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Imperfections in solids are important in the pinning of both superconducting filaments[1] and fluxoids,[2] the two limiting forms in the mixed-state. According to Anderson,[3] flux penetration in the mixed-state is a thermally activated process with an activation energy resulting from inhomogenities, strains, dislocations, and other physical defects. An external field aids the motion of "bundles" of flux over these energy barriers and plays a crucial role in determining the critical current density.[4] Friedel et.al.[5] and Silcox and Rollins[6] have investigated the pinning of fluxoids by cavities as found, for example, in sintered compacts. Recently, Webb[7] has analyzed the pinning forces due to forests of screw dislocations on fluxoids and single screw dislocations on coaxial and parallel superconducting filaments. Webb's model is based on the small differences in the elastic compliances and moduli between the normal and superconducting phases; these differences result in a small free energy density difference and a pinning force. Evidence for a dislocation stabilized filamentary network has been found recently.[8]

It is often assumed[9] that continuous defect structure such as a dislocation network is needed to pin a continuous filamentary phase. We show here that an array of isolated composition fluctuations[10] can result in localized stress fields capable of pinning a continuous filamentary phase.

Since the stress fields surrounding volume imperfections vary approximately with the inverse cube of the distance (as compared to inversely with the distance for dislocations)the strain energy will be highly localized. The magnitude of the strain energy and the pinning forces on both superconducting filaments and fluxoids will be estimated and compared to those estimated for other pinning mechanisms.

Our model material contains "regions of composition fluctuations"[10] which are represented as spheres of radius "a" separated from the other similar spheres by an average distance of 40 a. The superconducting filaments (or the fluxoids) are present in the form of cylinders with an effective radius "c" and the cylinders extend from the region of one sphere to the next. A stress field will necessarily surround regions of composition fluctuation when the types of atoms composing the alloy are of different sizes. Precipitation of a second phase would tend to relieve this stress. Although we are primarily interested in the effects of the stress field, it is also necessary to distinguish between three types of composition fluctuations: those with a localized critical temperatures equal, greater, or less than that of the material in the matrix. For the first two types, the superconducting filaments will be most stable when the center of the sphere coincides with the axis of the superconducting filament. In the last type the region of composition fluctuation will tend to trap flux while the strain field will tend to trap superconducting filaments in the same region. To approximate the last type we shall determine the pinning force on a superconducting filament when its axis is separated from the center of the sphere by a distance $b = a + c$.

To determine the interaction energy U between a superconducting filament and a stress field surrounding a spherical imperfection, it is convenient to express the strain energy in terms of the compressibility χ and Poisson's ratio ν. Friedel[11] has derived this relationship for spherical cavity. For the case $b \geqslant a + c$ the interaction energy is

$$U = 36 \frac{\Delta\chi}{\chi} \frac{(1-2\nu)}{(1+\nu)} \frac{a^4(a-r_s)^2}{\chi} \int_0^{z_{max}} dz \int_{b-c}^{b+c} r\,dr \int_0^{\cos^{-1}\left[\frac{r^2+b^2-c^2}{2rb}\right]} \frac{d\theta}{(z^2+r^2)^3} \qquad (1)$$

where the integration is over the superconducting volume. Here the origin is taken at the center of the sphere and z is parallel to the filament. We define r_s as the radius the sphere would assume if it contained an equal number of stress-free atoms of the same composition as the matrix and $\Delta\chi = \chi_s - \chi_n$.

