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PREDICTION OF A MINIMUM IN THE HIGH ENERGY nN -> w N DIFFERENTIAL CROSS SECTION

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### **Author**

Wang, Ling-Lie.

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**University of California  
Ernest O. Lawrence  
Radiation Laboratory**

PREDICTION OF A MINIMUM IN THE HIGH ENERGY  
 $\pi^+ N \rightarrow \omega N$  DIFFERENTIAL CROSS SECTION.

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Ling-Lie Wang

February 8, 1966

## PREDICTION OF A MINIMUM IN THE HIGH ENERGY

 $\pi N \rightarrow \omega N$  DIFFERENTIAL CROSS SECTION\*

Ling-Lie Wang

Lawrence Radiation Laboratory  
 University of California  
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The energy dependence of the high-energy charge-exchange  $\pi^- p \rightarrow \pi^0 n$  differential cross sections<sup>1</sup> can be fitted well by single- $\rho$  Regge-pole exchange.<sup>2</sup> The fit for the  $\rho$  trajectory is  $\alpha_\rho(t) = (0.56 \pm 0.03) + (0.81 \pm 0.08)t$  with  $t$  in units of  $(\text{GeV}/c)^2$ , so  $\alpha_\rho(t)$  has a zero at  $t = -0.69 (\text{GeV}/c)^2$ .<sup>3</sup> The experimental data for the differential cross sections of  $\pi^- p \rightarrow \pi^0 n$  shows a minimum around  $t = -0.6 (\text{GeV}/c)^2$  which can be explained by the fact that spin-flip amplitude dominates the spin-nonflip amplitude and the spin-flip amplitude contains a factor  $\alpha_\rho(t)$ . Thus the major component in the amplitude vanishes when  $\alpha_\rho(t) = 0$ , producing a minimum here in the differential cross section. In this Letter we shall show that a similar but even more pronounced minimum should appear in the high-energy  $\pi N \rightarrow \omega N$  differential cross sections.<sup>4</sup>

For experiments with no polarization, the differential cross section for interactions  $a + b \rightarrow c + d$  in terms of helicity amplitudes is

$$\frac{d\sigma}{dt} = \frac{1}{4\pi s p_{ab}^2} \frac{1}{(2J_a + 1)(2J_b + 1)} \sum_{a,b,c,d} |f_{cd;ab}^s(s,t)|^2 \quad (1)$$

where  $s$  is the square of the invariant mass for the direct channel;  $p_{ab}$  is the c.m. momentum of particle  $a$ ;  $J_a, J_b$  are the spins of particles  $a$  and  $b$ , respectively; and  $f_{cd;ab}^s(s,t)$  are the direct-channel helicity amplitudes with  $a, b, c$ , and  $d$  denoting the helicity states of the corresponding particles.<sup>5</sup> The  $f_{cd;ab}^s(s,t)$  are related to the  $t$ -channel (i.e.,  $D + b \rightarrow c + A$ ) helicity amplitudes  $f_{cA;Db}^t(s,t)$  by an orthogonal crossing matrix.<sup>6</sup> Thus the differential cross section in the  $s$  channel can be simply related to the  $t$ -channel helicity amplitudes by

$$\frac{d\sigma}{dt} = \frac{1}{4\pi s p_{ab}^2} \frac{1}{(2J_a + 1)(2J_b + 1)} \sum_{c,A,D,b} |f_{cA;Db}^t(s,t)|^2, \quad (2)$$

where we use  $A, B, C$ , and  $D$  to denote the corresponding antiparticles and their helicity states. The Regge poles in the  $t$  channel can now be easily put into the  $s$ -channel differential cross section through Eq. (2).

