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PAGE

PAGE	
5	Add minus sign before $\|\pi\rangle$ and $\|K\rangle$ on right hand side of (5a) and (5b)
10	Equation 19 - add cross - $M \propto (\epsilon_1 \times \epsilon_2)$ etc.
14	Equation 29 $\frac{G_i}{G_f} = -(-1)^{(I_f - I_i)}$
20	Equation 34 - the middle factor should read: $\left(\frac{\Delta/m_\pi}{\Delta^2 - M^2_\pi}\right)^2$
24	First sentence after equation 38 add a small zero over the π. $M \to 3\pi^0$.
27	Equation 47 - add delta sign before C and G. i.e. ΔC= No; Δ G = Yes, etc.
29	Fifth line down in paragraph C - (47a) change to (48a). Ninth line down in Paragraph C, L = ℓ, not L = 1.
41	Table III change N (1520) to (1512); also change N_a (1588) to (1688).

STRONGLY INTERACTING PARTICLES AND RESONANCES

Contents

STRONGLY INTERACTING PARTICLES AND RESONANCES*

Arthur H. Rosenfeld

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August 1962

INTRODUCTION

I want to start with a few words about terminology. I will use the word "particle" to include both stable particles and "resonant states" which can decay rapidly via the strong interaction into other particles. Hence a precise but less conventional title for this course would be "Strongly Interacting Particles: Bound and Unbound." Notice that I have altogether avoided the word "elementary."

The most familiar example of an unbound state is the I = 1/2, J = 1/2 pion nucleon resonance. In this case there is only one decay channel, and we can show that the pion-nucleon scattering phase shift goes through 90° at the resonance; if there were more than one channel we could still show that the scattering amplitude becomes pure imaginary at the resonance.

An example of a slightly bound system is the deuteron. Precisely because it is slightly bound its properties tend to be those of the sum of its consitutents, and we tend to think of it as a "composite" system.

An example of a tightly bound system is the pion, considered as a bound state of a nucleon and an antinucleon. Its binding energy is so great ($m_\pi \ll 2m_N$) that the new system has properties completely different from its constituents, and at this moment in history tends to be thought of as "elementary."

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* Lectures given in the course on "Elementary Particles," Enrico Fermi International School of Physics, July 23 through August 4, 1962, at Varenna, Como, Italy. (To be published by the Italian Physical Society in the Proceedings of the Varenna summer school.)

It is best to classify a particle by properties other than its decay via any particular channel; thus it would be an incomplete statement to say that th Y_0^* of mass 1520 MeV is a $\Sigma\pi$ state, because this neglects all other possible final states. In fact, the Y_0^* (1520) decays into $\Sigma\pi$ (about 60%), $\bar{K}n$ (about 30%), and $\Lambda\pi\pi$ (about 10%).

I. THE FOUR BASIC INTERACTIONS AND THEIR QUANTUM NUMBERS

Since there are many particles and only four interactions it is better to discuss first the interactions. Anyway I feel that eventually the interactions will explain the particles rather than vice versa.

In nature only four fundamental interactions exist: gravitational, weak, electromagnetic, and strong. It is generally assumed that all four interactions obey the following symmetries and conservations laws,[1] even if they are not experimentally tested for all the interactions:

(a) conservation of linear and angular momentum.

(b) conservation of electric charge.

(c) conservation of baryon number B and lepton number L.

(d) invariance under CPT.

The operator C when applied to a single particle in its own rest frame transforms the particle into the corresponding antiparticle. (Thus we assume that every particle has an antiparticle which may or may not be distinct). For some purposes it is convenient to think of an antiparticle as a hole in a negative-energy particle sea. The C concept is useful mainly for a system of non-strange neutral mesons, discussed more fully later.

(e) Invariance uner T (or CP, because of d).

The notion of a moving picture offers a simple physical model of the time-reversal operation. The time-inverted situation is obtained by running the movie backward. Time-reversal invariance requires that to an observer who does not know the initial conditions the inverted situation makes sense.

The time-reversal operation is meaningful only for microscopic systems, not for the large ensembles that are governed by statistical as well as microscopic mechanics.

Additional conservation laws are obeyed by some but not all of the four interactions. These we take up next.

