneous distance between a pre-determined unit k to a specified generator g. Specifically, consider:

$$d_{kg}^{H} = Minimize \sum_{i} \sum_{j \in \Phi_i} \delta_{ij} X_{kij} \tag{2.11}$$

Deterministic dynamic programming offers potential for efficient solution. Dynamic programming is directly formed to estimate the shortest heterogeneous distance between unit k and any generator g. This is exactly what is needed for constructing the heterogeneous Voronoi diagram, V^H. The shortest heterogeneous distance value is initialized as zero for each generator and infinitely large for others. At the iteration n, updating of distance is as follows:

$$\min_{g\in G} d_{kg}^{H\ (n)} = \min\ \{\min_{g\in G} d_{kg}^{H\ (n-1)},\ \min_{g\in G} d_{\bar{k}g}^{H\ (n-1)} +\ d_{k\bar{k}}^{H}\} \qquad (2.12)$$

where $\bar{k} \in \psi_k$, and $d_{k\bar{k}}^{H}$ is equal to $\delta_{k\bar{k}}$, the heterogeneous cost to use directional arc from neighboring unit $\bar{k}$ to k. The above interactive function means that the shortest heterogeneous distance for k in iteration *n* is equal to the minimal value between the values for iteration *n-1*. The shortest heterogeneous distance values d_{kg}^{H} after all the iterations can be regarded as a sequence that is monotonically decreasing (based on equation 2.12) and bounded (greater or equal to zero). After a finite number of iterations, convergence is achieved based on monotone convergence theorem when. Specifically, this occurs when there is no change of d_{kg}^{H}. Formally, this reflects that the difference between heterogeneous Voronoi diagram $V^{H^{(n)}}$and $V^{H^{(n-1)}}$ is within some tolerance τ. Conceptually, this approach bears some resemblance to Dijkstra's algorithm and Floyd's algorithm (see Winston & Goldberg, 2004), which aim to find shortest paths from source to destination vertices in a given network using dynamic programming. Units in a raster could be represented as vertices in a network, and the network is built based on curves connecting neighboring units. When generators are regarded as several specified vertices in the network, equation (2.12) is used to calculate the minimal value of the shortest heterogeneous distances from a source vertex to these generator vertices.

A flowchart representing the above process for deriving a heterogeneous Voronoi diagram in real time planning and analysis is detailed in Figure 2.2. The process highlights the interaction of decision making, geographic information, and spatial analytics. Specifically, GIS provides the ability to capture, store, manipulate, analyze, and display all types of spatial/geographical data (Church and Murray 2009, Clarke 2011). What we focus on in this chapter is spatial heterogeneity in constructing the Voronoi diagram in order to support appropriate allocation. Therefore, detailed spatial information is required, including road network, travel patterns, population density, behavioral characteristics, boundary of restriction area, etc. Further, real time context knowledge, such as traffic conditions, weather, accidents, constructions, etc., are also essential since they may influence travel behavior in certain conditions. GIS facilitates creation, integration and management of these different kinds of data as well as transfer of attribute values. After specification of the heterogeneous Voronoi diagram model, the solution approach is detailed. This approach is based on iterative approximation using deterministic dynamic programming. Visualization and display of allocation is a straightforward task in GIS.

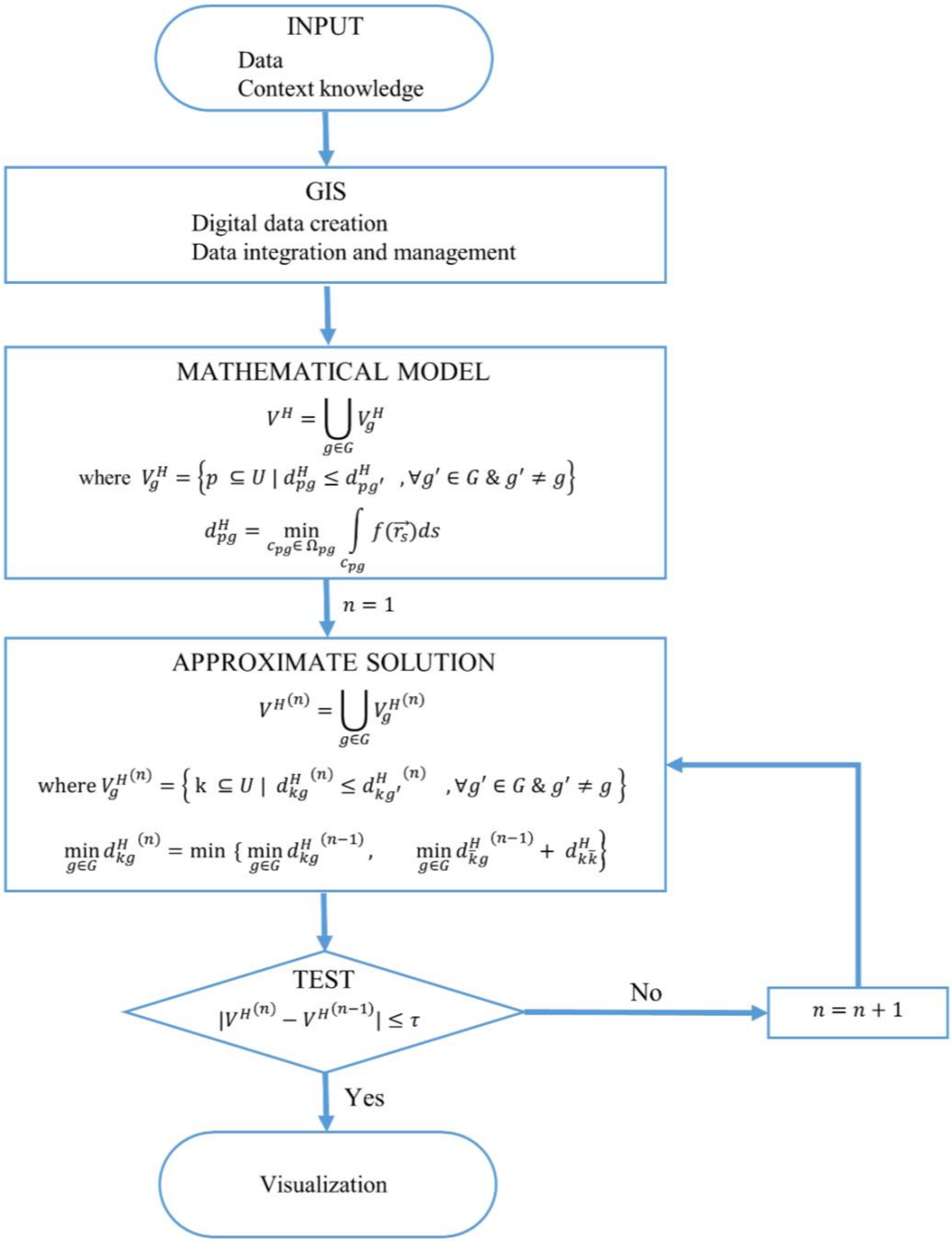


Figure 2.2 Modeling flowchart for deriving the heterogeneous Voronoi diagram

2.4 Case Study Context

Allocation involving heterogeneous demand is explored for drone delivery of EMS from hospitals. Unmanned Aerial Vehicle (UAV), often called a drone, has rapidly developed for commercial and personal use in recent years (Clarke, 2014; Finn & Wright, 2012). UAV applications have expanded from military operations to remote sensing and aerial imagery collection, scientific research, emergency response, recreational activities, etc. (Finn & Wright, 2012; Clarke, 2014). Drones for package delivery service have attracted much attention and have been deployed and tested by companies and public agencies (Hern, 2014; Welch, 2015). Using drones to respond to medical emergencies has appeared on the public horizon recently (Thiels et al., 2015; Pulver, Wei, & Mann, 2016). Drones, equipped with certain medical supplies, are planned for flying directly to the patient's vicinity, with bystanders provided directions for using the medical supplies on the patient (Communication, 2014). Drones have the potential to become a promising and effective transportation tool for emergency medical service delivery since most medical supplies and blood samples are small, light, valuable and time-sensitive; cargo easily delivered by small drones. GPS based technology can help to accurately navigate drones to the target, possibly supported by computer vision and machine learning algorithms (Lugo & Zeil, 2013). The key advantage in using drones for EMS is that they can travel faster than traditional ground based EMS vehicles, and therefore significantly reduce travel time in order to increase the survival chances of patients.

Emergency drone delivery exemplifies allocation issues in heterogeneous space because it reflects complications associated with aspects of perceived closest assignment. Medical supplies are usually stored at fixed hospitals/EMS stations, while potential patients are distributed continuously in space. Of course, the question is what the associated services areas of drone equipped hospitals/EMS stations should be - the allocation problem. Allocating medical supplies to patients could be carried out by constructing a (homogeneous) Voronoi diagram, V, detailed in equation (2.1) if straight-line based travel is appropriate.

For EMS, the most critical factor is response time, not distance. The shorter the response time, the higher the probability that a patient can survive. Assigning closest hospitals/EMS stations is based on an assumption that the shortest distance corresponds to the shortest response time. This is simply not true in general, as local factors and conditions may impact travel times in various ways. Drones very much reflect the situation of local impacts and conditions. According to the Federal Aviation Administration (FAA), there are many types of airspace restrictions that commonly affect drones; these includes airports (flight within five miles of an airport), restricted airspace (e.g., military bases), stadiums, sporting events, etc. These "no drone zones" may be regarded as obstacles (speed equal to zero) for drone delivery. Beyond this, the speed of a drone is not only determined by device configuration but also wind direction and wind speed. Imagine a drone's top air speed is 50 mph, the wind speed is 20 mph, and drone is flying directly into the wind. Therefore, the drone is flying at 50 mph within an enormous mass of air moving in the opposite direction at 20 mph. In this case, the drone's ground speed will be at most 30 mph. If you take the opposite scenario, where the drone is flying downwind, the drone's air speed remains 50 mph, while its speed over the ground could be as much 70 mph. Since wind direction and speed are varied in space and time, a drone's ground speed cannot be fixed. The spatial heterogeneity in a drone's ground speed necessitates accounting for local condition in any service allocation process, particular considering something as critical as EMS.

Our case study details planning for delivery of automated external defibrillator (AED) from certain fixed agencies to out-of-hospital patients who are suffering a cardiac arrest. Multiple studies have shown the significance of AED and its influence on survival of cardiac arrest, particularly during the first several minutes (Cummins et al., 1984; Caffrey et al., 2002; Dao et al., 2012). Allocation is important here because it can shorten AED delivery time and increase survival to the greatest extent possibility. The study area for this project includes Santa Barbara County, Ventura County, and parts of Los Angeles County, San Luis Obispo County, and Kern County. Associated data was obtained from several sources. The U.S. Geological Survey maintains the National Structures Dataset where data on Fire Station/EMS Stations, Hospital/Medical Centers, Ambulance Services, etc. is accessible for public consumption. Figure 2.3 indicates the 322 hospitals/EMS stations in the study area (those within drone no fly zones are not included). No drone zones are derived from the FAA's U.S. Air Space Map. A continuous raster-based wind map (Figure 2.4), with a one-kilometer spatial resolution from April 27th, 2004, was provided by the Climate Variations and Change lab at University of California, at Santa Barbara. The mean and standard deviation of the wind speed is 4.55 and 3.84 miles per hour, respectively. For the study area, there are 276*321= 88,596 spatial units, with approximately 16 neighbors for each unit ($|\Phi_i| = |\Psi_i| \approx 16$). Suitability of sites was evaluated for potential to provide AED, accessible area for a drone to fly through, and the varied maximal speed the drone can reach across space.