Integrating Eq. (1) yields

$$U = 2.25\,\Pi \frac{\Delta\chi}{\chi} \frac{(1-2\nu)}{(1+\nu)} \frac{a^4(a-r_s)^2}{\chi} \left\{ \left[\frac{b^2+7c^2}{3(b^2-c^2)^3}\right] \left[2bE\left(\frac{\Pi}{2}, \frac{c}{b}\right) - \left(\frac{b^2-c^2}{b}\right) F\left(\frac{\Pi}{2}, \frac{c}{b}\right)\right] - \frac{1}{3(b^2-c^2)b} F\left(\frac{\Pi}{2}, \frac{c}{b}\right) \right\} \qquad (2)$$

Here F and E are Legendre normal integrals of the first and second kind. In performing the integration we introduced a small approximation by changing $Z_{max} = 20$ a to infinity. The pinning force $F_c = \frac{\partial U}{\partial b}$ is

$$F_c = 2.25\Pi \frac{(1-2\nu)}{(1+\nu)} \frac{\Delta\chi}{\chi} \frac{a^4(a-r_s)^2}{\chi} \left\{ \frac{2b^2+14c^2}{(b^2-c^2)^3} F\left(\frac{\Pi}{2}, \frac{c}{b}\right) - \frac{2b^4+28b^2c^2+2c^4}{(b^2-c^2)^4} E\left(\frac{\Pi}{2}, \frac{c}{b}\right) \right\} \qquad (3)$$

For the region $b \geqslant a + c$, F_c has its maximum value at $b = a + c$. The magnitude of F_c can be estimated using typical values of $a = 10^{-5}$ cm, $c = 10^{-5}$ cm, $\Delta\chi/\chi = 4 \times 10^{-6}$ (Aler's and Waldorf's[12] values for Pb at 4.2°K) $\chi = 5 \times 10^{-13}$ cm^2/dyne, $\nu = 1/3$, and $|\frac{a-r_s}{a}| = .02$. Substitution of these values into Eq. (3) yields $F_c = 1.8 \times 10^{-3}$ dyne/cm. when the spacing of these spheres is 40 a. The larger the sphere and the greater the difference in atomic size, the larger F_c becomes.

For the case where a = c and b = 0, we find

$$U = 4.9\pi \frac{(1-2\nu)}{(1+\nu)} \frac{a(a-r_s)^2}{\chi} \frac{\Delta\chi}{\chi} + \frac{6\pi a(a-r_s')^2}{\chi'} \frac{\Delta\chi'}{\chi'} \qquad (4)$$

Here χ' is the compressibility within the sphere, $\Delta\chi' = \chi'_s - \chi'_n$, and r_s' is the radius the sphere would assume in its stress-free state. The first term on the right is the difference in strain energy contained in the matrix and the last term is that difference in the sphere.[11] To avoid geometric complications, we shall underestimate the pinning force and define

$$\overline{F}_c = \frac{U(d=2a) - U(d=0)}{2a} .$$

Setting $\chi' = \chi$, $\frac{\Delta\chi'}{\chi} = \frac{\Delta\chi}{\chi}$, $|\frac{a-r_s'}{a}| = \frac{1}{2} |\frac{a-r_s}{a}|$ and using the above values yields $F_c = 3.2 \times 10^{-3}$ dyne/cm.

These values for F_c are of the same order of magnitude as those found for dislocations.[7] Here the regions of composition fluctuations are separated from one another by 4 microns, while in the dislocation model the dislocations and filaments are coaxial.

Pinning of fluxoids[13] in the critical state results from their repulsion from the stress barriers of the imperfections. Webb estimates that the value of the pinning force on fluxoids by a "forest" of dislocations is of the same order of magnitude as that due filament pinning by a single screw dislocation. Although we expect the pinning by the stress field due to groupings of these regions of composition fluctuations to be of comparable value, there is, however, a significant difference. The fluxoid may become trapped on the region of composition fluctuation when the T_c of the material in this region is less than that of the matrix. Livingston[14] has explained his magnetization results on the Pb-Sn system in this manner.[15]

It has been suggested on the basis of experiments with heat treated niobium-zirconium alloys[16] and age-hardened aluminum[17] that the stress field around pre-precipitation zones (a more specific form of composition fluctuation) is important in determining critical current densities and hysteresis effects. Our calculations would seem to substantiate these suggestions.

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