Consider the Reggeization of the  $t$ -channel (i.e.,  $\bar{N}N \rightarrow \omega\pi$ ) helicity amplitudes for the  $\pi N \rightarrow \omega N$  interaction.<sup>7</sup> Of the known high-ranking trajectories, only the  $\rho$  has the necessary quantum numbers. Combinations of partial-wave helicity states with parity  $(-)^J$  and thus with  $(J \text{ parity}) \times \text{parity} = +$  can communicate with the  $\rho$ , so the  $\rho$  pole will appear in the partial-wave amplitudes  $F^{J,+}(t)$  which are associated with the two linear combinations of helicity amplitudes having these quantum numbers:

$$\begin{aligned}
 f_{10; \frac{1}{2} \frac{1}{2}}^{t,+}(s, t) &\equiv \frac{1}{\sin \theta_t} f_{10; \frac{1}{2} \frac{1}{2}}^t(s, t) + \frac{1}{\sin \theta_t} f_{-10; \frac{1}{2} \frac{1}{2}}^t(s, t) \\
 &= \sum_J (2J + 1) F_{10; \frac{1}{2} \frac{1}{2}}^{J,+}(t) e_{01}^{J,+}(\cos \theta_t) \\
 &+ (\text{unimportant term for large } \cos \theta_t), \quad (3)
 \end{aligned}$$

$$\begin{aligned}
 f_{10; \frac{1}{2} -\frac{1}{2}}^{t,+}(s, t) &\equiv \frac{1}{2} [\cos(\theta_t/2)]^{-2} f_{10; \frac{1}{2} -\frac{1}{2}}^t(s, t) \\
 &- \frac{1}{2} [\sin(\theta_t/2)]^{-2} f_{-10; \frac{1}{2} -\frac{1}{2}}^t(s, t) \\
 &= \sum_J (2J + 1) F_{10, \frac{1}{2} -\frac{1}{2}}^{J,+}(t) e_{11}^{J,+}(\cos \theta_t) \\
 &+ (\text{unimportant term for large } \cos \theta_t); \quad (4)
 \end{aligned}$$

where

$$e_{01}^{J,+}(\cos \theta) = P'_J(\cos \theta) / [J(J + 1)]^{\frac{1}{2}},$$

$$e_{11}^{J,+}(\cos \theta) = [P'_J(\cos \theta) + \cos \theta P''_J(\cos \theta)] / J(J + 1),$$

and

$$\cos \theta_t = [2s + t - (m_\pi^2 + m_\omega^2 + 2m_N^2)] / 4 p_{\omega\pi} p_{NN},$$

if  $\theta_t$  is the scattering angle in the t-channel center-of-mass system. Notice that  $\pi\omega$  can couple to the  $\rho$  pole only when  $\omega$  has helicity state 1. Therefore there is only one coupling function at the  $\pi\omega\rho$  vertex. As shown in Ref. 5 the sines and cosines of  $\theta_t$  in Eqs. (3) and (4) ensure that the  $f^{t,+}$  should be free of s-kinematic singularities. There are still pure t-kinematic singularities, but using the result of Ref. 5, we find that

$$f_{10; \frac{1}{2} \frac{1}{2}}^{t,+}(s, t) (\mathcal{T}_{\pi\omega})^{-1} \text{ and } f_{10; \frac{1}{2} -\frac{1}{2}}^{t,+}(s, t) t^{-\frac{1}{2}} (\mathcal{T}_{\pi\omega})^{-1}$$

are completely free of kinematic singularities and zeros if

$$(\mathcal{T}_{\pi\omega})^2 = [t - (m_\pi + m_\omega)^2] [t - (m_\pi - m_\omega)^2]$$

After Reggeizing according to the method of Ref. 7 and considering only the  $\rho$  pole, we obtain for large  $\cos \theta_t$

$$f_{10; \frac{1}{2} \frac{1}{2}}^{t,+}(s, t) \approx \frac{2\alpha(t) + 1}{\sin \pi \alpha(t)} \frac{1}{2} [1 - \exp(-i\pi\alpha)] \beta_{01}(t) E_{01}^{\alpha(t),+}(\cos \theta_t), \quad (5)$$

$$f_{10; \frac{1}{2} -\frac{1}{2}}^{t,+}(s, t) \approx \frac{2\alpha(t) + 1}{\sin \pi \alpha(t)} \frac{1}{2} [1 - \exp(-i\pi\alpha)] \beta_{11}(t) E_{11}^{\alpha(t),+}(\cos \theta_t), \quad (6)$$

where  $\beta_{01}(t)$ ,  $\beta_{11}(t)$  are the residue functions of the  $\rho$  pole of

$$F_{10; \frac{1}{2} \frac{1}{2}}^{\alpha,+}(t) \quad \text{and} \quad F_{10; \frac{1}{2} -\frac{1}{2}}^{\alpha,+}$$

in the  $\alpha$  plane respectively. They contain the pure t-kinematic factors mentioned in the last paragraph. In Eqs. (5) and (6), the  $E^{\alpha,+}(\cos \theta)$ 's are the  $e^{\alpha,+}(\cos \theta)$ 's with  $P_\alpha(\cos \theta)$  replaced by

$$P_\alpha(\cos \theta) = -Q_{\alpha-1}(\cos \theta) \pi^{-1} \tan \pi \alpha \approx \frac{\Gamma(\alpha + \frac{1}{2})}{\cos \theta \gg 1} \frac{(2 \cos \theta)^\alpha}{\Gamma(\alpha + 1) \pi^2}.$$