A. Strong Interaction

1. Orders of Magnitude

The strong (nuclear) interaction has the following characteristics:

a. The range is short. Its order of magnitude is given by the pion Compton wavelength,

$$\lambda_\pi = \frac{\hbar}{m_\pi c} = 1.4 \times 10^{-13} \text{ cm} = 1.4 \text{ Fermi.} \tag{1}$$

b. The energy is large. For example, nuclear bindings run in tens of MeV and the production of mesons in hundreds of MeV.

c. The natural unit of time for strong interactions is given by the time it takes for a light signal to cross a distance equal to the range of nuclear forces:

$$\tau = \frac{\hbar}{m_\pi c^2} \approx \frac{1}{3} \times 10^{-23} \text{ sec.} \tag{2}$$

d. If a reaction takes place in a time τ the corresponding full width at half maximum Γ of its Fourier transform is

$$\Gamma = \frac{\hbar}{\tau} = \frac{2/3 \times 10^{-21} \text{ MeV sec}}{\tau}.$$

A more useful form is

$$\Gamma = \frac{\hbar c}{\tau c} = \frac{197 \text{ MeV F}}{\tau c}. \tag{3}$$

If we take, for example, $c\tau = 1$ F, we get $\Gamma = 200$ MeV. (Notice that if the particle has mass m and is produced with momentum $p = \eta mc$, then it actually goes an average distance $\eta c\tau$ from its point of prod[illegible] If η

is large, this factor can help the resonance get out of the range of nuclear forces before it decays.) The $\frac{3}{2}, \frac{3}{2}$ pion-nucleon resonance Δ has $\Gamma_\Delta \approx 100$ MeV (i.e., $c\tau \approx 2$ F). Similarly the ρ meson has $\Gamma_\rho \approx 100$ MeV. But the ω meson has $\Gamma_\omega \leq 15$ MeV and probably $\Gamma_\omega \approx 1$ MeV. This means a corresponding $c\tau \approx 200$ F, which is "outer space" in comparison with the dimensions involved in nuclear interactions.

2. Additional Conservation Laws for the Strong Interactions

The student who needs further basic information on these conservation laws will find a helpful exposition in a review article by Wick.[2]

a. Conservation of Isotopic Spin I (both $|I|$ and I_z).

Figure 1 shows the particles that are stable against decay via the strong interaction. Most of them have been known for many years, although the η meson was not discovered until 1961, and its quantum numbers were sorted out only in 1962. The particles without any strong interaction (photon and leptons) are shown as thin bars; thick bars represent the strongly interacting particles (mesons and baryons); whose grouping into multiplets is evident. This grouping suggested that all members of the multiplet shared a new quantum number I, called isotopic spin. For the rest of this text we refer to it for short as ispin, a conserved vector in "ispace." The projection of I along the "charge axis" was called I_z, and gave the electric charge in units of $|e|$,

$$Q = \frac{Y}{2} + I_z. \tag{4}$$

The constant Y is called the "hypercharge," since it measures the "center of charge" of a multiplet. For mesons the hypercharge and the "strangeness" are the same thing. For the nucleon doublet, though, we have

$$Q = \frac{1}{2} + I_z,$$

i.e., Y = 1. However, since nucleons are familiar, we like to say they have zero strangeness, i.e., we invent the relation

$$Y = S + B,$$

where S is the strangeness and B the baryon number.

In summary, both Y and S are used to describe the position of the center of charge of a multiplet, but Y is used to relate its displacement from zero charge, and S is used to relate its displacement from the center of charge of the "nonstrange" particles.

b. Separate Invariance Under C and P; G Parity

We define parity P as the operation that reflects space coordinates. Thus the parity of a wave function of orbital angular momentum ℓ is $(-1)^{\ell}$. As discussed in any text on particle physics, the intrinsic parity of both the π meson and the K meson has been determined experimentally to be odd.[1b] We discuss the parity of fermion-antifermion pairs in connection with Eq. (18).

$$P\,|\pi\rangle = |\pi\rangle \,, \tag{5a}$$

$$P\,|K\rangle = |K\rangle \,. \tag{5b}$$

We define charge conjugation, C, as the operation that transforms particles into antiparticles and vice versa; thus electrons e^- transform into positrons, e^+, while π^+ transforms into π^- and π^0 into itself. As W.S.C. Williams has pointed out,[1a] a more satisfactory name for this operation would be particle-antiparticle conjugation, as there is not always a change in electric charge.