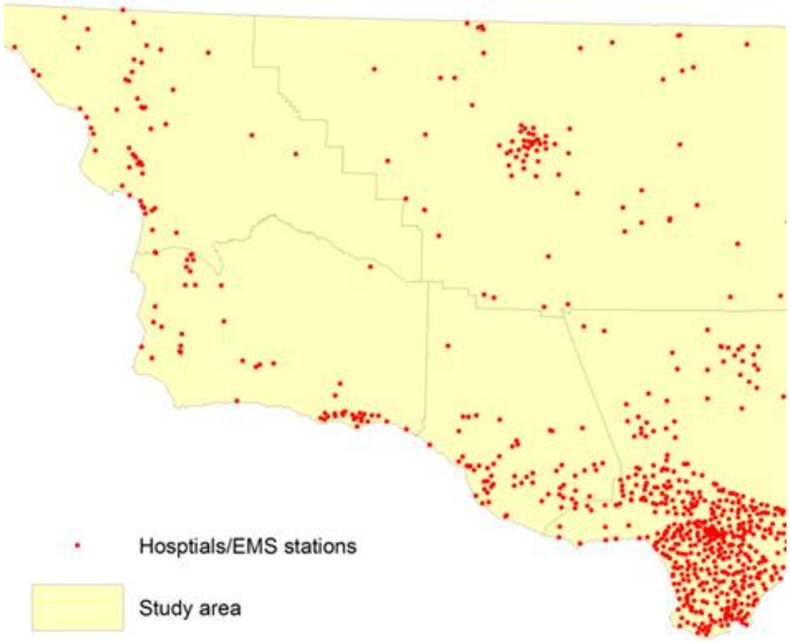


Figure 2.3 Hospital/EMS stations in the study region

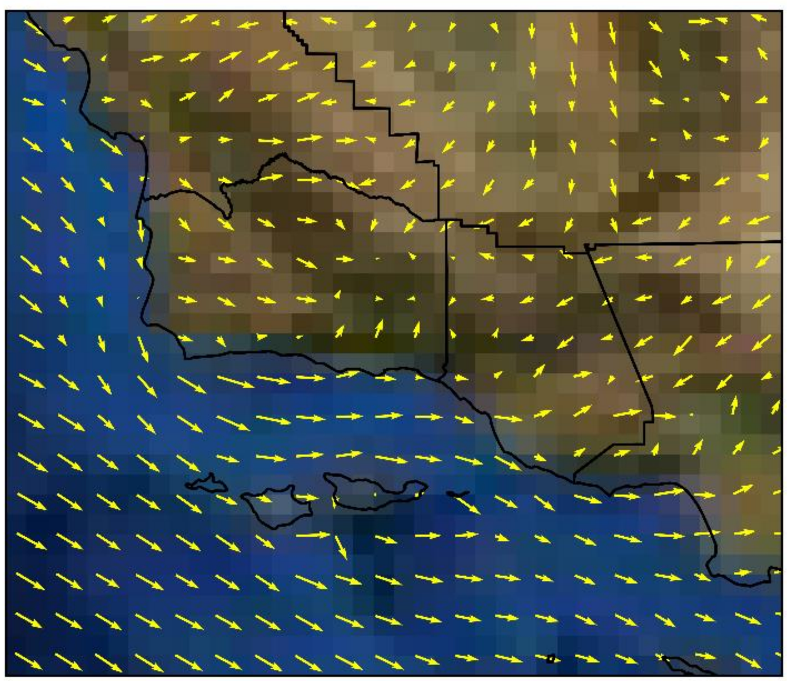


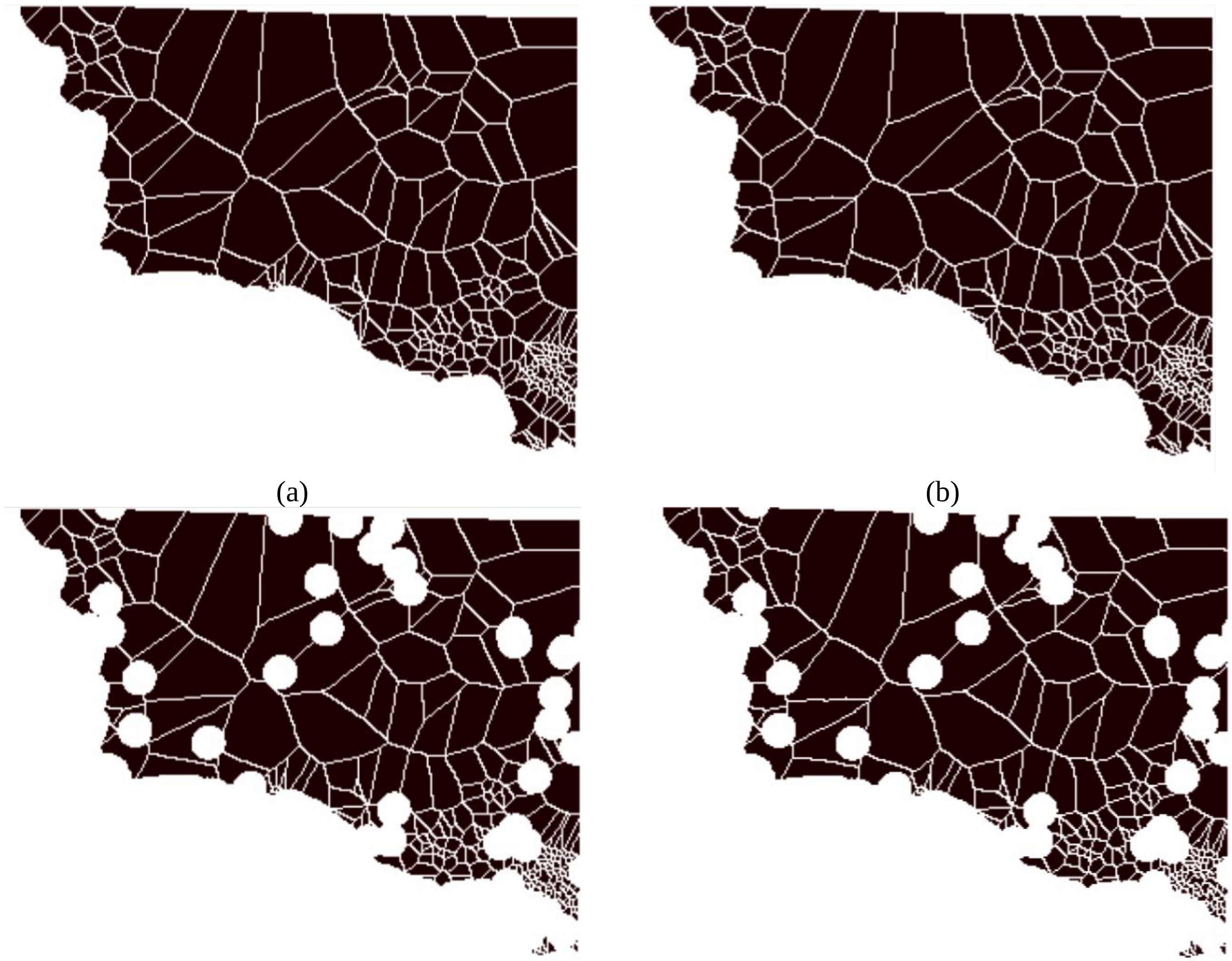
Figure 2.4 Wind map in the study area

Wind was represented as a two-dimensional vector since the impact of wind on UAV flight is most significant in the horizontal plane (see Selecky et al., 2013). A drone's air speed is defined as a fixed value, 50 miles/hour, in all directions, and it is recorded as a list of vectors for each unit. Ground speed is therefore vector summary of air speed and wind speed. Based on the relationship between ground speed and travel time, the time cost for flying from unit k to a neighboring unit i can be derived. The iterative algorithm, equation (2.12) and shown in Figure 2.2, is used to calculate the shortest travel time between each unit and all potential AED equipped drone staging sites, hospital/EMS facilities. We initialize the shortest time value as zero for each unit where a hospital/emergency center is located and an infinitely large value for all others. The models summarized in Figure 2.2 were implemented in MATLAB and run on an Intel(R) Core (TM) i5-4670K (3.40GHz) computer running Windows 7 Enterprise 64 bit with 8 GB of RAM. ArcGIS was utilized for data creation, management, manipulation, analysis and display. The derivation of V^H, the heterogeneous Voronoi diagram shown in Figure 2.5d, required approximately 20s processing time using proposed method in this chapter. The computing time is highly dependent on the level of granularity of wind raster, which is used to represent the heterogeneous local environment. If the study area is aggregated to 27*32=864 spatial units, the computing time shrinks to less than 1s. In addition to the approach proposed in the chapter, a commercial optimization package, Gurobipy, was used to solve the aggregated case study instance. Following (2.6) - (2.10), the optimization model has approximately 123,000 rows and 63 million columns. The package required around 15 minutes for problem input and initialization, and 140s to solve.

2.5 Application Results

Figure 2.5a shows the homogeneous Voronoi diagram derived by the traditional approach, while figure 2.5b depicts the heterogeneous Voronoi diagram, accounting for wind magnitude and direction. Figure 2.5c is the homogeneous Voronoi diagram where “no drone zones” are considered as obstacles and Figure 2.5d is the corresponding heterogeneous Voronoi diagram.

The four Voronoi diagrams, homogeneous or heterogeneous, look quite similar in many ways. However, there are important and significant differences. Figure 2.6a highlights the allocation changes (6.18% units in the study region) where different hospitals/EMS stations are assigned because of the consideration of wind magnitude and direction by comparing two different Voronoi diagrams in Figure 2.5d and 2.5c. Figure 2.6b depicts changes (0.61% units in the study region) when obstacle impact is accounted for comparing Figure 2.5d and 2.5b. Since wind and “no drone zones” are operationally essential, these highlighted areas are not appropriately allocated to hospitals/EMS stations (providing the shortest response time) in the homogeneous case (like Figure 2.5c and 2.5b). The inefficiency in allocation is caused by the unrealistic assumption of spatial homogeneity and failure to account for obstacles. The distributions of erroneous response time are presented in Figure 2.6c and 2.6d. The average time difference between accounting for wind or not is 54.16s. Compared to shortest response time, an addition of almost one minute will be spent on average for each unit in highlighted areas when wind is ignored. The average time difference for considering obstacles is 433.43s. Even though the proportion of obstacle impact highlighted areas is relatively small, their time variations are relatively large. In sum, critical response time could be saved by determining hospitals/EMS stations assingment using the Heterogneous Voronoi diagram, enabling both wind and obstacles to be considered.



(a) (b)

(c) (d)

Figure 2.5 Four types of Voronoi diagram

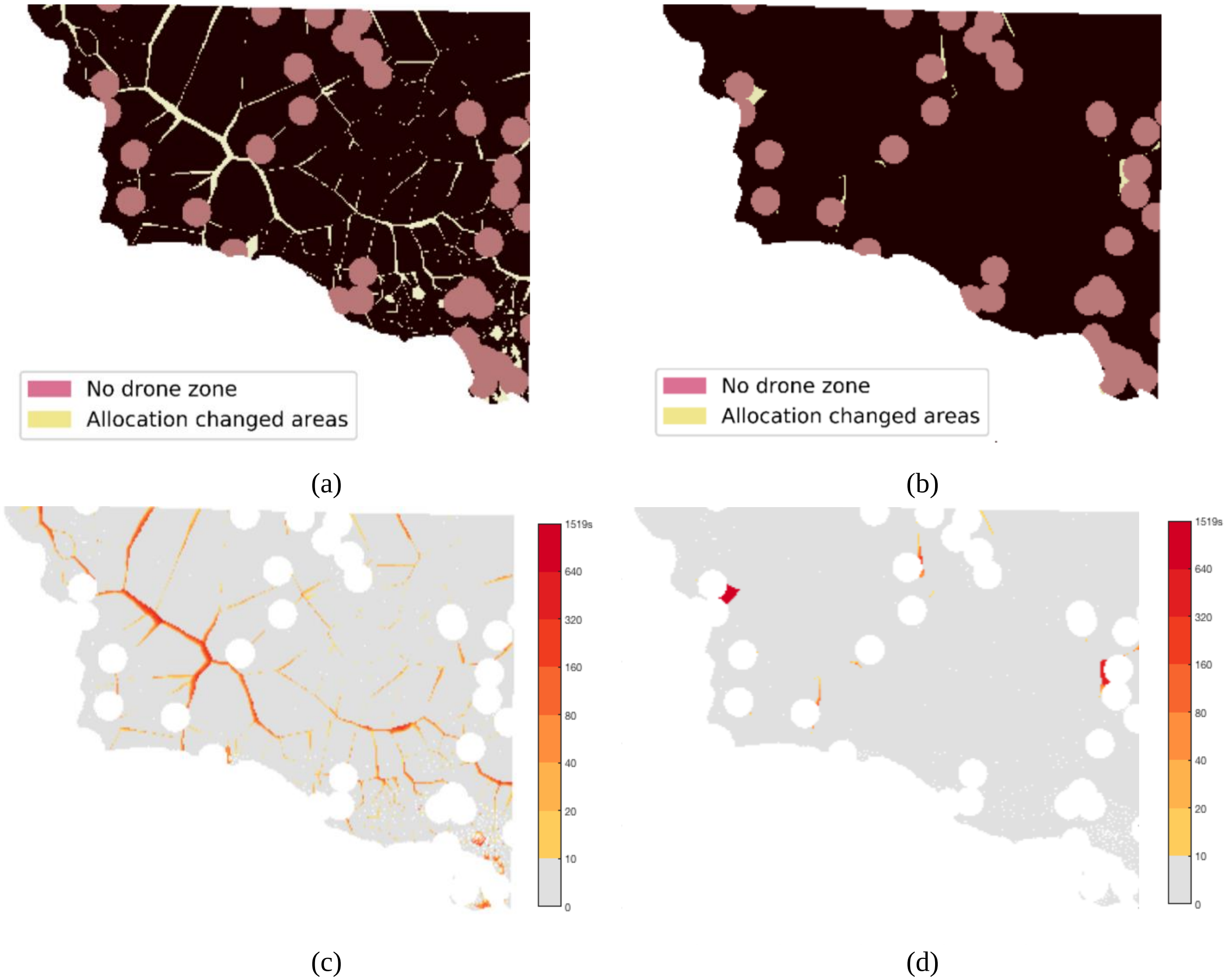
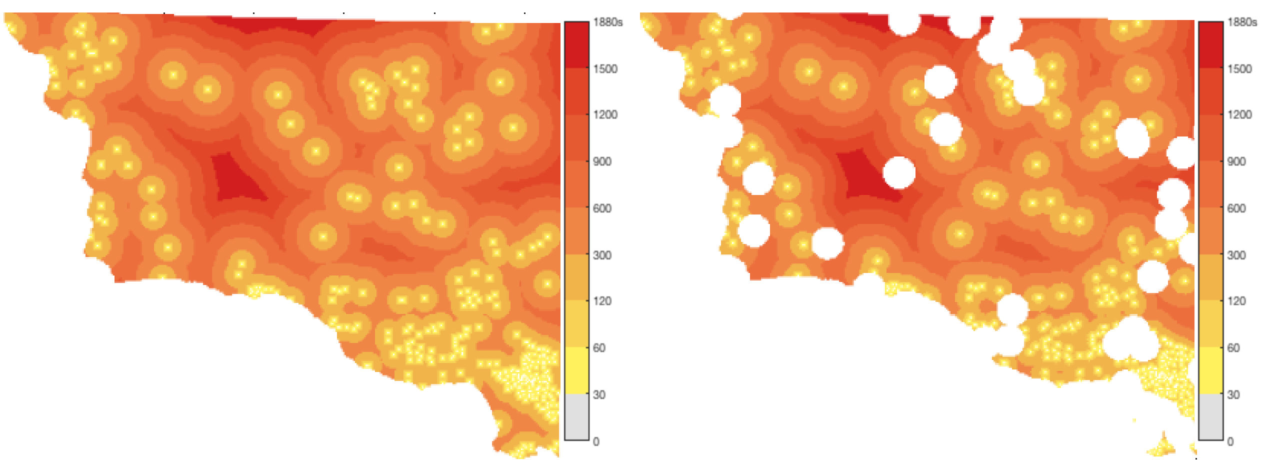


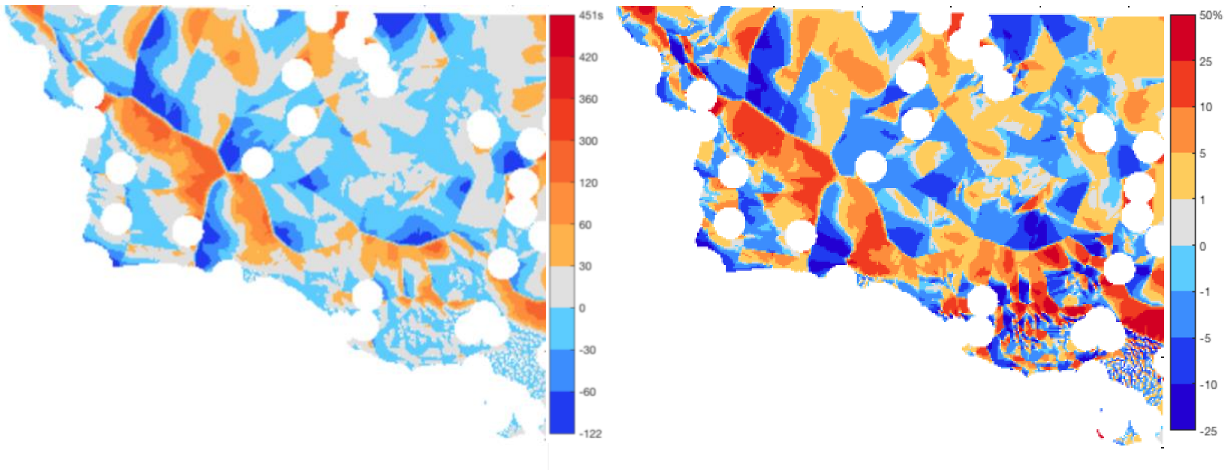
Figure 2.6 Allocation changed areas and their corresponding distribution of wasted response time

Response time is summarized in Figure 2.7 based on the spatial partition of the Homogeneous Voronoi diagram (Fig. 2.5a) and the Heterogeneous Voronoi diagram (Fig. 2.5d). The average response times for the two scenarios are 540.50 and 554.20s with a standard deviation of 369.68s and 376.10s. By comparing the response time for each pair of units in the two diagrams, the distribution of errors in allocation is summarized in Figure 2.8a. The two sets of response time are significantly different (pairwise t-test, $p<0.00001$). The mean and standard deviation of time difference are 27.51s and 37.57s, respectively. That is, an average of almost half a minute is over or under estimated by allocation using a homogeneous Voronoi diagram for each unit in the study region. Figure 2.8b presents the percentage distribution of over or under estimated response time, having a mean of 5.19% and a stand deviation of 6.13%. For some areas, which have a relatively small time difference, like the light blue and grey areas in Ventura County, the percentage of change is relatively high. One can therefore get a sense of the spatial bias attributed to not accounting for heterogeneity.



(a) (b)

Figure 2.7 The distribution of response time associated with the Homogeneous Voronoi diagram (no obstacles and obstacles included)



(a) (b)

Figure 2.8 The distribution of errors and their percentage in allocation for response time if spatial heterogeneity is considered

2.6 Conclusions

Spatial allocation is a fundamentally important process reflecting customer behavior, efficient service assignment, districting, etc., and is at the heart of many spatial analytical methods and processes. The Voronoi diagram has proven to be an important mathematical and geometric construct and has been widely applied in various fields because it is intuitive and efficient in the allocation and/or partitioning of space. However, existing Voronoi diagram approaches rely on the assumption that the attribute(s) of continuous space (non-generator points) is homogenous, which often is not the case for many application contexts. This chapter proposes the concept of a Heterogeneous Voronoi diagram, describes associated properties and develops a raster-based solution method to derive it for a general 2D bounded region. A drone emergency delivery case study was detailed, using the heterogeneous Voronoi diagram to identify the best allocation scheme. The results demonstrate the significance of spatial heterogeneity. Combining wind and "no drone zones", the Heterogenous Voronoi diagram can optimally assign demand to hospitals/EMS stations. Failure to accurately account for heterogeneity will result in significant over- and under-estimates. Unfortunately, errors in response time will result in loss of life in the case of EMS response.

2.7 Pseudo-code

Data: $I, G, \Psi_i, \delta_{ij}, \varepsilon$

Result: $\mathrm{min\_cost}_i$ and $\mathrm{allocation}_i$

Initializing minimal cost and allocation assignment for each unit

$n \leftarrow 0$

for $i \in I$ **do**

  $min\_cost_i^n \leftarrow \mathrm{INF}$

  $allocation_i^n \leftarrow NULL$

  if $i \in G$ **do**

    $min\_cost_i^n \leftarrow 0$

    $allocation_i^n \leftarrow i$

  end if

end for

Starting iteration and exiting if converge

while True do

  $n \leftarrow n + 1$

  for $i \in I$ **do**

    for $j \in \Psi_i$ **do**

if $min\_cost_i^{n-1} > min\_cost_j^{n-1} + \delta_{ji}$ **then**

$min\_cost_i^{n} \leftarrow min\_cost_j^{n-1} + \delta_{ji}$

$allocation_i^{n} \leftarrow allocation_j^{n-1}$

else

$min\_cost_i^{n} \leftarrow min\_cost_i^{n-1}$

$allocation_i^{n} \leftarrow allocation_i^{n-1}$

end if

end for

end for

if $\sum_i (min\_cost_i^{n} - min\_cost_i^{n-1})^2 < \varepsilon$ **then**

break

end if

end while

Chapter 3 Spatiotemporal Heterogeneous Allocation to Support Service Area Response[2]

3.1 Introduction

It has been well documented that how we represent spatial phenomena will inevitably influence the findings of spatial analytical methods (Miller and Wentz, 2003; Goodchild and Haining, 2004; Church and Murray, 2009). In geographic information systems (GIS), a field is one of the basic conceptual models of viewing geographic space (Longley et al. 2011), and considers a phenomenon continuously distributed across space. Two spatial representation approaches, vector and raster, have generally been used to reflect a continuous field. What is known is that the possibilities for a continuous field are infinite, yet representation in a computational environment is finite (Winter 1998, Cova and Goodchild 2002). The abstraction of geographic space therefore necessarily reduces the complexities of the real world to something more manageable while introducing unintended errors (Tong & Murray, 2009). Cumulative measurement and interpretation errors will inevitably be included into subsequent results if a continuous surface is approximated (Goodchild, 1992; Yao & Murray, 2013; 2014). In order to model processes better, analytical methods need to be improved through enhanced spatial description.