At high  $s$ , we have

$$E_{01}^{\alpha,+}(\cos \theta_t) \approx \frac{\Gamma(\alpha + \frac{1}{2})}{\pi^{\frac{1}{2}} \Gamma(\alpha + 1)} \frac{2\alpha}{[\alpha(\alpha + 1)]^{\frac{1}{2}}} \left( \frac{s}{p_{\pi\omega} p_{\bar{N}N}} \right)^{\alpha-1} \quad (7a)$$

and

$$E_{11}^{\alpha,+}(\cos \theta_t) \approx \frac{\Gamma(\alpha + \frac{1}{2})}{\pi^{\frac{1}{2}} \Gamma(\alpha + 1)} \frac{2\alpha^2}{\alpha(\alpha + 1)} \left( \frac{s}{p_{\pi\omega} p_{\bar{N}N}} \right)^{\alpha-1}. \quad (7b)$$

Using the method of Ref. 7 to study the partial-wave helicity amplitudes  $F^{J,+}$  in detail, one finds that in addition to the t-kinematic factors, the residue functions should have the following threshold behavior and  $\alpha$  factors:

$$\beta_{01}(t) \propto \mathcal{T}_{\pi\omega} (p_{\pi\omega} p_{\bar{N}N})^{\alpha-1} [\alpha(\alpha + 1)]^{\frac{1}{2}} \quad (8)$$

$$\beta_{11}(t) \propto t^{\frac{1}{2}} \mathcal{T}_{\pi\omega} (p_{\pi\omega} p_{\bar{N}N})^{\alpha-1} \alpha(\alpha + 1). \quad (9)$$

Roughly speaking, the  $[\alpha(\alpha + 1)]^{\frac{1}{2}}$  of Eq. (8) comes from the fact

that  $\alpha = 0$  is a sense-nonsense<sup>9</sup> value for  $F_{10; \frac{1}{2} \frac{1}{2}}^{\alpha, +}(t)$ , while the  $\alpha(\alpha + 1)$  factor of Eq. (9) comes from the fact that  $\alpha = 0$  is a nonsense-nonsense value for  $F_{10; \frac{1}{2} -\frac{1}{2}}^{\alpha, +}(t)$ .<sup>10</sup> Substituting Eqs. (7) into Eqs. (5) and (6) and using Eqs. (8) and (9), one obtains

$$f_{10; \frac{1}{2} \frac{1}{2}}^{t, +}(s, t) \approx \frac{2\alpha + 1}{\sin \pi \alpha} \frac{1}{2} [1 - \exp(-i\pi\alpha)] \mathcal{T}_{\pi\omega} \quad (10)$$

$$\times \left\{ \frac{\beta_{01}(t) (\mathcal{T}_{\pi\omega})^{-1}}{(p_{\pi\omega} p_{\bar{N}\bar{N}})^{(\alpha-1)} [\alpha(\alpha + 1)]^{\frac{1}{2}}} \right\} 2\alpha \frac{\Gamma(\alpha + \frac{1}{2})}{\pi^{\frac{1}{2}} \Gamma(\alpha + 1)} (s)^{\alpha-1}$$

and

$$f_{10; \frac{1}{2} -\frac{1}{2}}^{t, +}(s, t) \approx \frac{2\alpha + 1}{\sin \pi \alpha} \frac{1}{2} [1 - \exp(-i\pi\alpha)] t^{\frac{1}{2}} \mathcal{T}_{\pi\omega} \quad (11)$$

$$\times \left\{ \frac{\beta_{11}(t) t^{-\frac{1}{2}} (\mathcal{T}_{\pi\omega})^{-1}}{(p_{\pi\omega} p_{\bar{N}\bar{N}})^{(\alpha-1)} \alpha(\alpha + 1)} \right\} 2\alpha^2 \frac{\Gamma(\alpha + \frac{1}{2})}{\pi^{\frac{1}{2}} \Gamma(\alpha + 1)} (s)^{\alpha-1}$$