We want the photon to have the same behavior under C as electric or magnetic fields and currents, namely

$$C|\gamma\rangle = -|\gamma\rangle \,. \tag{6}$$

Since π^0 decays electromagnetically into two identical particles (two γ rays), and since C is conserved by the electromagnetic (em) interaction as well as by the strong interaction, we have

$$C|\pi^0\rangle = +|\pi^0\rangle. \quad (6a)$$

C applied to charged particles is not an obviously useful operation, since they are not in eigenstates of C, and hence C alone can yield no new selection rules. Therefore for most charged particles we do not bother with C except to note $C^2 = +1$, so that $[C, H] = 0$, where H is the strong or electromagnetic Hamiltonian. But because of G, which is defined in (7) explicitly for pions, both charged and neutral, we want to adopt the convention

$$C|\pi^{\pm}\rangle = +|\pi^{\mp}\rangle, \quad (6b)$$

in analogy with (6a). This choic4 is arbitrary; we could have chosen $C|\pi^{\pm}\rangle = e^{\pm i\delta}|\pi^{\mp}\rangle$, but (6b) is simpler.

(i) G Parity

Next we wish to take advantage of simultaneous conservation of C and I to derive a new conserved quantity G, first introduced by Lee and Yang[3] and Michel.[4] They defined G as $Ce^{i\pi I_y}$, i.e., a charge conjugation and a 180° rotation in ispace around I_y. Actually, with C defined as it is above, we must redefine G as

$$G = C \exp(i\pi I_x). \quad (7)$$

The reason for introducing G is readily seen. Given a multiplet with $B = S = 0$, only the neutral component can be in an eigenstate of C, since C reverses charges. However, the rotation about I_x or I_y again reverses charges, so that the whole multiplet can be an eigenstate of G. Notice that C performs a reflection in ispace, and $\exp(i\pi I_x)$ is a rotation. Hence G has the properties of a parity operation in ispace--and Wick calls it "isotopic parity."[2]

Next we wish to prove Eq. (8), i.e., that for any system of n_π pions, $G = (-1)^{n_\pi}$:

To do this it is convenient to represent the pion (which has unit angular momentum in ispace) by the spherical harmonics in ispace, i.e.,

$$|\pi^0\rangle = Y_1^0 \propto \cos\theta = z,$$

$$|\pi^+\rangle = Y_1^1 \propto -\sin\theta\, e^{i\phi} = -x - iy,$$

$$|\pi^-\rangle = Y_1^{-1} \propto +\sin\theta\, e^{-i\phi} = +x - iy.$$

Using the C parity of π from (6a) and (6b), we then have

$$G|\pi^0\rangle = C\exp(i\pi I_x)\ |z\rangle = C|-z\rangle = C|-\pi^0\rangle = -|\pi^0\rangle, \text{ and}$$

$$G|\pi^\pm\rangle = C\exp(i\pi I_x)\ |\mp x - iy\rangle = C|\mp x + iy\rangle = C|-\pi^\mp\rangle = -|\pi^\pm\rangle.$$

Then G changes the sign of each pion, and consequently

$$G = (-1)^{n_\pi}. \qquad \text{Q.E.D.} \tag{8}$$

Since C and I are conserved in strong interactions, so is G; consequently an even number of pions cannot transform into an odd number (and vice versa), therefore pion vertices in Feynman diagrams must consist of an even number of pions. We can now repeat the above discussion of the effect of operating with $\exp(i\pi I_x)$ on $Y_I(z)$ to obtain a different result for <u>any</u> particle in an eigenstate of C(i.e., any nonstrange neutral meson with arbitrary ispin I). The symmetry of $Y_I^0(z)$ is $(-1)^I$, i.e., $\exp(i\pi I_x)\ Y_I^0 = (-1)^I\ Y_I^0$; thus we have the result

$$G = C(-1)^I \tag{9}$$

for neutral nonstrange mesons. Once established for the neutral member of a multiplet, G applies for the whole multiplet.

c. Particle-Antiparticle Systems

(The rule $CPX_s = +1.$)

