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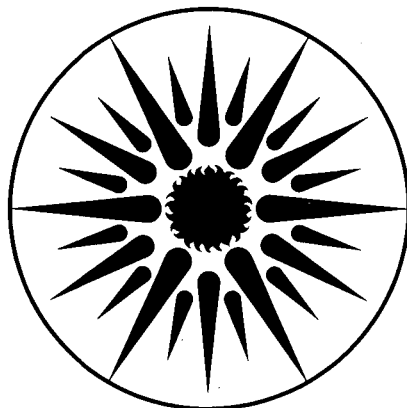
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M.H. Rothkopf, T.J. Teisberg,  
and E.P. Kahn

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by

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December 16, 1987

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# Why are Vickrey Auctions Rare?

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Michael H. Rothkopf,\* Thomas J. Teisberg,\*\* and Edward P. Kahn\*

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## Abstract

In 1961, Vickrey showed that, in an independent private values context with symmetric risk neutral bidders, sealed second-price auctions have dominant truth revealing equilibrium strategies, that with such strategies they are perfectly efficient economically and that they produce the same expected revenue for bid takers as would equilibrium strategies in oral progressive auctions, Dutch auctions or standard, first-price sealed bidding. Yet sealed second-price auctions seldom occur. We consider seven possible explanations for their rarity: the effects of bidder risk aversion, bidder asymmetry, inertia, the effect of the possibility of multiple offers from a single bidder, the possibility of bid taker cheating, the presence of affiliated or common values, and the behavioral disinclination and nonauction disincentives for bidders to follow truth revealing strategies. We argue that the possibility of bid taker cheating and the disincentives for bidders to follow truth revealing strategies are important explanations.

We present a simple model of auctions in which third parties capture a fraction of the economic rent revealed by the second-price procedure. In it, equilibrium strategies are modified so that on the average all of the cost of the captured economic rent is passed on to the bid taker. Furthermore, we point out that the logic that led to Myerson's revenue equivalence theorem applied to this model requires that with risk neutral bidders the average cost of revealed economic rent captured by third parties always be passed on to the bid taker at equilibrium in any symmetric independent private values model.

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## Introduction

Over a quarter of a century ago, Columbia University economics professor William Vickrey [1961] analyzed and compared four kinds of auctions. Three of them, oral progressive auctions, standard first-price sealed bidding and Dutch auctions, were in common use. The fourth was a sealed second-price procedure (now sometimes called Vickrey auctions) that he devised in order to have a sealed bid procedure that, in some ways, was "logically isomorphic" to oral auctions. In this procedure, the best bid would win, but the payment to its maker would be the amount of the best losing bid. His analysis showed that in an independent private values model with risk neutral bidders such sealed second-price procedures have several desirable properties.

First of all, in this context the equilibrium strategies are truth revealing. That is, the equilibrium strategy is for the bidder to bid his or her true cost or value. In addition, these truth revealing strategies are not only equilibrium strategies, they are dominant strategies. That is, it is optimal for a bidder to follow the truth revealing strategy even if he or she assigns a positive probability to the possibility that his or her competitors will deviate from their equilibrium strategies. Furthermore, truth revealing strategies simplify bid preparation. Because they are dominant strategies, they do not require the gathering or analysis of any information about the situation or intentions of competitors.

Second, at equilibrium the auction always leads to complete economic efficiency. The bidder with the highest value or the lowest cost always wins. There is no chance that a bidder with a higher value will misestimate the level of competition and lose the auction to a bidder with a lower value.

Third, Vickrey showed, in a result that has since been generalized by Myerson [1971], that if the bidders are symmetric, i.e. draw their independent private values from the same distribution, the expected revenue to the bid taker in all four kinds of auctions is the same with equilibrium behavior.

Vickrey's paper was a seminal work that led to a great deal of research by theoretical economists. McAfee and McMillan [1987] have recently written an able and extensive summary of the results of this research in the *Journal of Economic Literature*. In it, they write "William Vickrey's remarkable 1961 paper, two decades ahead of its time, is still worth reading as an introduction to the analysis of auctions." Yet in spite of the attention this paper has received since 1961 and in spite of the advantages of sealed second-price

procedures that Vickrey found, such procedures remain extremely rare, a fact discussed in more detail in the next section.