The quantities in the braces of Eqs. (10) and (11) are now free of all kinematic factors. Their  $t$  dependence is purely from dynamics, there being no singularities for  $t$  negative. To absorb other factors that have smooth  $t$  dependence, we define modified residue functions by

$$\gamma_{01}(t) = \left[ \frac{\beta_{01}(t) (\tilde{T}_{\pi\omega})^{-1}}{(p_{\pi\omega} p_{NN})^{\alpha-1} [\alpha(\alpha+1)]^{\frac{1}{2}}} \right] \left[ \tilde{T}_{\pi\omega} \frac{\Gamma(\alpha + \frac{1}{2})}{\pi^{\frac{1}{2}} \Gamma(\alpha+1)} \frac{\alpha + \frac{1}{2}}{\alpha+1} \right] (s_0)^{\alpha-1}, \quad (12)$$

$$\gamma_{11}(t) = \left[ \frac{\beta_{11}(t) (\tilde{T}_{\pi\omega})^{-1} t^{-\frac{1}{2}}}{(p_{\pi\omega} p_{NN})^{\alpha-1} \alpha(\alpha+1)} \right] \left[ \tilde{T}_{\pi\omega} \frac{\Gamma(\alpha + \frac{1}{2})}{\pi^{\frac{1}{2}} \Gamma(\alpha+1)} \frac{\alpha + \frac{1}{2}}{\alpha+1} \right] (s_0)^{\alpha-1},$$

where  $s_0$  can be chosen at a convenient value. Notice that the quantities in the second brackets of Eqs. (12) are all smoothly varying functions of  $t$  for  $t < 0$  and  $\alpha(t) > -\frac{3}{2}$ . Expressed through the modified residue functions, Eqs. (10) and (11) become

$$f_{10; \frac{1}{2} \pm \frac{1}{2}}^{t,+}(s, t) \approx [1 - \exp(-i\pi\alpha)] \frac{\alpha + 1}{\sin\pi\alpha} \alpha \gamma_{01}(t) (s/s_0)^{\alpha-1} \quad (13a)$$

and

$$f_{10; \frac{1}{2} \pm \frac{1}{2}}^{t,+}(s, t) \approx [1 - \exp(-i\pi\alpha)] \frac{\alpha + 1}{\sin\pi\alpha} \alpha^2 \gamma_{11}(t) (s/s_0)^{\alpha-1} \quad (13b)$$

Finally we can write the differential cross section in the form

$$\begin{aligned} \frac{d\sigma}{dt} &= \frac{1}{4\pi s p_{ab}^2} |1 - \exp(-i\pi\alpha)|^2 \left( \frac{\alpha(t) + 1}{\sin\pi\alpha(t)} \right)^2 \\ &\times \left( 2 |\sin\theta_t|^2 \alpha^2 [\gamma_{01}(t)]^2 + 4 \left[ [\cos(\theta_t/2)]^4 + [\sin(\theta_t/2)]^4 \right] \right. \\ &\times \left. |t| \alpha^4 [\gamma_{11}(t)]^2 \right) \left( \frac{s}{s_0} \right)^{2\alpha-2}, \end{aligned} \quad (14)$$

remembering that  $\alpha(t)$  and the  $\gamma(t)$ 's are real in the s-physical region.

We see that both terms in the differential cross section of Eq. (14) vanish at the value of  $t$  where  $\alpha(t) = 0$ . Since there are background terms,  $\frac{d\sigma}{dt}$  will not be exactly zero at this point, but there should be a deep minimum in  $\frac{d\sigma}{dt}$ . Experimental observation of this minimum at high energy would constitute an important verification of the Regge-pole model.