(i) Case I. Boson-antiboson pairs

For two identical spinless bosons such as $2\pi^0$ the wave function must be symmetric under the operator X, which exchanges these two particles; i.e.,

$$X = +1. \tag{10}$$

If the bosons are charged, with charge Q, and have spins, we can write a wave function of three variables,

$$\psi = \psi(r)\psi(Q)\psi(S), \tag{11}$$

where $\vec{S} = \vec{S}_1 + \vec{S}_2$ is the diparticle spin. Then

$$X\psi = X_r\psi(r)\, X_Q\psi(Q)\, X_S\psi(S) = P\psi(r)\, C\psi(Q)\, X_S(\psi_S),$$

i.e.,

$$X = PCX_S.$$

This generalized ψ must again be symmetric under X, since boson field operators commute. Therefore

$$X = PCX_S = +1. \tag{12}$$

If $\psi(r)$ is an ℓ wave,

$$P = (-1)^{\ell}. \tag{13}$$

Questions of intrinsic parity and C do not arise, since we have two bosons, and $P^2 = C^2 = +1$.

The symmetry of $\psi(S)$ is $(-1)^{S_1+S_2-S} = (-1)^S$, so

$$X_S = (-1)^S; \tag{14}$$

(12) then becomes

$$C = (-1)^{\ell + S}. \tag{15}$$

We can use (15), for example to calculate C for the ρ^0 meson, which decays into $\pi^+ + \pi^-$ in a p wave. For spinless boson-antiboson pairs (15) is

$$C = (-1)^{\ell}, \tag{15a}$$

i.e., $C_{\rho^0} = -1$.

Application of $C = (-1)^{\ell}$ to K_1K_1 vs K_1K_2 pairs

Consider a nonstrange $K^0\overline{K}^0$ pair produced with relative angular momentum ℓ. The K meson is spinless, therefore (12) becomes

$$PC = +1. \tag{16}$$

Now, it is well known that neutral K mesons decay via the weak interaction not as K^0 or $\overline{K}^0$ but in eigenstates of CP called K_1 and K_2. Thus

$K_1 \rightarrow 2\pi$ in an s-wave, therefore $P = +1$, $C = +1$, $CP = +1$;

K_2 cannot $\rightarrow 2\pi$ and has $CP = -1$.

If CP is applied to $|K_1K_1\rangle$ or $|K_1K_2\rangle$ we have

$$CP\,\psi_1(r)\,|K_1K_1\rangle = (-1)^{\ell}\ CP\,|K_1\rangle\ CP\,|K_1\rangle = +(-1)^{\ell}, \tag{17a}$$

$$CP\,\psi_1(r)\,|K_1K_2\rangle = (-1)^{\ell}\ CP\,|K_1\rangle\ CP\,|K_2\rangle = -(-1)^{\ell}. \tag{17b}$$

Combining (17) and (16), we see, for $K^0\overline{K}^0$ systems,

even P (= even C) requires decay via K_1K_1 and K_2K_2

odd P (= odd C) requires decay via K_1K_2.

For further discussion, see Reference 5.

(ii) Case II. Fermion-Antifermion ($f\overline{f}$) Pairs

It so happens that Eqs. (12) and (15) also apply to $f\overline{f}$ pairs. To explain this we must remind the reader of an important minus sign that enters in the parity of $f\overline{f}$ pairs. For $f\overline{f}$ pairs in an orbital ℓ wave, the parity is

$$P = -(-1)^{\ell}. \tag{18}$$

There are two ways that we can understand this minus sign:

(a) A modern theoretical explanation is given by Stapp.[6]

(b) Purely experimentally, we can note that when positronium annihilates from the $^1S_0$ state into two photons, they are linearly polarized perpendicular to each other.[7] We shall now show from this fact that $^1S_0$ must be a pseudoscalar (0^-). We shall use this sort of argument many times. It runs as follows. The matrix element M for the process must involve linearly a vector for each photon created, and a scalar or pseudoscalar $0^{\pm}$ representing the annihilated $^1S_0$ state. For the two photon vectors we choose their polarization directions $\underline{\epsilon}_1$, $\underline{\epsilon}_2$. The matrix element M must be a scalar quantity. In addition to $\underline{\epsilon}_1$, $\underline{\epsilon}_2$, $0^{\pm}$ it can contain the photon momentum $\underline{k}$ to any power. If experimentally $\underline{\epsilon}_1$ and $\underline{\epsilon}_2$ are perpendicular to each other, M must contain $(\underline{\epsilon}_1 \times \underline{\epsilon}_2)\ 0^{\pm}$, where $\underline{\epsilon}_1 \times \underline{\epsilon}_2$ is an axial vector. To make a scalar quantity we must write

$$M \propto (\underline{\epsilon}_1 \quad \underline{\epsilon}_2) \cdot \underline{k}\ 0^- . \qquad (19)$$

i.e., the parity of $^1S_0$ positronium must be negative. Q.E.D.