Spatial and temporal heterogeneity do exist and make a difference. Spatial heterogeneity has been an important and meaningful concept in research related to populations, communities, ecosystems, and landscapes (Shaver, 2005). Spatial heterogeneity is defined as

[2] This chapter represents a revised version of a paper submitted to *Computers and Geosciences*, co-authored with Dr. Alan Murray.

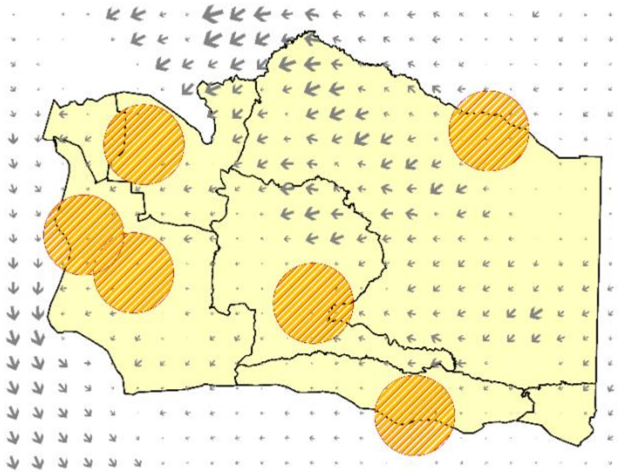
the uneven distribution of an attribute/variable. An example is the prevailing wind map shown in Figure 3.1. The magnitude and direction of wind changes locally in different time periods. Orange circles represent either fixed or temporal obstacles affecting travel accessibility of vehicles over space. Time is also fundamental for both human activities and physical processes. The work of Hägerstrand (1970) makes this abundantly clear. With the increase in availability of temporal data, such as remote sensing images, location based data, etc., more temporal analysis has been used to explore dynamic processes, including urban growth, territorial changes, migration, social interaction, disease diffusion, etc. (e.g. Herold et al., 2003; Plumejeaud et al, 2011; Wen et al, 2012; Davis et al, 2013; Wang et al, 2018). Modeling processes over space and time is essential, but methods are needed to explicitly account for spatial and temporal heterogeneity of spatial phenomena.

Allocation is the process of assigning the best facility (e.g., factory, store, warehouse, depot, etc.) to provide service to a demand area. It has proven important and useful in reflecting customer behavior, provider response, efficient service assignment, districting, etc. Thus, many spatial analytical methods embed allocation, either implicitly or explicitly. In location analytics, allocation has received limited interest and emphasis. One reason for this is that computation/derivation of assignment is often viewed as relatively easy and straightforward. This is because of default assumptions, such as straight-line distance and travel over a network. To ensure fast response and maximize the chance of saving lives, the nearest medical personnel are typically dispatched (Gerrard & Church, 1996), as an example. Shortest distance therefore reflects a proxy for "best" accessibility or "minimal" cost travel. However, this assumes spatiotemporal homogeneity in accessibility/cost. In many application and analysis contexts, accessibility is heterogeneous because of varying local impacts and conditions. Allocation therefore is more complicated when there is heterogeneity over space and/or time.

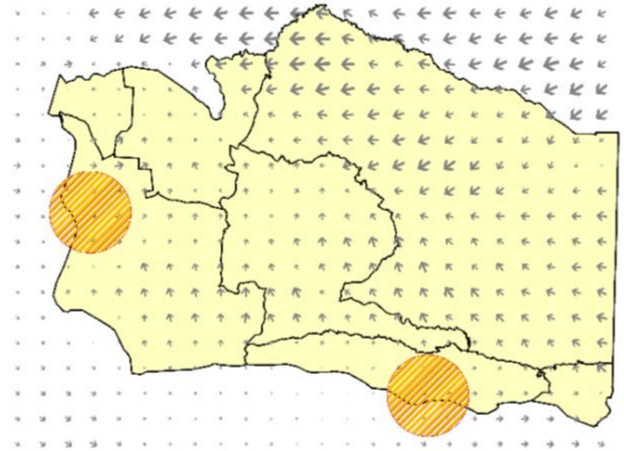
In this chapter, an approach is developed to derive the best allocation of demand to facilities over space and time. The contributions of the research are twofold. First, the allocation problem is formalized to account for spatiotemporal heterogeneity in accessibility. Second, a solution approach is devised to construct service areas that minimize assignment costs. In what follows, relevant literature is reviewed. Then, a mathematical model is structured. An algorithm is then detailed for solving this model. A case study follows concerning the allocation of emergency drone delivery service. Application results highlight the significance of considering spatiotemporal heterogeneity in accessibility as well as demonstrate the computational feasibility of the proposed approach to support planning and decision making.

3.2 Background

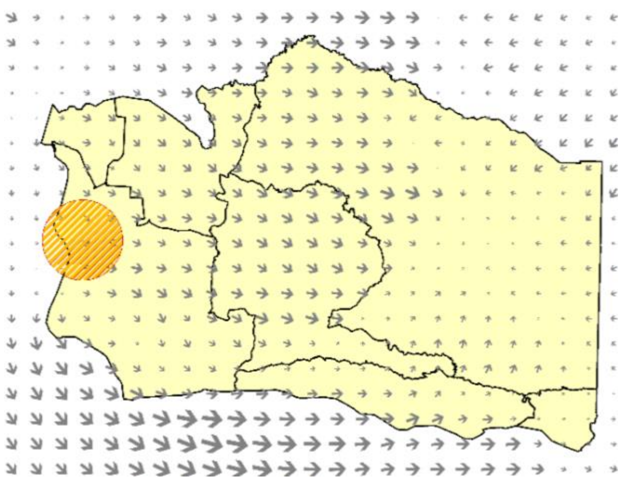
The effects of spatiotemporal heterogeneity are often considered as an extension of classic spatial analysis models. One example is that of regression where both time and space relationships are accounted for using a weights matrix (Huang et al., 2010; Fotheringham et al., 2015). In geostatistics—the spatiotemporal variogram and space-time kriging—take into account both spatial and temporal variation (Janis & Robeson, 2004). In spatial optimization, the p-median scheduling and location problem simultaneously determines optimal facility locations and schedules facility operations over time (Bloxham & Church, 1991). By accounting for temporal effects in these classic spatial analysis approaches, the time-varying nature of processes can be represented and analyzed.



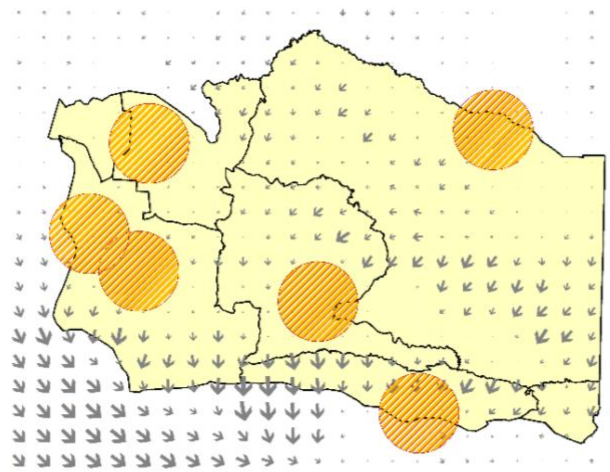
(a) 12 PM ~ 6 PM



(b) 6 PM ~ 12 AM



(c) 0 AM ~ 6 AM



(d) 6 AM ~ 12 PM

Figure 3.1 Varying spatiotemporal heterogeneity impacting travel accessibility

Unfortunately, extension of heterogeneous effects in allocation modeling has been limited to a pre-defined network. Computation/derivation of assignment for a network is relatively easy—simply sum up the costs along the segments of a route. These assignment costs by segment can be either fixed or time varying. Assignment cost may be the result of a variety of local conditions and attributes, including real-time traffic, speed limit, lane width, the slope and volume of the road, if/how the road winds/bends, etc. (see Toregas et al., 1971; Singh et al., 1998). For example, the toll fees for highways and bridges in some metropolitan cities, like Seattle, fluctuate based on traffic, usually reaching their peak during rush hour (Washington State Department of Transportation, 2019). The use of networks in allocation is understandable because most vehicles currently used for transportation travel over roads. However, for planes, helicopters, unmanned aircraft vehicles, watercraft (boats, ships, etc.), access is not restricted to a network. Rather, travel over continuous space has an infinite number of potential trajectories between demand and facilities. Calculating assignment cost and deriving an appropriate allocation becomes complex, especially when local conditions and attributes are heterogeneous over space and time.

The allocation problem has yet to be considered with respect to spatial and temporal heterogeneity for vehicles that are unrestricted in travel path. The rise and significance of Unmanned Aerial Vehicles (UAV), or drones, highlights problem with existing approaches and associated assumptions. Vehicles are increasingly relied upon for a range of service functions, including military operations, emergency response, environmental monitoring, scientific research, etc. (see Finn & Wright, 2012; Clarke, 2014; Pulver et al, 2018). Various airspace conditions affect the vehicle performance, and accessibility, including proximity of airports, stadiums, and other venues that are frequented by large crowds. Further, vehicle flight is also impacted by wind direction and speed. The movement of a vehicle plays an important role in determining its efficiency to accomplish a mission. In sum, it is essential to deal with spatiotemporal heterogeneity when modeling accessibility. Failure to do so will result in error and uncertainty in spatial representation, biasing findings and results.

3.3 Methods

In order to delineate continuous space for efficient allocation, methods to support this are needed. The Voronoi diagram, has been widely used for service allocation, but has strict assumptions. Given a set of service facilities $g \in G$ and a region S, the Voronoi diagram is defined as a set of polygons $\mathrm{V} = \{V_1, \ldots, V_{|G|}\}$, where polygon V_g associated with generator g contains all points $j \in S$ having the shortest distance d_{jg} to generator g than any other generator g'. Formally, this can be stated mathematically as follows:

$$V_g = \left\{ j \subseteq S \mid d_{jg} \leq d_{jg'}, \quad \forall g' \in G \,\&\, g \neq g' \right\} \tag{3.1}$$

where $\bigcup_g V_g = S$ and $V_g \cap V_{g'} = \emptyset$. The Voronoi diagram is a tessellation of a plane, reflecting demand assigned to its closest facility (generator). The major assumption is homogeneous accessibility across continuous space, represented as distance here, not accounting for spatial and temporal variability.

The family of existing weighted Voronoi diagrams expands the measurement of distance d_{jg} in several ways. However, the weights in the weighted Voronoi diagrams are only defined for facilities (generators) to reflect differing importance (e.g. Boots, 1980; Dong 2008; Okabe et al., 2009). As a result, spatial and temporal homogeneity accessibility between generators is assumed in constructing the weighted Voronoi diagram. What is missing is to be able to handle spatiotemporal heterogeneity of non-generator locations. This is the goal of this chapter.

Consider a set of facilities $g \in G$ again provide service to demand across continuous space S. Let c_{jg}^{p} be the assignment cost (or travel time) between a facility g and a point j in time period $p \in P = \{p_1, p_2, \dots, p_{|P|}\}$. The service area in time period p is therefore defined as the demand allocated to facility $g \in G$, as follows:

$$\tilde{V}_g^p = \left\{ j \subseteq S^p \;| c_{jg}^p \le c_{jg'}^p\,, \quad \forall g' \in G \;\&\; g \neq g' \right\} \tag{3.2}$$

Each service area associated with facility g, based on all points sharing the minimal assignment cost to facility g, varies over time. The entire space consists of the service areas for all facilities, and none overlap:

$$S^p = \bigcup_{g} \tilde{V}_g^p\,, \qquad \forall p \tag{3.3}$$

$$\tilde{V}_g^p \cap \tilde{V}_{g'}^p = \emptyset, \qquad \forall p, g, g' \in G \;\&\; g \neq g' \tag{3.4}$$

Without loss of generality, the assignment cost c_{jg}^{p}, which considers heterogeneous accessibility over space and time, is defined as the minimal travel time between a facility g and an arbitrary point j during time period p. Formally, this may be stated mathematically as:

$$c_{jg}^p = \min_{\boldsymbol{\theta}} \int_{t_0}^{t_n} 1 dt \tag{3.5}$$

where $g = (\,x(t_0),\, y(t_0)\,)$, $j = (\,x(t_n),\, y(t_n)\,)$, and $[t_0, t_n] \subseteq p$. In equation (3.5), $(\,x(t_0),\, y(t_0)\,)$ and $(\,x(t_n),\, y(t_n)\,)$ represent the positions of a vehicle traveling from facility g at time t_0 to a point j at time t_n. $\boldsymbol{\theta}$: $[t_0, t_n] \to [0,\ 2\pi)$ is a set of angles indicating the direction of motion of travel. The process of tracking $\boldsymbol{\theta}$ must account for local environment and conditions, including travel, traffic, weather, and others, which vary spatially and temporally. The set of angles indicates direction of movement to reach the target position in the minimum amount of time. They can be found using the following ordinary differential equation:

$$\frac{d\theta}{dt} = -\frac{\partial u^p(x,y)}{\partial y}\ cos^2\theta + \left(\frac{\partial u(x,y)}{\partial x} - \frac{\partial v^p(x,y)}{\partial y}\right) sin\theta cos\theta + \frac{\partial v^p(x,y)}{\partial x} sin^2\theta \qquad (3.6)$$

where $\left(u^p(x,y), v^p(x,y)\right)$ is a two-dimensional time varying vector field defined by a pair of coordinates (x, y) and time period p. Equation (3.6) is the well-known Euler-Lagrange equation, for the special case of calculating minimum time. This equation is the necessary condition for the solution of optimization problem (3.5) and can be derived using Pontryagin's minimum principle from the calculus of variations. Similar equations are used in Mahoney et al. (2012) in the context of modeling the dynamics/propagation of fluid flows, and in Rhoads et al. (2013) in the context of minimum time control of vessels in ocean currents.

Assume the vehicle traveling in field $\left(u^p(x,y), v^p(x,y)\right)$ with speed S will stay at its maximum speed in order to minimize travel time. Combined with θ found using equation (3.6), the movement of the vehicle can be formulated as follow:

$$\frac{dx}{dt} = u^p(x,y) + S * \cos\theta \qquad (3.7)$$

$$\frac{dy}{dt} = v^p(x,y) + S * \sin\theta \qquad (3.8)$$

Based on (3.7) and (3.8), the speed of the vehicle at location (x, y) is equal to the vector summary of the horizontal and vertical speeds of the vehicle plus the value of the local varying vector field.

In what follows, Figure 3.2 summarizes the general routine of a vector-based approach to solve for a spatiotemporal heterogeneous allocation. The detailed solution process used here is a marker particle method for front tracking. A large number of marker particles is generated using different head angles from each facility, ranging from [0, 2π). Of interest is the reachability front of these marker particles at each time interval. Along the front is the service area of certain facility, which could also be regarded as a set of all positions that can be reached or passed through at a given time t_n by a vehicle traveling from facility f at time t_0 in period p. The algorithm is as follows:

i) INPUT: provide the set of all the facilities G, the location coordinates of the facilities $(X_G(t_0), Y_G(t_0))$, the vector field $\left(u^p(x,y), v^p(x,y)\right)$, the time range of each iteration Δt, the boundary of study region B and the maximal travel time T_{max}. $I_g(T)$ is a set of indexes of marker particle for facility f at time T.

ii) INITIALIZATION: initialize the positions of the marker particles ($x_g^{(i)}(T)$, $y_g^{(i)}(T)$), their head angles $\theta_g^{(i)}(T)$, as well as the allocation surface $\hat{V}_g^p$ that records the facility index that each point in the study area belongs to.

iii) INTEGRATION: track the positions of the marker particles ($x_g^{(i)}(T)$, $y_g^{(i)}(T)$) and their head control $\theta_g^{(i)}(T)$ along the time in each iteration. The status of the marker particles ($x_g^{(i)}(T)$, $y_g^{(i)}(T)$, $\theta_g^{(i)}(T)$) are the integration of the Euler-Lagrange equation from time T till $T + \Delta t$.

iv) DECISION: pass the process when the travel time exceeds the maximal time T_{max}. Otherwise, go back to step iii) after updating $T = T + \Delta t$.