Note also that, since at the  $\pi\omega p$  vertex there is only one coupling function, by factorizability of the residues (unmodified ones),  $\beta_{01}(t)/\beta_{11}(t)$  will exactly equal the ratio of spin-nonflip and spin-flip residue functions in  $\pi N$  charge-exchange scattering.<sup>11</sup>

Arbab and Chiu found that with an appropriate choice of  $s_0$  they can fit the  $\pi^- p \rightarrow \pi^0 n$  differential cross section by setting the residue functions equal to constants.<sup>3</sup> To have some feeling about how the minimum should look in the  $\pi N \rightarrow \omega N$  differential cross section we plot a graph (Fig. 1) of Eq. (14) with the same choice of  $s_0$  and setting the residue functions equal to constants with the ratio equal to that of  $\pi^- p \rightarrow \pi^0 n$ . Of course the actual t-dependence of the  $\pi\omega p$  coupling function has to be determined by the fitting experiments on  $\pi N \rightarrow \omega N$ .

The essential points in the foregoing argument are that, because of G-parity, the  $\pi\omega$  system can couple only to the  $\rho$  Regge pole and can couple to  $\rho$  only when  $\omega$  has helicity 1 and

$\pi\omega$  is in the ( $J$ -parity)  $\times$  parity = + state. The result is that  $J = 0$  is always a nonsense value for  $\pi\omega\rho$  coupling. By the general mechanism of Reggeization, every helicity amplitude for interactions  $aB \rightarrow \rho \rightarrow \pi\omega$  will have a factor  $\alpha_\rho(t)$ , no matter what the particles  $a, B$  may be. Therefore all high-energy differential cross sections for  $\pi a \rightarrow \omega b$  should have a minimum at  $\alpha_\rho(t) = 0$ .

Similar arguments can be applied to vertices similar to  $\pi\omega\rho$ , e.g.,  $\pi\phi\rho$ ,  $K\omega K^*(1^-)$ , or  $K\omega K^*(2^+)$ . For example, in the  $K^- p \rightarrow \omega\Lambda$  interaction,  $K^*(1^-)$  and  $K^*(2^+)$  can be exchanged. If one is higher ranking than the other, it alone will dominate the high-energy  $K^- p \rightarrow \omega\Lambda$  interaction, and one expects to see a minimum at the zero of that trajectory.

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FOOTNOTES AND REFERENCES

- \* Work done under the auspices of the United States Atomic Energy Commission.
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  2. R. K. Logan, Phys. Rev. Letters 14, 414 (1965); R. Phillips and W. Rarita, Phys. Rev. 139, 1336B (1965).
  3. F. Arbab and C. B. Chiu, Association Between the Dip in the  $\pi^- p \rightarrow \pi^0 N$  High-Energy Angular Distribution and the Zero of the Rho Trajectory, Lawrence Radiation Laboratory Report UCRL-16686, February 1966, (submitted to Phys. Rev. Letters); G. Hohler, J. Baacke, H. Schlaile, and P. Sonderegger, Phys. Letters 20, 79 (1966).
  4. For the  $\pi N \rightarrow \omega N$  interaction, energies greater than 5 GeV/c of the  $\pi$  momentum in the laboratory system can be considered high.
  5. The convention and notation used here are the same as in a previous paper by the author (Lawrence Radiation Laboratory Report UCRL-16240, June 1965), to be published in Phys. Rev.
  6. T. L. Trueman and G. C. Wick, Ann. Phys. 26, 322 (1964).
  7. We use the method of Reggeization constructed by M. Gell-Mann, M. Goldberger, F. Low, E. Max, and F. Zachariasen in Phys. Rev. 133B, 145 (1964), Appendix B.