It can further be shown that (15) holds not only for e^+e^- but also for all $f\bar{f}$ pairs.[8]

Equation (18) comes up particularly often because antiprotons coming to rest in hydrogen are captured in s states. Hence we know that they are captured from negative parity states. The prediction that π^-, K^-, and $\bar{p}$ should be captured from s states was first made by Day, Snow, and Sucher, in 1959.[9] It was tested experimentally for π^- in 1960 by Fields et al.[10] and more recently by Hildebrand.[11] In 1961 M. Schwartz[12] pointed out that one could use the constraint of s-wave capture to calculate the spin of $K^*(888)$ when sufficient data became available. In 1962 Armenteros et al.[13] confirmed the predicted s-wave capture and gathered enough data to suggest strongly that K^* (888) had spin $S > 0$. This experiment was well covered here at Varenna in the lectures by Harold K. Ticho. G. A. Snow[14] has recently published a table of other particles whose quantum numbers might be determined in similar experiments.

Having established (18) we can derive (12) exactly as for boson-antiboson pairs, except that for two fermions we want $X = -1$. Here X is still $X_r C X_S$, but this time $X_r = (-1)^\ell$ is not P, but rather -P, so we still have

$$P C X_S = +1. \tag{20}$$

This time, since spins are half integral,

$$X_S = -(-1)^S. \tag{21}$$

Since $P = -(-1)^\ell$, (20) and (21) combine to give

$$C = (-1)^{\ell + S}, \tag{22}$$

which happens to be identical with (15). Q. E. D.

(iii) Summary of Particle-Antiparticle Systems

Note that in this discussion of particle-antiparticle systems we have not yet used the concept of ispin. In fact, for Case II, $f\bar{f}$, we used for our example e^+e^-, for which ispin is not defined. If I, and therefore G, is defined, we can combine (15) or (22) with (9) to form

$$G = (-1)^{\ell + S + I}. \tag{23}$$

This applies for both boson-antiboson and fermion-antifermion pairs. For pairs of pions ($\pi^+\pi^-$, $\pi^\pm\pi^0$) in a pure istate, we can write $\psi = \psi(r)\psi(\text{ispin})$. The symmetry of $\psi(\text{ispin})$ is $(-1)^I$. For these two bosons we can as usual require

$$X = (-1)^\ell X_I = +1; \tag{24}$$

i.e., for dipions we have

$$l + I = \text{even}. \tag{25}$$

Thus the strong p-wave decay $\rho \to \pi\pi$ shows that ρ has unit ispin.

It is not easy to generalize the approach above to other particles because one has to introduce both ispin and charge coordinates. Thus the $K\bar{K}$ istate $\uparrow\downarrow + \downarrow\uparrow$ could describe either K^+K^- or $K^0\bar{K}^0$.

It is interesting, and probably significant, to note the following relationship, illustrated in Table I: Established so far are four nonstrange mesons--two pseudoscalars (one each with I = 0 and 1) and two vectors (again with I = 0 and 1). The pseudoscalar mesons both have C = +1, as illustrated by the fact that they both decay into two γ rays; the vector mesons both have C = -1, as we shall see in Chapters IV and V when we discuss their decay modes. The relationship is that these are precisely the characteristics of the possible nucleon-antinucleon states as summarized in (22) and (23).

Finally we may note that K and K* (888) are also 0^- and 1^- respectively, and could have S-wave $\bar{\Lambda}N$ dissociation products.