This paper inquires into the reasons for the infrequent use of Vickrey's proposed sealed second-price auctions. This concern is not just the result of intellectual curiosity, but arose in the course of attempting to aid government agencies to design an effective auction mechanism for the purchase by electric utilities of power from cogenerators and small renewable power sources [Rothkopf, *et al.*, 1987].

In this paper, we consider seven different possible reasons why Vickrey auctions might be rare. First, we argue that five of them are unpersuasive. One concern, considered by Vickrey himself, arises when multiple objects are to be sold in a single auction (as, for example, with treasury bills) and a single bidder wishes to bid on more than one item. Another concern, also considered by Vickrey, is the breakdown of the revenue equivalence result (but not the dominance of truth revealing strategies or of the economic efficiency result) in the face of asymmetry among bidders. Two other concerns, the effect of bidder risk aversion and the presence of nonindependent information have been discussed extensively in the literature that followed Vickrey's paper. We also consider the possible role of inertia.

Next, we consider two concerns that we do believe to be important explanations of the scarcity of Vickrey auctions. One of these is bidder concern about bid taker cheating, a factor discussed but not regarded as critical by Vickrey. The other, which we believe to be new to the literature, is bidder reluctance for both behavioral reasons and economic reasons related to subsequent transactions to use truth revealing strategies.

In order to study the economic disincentives for truth revealing strategies, we imbed a simple auction model in a context in which a fraction of the revealed economic rent of a second-price auction is captured by third parties. In this model, the presence of the loss of rent to third parties results in an adjustment to the equilibrium bidding strategies. The effect of this revision, on the average, is to pass on to the bid taker all of the loss of rent to third parties.

This result in our simple model is not driven by its simplicity. We point out that the logic of Myerson's powerful revenue equivalence theorem [1981] properly applied in the context of partial or total third party capture of revealed economic rent requires that in any independent private values model the average cost of that capture always be passed on the bid taker at equilibrium. The

revenue that is "equivalent" is the combined revenue of the bid taker and the third parties.

Before concluding this paper, we discuss the implications of our work about the kind of research needed to support the application of economic theory to the design of market mechanisms. We argue that the "low theory" task of including important factors in auction design models merits attention.

### **Reports of the Use of Vickrey Auctions**

In addition to the casual observations of people interested in auction theory, there are other reasons for believing that sealed second-price bidding is rare. In the introduction to his 1961 paper, Vickrey refers to "modification of current practices" in single item sealed-bid auctions and "departures from currently prevalent practices" in sealed-bid sales of multiple identical items. Nowhere in the paper does he indicate an awareness of any sealed-bid sales in which the award is at the price of the best losing bid. In section V. of his paper, in which he discusses the sale of multiple identical items, he does refer to and criticize an "alternative method" (to the usual first-price method) in which the price for all successful bidders is set at the level of the worst bid accepted. (Where there are many finely graduated bids, this procedure may in fact approximate closely Vickrey's preferred best-rejected-bid nondiscriminatory procedure.)

While Cassady's [1967] extensive survey of the usage of various kinds of auctions concentrates on oral procedures, it does discuss sealed bidding and it does not mention any sealed second-price procedures. Finally, many current sealed bid auctions of economic importance are first-price auctions. This includes federal oil lease and coal lease auctions, federal timber sales when done by sealed bid, sales of federal debt including treasury bills and a wide variety of transactions by the California State Lands Commission and the federal General Services Administration.

There are, however, some auctions that are essentially sealed second-price auctions. In some auctions of collectable items such as stamps and autographs, the auction involves both mailed in sealed bids based on a catalog listing as well as oral bids. In at least some of these, the mailed in sealed bids are explicitly upper limits to which the auctioneer, acting as the mail bidder's agent, may advance the bid, rather than a "standard" first-price bid. (See, e.g. Stampazine [1987], which states "Terms of Sale ... Each bid is executed at the indicated advance over the next lowest bid or the



starting bid....") In addition, at least one seller of collectable items conducts sales by catalog with second-price sealed bids and the only oral bids being by telephone [Americana Arts Auction, 1985].

In 1973 and 1974, under Secretary George Schultz (who has an economics degree from the University of Chicago), the U.S. Treasury experimented with nondiscriminatory sealed bidding in seven sales of treasury bonds. An important motive for using nondiscriminatory auctions was to attract into the competition relatively small buyers who normally purchased treasury bonds at a small markup from auction winners. Apparently, this did not occur. The use of nondiscriminatory auctions was abandoned after Schultz was replaced by William Simon and has not been resumed since. However, bidders interested in small quantities of treasury bonds may now submit "noncompetitive bids." These bids are filled at the average price obtained for the bonds sold competitively. Most bonds sold in each sale are sold competitively.