8. Because of the unequalness of the masses,  $(\cos \theta_t)^2$  is +1 on the physical boundary. Therefore  $(\cos \theta_t)^2$  is always 1 at the exact s-channel forward direction no matter how high the energy is. However, the amplitudes in Eqs. (5) and (6) will behave like  $(s)^{\alpha-1}$  in all directions near the forward direction. Since the amplitudes in Eqs. (5) and (6) are analytic on the physical boundary, the  $(s)^{\alpha-1}$  dependence is expected to be true at the s-channel forward direction. The same situation happens in the  $\pi N$  backward scattering. J. Stack, (private communication (Department of Physics, University of California, Berkeley), to be published in Phys. Rev. Letters; G. Chew and J. Stack, Lawrence Radiation Laboratory Report, UCRL-16293, July 1965, unpublished.
9. Note that the sense-honsense distinction is in addition to the signature distinction between even and odd  $J$ .
10. Here the factorizability of the residue functions has been used. The modified residue function  $\beta_{11}(t)$  will behave like  $\alpha(\alpha + 1)$  if we assume the residue function of  $F_{\frac{1}{2} \frac{1}{2}; \frac{1}{2} \frac{1}{2}}^{J,+}$  for the  $N\bar{N} \rightarrow N\bar{N}$  interaction does not behave like  $\alpha(\alpha + 1)$ . If the residue function for  $F_{\frac{1}{2} \frac{1}{2}; \frac{1}{2} \frac{1}{2}}^{J,+} (\bar{N}N \rightarrow \bar{N}N)$  does behave like  $\alpha(\alpha + 1)$ , which also implies  $F_{\frac{1}{2} \frac{1}{2}; 0 \ 0}^{J,+} (\bar{N}N \rightarrow \pi\pi)$  behaving like  $[\alpha(\alpha + 1)]^{\frac{1}{2}}$ , then  $\beta_{11}(t)$  will not have the  $\alpha(\alpha + 1)$  behavior, and there will be only the first power of  $\alpha$  in Eq. (11).

11. The differential cross section of charge-exchange  $\pi N$  scattering with  $\rho$  Regge-pole exchange is related to residue functions, to which factorizability applies, by

$$\begin{aligned} \frac{d\sigma}{dt} &= \frac{1}{4\pi s p_{\pi N}^2} \\ &\times \left\{ \left[ \frac{2\alpha + 1}{\sin \pi \alpha} \right]^{\frac{1}{2}} [1 - \exp(-i\pi\alpha)] \beta_{00}(\bar{N}N \rightarrow \pi\pi) \frac{\Gamma(\alpha + \frac{1}{2})}{\pi^{\frac{1}{2}} \Gamma(\alpha + 1)} \left( \frac{s}{p_{\pi\pi} p_{\bar{N}N}} \right)^{\alpha} \right|^2 \\ &+ \left[ \sin \theta_t (\bar{N}N \rightarrow \pi\pi) \frac{2\alpha + 1}{\sin \pi \alpha}^{-\frac{1}{2}} [1 - \exp(-i\pi\alpha)] \beta_{10}(\bar{N}N \rightarrow \pi\pi) \right. \\ &\quad \left. \times \frac{2\alpha}{[\alpha(\alpha + 1)]^{\frac{1}{2}}} \frac{\Gamma(\alpha + \frac{1}{2})}{\pi^{\frac{1}{2}} \Gamma(\alpha + 1)} \left( \frac{s}{p_{\pi\pi} p_{\bar{N}N}} \right)^{\alpha-1} \right] \end{aligned}$$

Factorizability says

$$\beta_{01}(\bar{N}N \rightarrow \pi\omega)/\beta_{11}(\bar{N}N \rightarrow \pi\omega) = \beta_{00}(\bar{N}N \rightarrow \pi\pi)/\beta_{10}(\bar{N}N \rightarrow \pi\pi)$$

In terms of Arbab's and Chiu's notation, we have

$$\begin{aligned} \frac{d\sigma}{dt} (\pi N \rightarrow \pi N) &= \frac{1}{\pi s} \left( \frac{m_N}{4 p_{\pi N}} \right)^2 \\ &\times \left\{ \left( 1 - \frac{t}{4 m_N^2} \right) \left| c_0 \frac{\alpha + 1}{\sin \pi \alpha} [1 - \exp(-i\pi\alpha)] \left( \frac{E}{E_0} \right)^\alpha \right|^2 \right. \\ &\left. + \frac{t}{4 m_N^2} \left( s - \frac{s + p_\pi^2}{1 - \frac{t}{4 m_N^2}} \right) \left| D_0 \alpha \frac{\alpha + 1}{\sin \pi \alpha} [1 - \exp(-i\pi\alpha)] \left( \frac{E}{E_0} \right)^{\alpha-1} \right|^2 \right\}. \end{aligned}$$

The ratio of our modified residue functions is related to  $c_0$   
and  $D_0$  by