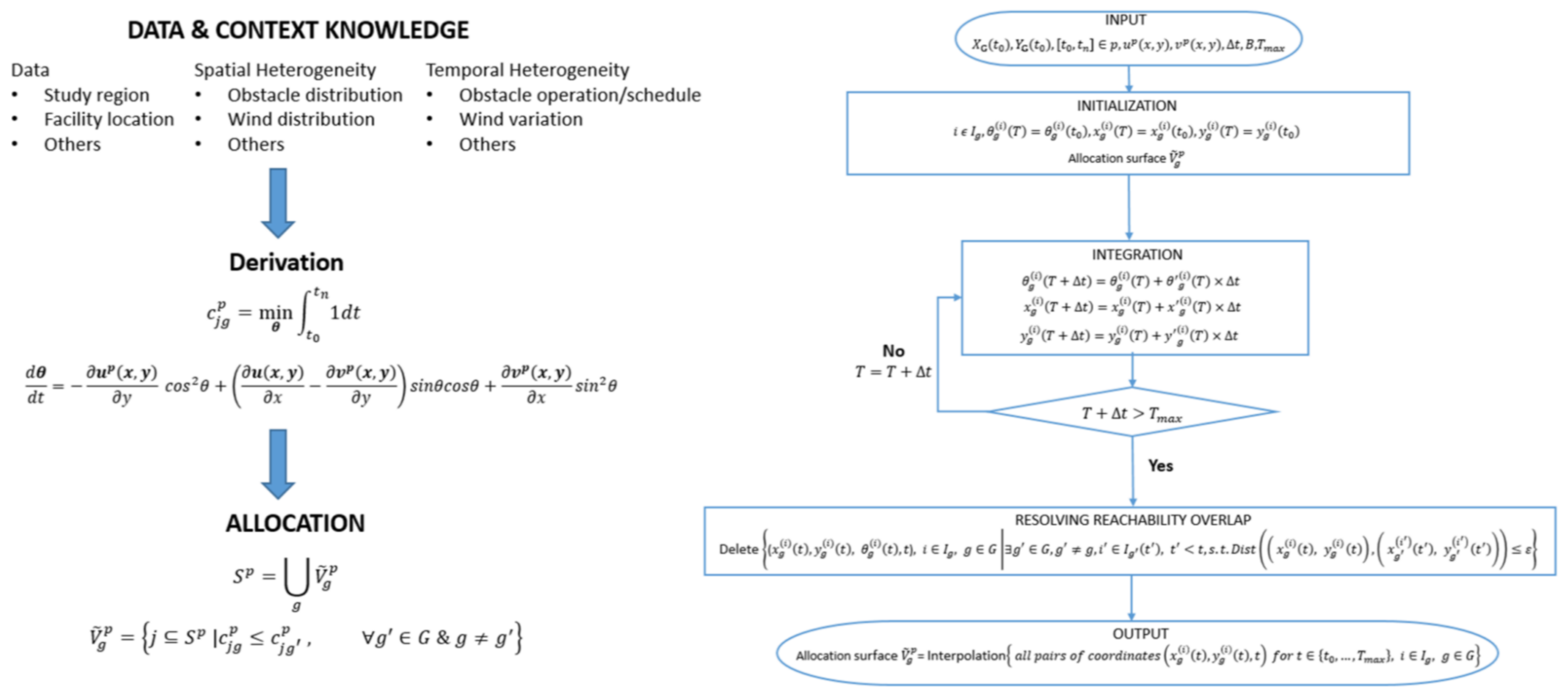


Figure 3.2 Modeling flowchart for deriving the allocation solution surface

v) RESOLUTION: trim away markers and their associated segments when they are out of bounds or sufficiently close to other markers. To be more specific, a marker is deleted if it travels to a location where another marker—regardless of whether it originated from the same facility—has already arrived or is within a tolerable distance. We only recorded the facility index from which the marker particle travels in the shortest arrival time.

vi) OUTPUT: update the allocation assignment $\widehat{\mathrm{V}}_{\mathrm{g}}^{\mathrm{p}}$ based on the interpolation result of markers. Interpolation methods can include Inverse distance weighted (IDW) interpolation, Nearest-neighbor interpolation, Delaunay triangulation, etc.

Figure 3.3 depicts one-facility case resulting from these steps. The heading angle of the vehicle θ is determined by equation (3.6) and then its corresponding location ($x(t)$, $y(t)$) at time t is acquired by equations (3.7) and (3.8). After repeating the process by increasing t by Δt, the moving path of the vehicle—from the given origin with the initial moving angle to the farthest location in a certain time—is identified. Assuming there are multiple vehicles moving from the same origin at different initial angles, the reachability surface can be estimated.

3.4 Case Study

Emergency drone delivery is an increasingly important service and involves allocation. It is assumed that medical supplies are stored at fixed stations and patients, who are in need of these supplies, are dispersed across continuous space. The solution to the allocation problem answers the question of which areas should be best served by which drone-equipped stations in order to ensure the most rapid response possible.

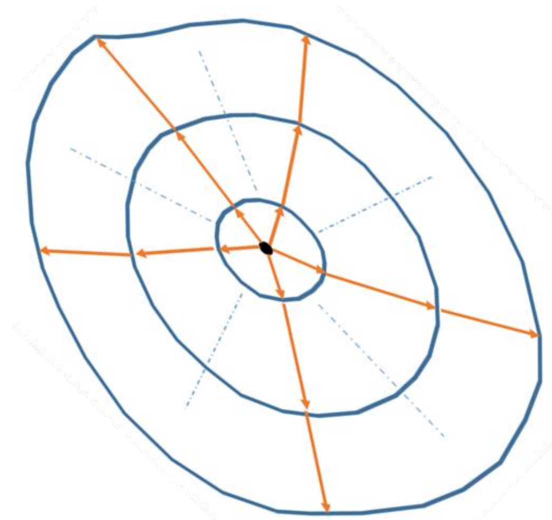


Figure 3.3 Visualization of deriving a reachability surface

As previously stated, both airspace restrictions and wind affect spatiotemporal accessibility for drone delivery. “No drone zones” are essentially obstacles, but other restrictions could be temporary/negotiable. Flying drones in and around stadiums or other large-scale event venues is typically prohibited during events. Without specific air traffic permission and coordination, flying drones recreationally within five miles of an airport is forbidden. However, EMS drone operators may be granted permission to fly within the five-mile radius if the nearby airport has a tower and an air traffic controller to assist. Whether the EMS drones are permitted to fly through “no drone zones”, however, is dependent on the time of the flight. “No drone zones” as fixed obstacles may not be an appropriate way to represent airspace in the process of allocating emergency drone delivery. In addition, wind direction and wind speed affect the flight of a drone. The top air speed of a drone is fixed, while its ground speed is the vector sum of the air speed and wind speed. Since wind speed varies over space and time, the ground speed of a drone is not homogeneous in all directions.

The case study involves automated external defibrillator (AED) delivery from certain fixed agencies to out-of-hospital patients experiencing cardiac arrest. Previous research has shown that the use of AEDs, especially during the first several minutes of cardiac arrest, yields higher survival rates (Cummins et al., 1984; Caffrey et al., 2002; Pulver et al, 2016; 2018). Spatiotemporal heterogeneous allocation is utilized to ensure rapid AED delivery time to support this. The study area is Santa Barbara County shown in Figure 3.4. Associated data was obtained from several sources. The U.S. Geological Survey maintains the National Structures Dataset where data on Fire/Police Stations, Hospital/Medical Centers, Ambulance Services, etc. is accessible for public consumption. "No drone zones" are derived from the FAA's U.S. Air Space Map. Figure 3.4 indicates five airports (Santa Maria Airport, Lompoc Airport, Santa Ynez Airport, Santa Barbara Airport and New Cuyama Airport), one military area (Vandenberg Air Force Base) and thirty-two stations (fire, EMS, national conservation area visitor centers, etc.) in the study area. The area within five miles of Vandenberg Air Force Base is regarded as a fixed obstacle ("No drone zone") for travel, while the associated buffer areas for airports are temporary obstacles depending on time of travel. The Santa Barbara airport operates from 6 am to midnight and the other four airports from 6 am to 6 pm. If a station is within five miles of an airport or the Air Force base during these periods, it is unable to dispatch.

Raster-based wind maps (shown in Figure 3.1), with a one-kilometer spatial resolution and six-hour time resolution from April 26-27, 2004, were acquired. The mean wind speed is 3.02 mph, 3.49 mph, 4.42 mph, and 2.9 mph in four time periods, respectively. Generally, the wind blows first from east to west, then from northwest to southeast, and lastly from northeast to southwest. Wind is represented as a two-dimensional vector since the impact of wind on UAV flight is most significant in the horizontal plane (see Selecky et al., 2013).

The air speed of a drone is defined as a fixed value of 50 mph in all directions. The marker particle-based front tracking method was used to calculate the shortest travel time between each point and potential AED-equipped drone staging stations. The location and head angles of marker particles were initialized using the method detailed previously. The model summarized in Figure 3.2 was implemented in MATLAB and run on an Intel (R) Core (TM) i5 (1.8GHz). ArcGIS was utilized for data creation, management, manipulation, analysis, and display. The derived allocation solution was summarized in Figure 3.5 and required approximately twenty minutes processing time.

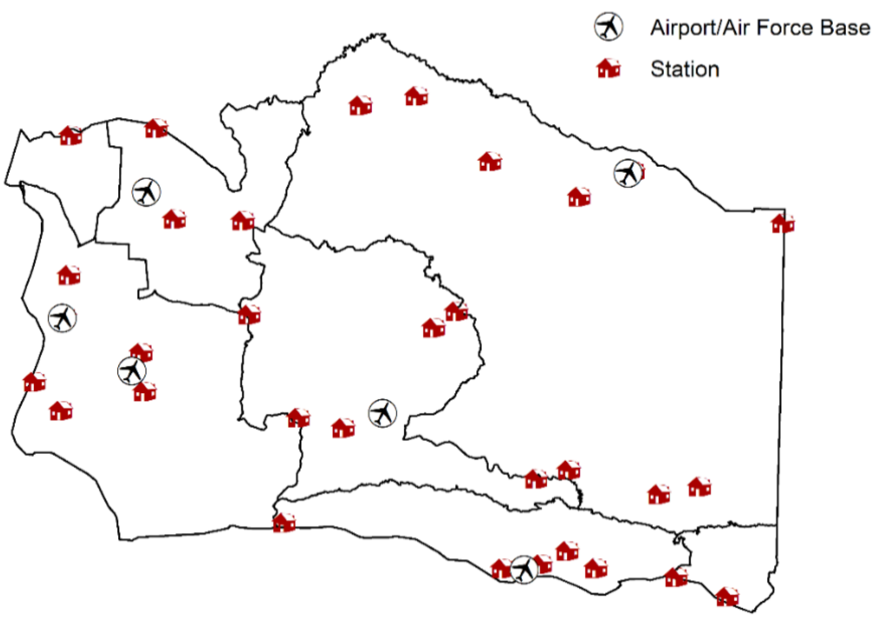
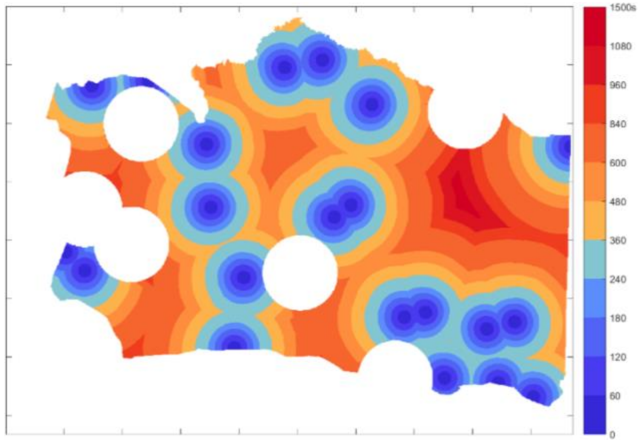
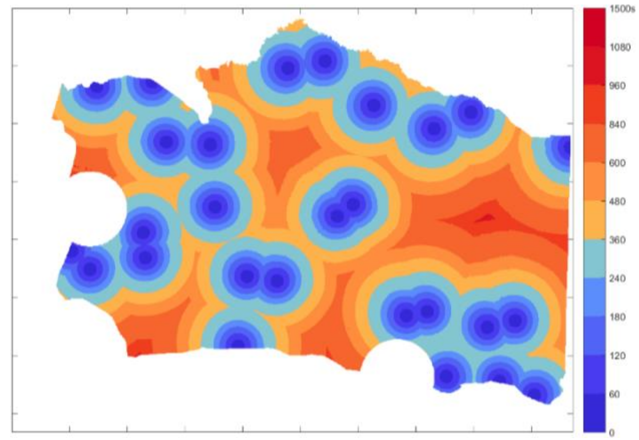


Figure 3.4 Study area – Santa Barbara County

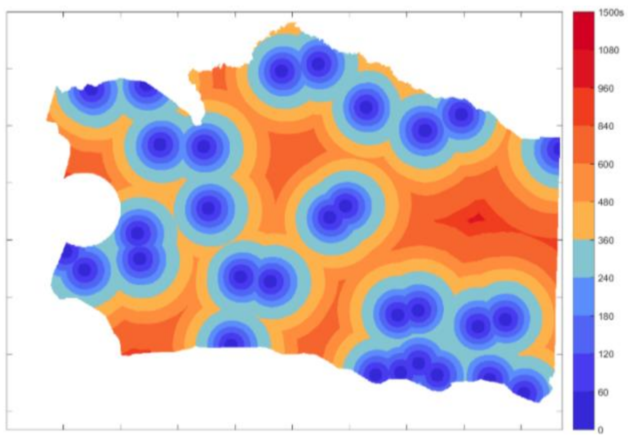
The four sections in Figure 3.5 depict the response time in four time periods for multiple stations in our study area. There are important differences between them, since the distribution of both obstacles and wind varies spatially and temporally. For the four scenarios, the total reachable areas make up 84.68%, 95.73%, 97.51%, and 84.68% of the entire study region. The average response times are 428.03, 368.31, 357.14 and 431.85 seconds with a standard deviation of 238.27, 199.42, 194.27 and 239.77 seconds. The maximum response times are 1146.13, 1063.32, 993.00, and 1149.71 seconds, respectively. The average response time is relatively higher during daytime periods (6 AM ~ 12 PM and 12 PM ~ 6 PM) than nighttime periods due to larger obstacle areas and fewer accessible stations. The four sets of response times vary for points at the same location. For example, a random selection of two hundred points in the study area confirmed a mean absolute difference in response time for Figures 3.5a and 3.5b of 62.48 seconds. That is, an average of one minute for response is due to the time varying local environment. The differences in response time increase if there is a drastic change in local reachability, such as the wind becomes stronger or the restricted areas become temporarily accessible.



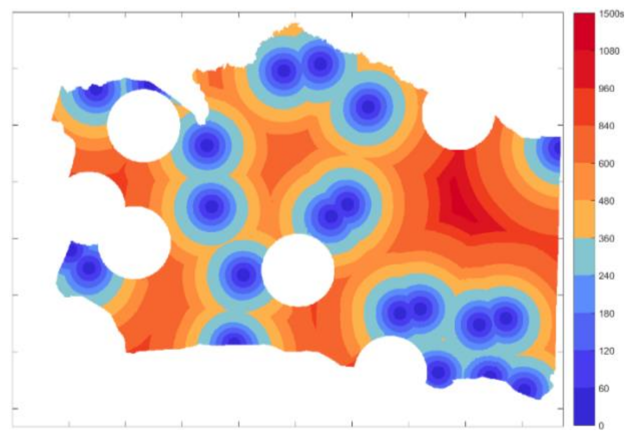
(a) 12 PM ~ 6 PM



(b) 6 PM ~ 12 AM



(c) 0 AM ~ 6 AM

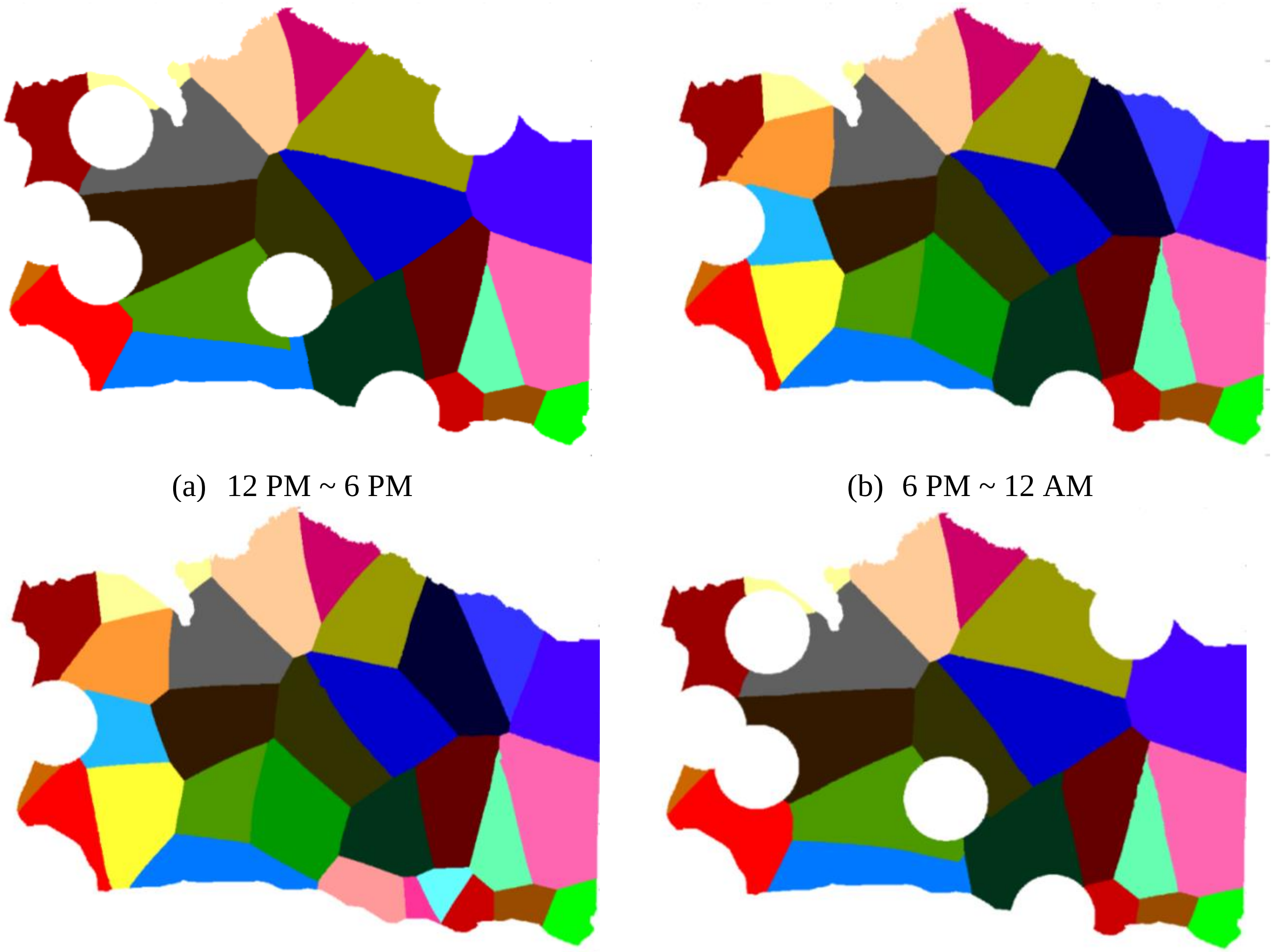


(d) 6 AM ~ 12 PM

Figure 3.5 Reachability surfaces in four time periods

Figure 3.6 depicts the allocation results, with total allocation faces of 21, 27, 30 and 21 (corresponding to the same amount of working stations) for the four time periods. The faces with same color in the four subfigures represent the areas served by the same drone-equipped station. Considering the 21 stations that are available in the entire 24 hours, their service areas change significantly over time. Using the service areas in Figure 3.6d as a baseline, the 21 stations' median percentages of service area changes in the other three time periods for the twenty-one stations are 4.4%, 11.5%, and 16.3%. The maximum percentages of service area changes are 13.6%, 42.6%, and 42.3%.