Recently, the California Public Utilities Commission considered the form of auctions to be used in the future for the purchase under long term contracts of electric power by California utilities from cogenerators and small power producers qualifying under the federal Public Utilities Regulatory Policy Act of 1978. They selected the sealed second-price auction format [California PUC, 1986] after hearing arguments based on those of Vickrey [Jurewitz, 1986; Vail, 1986]. However, since California utilities became committed to a large quantity of cogeneration power under a previous posted price procedure, no auctions have yet been held in California and no auctions are likely to be held for, at least, several years. In addition, auctions for similar purposes held in Maine and in Massachusetts have been first-price auctions.

Aside from the few examples mentioned above, we are not aware of any use of sealed second-price auctions.

### **Five Non-Reasons for the Rarity of Vickrey Auctions**

We have identified seven different potential reasons to account for the fact that Vickrey auctions are unusual. In this section, we discuss and reject five of them that we find unconvincing.

#### *Multiple Objects for Sale*

One potential objection to the use of sealed second price auctions is that two of their desirable properties--they have dominant truth revealing strategies and they are economically efficient--both break down if there are multiple items involved in

the auction and if any bidder wishes to bid for more than one item. This concern was discussed by Vickrey himself [1961] as well as by Dubey and Shubik [1980]. Dubey and Shubik have specified a modification of the sealed second-price procedure that restores the truth revealing nature of optimum strategies. However, the modification amounts to little more than an explicit recognition of the market power of bidders interested in more than one item, and, to our knowledge, it has not been tried. In particular, it has not been implemented in the trial treasury bill auctions and in the proposed California cogeneration auctions in spite of the interest of bidders in making multiple bids.

More fundamentally, we are convinced that it is not the key reason for the scarcity of Vickrey auctions because, if it were, we would expect to see many Vickrey auctions in which only single items are sold and, hence, in which it can be of no force.

#### *Bidder Risk Aversion*

Vickrey's results depend upon his assumption that bidders are risk neutral. It is now well established that, in an independent private values model with risk averse bidders, the bid taker can expect more revenue with a first-price auction than with a second-price auction [Harris and Raviv, 1984; Holt, 1980; Maskin and Riley, 1980; McAfee and McMillan, 1987; Riley and Samuelson, 1981]. Could this account for the rarity of sealed second-price auctions?

We think not. We do not doubt that bidders are often risk averse and that many bid takers would prefer more expected revenue to less. However, the interpretation of von Neumann Morgenstern risk aversion in this context is perverse. In particular, risk averse does not mean "cautious." Because of the independent private values context, the "risk" to which risk averse bidders are averse is the risk of not winning the auction. That outcome with its zero profit is assumed to be the worst possible event. There is no allowance, except, perhaps, in the private value, for any chance that the subject of the auction will be worth less to the bidder than he anticipated. (This suggests that a guaranteed shill-free oral auction might be reassuring to a risk averse bidder and produce higher bids.) In the context of sealed bidding, the cautious bidder may feel comforted by the safety margin built into his optimal bidding strategy in a first-price auction, but panicky about the chance that he will actually have to pay the true value his optimal second-price strategy calls on him to make. If he is more concerned about losing after winning the auction than about merely losing the auction, the standard theory for

risk averse bidders with independent private values may not describe his behavior. Thus, even if bidders are risk averse (and bid takers choose the auction form), it is not at all clear that they would actually choose sealed second-price procedures.

However, not only is it unclear that bidder risk aversion would lead bid takers to prefer sealed second-price auctions, it is not clear that bid takers get to choose the auction form without any other considerations. Bidders may be able to choose whether to participate, and that choice may be influenced by auction form. Most of the "high theory" on auction form summarized so well by McAfee and McMillan [1987] *assumes* that bidder participation is unaffected by the choice of form. Engelbrecht-Wiggans [1987] has recently argued that there are reasons for questioning that assumption. He shows the potential impact of a shift in a bidder's auction participation decision on the bid taker's decision on choice of reservation price. Furthermore, Rothkopf [1986] unearthed some old empirical evidence [Albion, 1961] that bid takers have indeed profited from increased auction participation due to their choosing an auction form that appealed to bidders. Hence, we are not persuaded that bidder risk aversion is an important reason for the scarcity of Vickrey auctions.