B. Electromagnetic Interaction

1. Order of Magnitude

The electromagnetic (em) interactions have the following characteristics:

a. The range is defined by the 1/r dependence of the Coulomb potential $U = e/r$;

b. The strength of the interaction is given by the fine-structure constant $\alpha = e^2/\hbar c = 1/137$. In the same units the strong-interaction coupling constant is of the order unity. Therefore the em interaction is only about one-hundredth as strong as the strong interactions. Another way of comparing the em interaction with the nuclear one is computing the potential energy U at the range of the nuclear forces:

$$U = \frac{e^2}{1.4F} = 1 \text{ MeV}.$$

2. Conservation Laws and Selection Rules

a. The em interaction has long been known to conserve C and P separately.

b. Early in the study of strange particles it was observed that photon emission conserved strangeness, i.e., decays such as $\Lambda \to p\gamma$ did not occur.

c. By use of the postulate of "minimal electromagnetic interaction" we can show that a single photon can carry away only zero or one unit of ispin. The "minimal interaction" assumption is that the photon is coupled only to electrical currents, whose time component is the Q of Eq. (4), which we rewrite as

$$Q = \underline{I} \cdot \underline{e}_Z + \frac{1}{2} Y \qquad (\underline{e} = \text{unit vector}). \tag{27}$$

The first term on the right-hand side contains an ispin vector: the second term is scalar. Hence the photon's ispin transformation properties cannot be more complicated that those of a scalar and a vector quantity; thus we have proved that, for a photon, $\Delta I = 0$ or 1.

d. When a single photon is emitted by a strongly interacting system either G changes or I changes by one unit: i.e., ΔG = Yes, $\Delta|I| = 1$. To see this, call the initial state $|i\rangle$, the final $|f\rangle$ and write the reaction as

$$|i\rangle \to |f\rangle + |\gamma\rangle \quad ,$$

where

$$C|i\rangle = C|f\rangle\, C|\gamma\rangle = -C|f\rangle. \tag{18}$$

For the initial state, (9) becomes

$$G|i\rangle = C\,|i\rangle\,(-1)^{I_i} = -C|f\,\,(-1)^{I_i}. \tag{28}$$

Dividing (28) by $G|f\rangle = c|f\rangle (-1)^{I_f}$, we have

$$\frac{G|i\rangle}{G|f\rangle} = -(-1)^{(I_f - I_i)}, \qquad \text{Q.E.D. (29)}$$

where $(I_f - I_i) = 0$ or $= 1$.

In this sense a photon behaves either like a ρ meson (γ_ρ) (i.e., it "carries off" $I = 1$, and does not change G), or like an ω meson ($I = 0$, $G = -1$) which can be written γ_ω. In our discussion of the η meson we use this reasoning extended to emission and reabsorption of a virtual photon to derive Eq. (47).

We next want to prove two other important selection rules:

First, $(J = 0) \not\to (J = 0) + \gamma$. Thus, for example,

$$\eta \not\to \pi^0 + \gamma, \tag{30}$$

because gauge invariance requires that a γ can exist only in the substate $J_z = \pm 1$.

Second, $(J = 1) \not\to 2\gamma$. Thus, for example,

$$\omega \not\to \gamma + \gamma. \tag{31}$$

Proof: The two γ can have a total $J_z = 0$ or ± 2. Since the $(J = 1)$ system cannot have $J_z = \pm 2$, only the states in which the photons have $J_z = 0$ need be examined further. There are two such states, one with both photons polarized left-handedly, and one with both right-handedly. A rotation of either state through 180° about the x axis merely interchanges the two photons which multiplies the state by +1 because the photons obey Bose-Einstein statistics. However, the same rotation of a $(J=1, J_z=0)$ system must multiply it by -1, as may be seen by considering $Y_1^0(z) \propto \cos\theta$. Thus the two γ's do not have the same angular momentum properties as a $J=1$ system, and the decay is forbidden. Q.E.D.

C. Weak Interaction

The weak interaction is so short-ranged and weak that it binds nothing, so it is responsible only for decays (π decay, Λ decay, β decay, etc.). Its exact range is not known.

For processes of comparable momentum, weak interaction rates are only 10^{-14} as fast as strong rates. Thus where the strong decay

$$K^*(888) \rightarrow K\pi \text{ (with a momentum of 286 MeV/c)}$$

takes 10^{-23} sec, the weak decay

$$K_1^0 \rightarrow \pi^+\pi^- \text{ (with a comparable momentum of 206 MeV/c)}$$

takes about 10^{-10} sec.

The weak interaction is well known to violate C and P separately. Its charactistics are discussed in the lectures by Professor Frank S. Crawford.