Figure 3.7 attempts to address the question of why temporal heterogeneity is important in the allocation process. The results in Figure 3.7 are derived based on the assumption that the allocation surface in Figure 3.6d is fixed to be used in the early morning from 12am to 6am. Figure 3.7a highlights the allocation changes—18.72% of the entire study region—where different stations are assigned by comparing the allocation surfaces in Figure 3.6c and 3.6d. Since wind and "no drone zones" are operationally essential and do change over time, these highlighted areas are not allocated to stations (providing the shortest response time) assigned in Figure 3.6d. This inefficiency in allocation is caused by the unrealistic assumptions of temporal homogeneity about local accessibility conditions. The distribution of response time savings is presented in Figure 3.7b. The average time difference (flight time saving) is 184.34 seconds with a maximum time difference of 1,014.40 seconds. Compared to the shortest response time, an additional three minutes is spent on average in highlighted areas when temporal heterogeneity is ignored. In sum, critical response time is saved by determining the allocation of drone delivery that considers spatial and temporal heterogeneity.



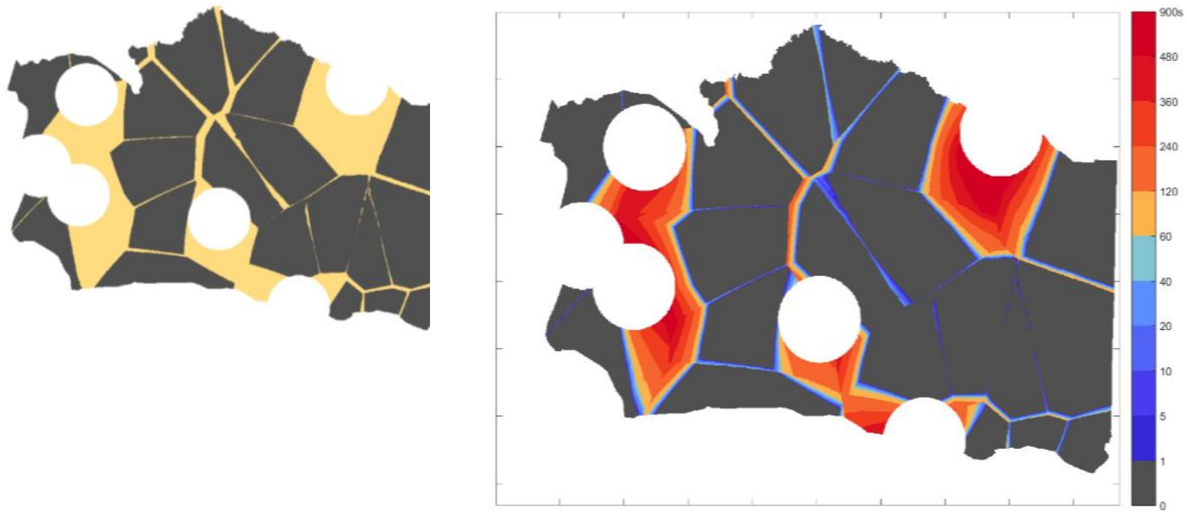
(a) 12 PM ~ 6 PM

(b) 6 PM ~ 12 AM

(c) 0 AM ~ 6 AM

(d) 6 AM ~ 12 PM

Figure 3.6 Allocation surface in four time periods



(a) (b)

Figure 3.7 Areas allocated to different stations and associated response time difference for allocation changed areas

3.5 Conclusions

Uncertainty and error are inevitable when continuously varying geographic phenomena is approximated. Describing geography and spatial relationships is therefore challenging using analytics for planning and management. Even though spatial and temporal heterogeneity are part of assignment processes formalized in allocation problems, previous studies often assume a pre-specified road network. How to structure and solve an allocation process is particularly challenging when heterogeneity must be taken into account across continuous space and through time. In this chapter, a vector-based solution method is developed to construct service areas in order to minimize assignment cost in a continuous region where accessibility is spatially and temporally heterogeneous. Application findings are reported for planning problems involving emergency drone delivery. Results show that response time (or system costs) can be saved by taking into account time varying local environments, including wind and travel obstacles. Incorporating spatial and temporal heterogeneity in continuous space allocation processes is useful and significant.

3.6 Pseudo-code

Data: $G, I_g, X_G(t_0), Y_G(t_0), u^p(x,y), v^p(x,y), \Delta t, B, T_{max}$

Result: allocation assignment $\widehat{\mathrm{V}}_{\mathrm{g}}^{\mathrm{p}}$

Initializing the positions of the marker particles and their head angles

for $g \in G$ **do**

for $i \in I_g$ **do**

$x_g^{(i)}(T) \leftarrow x_g^{(i)}(t_0)$

$y_g^{(i)}(T) \leftarrow y_g^{(i)}(t_0)$

$\theta_g^{(i)}(T) \leftarrow \theta_g^{(i)}(t_0)$

end for

end for

Generate an initial allocation surface $\hat{V}_g^p$

Integration

while $(T \leq T_{max})$ **do**

for $g \in G$ **do**

for $i \in I_g$ **do**

$x_g^{(i)}(T+\Delta t) \leftarrow x_g^{(i)}(T) + {x'}_g^{(i)}(T) \times \Delta t$

$y_g^{(i)}(T+\Delta t) \leftarrow y_g^{(i)}(T) + {y'}_g^{(i)}(T) \times \Delta t$

$\theta_g^{(i)}(T+\Delta t) \leftarrow \theta_g^{(i)}(T) + {\theta'}_g^{(i)}(T) \times \Delta t$

end for

end for

$T \leftarrow T + \Delta t$

end while

Resolving reachability overlap

for $(t = t_0,\ t \leq T_{max},\ \Delta t++)$ **do**

for $g \in G$ **do**

for $i \in I_g$ **do**

if marker $\left(x_g^{(i)}(t), y_g^{(i)}(t)\right)$ has not been deleted **then**

create a circle centered at $\left(x_g^{(i)}(t), y_g^{(i)}(t)\right)$ with radius ε

for each marker $\left(x_{g'}^{(i')}(t'),\ y_{g'}^{(i')}(t')\right)$ in the circle **do**

if $(t' > t)$ and $(g' \neq g)$ **then**

delete marker $\left(x_{g'}^{(i')}(t'),\ y_{g'}^{(i')}(t')\right)$

end if

end for

end if

end for

end for

end for

Generate final allocation assignment $\widehat{\mathrm{V}}_{\mathrm{g}}^{\mathrm{p}}$ by interpolating all pairs of coordinates $\left(x_g^{(i)}(t), y_g^{(i)}(t)\right)$

Chapter 4 Medical Drone Service Response: Spatiotemporal Heterogeneity Implications[3]

4.1 Introduction

A location problem reflects an abstraction of a number of factors that must be taken into account in the design of a service system. Major components of a location problem include identifying the best sites for service facilities and the associated routing and/or service allocation (Church & Murray, 2009). Examples of service systems include stores, banks, restaurants, telecommunication, entertainment and many others. Additionally, public service systems too are vital, including education, waste processing, emergency response, mail, etc. Location models are often key to ensuring efficient investment and operation, when used in a prescriptive manner (Murray, 2010). The operational efficiency of these systems directly results from the geographical arrangement of a given set of facilities, the manner in which their services are provided, and the spatial distribution of demand. Location analysis and modeling to support this therefore plays an important role for not only ensuring that a given system is sustainable, but also can be the difference between a successful service venture and a failure.

The oft spoken mantra "location, location, location" highlights the significance of siting decisions. Allocation, or service assignment, on the other hand is often relegated to be an afterthought, perceived to be the byproduct of the siting selection. The reason for this is that

[3] This chapter represents a revised version of a paper submitted to *International Journal of Geographical Information Science*, co-authored with Dr. Alan Murray and Dr. Richard Church.

for many location-allocation problems the best service assignment strategy is simply the closest or least cost option once good siting selections have been made. This may well be true for a given problem context, provided that an assumption of spatial and temporal homogeneity holds for proximity and/or travel cost. This implies, however, that service assignment distance or cost is not impacted by instantaneous travel movement changes. Causes for such change may be attributed to varying weather conditions, such as temperature, wind speed and direction, as well as the impact of topography, including slope and tunneling effects of buildings and valleys. Further complicating matters is that demand for service can also vary over time and space because of daily routines in human movement. This means that there is heterogeneity in both demand for service as well as the assignment or allocation of service based upon the fastest or least energy route from one or more dispatching locations, all of which confounds processes of allocation. If allocation is complicated, then the location component is complex as well because it is dependent on simultaneously making the best allocations possible.

In what ways does this matter? An emerging issue that has been encountered with an unmanned aerial vehicle (UAV), also called a drone, is the problem of heterogeneity in demand and travel time over space and time. The use and reliance on UAVs has expanded significantly from their initial support of military operations. Remote sensing (including LiDAR), aerial imagery collection, goods delivery, surveillance, etc. applications abound (Finn & Wright, 2012; Clarke, 2014; Hern, 2014; Welch, 2015; Feng & Murray, 2018). An ever more important usage of UAVs involves the delivery of medical services, where they are dispatched with small, light, valuable and time-sensitive supplies (Communication 2014; Thiels et al. 2015; Pulver et al. 2016). Businesses are developing and testing the use of drones for package delivery with the objective of decreasing unit costs in shipping small packages. While the typical package delivery system is not designed for immediate usage of items, medical service must be rapid. For example, Pulver et al. (2016) detail the use of drones for medical aid, and in particular the ability to equip a UAV with an automated external defibrillator (AED) in order to respond to cases of cardiac arrest. In the case of cardiac arrest, response time is vital, with evidence indicating that there is a greater likelihood of survival when response times can be reduced (Cummins et al., 1984; Caffrey et al., 2002). This means that drones with defibrillators need to be located so that patients can be reached within a desired response time. This is not a simple problem as we cannot assume homogeneity in either temporal demand or spatial accessibility. Disregarding temporal variation or assuming a fixed distribution of patients can result in significant decision making errors. Further, Feng and Murray (2018) demonstrated that a spatial homogeneity assumption is problematic for service allocation, especially when drone delivery is considered. Airspace restrictions and local environmental conditions, such as wind direction and speed, can significantly affect travel accessibility and overall response time.

This chapter mathematically formalizes a location problem to address spatiotemporal heterogeneity across a continuous terrain surface. The next section provides a review of location-allocation problems and UAV service systems. This is followed by extensions of a general continuous-domain location-allocation problem that accounts for heterogeneity in travel accessibility, demand distribution, and temporal variability. A solution approach is then derived. Application results are presented to highlight capabilities of this model, focusing on drone equipped emergency medical service facilities. The chapter ends with a discussion and concluding comments.

4.2 Background

Location decisions are extremely important and directly influence the efficiency of a given service. Primary location decisions involve where to site a facility within a service system. When more than one facility exits in the system, then service demand will naturally be divided between facilities. The process of determining who will be served by a given facility is typically called allocation. Locating facilities and allocating services are inter-dependent, and must be done simultaneously to ensure system efficiency (Church and Murray, 2009). This is the essence of location-allocation: site multiple facilities and assign demand to be served in an optimal manner.

Spatial heterogeneity is well documented/studied in many spatial process models (Anselin, 1988; 2013). It is associated with contextual variation over space, and the inherent non-uniformity in spatial unit delineation. Given a directed network, a location-allocation problem that considers heterogeneity in travel accessibility can be readily structured and solved (see Toregas et al., 1971; Singh et al., 1998). The assignment cost in terms of distance, time, energy, etc., is usually represented as attributes on arcs or path segments, which can vary over time along a path (Wang et al., 2018). However, in continuous space, spatial heterogeneity in travel has rarely been taken into account, except in large scale systems involving distances of hundreds of miles and transport such as marine shipping and air travel (see for example, Patron et al., 2013 and Lee et al., 2015). Without the restriction of a network, deriving the minimal assignment cost for smaller areas involving drones is not straightforward, with infinite trajectory options between demand and facility. The work of Feng & Murray (2018) is an exception in the literature, demonstrating that allocation can be structured as a heterogeneous

Voronoi diagram. However, no attempt was made to simultaneously address combined location-allocation decision making for this complex problem.

Temporal heterogeneity too is important in spatial analysis. Variability in accessibility, travel and demand is common for most service systems. In a location-allocation problem, one classic objective is to minimize total weighted cost (or average cost), defined as the total cost associated with each demand traveling to or served by its closest facility (Church & Murray, 2009). Both travel cost and amount of service demand are often temporally heterogeneous. Traffic situations are significantly different during commuting rush hours and other time periods. Humans move from their residences and temporarily relocate for daytime activities, including places of education (e.g., schools, universities), employment, businesses (e.g., restaurant, grocery shopping), and recreation (e.g., parks, national monuments, wildlife refuges) over the course of a day (US Census Bureau 2000; Bhaduri et al., 2007). Facets of temporal heterogeneity have been represented in location-allocation models. For example, demand at a position in a given time period will be served based upon the best siting, route and facility scheduling possible (Mirchandani, 1980; Weaver and Church, 1983; Bloxham & Church, 1991). However, spatial and temporal heterogeneity have not been simultaneously considered in location application studies involving continuous space, especially in the case of emergency response (e.g., Pulver & Wei, 2018, Yao et al., 2019).

Research related to issues of space-time has been of central interest in the field of time geography, focusing on evaluating/measuring accessibility, feasible opportunity set and possible activity duration (Kim & Kwan, 2003; Miller, 2017). Concepts of multiple space-time prisms within GIS were proposed to describe the accessibility of human activities and interactions with spatiotemporal restrictions (Miller, 1991; 2005). The prism represents potential ability to move in space over time. Space-time prisms have been extended and applied in various fields, including transportation, urban science, social sciences, environmental sciences, etc. (Kuijpers et al., 2010; Song et al. 2017; Miller, 2017). Research associated with space-time prisms deals with many important questions like where and when people are able to access certain services within a spatial and temporal limitations (Song et al. 2017; Lee & Miller, 2018). Most of the associated analyses are based on a given network, not continuous space travel, like that used in UAVs. Thus, spatiotemporal heterogeneity in accessibility has not been defined without road segments along a trajectory.

Siting drone-equipped EMS stations exemplifies location and allocation challenges under spatiotemporal heterogeneity because complications associated with travel accessibility and varying demand are evident. Various local situations and conditions have significant impact on the flight of drones (Federal Aviation Administration, 2019) as well as the distribution of potential patients. Drone flight is influenced by wind direction and wind speed (McNeely et al, 2007; Anderson et al, 2013), which in turn are impacted by atmospheric pressure, the rotation of earth, seasons, sea and land interactions, terrain variability, etc. A drone's ground speed is not fixed since wind varies in space and time. The heterogeneity in drone flight accessibility necessitates accounting for local environmental conditions. Another important factor is the dispersion of potential patients over the course of the day. This variation is important for location selection because facilities must be close enough to demand to respond quickly. Because of limited flight distance capabilities, drones can only serve neighboring demand and must be re-charged regularly (Pulver et al., 2016; Hong et al., 2017). Therefore, the representation of variable demand plays an important role in the process of optimally siting drone-equipped EMS stations. Previous work has not considered spatiotemporal heterogeneity in location and allocation processes involving a continuous space domain. Optimizing drone location and service allocation is an interesting and meaningful problem context requiring much needed research.