#### *Bidder Asymmetry*

Vickrey's revenue equivalence results do not hold if bidders are asymmetric in the sense that *a priori* one can make statements distinguishing bidders' relative value for the subject of the auction or their relative information situation. Vickrey analyzed and discussed this. He argued that since auction forms were not usually varied from auction to auction, there was unlikely to be a long term allocative difference. However, the Pareto optimality of the second-price auction becomes more important when the situation is asymmetric. We see no flaw in Vickrey's discussion. We do not believe that bidder asymmetry is likely to be a significant reason for bid takers to prefer first-price auctions. Furthermore, as we argued above in the discussion of risk aversion, we do not believe that bid taker preference in the context of models with a fixed number of bidder is necessarily controlling as to auction form.

#### *Nonindependent Values*

When Vickrey wrote his paper, the only academic discussions of auctions that considered the issue assumed independent private values. Later, other work appeared that developed models based upon an assumption that bidders had a common (but unknown) value

[Rothkopf, 1969; Capen, Clapp and Campbell, 1971; Oren and Williams, 1975]. Due particularly to the persuasiveness of Capen, Clapp and Campbell, this became the preferred form for modeling many auctions, especially offshore oil lease sales. In 1982, Milgrom and Weber generalized the range of assumptions considered by defining and obtaining results for a class of "affiliated" values. Roughly, if values are affiliated, then a high value estimate by one bidder is evidence for a higher value for all bidders. Common value models are a special case of affiliated value models.

Milgrom and Weber [1982] have shown that in an affiliated values model auctions can be rank ordered as to expected revenue to the bid taker at equilibrium. The rank order is first, (a somewhat artificial version of) an oral auction; second, sealed second-price bidding; and third, a tie between first-price sealed bidding and Dutch oral auctions. Hence, it is hard to argue that Vickrey auctions are rare because bid takers avoid them due to nonindependent values by bidders.

### *Inertia*

It can be argued that Vickrey auctions are rare because institutions are slow to learn and change. In other words, the rarity of Vickrey auctions is evidence of a kind of implementation problem. We have two reasons for disbelieving this argument. First, while institutions *are* slow to change, we doubt that they are *that* slow purely for reasons of inertia. It has been over a quarter century since Vickrey's paper appeared. During that time, there have been some experiments and some modifications of particular auction practices. Many more changes have been considered seriously. In addition, some completely new auction markets have been started.

Secondly, even the quarter century time scale is misleading. Most auction procedures that are common developed before any formal analysis recommended them. Hence, one must wonder why, if there are no problems with it, some auction market didn't stumble or evolve into sealed second-price bidding and recognize its advantages even before 1961.

### **Bidder Fear of Bid Taker Cheating**

There is evidence that robustness with respect to the possibility of cheating is more influential than optimality in the absence or cheating in determining auction form. Marc Robinson [1984, 1985] makes this case with respect to cheating by bidders. In his 1985 paper, he argues that standard sealed first-price bidding is sometimes used where, from the point of view of theory

developed on the assumption of no cheating, one would expect to find oral progressive auctions. This happens because agreements by bidders to collude in oral auctions are stable while in sealed first-price bidding they are not. While he discusses that argument primarily with respect to a comparison between oral auctions and sealed first-price bidding, he points out in a footnote that it still applies if sealed second-price bidding is substituted for oral auctions or if Dutch oral auctions are substituted for sealed first-price bidding. Presumably, a bid taker in an oral auction who fears collusion by bidders can switch to a first-price sealed bid system. If only some bid takers have that fear, then only some would switch, and there would be both oral and sealed first-price bidding.

In addition to the problem of bidder collusion discussed by Robinson, price enhancing activities by bid takers can be a concern of bidders. In some oral auctions, the use by auctioneers of shills and imaginary bids to force the price above the second highest bidder's value is notorious [Cassady, 1967, Chapter 12]. If a bidder fears that such tactics are being used against him, he may be reluctant to bid to his full value. Such strategic reluctance may be a wise strategy if the bidder has reason to believe that his intentions can be read by the auctioneer or will affect the auctioneer's behavior in future auctions. In an oral auction, a bidder at least has the opportunity to observe the proceedings while he is bidding, and he can drop out at any time if he suspects he is being victimized. In sealed second-price bidding, a bidder has no such ongoing protection. He must, if he follows his no-cheating equilibrium strategy, reveal his ultimate reservation price. If he fears that the bid taker will, after observing this price, insert an imaginary losing bid or a real losing bid from a confederate, then he has an incentive to bid strategically. Notice that actual cheating by the bid taker is not required to produce this result; mere fear of it (i.e. assigning a positive probability to it) will suffice.