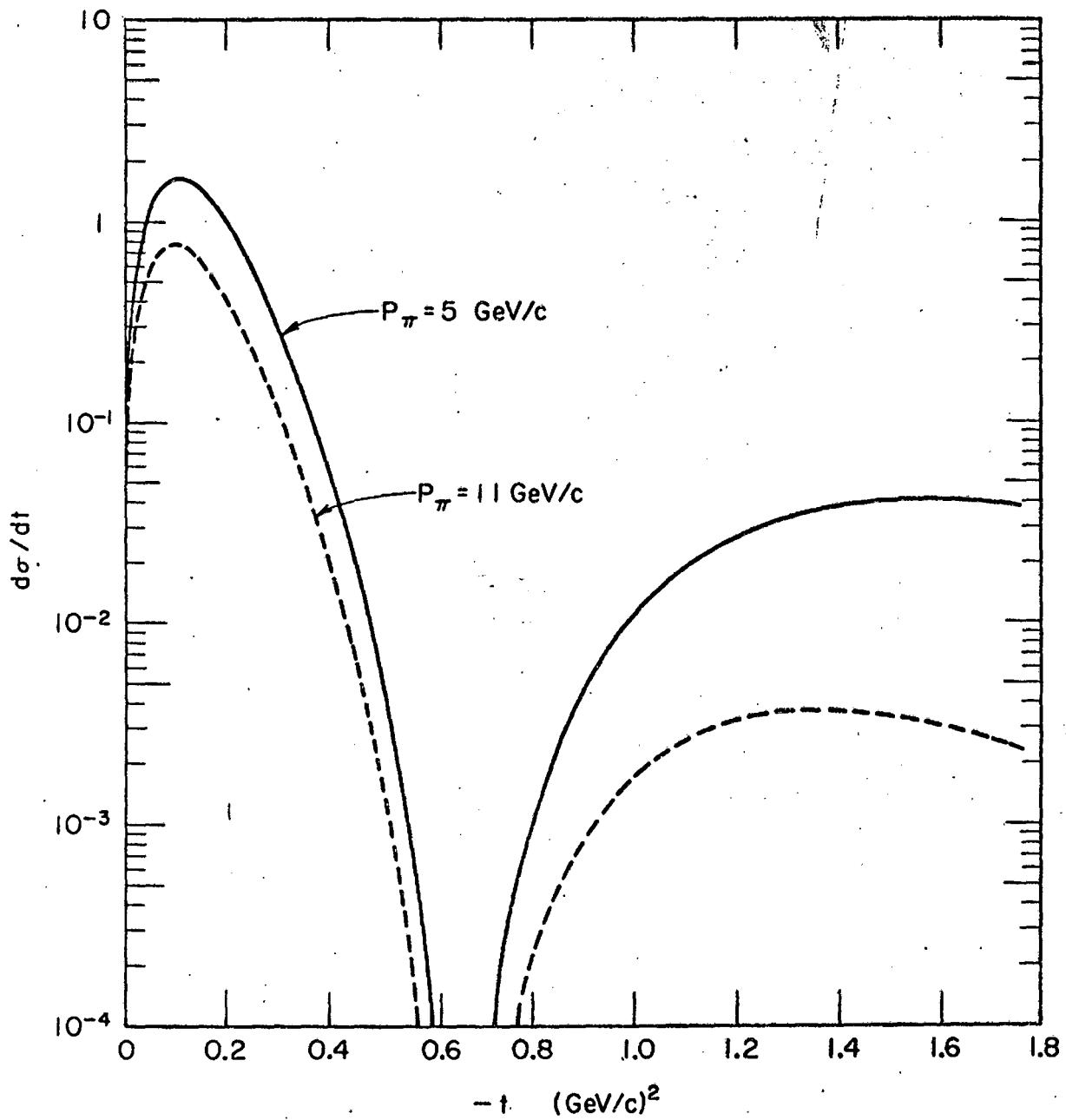
$$\gamma_{01}(\bar{N}N \rightarrow \pi\omega)/\gamma_{11}(\bar{N}N \rightarrow \pi\omega) = (c_0/D_0) (4 m_N^2 - t)/2s_0.$$

FIGURE CAPTION

Fig. 1. Differential cross sections for  $\pi N \rightarrow \omega N$  with

$$(\gamma_{01})^2(4/\pi) = .20, (\gamma_{11})^2(4/\pi) = 6272, \text{ and } s_0 = 2m_N^2(\text{GeV})^2$$

in Eq. (14). In the figure,  $p_\pi$  is the pion momentum in  
the laboratory system.



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