4.3 Model Development

As suggested previously, drone based medical service delivery presents unique challenges. A drone is relatively small, making it easy to store and mobilize at different EMS stations. Of course, supporting infrastructure and oversight are also necessary. An important characteristic, however, is that the travel time of a drone can be impacted by local spatiotemporal conditions, making accessibility and access heterogeneous. The basic location-allocation problem associated with drone siting and system configuration is the following:

Locate a fixed number of drones in order to maximize demand accessibility over space and time.

One approach for addressing this problem is to assume that travel time/costs can be derived for discrete locations at given times of the day. Accordingly, this would reflect an extension of a classic location-allocation approach, the *p*-median model of ReVelle and Swain (1970). To begin, assume that there is only one time period. Formulation of an associated location-allocation model relies on the following notation:

j = index of demand areas $(1, 2, \ldots, n)$

g = index of potential facility sites $(1, 2, \ldots, m)$

$p =$ the number of stations to be located

a_j = amount of demand in area j

d_{jg} = shortest distance from demand area j to potential facility site g

$$Z_g = \begin{cases} 1 & \text{if facility at site } g \text{ is located} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{jg} = \begin{cases} 1 & \text{if demand } j \text{ assigns to facility } g \\ 0 & \text{otherwise} \end{cases}$$

The classic p-median location-allocation model is as follows:

$$Minimize \sum_j \sum_g a_j d_{jg} X_{jg} \tag{4.1}$$

Subject to:

$$\sum_g X_{jg} = 1 \qquad \forall j \tag{4.2}$$

$$X_{jg} \leq Z_g \qquad \forall j, g \tag{4.3}$$

$$\sum_g Z_g = p \tag{4.4}$$

$$X_{jg} = \{0, 1\} \quad \forall j, g \qquad Z_g = \{0, 1\} \quad \forall g \tag{4.5}$$

The objective, (4.1), seeks a minimum total weighted assignment distance, which is equivalent to minimizing average service cost. Constraints (4.2) indicate that each demand area *j* must to be served by a facility. Constraints (4.3) restrict allocations made for a given demand area *j* to only sites *g* that have been chosen for a facility. Constraint (4.4) specifies that p sites are to be selected for facilities. Constraints (4.5) indicate binary integer restrictions on decision variables.

There are two coefficients in (4.1): a_j and d_{jg}, one related to demand density and the other associated with travel cost. a_j reflects spatial heterogeneity of demand over space but ignores possible variations over time. This is problematic. Instead of a_j, suppose that $\alpha(\vec{j}, t)$

represents the instantaneous demand density at point $\vec{j}$ at time t. By doing so, the temporal heterogeneity of demand over continuous space can be considered, though it is a continuous function. The coefficient d_{jg} in (4.1) is also too limited. Suppose that $V_g(\vec{j},\mathrm{t})$ represents instantaneous travel over space from potential site g to a point $\vec{j}$ at time t, which is defined as:

$$V_g(\vec{j},\mathrm{t}) = \varphi\left(\delta_g(\vec{j},t)\right) = e^{-c\times\delta_g(\vec{j},t)} \tag{4.6}$$

where

$$\delta_g(\vec{j},t) = \min_{c_{\vec{j}g}\,\in\,\Omega_{\vec{j}g}} \int_{s\in c_{\vec{j}g}} f(s\,,t)ds \tag{4.7}$$

$V_g(\vec{j},\mathrm{t})$ is a continuous function of the quality decrement φ (see Figure 4.1) related to shortest travel time $\delta_g(\vec{j},t)$. In (4.6) the constant c represents the degree of the decrement (or decay) such that $0 < e^{(-\mathrm{c}*Max\_time)} \leq \varepsilon$, where ε is a sufficiently small positive value and Max_time is flight time assuming best case battery life. The quality decrement function simultaneously considers coverage and the level of service delivered to demand, making $V_g(\vec{j},\mathrm{t})$ infinitely close to zero when the shortest travel time $\delta_g(\vec{j},t)$ reaches the drone maximum service time Max_time. While small d_{jg} values represent low travel cost in (4.1), a larger $V_g(\vec{j},\mathrm{t})$ in (4.8) given below represents high service quality. Figure 4.2 illustrates a contrast in trajectories for d_{jg}(red lines) and possible $V_g(\vec{j},\mathrm{t})$ (blue and green lines) at different given times of the day. The dotted lines represent the real trajectories between demand $\vec{j}$ (or j) and facility g in two dimensions (horizontal space). The solid lines are their corresponding trajectories in three dimensions, with the third (vertical) axis representing travel time. The importance is that the travel cost, d_{jg}(red line), is fixed, while travel cost for other paths,

$V_g(\vec{j}, \mathrm{t})$, are time varying. The blue and green lines are two samples of trajectories connecting a demand at $\vec{j}$ (or j) and a facility at g with the maximum $V_g(\vec{j}, \mathrm{t})$ at different given times. These trajectories vary based on local environments and conditions at particular times.

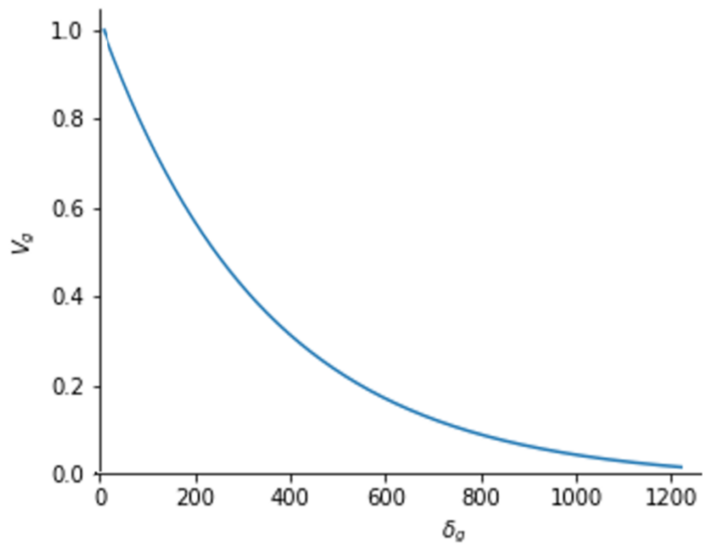


Figure 4.1 Quality decrement function φ

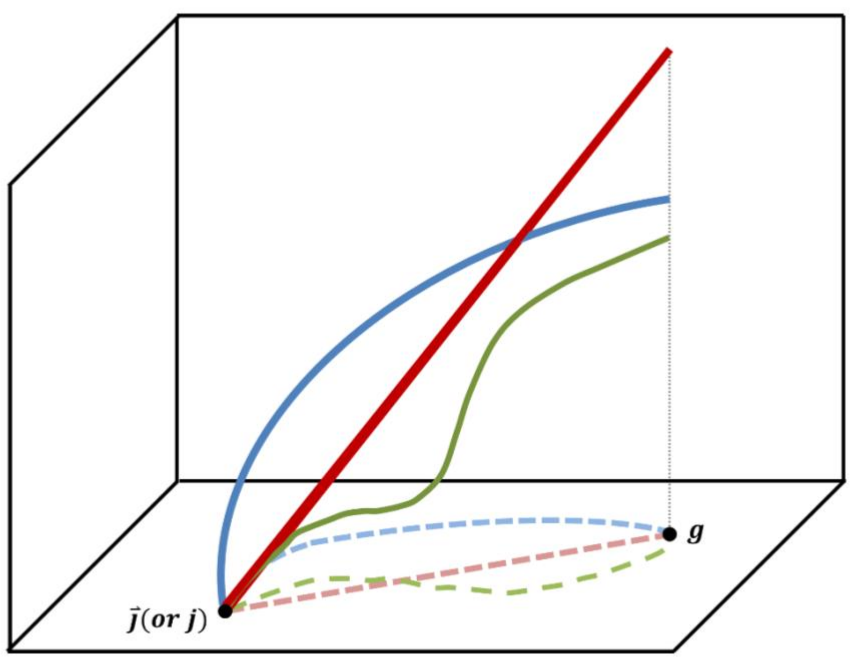


Figure 4.2 Different trajectories of minimum d_{jg} (red solid/dotted lines) and $V_g(j,t)$ (blue & green solid/dotted lines)

$\delta_g(\vec{j}, t)$ is the shortest travel time from g to $\vec{j}$ at time t calculated as an integral. Feng & Murray (2018) account for spatial heterogeneity in travel accessibility associated with an allocation process. Function (4.7) considers both spatial and temporal heterogeneity. $\Omega_{\vec{j}g}$ is the set of all paths $c_{\vec{j}g}$ from g to $\vec{j}$. A path $c_{\vec{j}g}$ is a specific piecewise continuous curve in the feasible domain R. s is a vector describing the instantaneous movement along $c_{\vec{j}g}$. $f(\,)$ accounts for attributes and movement, relating spatial and temporal accessibility and taking into account travel, congestion and other conditions. Accessibility is not only based on the attribute(s) of the current location along the path curve but also the direction of movement as one moves through a location at a given moment in time. This is the essence of $V_g(\vec{j}, \mathrm{t})$.

A generic location-allocation model can therefore be conceived of as an extension of the p-median problem, where heterogeneity is accounted for using $\alpha(\vec{j}, t)$ and $V_g(\vec{j}, t)$ as instantaneous measures over continuous space and time. Consider the following additional notation:

$$X_g(\vec{j}, t) = \begin{cases} 1 & \text{if demand } \vec{j} \text{ assigns to facility } g \text{ at the time } t \\ 0 & \text{otherwise} \end{cases}$$

With this notation, a location-allocation model under spatiotemporal heterogeneity can be formulated as follows:

$$Maximize \sum_{g} \int_{t \in T} \int_{\vec{j} \in R} \alpha(\vec{j}, t)\, V_g(\vec{j}, t)\, X_g(\vec{j}, t)\; d\vec{j}\, dt \tag{4.8}$$

Subject to:

$$\sum_{g} X_g(\vec{j}, \mathrm{t}) = 1 \qquad \forall\, \vec{j} \in R, t \in T \tag{4.9}$$

$$X_g(\vec{J}, \mathrm{t}) \leq Z_g \qquad \forall\, \vec{J} \in R, t \in T \tag{4.10}$$

$$\sum_g Z_g = p \tag{4.11}$$

$$X_g(\vec{J}, \mathrm{t}) = \{0, 1\} \qquad \forall\, \vec{J} \in R, t \in T \qquad Z_g = \{0, 1\} \qquad \forall g \tag{4.12}$$

Because a larger $V_g(\vec{J}, \mathrm{t})$ represents a better service for demand $\vec{J}$, the objective, (4.8), is to maximize service quality for all demand. In this case, the integrals over the time horizon T and study area R are necessary for accounting for heterogeneity. Constraints (4.9) require that demand at $\vec{J}$ is assigned to a facility at time t and this holds true for demand over the entire study region. Constraints (4.10) ensure that station assignments are limited to those where a station is located for drone response. Constraint (4.11) stipulates the number of stations to be located, and constraints (4.12) are the decision variable requirements.

4.4 Solution

This new model is complicated by the continuous functions and integrals in (4.8). Solution therefore must simultaneously consider two interrelated parts: travel cost at a given time and facility location. Accessibility can be derived in a couple of ways. One approach is vector-based, tracking the accessibility front and service areas within a time period using certain marker particle methods (Mahoney et al., 2012; Rhoads et al., 2013). One option in modeling this is to track the motion of particles using the following ordinary differential equations:

$$\frac{d\vec{J}_x}{dt} = u(\vec{J}, \mathrm{t}) + S * \cos\theta \tag{4.13}$$

$$\frac{d\vec{J}_y}{dt} = v(\vec{J}, \mathrm{t}) + S * \sin\theta \tag{4.14}$$

$$\frac{d\theta}{dt} = -\frac{\partial u(\vec{J}, \mathrm{t})}{\partial \vec{J}_y} \; cos^2\theta + \left(\frac{\partial u(\vec{J}, \mathrm{t})}{\partial \vec{J}_x} - \frac{\partial v(\vec{J}, \mathrm{t})}{\partial \vec{J}_y}\right) sin\theta cos\theta + \frac{\partial v(\vec{J}, \mathrm{t})}{\partial \vec{J}_x} sin^2\theta \qquad (4.15)$$

Assume that a number of marker particles travel in a spatiotemporally heterogeneous field $(u(\vec{J}, \mathrm{t}), v(\vec{J}, \mathrm{t}))$ with speed S staying at its maximum speed to minimize travel time. Equation (4.15) then is the well-known Euler-Lagrange equation, providing the necessary condition for an optimal solution in the case of minimum travel time. The position of an infinitesimal front particle $(\vec{J}_x, \vec{J}_y)$ can be achieved using equations (4.13) and (4.14), combined with θ, the local orientation of the front particles found in equation (4.15). This vector-based approach could identify an optimal solution, but is likely computationally infeasible for real-time application, especially in the case of emergency response.

Another solution approach is raster-based (Figure 4.3). Raster cells, serving as an approximation of continuous space, are capable of summarizing a range of phenomena. If travel across space is defined using neighboring relationships (e.g., orthogonal directions, diagonal directions, etc.) in a raster, accessibility can be derived based on attributes of cells encountered during travel at a given time, as well as the distance and direction between neighboring cells. An approach to derive travel cost across a raster includes Dijkstra (used in this chapter), an algorithm like A* (Zeng and Church, 2009), or others (Smith et al, 1989, Feng & Murray, 2018). The raster-based approach has clear computational advantages since raster models are efficient for storing and managing data that vary continuously over space.

After acquiring travel costs using a raster-based discretization approach, an optimal service system configuration can be identified using the location-allocation model (4.8) - (4.12), solved via a commercial optimization package, such as Xpress, Cplex, or Gurobi. An obvious challenge in the solution process is dealing with the possibility of a large number of decision variables, constraints (4.12). There are a number of ways to limit them in order to ensure the model is solvable within an acceptable time. One is to control the size of raster cells, which is the unit within which demand is summarized. A second way is that decision variables $X_g(\vec{j}, t)$ are automatically set to zero where demand is zero. This effectively decreases the total decision variables from more than 100 million to around 30 million in the analysis that follows. A third approach might consider introducing certain spatial filters to indicate that a demand cannot be assigned to a facility if it is too far away (e.g., further than a drone's maximum flight distance). Similar ideas can be found in Church (2003; 2008). All three have been used in the study case that follows.

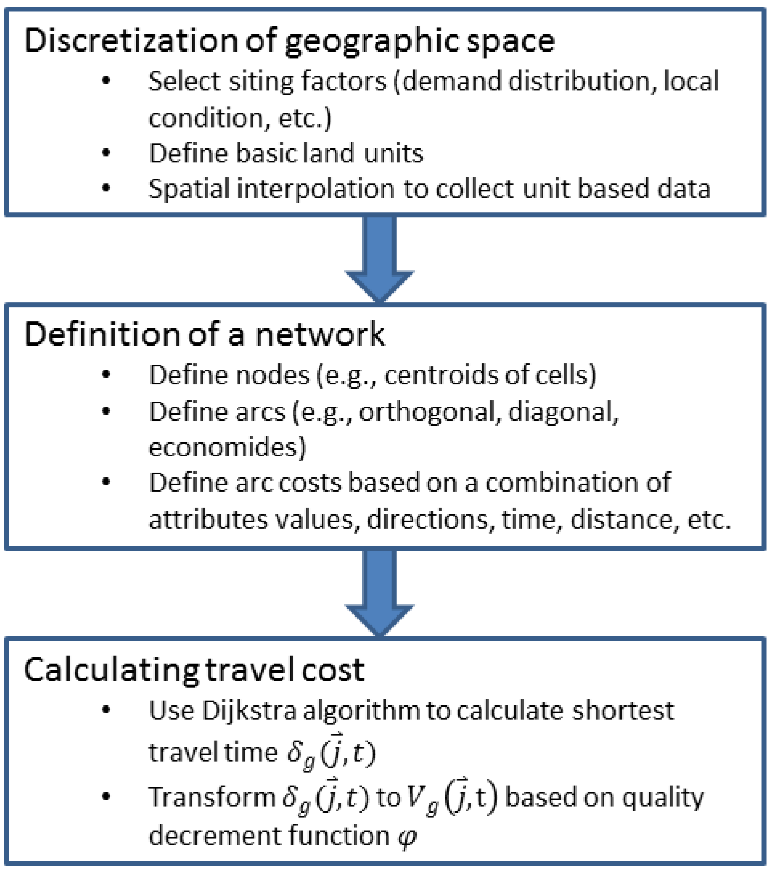


Figure 4.3 A raster-based approach to derive instantaneous travel cost

To support real time planning and analysis, the integration of spatial analytics, geographic information, and decision-making is required. The capabilities of GIS are important for analysis, modeling, and potential decision making involving facility location. The developed approach accounts for spatiotemporal heterogeneity in siting drone-equipped stations in order to optimize emergency response. In this case, GIS enables the extraction of real-time context knowledge, such as weather, traffic conditions, and airspace restrictions, and detailed spatial information, such as population distribution, behavioral characteristics, travel patterns, potential locations for storing drones (e.g., fire stations, health clinics, airports), land parcels, etc. GIS facilitates integration and management of these different kinds of data. Visualization and display of the distribution of facility locations and their associated service coverage is a straightforward task in GIS.