Vickrey anticipated this concern and suggested that it might be countered by having the bids delivered to and certified by a trusted third party. However, even if the bid taker is scrupulously honest in his opening of the bids, a bid taker anticipating a bidder's intentions could solicit an insincere or artificially increased bid from a confederate. Such a bid would cost nothing if it loses. If, by miscalculation, it were to win (and do so at too high a price), the winning bidder may be able to withdraw it or the bid taker may be able to find grounds to reject it or to somehow compensate its maker. Again, even groundless fear of such behavior by a bid taker would be enough to induce bidders to abandon the truth revealing

dominant strategy of the no-cheating model in favor of strategic behavior.

### **Bidder Resistance to Truth Revealing Strategies**

People of our acquaintance with experience in conducting business are reluctant to reveal their true costs or valuations. They are strongly conditioned to keep such information confidential. Even in a situation in which such conditioning is maladaptive, it would have to be overcome. However, we believe that such conditioning will not normally lead bidders to err. Vickrey's model considers the auction as an isolated event. However, economically important auctions are seldom completely isolated events. A truth revealing strategy may give away valuable information. It could reveal to potential competitors the extent to which a firm's technology was superior. Most important, it could reveal to others with whom the firm must subsequently negotiate precisely how much it can yield.

In our recent work on the design of auctions for the purchase by utilities of electric power from cogenerators and other facilities qualifying under the Public Utilities Regulatory Policy Act [Rothkopf *et al.*, 1987], we realized that successful bidders have reason to anticipate extensive negotiations after the auction. In addition to negotiating details and arrangements with the utility awarding them the contract, most winning bidders will have to negotiate for financing, construction, government permits, and labor. In these negotiations, the winning bidders would be at a distinct disadvantage if the other party knew their true cost, especially if it were much less than the amount they were to receive.

Winning bidders in other much analyzed auctions also face subsequent negotiations with parties possessing significant market power. Successful oil lease bidders must deal with drilling contractors, rig owners, etc. Successful coal lease bidders must deal with equipment suppliers, railroads, and coal purchasers. Successful construction contract bidders must deal with subcontractors and labor unions.

Keeping winning bids secret is a potential way around this difficulty. There are two problems, however. First, secrecy may defeat the public scrutiny that is needed to assure the bidders or the general public of the honesty and fairness of the process. Second, secrecy is never complete. Secret information tends to give power to its holder, and even a small chance of a breach of secrecy justifies a deviation from the dominant truth revealing strategies of the isolated auction model.

## A Model of a Vickrey Auction with Partial Loss of Revealed Rent

In the previous section, we argued that bidders may have good reasons for resisting the use of truth revealing strategies. One of the arguments we offered was that a truth revealing strategy imposes on a successful bidder a disadvantage in subsequent negotiations. This section considers a simple model of a low-bid-wins second-price auction in which the winning bidder must negotiate with third parties such as labor unions or permitting authorities. We assume that the third parties have some market power and that in addition to whatever else they may charge, they also extract some fraction of the economic rent of the winning bidder as revealed by the difference between his winning bid and the amount he gets paid under the second-price procedure. In our model, the bidders' equilibrium strategies take account of the effect of their bids on the winner's subsequent negotiations. After presenting the model, we point out that a key result in it, that at equilibrium all of the expected cost of the captured rent is passed on to the bid taker, could be anticipated from the application of the logic of Myerson's 1981 revenue equivalence theorem and that this result therefore applies to a broad class of symmetric auction models.

Our model is extremely simple. We assume a low-bid-wins auction with two bidders. Each bidder independently and privately learns his exact basic cost should he win the auction. *A priori*, the cost for each is independently and uniformly distributed from 0 to 1. Each bidder then uses a strategy for his bid that is an increasing function of his basic cost and that is independent of the still unrevealed cost of his competitor. The auction is a second-price auction that awards the contract to the low bidder at the price of the higher bidder. However, third parties with whom the bidder must negotiate may learn of the difference between the low bid and the contract price and, on the average, are able to extract some fraction,  $\alpha$ , of this difference from the winner. The bidders know that this may happen and take account of its possibility in deciding upon their bids. We assume that each bidder is risk neutral and, thus, chooses to maximize his expected profit from the auction. We seek a symmetric set of Nash equilibrium strategies in which neither bidder can unilaterally improve his expected profit.

Mathematically, we have basic cost  $c_i$ ,  $i = 1, 2$  for the two bidders. It is uniformly and independently distributed on  $[0, 1]$ . Bidders follow strategies  $b_i(c_i)$ ,  $i = 1, 2$ , that are increasing

functions of  $c_j$  with inverse functions  $b_j^{-1}(\cdot)$ . When bidder  $i$  has cost  $c_i$ , his expected profit is given by

$$E[\pi_i(c_i)] = \int_{b_j^{-1}(b_i(c_i))}^1 [b_j(c_j) - \alpha(b_j(c_j) - b_i(c_i)) - c_i] f(c_j) dc_j,$$

$i, j=1, 2; j \neq i$ .

In this expression, the square brackets contain bidder  $i$ 's profit if he wins with a bid of  $b_i$  when bidder  $j$ 's bid is  $b_j(c_j)$ . The quantity  $f(c_j)$  is the uniform probability density that bidder  $j$  has cost  $c_j$ . It is 1 on the interval  $[0, 1]$ . The integral is over those values of  $c_j$  that will lead to bidder  $i$  winning.

#### *The Model's Solution*

The derivative of  $E[\pi_i(c_i)]$  with respect to  $b_i$  is given by

$$\frac{dE[\pi_i(c_i)]}{db_i} = \int_{b_j^{-1}(b_i(c_i))}^1 \alpha f(c_j) dc_j - [b_j(b_j^{-1}(b_i)) - \alpha(b_j(b_j^{-1}(b_i)) - b_i(c_i)) - c_i] f(b_j^{-1}(b_i)) \frac{db_j^{-1}(b_i)}{db_i},$$

$i, j=1, 2; j \neq i$ .

Setting this derivative equal to zero for  $i = 1$  and  $i = 2$ , using the symmetry condition

$$b_1(c_1) = b_2(c_2) \equiv b(c),$$

and the relationships

$$b_i(b_i^{-1}(c_i)) = c_i, \quad i = 1, 2,$$

and

$$\frac{db_i^{-1}(b_i(c_i))}{db_i} = \frac{1}{b_i'(c_i)}, \quad i = 1, 2,$$

and simplifying gives the differential equation that a symmetric equilibrium strategy  $b(c)$  must satisfy:



$$\alpha(1-c)b'(c) = b(c) - c.$$

It may be verified that the solution of this equation is

$$b(c) = \frac{\alpha + c}{\alpha + 1}.$$

When both bidders follow this strategy, a bidder whose cost is  $c$  has an expected profit

$$E[\pi(c)] = (1 - c)^2/2.$$

This quantity is independent of  $\alpha$ . Hence, all of the rent captured by third parties is passed on to the bid taker.

*A priori*, before learning of his cost, each bidder has a 50% chance of winning and an expected profit, independent of  $\alpha$ , of

$$\int_0^1 \frac{(1-c)^2}{2} f(c) dc = 1/6.$$

The expected cost of the lower cost bidder is one third. When  $\alpha = 0$ , the expected payment of the bid taker is  $1/2 + 2(1/6) = 2/3$ .

With equilibrium bidding, the expected value of the higher bid is  $(3\alpha + 2)/3(\alpha + 1)$ , and the expected value of the lower bid is  $(3\alpha + 1)/3(\alpha + 1)$ . The expected difference between the bids is  $1/3(\alpha + 1)$ , and the expected payment to the third parties is  $\alpha$  times this amount:  $\alpha/3(\alpha + 1)$ . As a fraction of the cost to the bid taker, this cost is  $\alpha/2(\alpha + 1)$ . Thus, if third parties can extract 10% of the difference between the bids, the extra cost to the bid taker is 4.5%. If they can extract half of the difference, the extra cost is 16 2/3%, and if they can extract it all, the added cost is 25%.