4.5 Case Study

The siting of drone-equipped stations to deliver automated external defibrillators (AEDs) to out-of-hospital patients experiencing cardiac arrest is carried out in this chapter. Drones, equipped with AEDs, are planned for flying directly to the patient's location, with bystanders who are provided directions for using the medical equipment on the patient (Communication, 2014). The significance of AEDs and their influence on survival of cardiac arrest, particularly during the first several minutes, has been demonstrated in previous studies (Cummins et al., 1984; Caffrey et al., 2002; Dao et al. 2012).

The proposed location-allocation model, (4.8) - (4.12), is utilized to support planning for the best emergency delivery of AEDs possible in order to increase survival. The study area includes five counties in southern California: Santa Barbara County; Ventura County; and parts of Los Angeles County, San Luis Obispo County, and Kern County. The U.S. Geological

Survey maintains the National Structures Dataset that contains data on Fire/Police Stations, Hospital/Medical Centers, Ambulance Services, etc. We used this data in defining possible drone sites. Figure 4.4 shows the 520 potential stations in the study area. Out-of-hospital cardiac arrest (OOHCA) incidences are unknown, so population data is used as a proxy. The daytime and nighttime population in each block (Figure 4.5) was acquired from LEHD Origin-Destination Employment Statistics (LODES) Dataset. Raster-based wind maps (Figure 4.6), with one-kilometer spatial resolution and one-hour time resolution from April 26-27, 2004, were acquired. The mean wind speed is approximately 3.20 mph and 4.86 mph during daytime and nighttime, respectively. For the study area, there are 107,910 spatial units. Generally, the wind picks up at the coastline and then spreads inland. We consider wind in two dimensions because wind in the horizontal plane has the most significant impact on drone flight (see Selecky et al., 2013). The results that follow are based on air speed of a drone fixed at 50 mph in all directions with a maximum service time of 20 minutes given battery capacity.

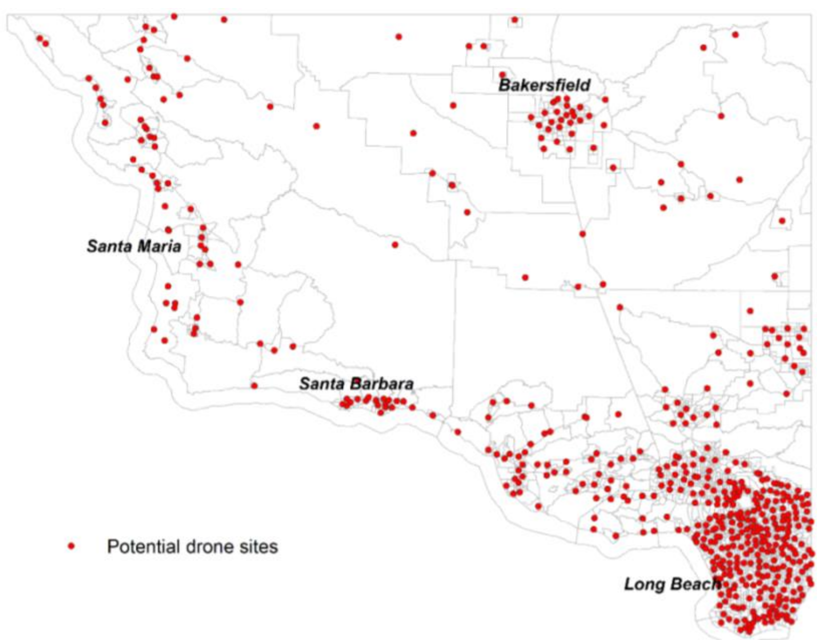
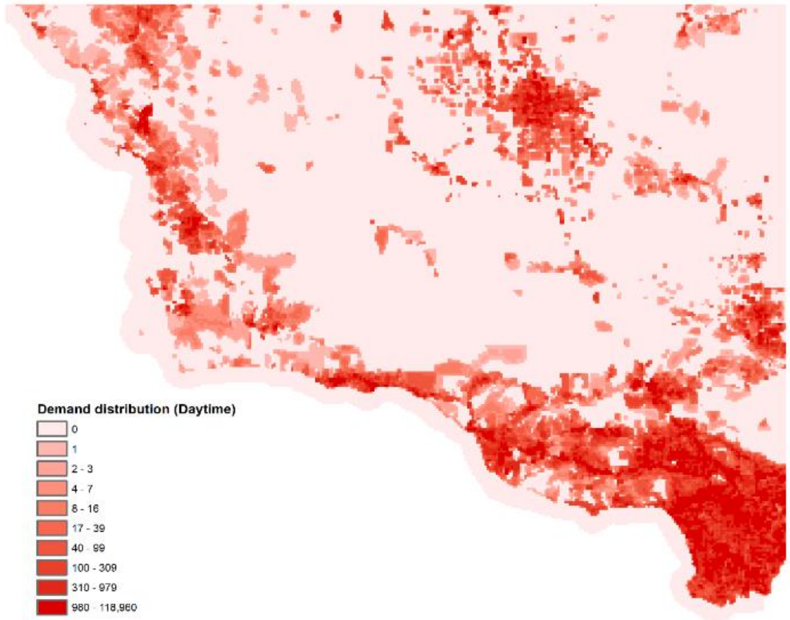
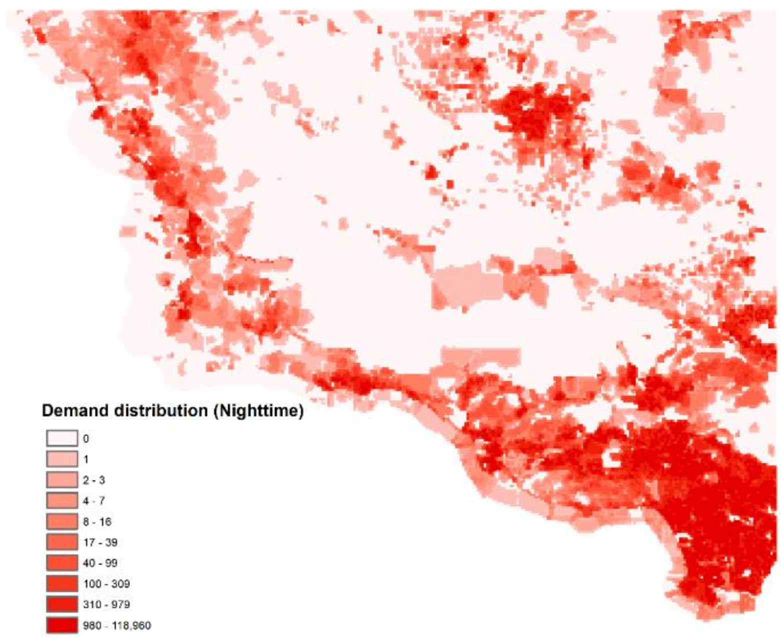


Figure 4.4 Study area



Daytime



Nighttime

Figure 4.5 Demand distribution

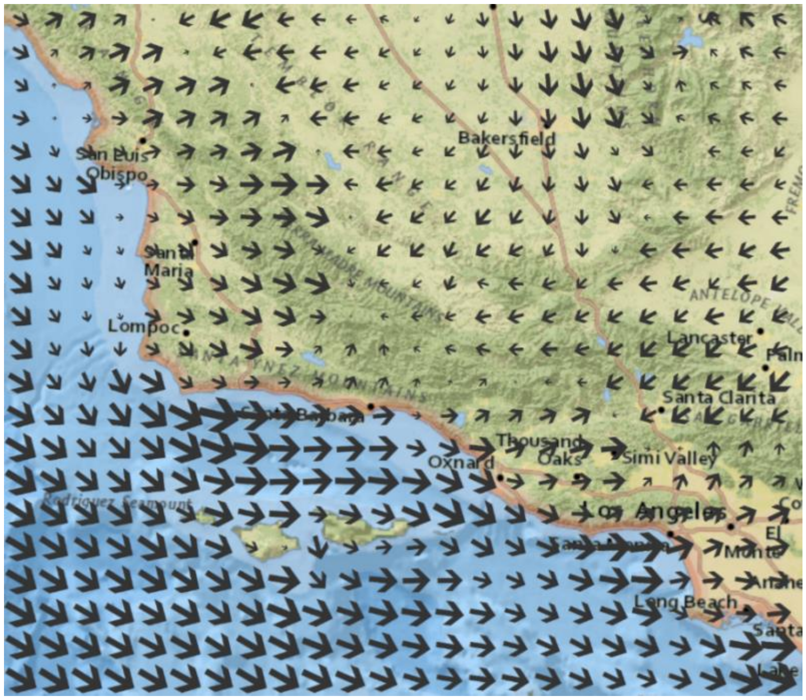


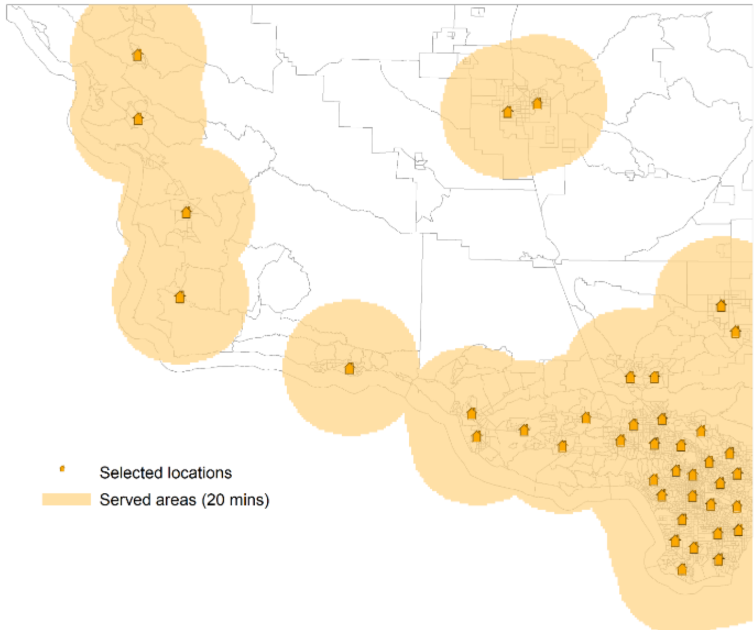
Figure 4.6 Prevailing wind conditions

The models were implemented in Python and run on an Intel (R) Xeon (R) CPU (2.3GHz) with 64GB of RAM. ArcGIS was utilized for data creation, management, manipulation, analysis and display. Gurobipy, a commercial optimization Python package, was used to solve the proposed location-allocation model, (4.8) - (4.12). The applied model has approximately 2 million rows, 2 million columns and 6 million nonzero decision variables associated with this study. Solution required approximately 20 mins of processing time. The computing time is dependent on the level of granularity of the wind raster, which is used to represent heterogeneous local conditions. The distribution of potential demand (day & night population) was resampled to have a consistent spatial resolution using spatial interpolation.

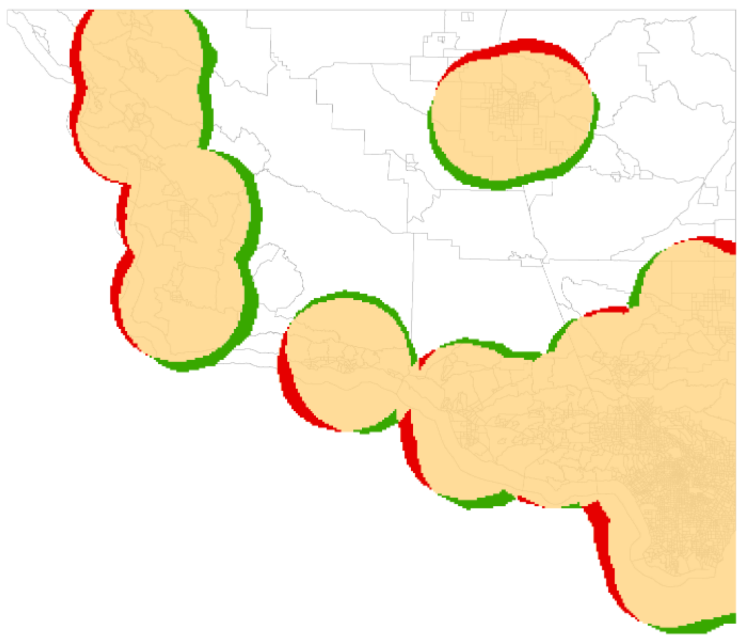
4.6 Application Results

To provide a basis for comparison, the generic p-median model, (4.1) - (4.5), was used to obtain initial results. Figure 4.7a shows the location of 40 selected base stations for drones and their corresponding service areas. With the assumption of homogeneity of travel accessibility, reachability is assumed to not be impacted by wind. In this case, the boundary of service areas for each selected station is essentially a perfect circle, which may be quite different from the real situation. Because of the various wind magnitudes and directions encountered over continuous space, the service area for each drone-equipped station at certain times may be irregular. Figure 4.7b indicates the impact of heterogeneous travel accessibility on service areas. The green areas can be reached within a drone’s maximal flight time but are not included as part of the service area. More importantly, the red areas cannot be reached. Thus, the solution of this generic p-median (homogeneous travel) model mistakenly regards these areas as served. This is a serious problem and represents a form of modeling error. Because of a drone’s limited battery, this miscalculation of service (allocation) will not only result in a flight mission failure, but also decreases patient survivability.

The siting in Figure 4.7a is not appropriate also because of the assumption of temporal homogeneity in demand distribution (i.e. daytime is same as nighttime). Since humans move routinely during the day (see Figure 4.5), demand heterogeneity over time must be simultaneously considered in the location-allocation process.



a



b

Figure 4.7 Analysis assuming homogeneity (with underestimates and overestimates of areas served)

The second scenario accounts for temporal variability. The solution when the new model, (4.8) - (4.12), is utilized, is depicted in Figure 4.8 when temporal heterogeneity is account for.

The location results differ significantly from the simple, homogenous model in Figure 4.7a. By incorporating spatiotemporal heterogeneity, greater performance is possible. Total weighted demand, equation (4.8), which considers both demands and service quality, is regarded as a good indicator for comparing results. The total demand served for Figure 4.8 is 1.107×10^7, including 6.343×10^6 for daytime and 4.728×10^6 for nighttime, while for Figure 4.7a, it is 1.068×10^7, including 5.862×10^6 for daytime and 4.823×10^6 for nighttime. Compared with Figure 4.8, the stations in Figure 4.7a cover 2.0% more demand at night, however, 7.6% less during the day. Thus, the solution of the new model (Figure 4.8) finds a balance serving demand over time and taking into account the changes in wind (speed and direction). In sum, the heterogenous model, (4.8) - (4.12), which considers time varying demands, served 3.7% more demand than when demand is assumed homogeneous over time.

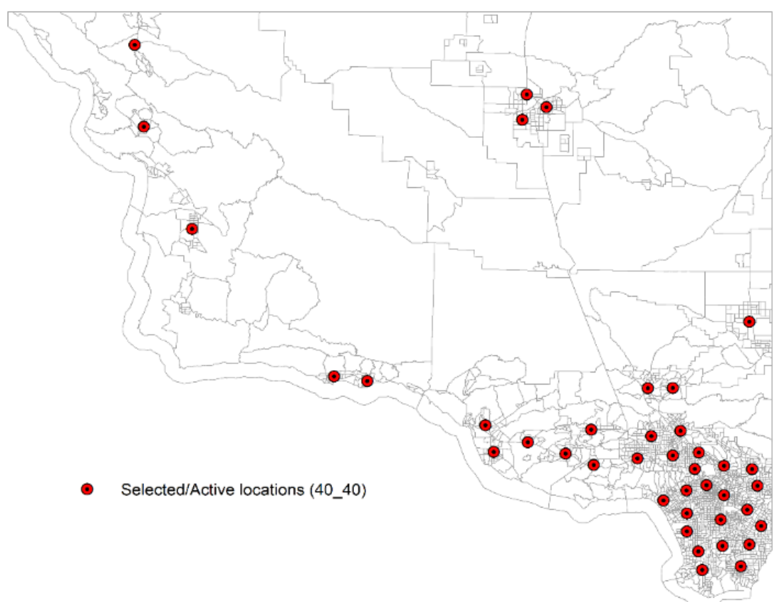


Figure 4.8 Resulting spatial pattern when considering temporal heterogeneity in distributed demand

The histogram curves displayed in Figure 4.9 are used to compare the performance of the models with and without consideration of heterogeneity (Figures 4.7a and 4.8). The x-axis represents the travel time between demand and its sited station, and the y-axis is the total weighted demand located at all units that can be reached/served in each flight time increment. The total weighted demand across all travel times can be measured by the area under the curve. In the first two minutes, the heterogeneous model serves 65.9%, and 17.7% more demand than the homogeneous model respectively. This is a significant improvement in emergency response since the first several minutes are particularly essential for the treatment of patients suffering from cardiac arrest. Moreover, the average response times are 195.95 and 213.41 seconds. That is, 8.9 % of average response time can be saved using our proposed heterogeneous model, with a clear difference shown in Figure 4.9.