### A General Result

As mentioned above, the highlighted result with this model--namely, that the entire cost of the revealed economic rent captured by third parties is, on the average, passed on to the bid taker--is not an artifact of some peculiarity of the particular model we have chosen to analyze. Rather, it is quite general. Myerson [1981]

considers a rather general single object auction model with  $n$  risk neutral bidders. In it, the bidders share a commonly known joint prior distribution on private value signals and have utility functions that depend upon three arguments: the private signal, the probability of winning the auction, and the payment to or by the bid taker. The joint prior distribution is unrestricted except that each bidder's signal is bounded above and below. Myerson considers auction mechanisms characterized by two vectors with one component for each bidder: one of win probabilities,  $p$ , and one of expected payments to the seller,  $x$ . Each of these vectors is a function of the vector of private signals. Myerson proves that for any feasible auction mechanism, there exists a feasible "direct revelation mechanism" (i.e. a scheme in which the outcome is based upon the revelation of his signal by each bidder and in which each bidder has the incentive to reveal his signal truthfully) which is equivalent in that it gives to the seller and each bidder the same expected utilities.

Restricting himself to such direct revelation mechanisms, Myerson then proves a theorem which implies that once we know who gets the object being auctioned in each situation (i.e. the vector  $p$ ) and how much utility each bidder would get if his value estimate were at its lowest possible level, then the seller's expected utility from the auction does not depend upon the payment function  $x$ . In particular, the seller must get the same expected utility from any auction mechanisms for which (1) the object always goes to the bidder with the highest value above a prespecified reservation price, and (2) any bidder with the lowest possible value signal expects zero utility. This implies that the seller gets the same expected revenue in any symmetric situation (in which zero value is considered a possible signal) regardless of the auction form provided only that it leads to equilibrium bidding strategies that increase with the value signal. This, of course, includes standard sealed bidding and Vickrey auctions. This is a crude summary of Myerson's "Revenue-Equivalence Theorem."

Myerson, however, does not consider models in which there are payments to third parties that depend upon the auction form. If such payments are included, then his Revenue-Equivalence Theorem still applies except that its application is to the combined revenue of the bid taker and the third parties. That is what is invariant to auction form. Hence, under the conditions considered by Myerson, modified for payments to third parties that depend upon the auction form, *the expected amount of any payment to third parties comes from the bid taker*. With such third party payments, it is the bidders' expected

revenue rather than that of the bid taker that is invariant across auction forms.

### **Thoughts on Research on Auction Design**

Our work designing auctions for the purchase of electric power and our consideration of the reasons for the scarcity of Vickrey auctions have led to some thoughts on fruitful directions for research in auction design. In recent years, there has been a magnificent flowering of mathematical analysis of models of single isolated auctions. We do not doubt the value of this research. In particular, this paper has benefitted greatly from Myerson's work. However, we believe that more emphasis on formulating rather than optimizing is called for and that the practical value of the conclusions of some mathematical research with respect to "optimal auctions" is suspect.

There are many critical assumptions in most auction models. For example, the assumption of a single isolated auction is almost in direct contradiction to an assertion that the auction is part of an important stream of commerce. It is useful to study threads in order to improve clothing, and it is useful to study bricks in order to improve buildings. However, clothing designers would have reason to be suspicious of any conclusions on "optimal design of threads" that were independent of the intended garment, and architects would have reason to question results on "optimal bricks" that were independent of the building design and planned construction methods. So too is it with auctions. It is useful to study the effects of varying auction rules in mathematical models of an isolated auction, but the "optimal auction" is likely to be context dependent.

We believe that the most important undone research of direct importance for the design of auctions has to do with identifying and including, even crudely, in auction models considerations currently neglected. We believe that this "low theory" will add to the practicality of auction design modeling and may well lead eventually to an enriched mathematical "high theory."

### **Conclusions**

We believe that Vickrey auctions are rare for two reasons. First, they are rare because robustness in the face of cheating and of fear of cheating is more important in determining auction form than are properties related to economic efficiency and allocation in the absence of cheating or fear of it. Vickrey auctions are not robust with respect to cheating and fear of cheating.

Second, Vickrey auctions are rare because bidders are reluctant to follow the truth revealing strategies that the "proper" operation of such auctions would require. We have shown that bidders have good reasons to be reluctant when they may lose a fraction of the economic rent revealed by the sealed second-price format in subsequent negotiations. We have pointed out that, in equilibrium in auctions with symmetric, risk neutral bidders, the entire cost of this capture of revealed rent is borne on the average by the bid taker.

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