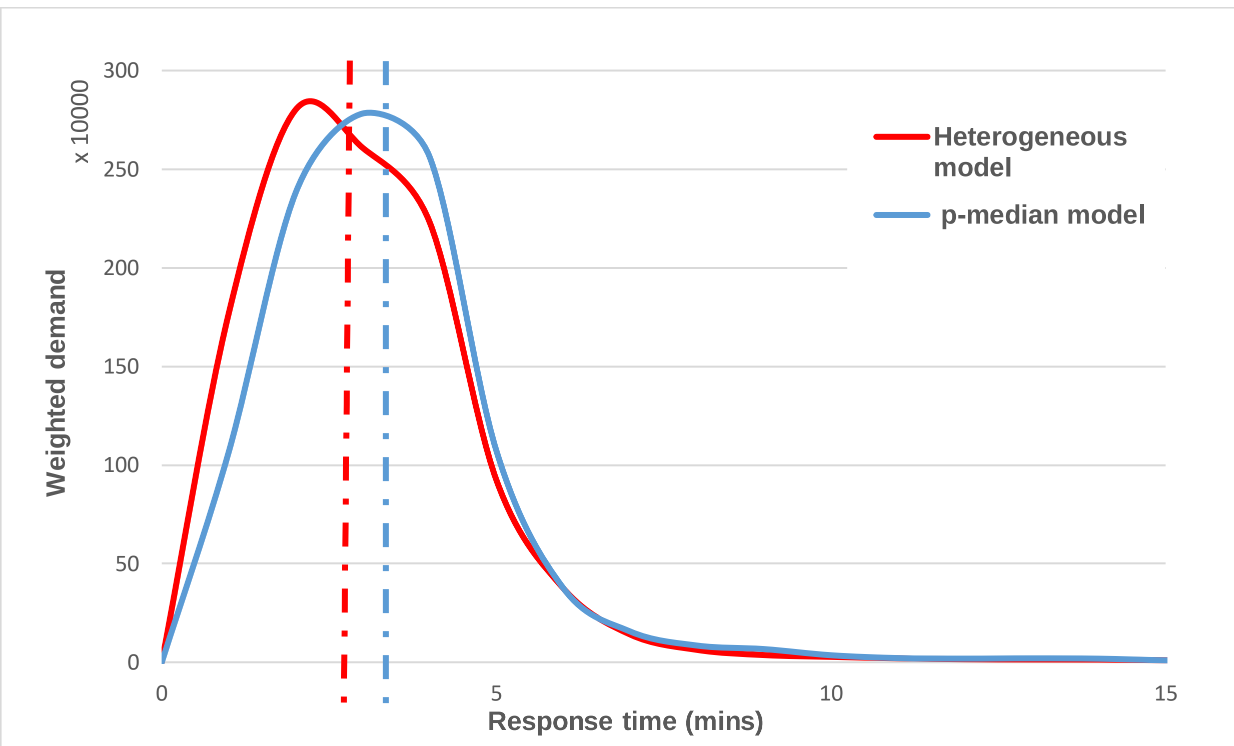


Figure 4.9 Service response comparison

4.7 Discussion

Different from traditional vehicles like trucks and ambulances, drones are relatively small sized, which makes them easy to store at different stations and different time periods. Because of the heterogeneous distribution of potential service populations, siting temporal varying drone-equipped stations is ideal for enhancing survivability. That is, it may be important to consider varying service provision over time, where some locations are operational during only specific periods of the day. To extend the proposed heterogeneous location-allocation model, the following model simultaneously determines $\bar{p}_t$ stations among the p stations that will operate at time t. Additionally, consider the following decision variables:

$$Y_g(t) = \begin{cases} 1 & \text{if a station located at } g \text{ is open at the time } t \\ 0 & \text{otherwise} \end{cases}$$

It is therefore assumed that drones may not be operable at some stations during all time periods of the day, due to staffing limitations, downtime for maintenance, or other considerations. The extended model is the following:

$$Maximize \sum_{g} \int_{t \in T} \int_{\vec{j} \in R} \alpha(\vec{j}, t)\, V_g(\vec{j}, t)\, X_g(\vec{j}, t)\ d\vec{j}\, dt \qquad (4.16)$$

Subject to:

$$\sum_{g} X_g(\vec{j}, t) = 1 \qquad \forall\, \vec{j} \in R,\ t \in T \qquad (4.17)$$

$$X_g(\vec{j}, t) \leq Y_g(t) \qquad \forall\, \vec{j} \in R,\ t \in T, \forall g \qquad (4.18)$$

$$Y_g(t) \leq Z_g \qquad \forall\, t \in T,\ \forall g \qquad (4.19)$$

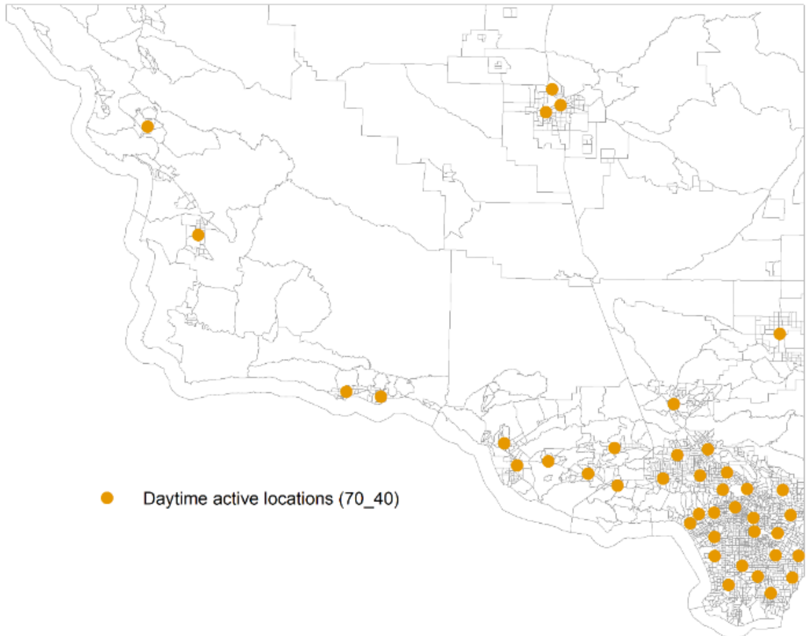
$$\sum_{g} Z_g = p \qquad (4.20)$$

$$\textstyle\sum_g Y_g(t) = \bar{p}_t \qquad \forall\, t \in T \qquad (4.21)$$

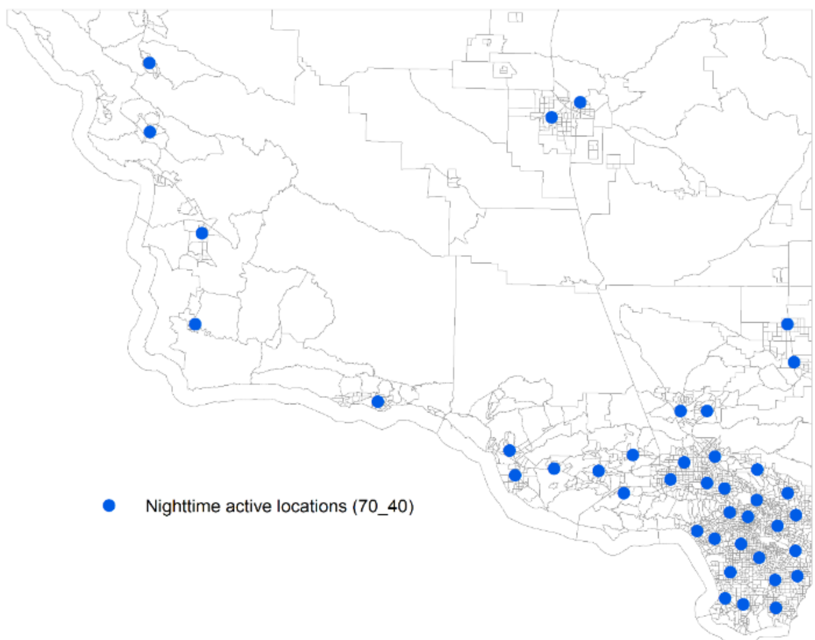
$$X_g(\vec{j}, t) = \{0,1\} \quad \forall \vec{j} \in R,\ t \in T \qquad Y_g(t) = \{0,1\} \quad \forall\, t \in T \qquad Z_g = (0,1) \quad \forall g \qquad (4.22)$$

Constraints (4.18) and (4.19) replace constraints (4.10) in the originally proposed model, ensuring that assignments for stations only when they have been scheduled to operate at time t. Similarly, a station is prevented from being scheduled to operate unless it is equipped with a drone. The number of selected stations, $\bar{p}_t$, to be operating at time t is specified in constraints (4.21). The remaining formulation remains consistent with that of model (4.8) - (4.12).

Figure 4.10 depicts the optimal locations over time (day & night) using this extension, (4.16) - (4.22). More than two time periods could be considered, but the day is separated into daytime (from 7am to 7pm) and nighttime (from 7pm to 7am) in this case study. The total number of selected stations (p) is 70, which means 70 locations are selected for drones. Among these 70 stations, 40 stations ($\bar{p}_t$) will be active during each time period. Due to the various demand and local environmental conditions, the optimal 40 active stations during the day might not be the same 40 stations for the night. In this scenario, the total demand is 1.125×10^7, including 6.407×10^6 at daytime and 4.845×10^6 at nighttime. The average response time is further reduced to 189.91 seconds. Compared with the results in Figure 4.8, a conclusion could be safely drawn that an increase in total demand (1.7%) and a decrease in average response time (3.1%) is possible if more locations are available, even though the number of active drones/stations, 40, is the same in each time period.



Daytime



Nighttime

Figure 4.10 Selected locations for drone siting considering spatiotemporal heterogeneity (daytime vs nighttime)

4.8 Conclusions

Spatiotemporal heterogeneity is an important issue but has been ignored in many location associated modeling contexts. Location-allocation is among the many spatial optimization problems impacted by spatiotemporal heterogeneity. This chapter proposes the concept of a spatiotemporally heterogeneous location-allocation problem and solves it for a general bounded region. Specifically, we formalize the problem of siting drone-equipped stations into a location and allocation model considering the spatiotemporal heterogeneity of demand distribution and travel accessibility. The results provided demonstrate that it is essential to account for the description of heterogeneity and show how the optimal solution of the location-allocation problem is influenced accordingly.

4.9 Pseudo-code

Data: $T, G, f, \varphi, Demand_t$

Result: Site option Z_g and allocation assignment $X_g(\vec{j}, t)$

Acquiring demand $\alpha(\vec{j}, t)$

for $t \in T$ **do**

    for $\vec{j} \in J$ **do**

        get $\alpha(\vec{j}, t)$ by interpolating with $Demand_t$

    end for

end for

Dijkstra algorithm

function Dijkstra (time t, origin g, destination $\vec{j}$, local environment graph f):

    return $\delta_g(\vec{j}, t)$

Acquiring accessibility $V_g(\vec{j}, \mathrm{t})$

for $t \in T$ **do**

    for $g \in G$ **do**

        for $\vec{j} \in J$ **do**

            if $\alpha(\vec{j}, t) \neq 0$ **then**

                $\delta_g(\vec{j}, t) \leftarrow$ Dijkstra $(g, \vec{j}, f)$

                $V_g(\vec{j}, \mathrm{t}) \leftarrow \varphi\left(\delta_g(\vec{j}, t)\right)$

            end if

        end for

    end for

end for

Integer Programming

set up decision variables $X_g(\vec{j}, t)$ and Z_g

set up constraints for the bounds of $X_g(\vec{j}, t)$ and Z_g

set up objective functions: Maximize $\sum_g \sum_t \sum_{\vec{j}} \alpha(\vec{j}, t)\, V_g(\vec{j}, t)\, X_g(\vec{j}, t)$

run optimization model

Chapter 5 Conclusions

5.1 Summary

This dissertation explored the impacts and implications of heterogeneity in allocation and location processes. Chapter 1 discussed the main motivation, listed important research objectives, provided important theoretical context, and gave an overview of the organization of the dissertation.

Chapter 2 introduced the concept of spatial heterogeneity in allocation. A new Voronoi diagram was defined – the heterogeneous Voronoi diagram. A raster-based solution method was developed to derive the heterogeneous Voronoi diagram using discretized spatial allocation properties. Application of the heterogeneous Voronoi diagram was reported for a planning problem involving emergency drone delivery. Results showed that response potential is over- and under-estimated when heterogeneity and travel obstacles are disregarded. Further, feasibility, usefulness and significance were demonstrated for incorporating geographic heterogeneity in the allocation process.

Chapter 3 proposed the concept of spatiotemporal heterogeneity in an allocation problem, describes associated properties, and developed a vector-based solution method for a general 2D-bounded region. The chapter illustrated that it is possible to account for spatial and temporal heterogeneity by describing continuous space using a vector approach. Moreover, it held that the allocation problem involving heterogeneous space can be addressed using the marker particle-based front tracking method. Drone emergency service delivery was used to highlight capabilities of the developed approach. The results demonstrated the significant influence of spatial and temporal heterogeneity in assigning demands to their ideal base-station facilities. This reduced the chances of loss of life in the case of emergency medical service response, making the developed approach invaluable.

Chapter 4 extended the p-median problem, introducing a new location-allocation model that considers spatial and temporal heterogeneity. The proposed location-allocation model was applied to aid in the deployment of medical drones. This chapter addressed the question of where to site medical drone base stations and how to allocate service in order to optimize response, given spatiotemporal heterogeneity in distributed demand with varying service response times/costs. Results showed that drone travel time to patients across a region can be significantly reduced by improved location and allocation decisions, supported by spatial optimization. Appropriate description of distributed demand and travel accessibility is critical and offers significant potential for improving planning and decision-making processes.

5.2 Theoretical Contributions

Spatiotemporal heterogeneity is an important issue, but it has been ignored in many location modeling contexts. Location-allocation (e.g., models summarized in Figure 1.1) is among the many spatial optimization problems impacted by spatiotemporal heterogeneity. This dissertation proposed heterogenous versions of allocation and location problems, described associated properties, and developed methods of solution, specifically focusing on the representation of spatiotemporal heterogeneity of demand and travel accessibility. The corresponding results demonstrated that the optimal solution of a location-allocation problem could be improved by accounting for such heterogeneity. The primary theoretical contribution, therefore, is the introduction of spatiotemporal heterogeneity in location and allocation processes. Of course, doing so introduces substantial practical and computational complexities, necessitating advancements in supporting analytical approaches. The reported research represents an initial step in this direction.

5.3 Future Work

There remain limitations and opportunities for future research, which are summarized as follows.

- Improving computational efficiency

The accuracy and computation time of allocation solutions using a Heterogeneous Voronoi diagram are dependent on the cell size of the raster data used to represent continuous space. Finer resolution provides more detail and may affect identified allocation areas. However, this will necessitate more computational effort. In the case study in Chapter 2 (88,596 spatial units), the heterogeneous allocation problem cannot be solved within 24 hours using a commercial optimization package. The deterministic dynamic programming approach proposed, however, takes approximately 20 seconds. This is acceptable for strategic planning, but still could be improved to support real-time dispatcher allocation associated with emergency response. In such situations, the first several minutes are essential, leaving little time to wait on model supported dispatching advice.

The processing time to derive allocation solutions using the vector-based method in Chapter 3 is quite long, approximately 20 minutes for only a single county. There are many factors, including the initial number of marker particles, the time range of each iteration, the size of study area, etc., on which increase or decrease of computational effort may depend. More work is needed to explore the possibility to enhance computational efficiency.

- Considering capacity for facilities

An important issue in facility planning involves capacity considerations. This has not been explicitly addressed in either the proposed allocation or location processes in the dissertation. Actually, considering the workload of individual facilities is a realistic problem in many cases. It is especially true when there is some limit on how many demands can be served by an individual facility, as in the drone emergency delivery context explored here. In many existing models that address capacity issues, certain constraints are added to track that the total demand assigned to each facility in order to ensure that they do not exceed their capacity. Some work in this area does exist, but assumes homogeneity. One example is the Capacitated Maximal Covering Location Problem with Closest Assignment (CMCLP-CA) detailed in Gerrard & Church (1996). The objective of the CMCLP-CA is to maximize total demand covered within the desired maximum service standard, but explicitly accounts for facility service constraints.

Extension of heterogeneous allocation and/or location accounting for the limitation of capacity is an interesting issue worth further investigation. The Backup Coverage Problem, a special case of the Vector Assignment P-Median Problem, has been applied in drone delivery (Pulver & Wei, 2018). Similar to the proposed extension of P-Median Problem in Chapter 4, an extension of Vector Assignment P-Median Problem may be a good option for introducing heterogeneity to the consideration of capacity for facilities.

- Incorporating drone delivery with other vehicles

The limited flight distance of drones poses a major restriction in the case of emergency delivery as only demand within the range of the base station can be served. Some existing research assumes both trucks and drones perform emergency deliveries simultaneously (Chowdhury et al., 2017), which effectively mitigates some drone-based limitations. One interesting issue worth further investigation is introducing the consideration of spatiotemporal heterogeneity into an integrated facility location and vehicle routing problem.

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