D. Gravity

Gravitational attraction is so weak as to have almost no applications to particle physics, therefore we ignore it.

II. NINETEEN MULTIPLETS

This section is limited to presenting and commenting on Fig. 2 and Table II, which display data on the currently known mesons and baryons.

Figure 2 shows the great increase since 1960 in the number of particles known. Thick colored bars represent those particles which are stable against strong decay; those which decay strongly are represented by thin colored bars, and their width Γ is shown as a vertical "flag."

The mass scale of Fig. 2 is so small that the mass differences within multiplets cannot be illustrated. There is a handy rule to help us remember which components are heavier--namely, that with the single exception of π, all multiplets in Fig. 2 (for which the mass differences are known) slope up to the left; i.e., the more negative the charge the greater the mass. Thus $m(K^0) > m(K)$, $m(n) > m(p)$, $m(\Sigma^-) > m(\Sigma^0) > m(\Sigma^+)$, $m(\Xi^-) > m(\Xi^0)$.

Not included in Fig. 2 are three particles not yet firmly established--ζ, $K^*_{1/2}(730)$, and $Y^*_1(1685)$--which are listed as questionable in Table II.

The notation of Table II for particle names (outlined in the notes to the table) is used henceforward.

1. Some comments on widths Γ

The widths listed in Table II are completely empirical half-widths at half maximum, even though the resonance may have $J > 0$ and not be described by a simple resonance curve. However, background has been subtracted and experimental resolution unfolded.

For all the well-established particles except Ξ^* all the widths seem to be rather reasonable. At first it may seem surprising that $Y^*_0(1520)$ has Γ only 15 MeV, but it should be remembered that before it can break up it has to penetrate a d-wave barrier. Why $\Xi^*_{1/2}(1530)$ has $\Gamma < 7$ MeV I do not understand, unless it also has a large J.

2. $\bar{K}^0 K^0$ attractions at about 1 GeV

Also not listed as resonances are: (a) the $K_1^0 K_1^0$ attraction [J and C both even, probably I = 0, e.g. $0(0^{++})$] reported recently (as indicated in Table II) by Erwin et al., Alexander et al., and Bigi et al; (b) the $K_1^0 K_2^0$ attraction [J and C both odd, probably I = 0, e.g., $0(1^{--})$]. reported by Bertanza et al.

In concluding our discussion of displays like that of Fig. 2, we would like to sketch, in Fig. 3, a fictitious quartet, $K^*_{3/2}$, and its antiquartet, $\bar{K}^*_{3/2}$. Although these particles do not exist (or, if they exist, they have certainly not been found), they help us visualize both new vertices which Crawford has to introduce in his Puppi tetrahedron as extended to allow for $\Delta|I| = 3/2$, with $\Delta S = \pm \Delta Q$ (see Chapter V of Crawford's lectures).

Note that the $K^*_{3/2}$, $\bar{K}^*_{3/2}$ mesons would contribute two new vertices to the Puppi tetrahedron with charge Q = +1. $K^{*+}_{3/2}$ corresponds to the one which, like the real $K^{*+}_{1/2}$, decays into leptons with $\Delta S = \Delta Q$; but $\bar{K}^{*+}_{3/2}$, with S = -1, must decay with $\Delta S = -\Delta Q$.

3. Determination of J

J for most of the particles has been established by data on angular distributions, but, for a few particles, lower limits have been set merely by general considerations of the total cross section; we now discuss this point.

Consider elastic scattering,

$$A + B \rightarrow A + B,$$

where the target B can be a nucleon or a peripheral pion.

If the A and B are spinless, then

$$\sigma \leq 4\pi\lambda^2 (2J+1) \sin^2\delta , \tag{32}$$

where δ = phase shift,

J = total angular momentum of the state.

(The equality sign refers to the case in which the elastic scattering channel is the only channel.)

For a resonant state, the phase shift is 90°, and

$$\sigma \leq 4\pi\lambda^2 (2J+1). \tag{33}$$

In the general case of particles with spin[15] but no isospin

$$\sigma \leq \frac{1}{(2S_1+1)(2S_2+1)} 4\pi\lambda^2 (2J+1) \sin^2\delta,$$

where S_1 and S_2 are the spins of the two incoming particles.

The appropriate Clebsch-Gordan coefficients must still be applied for taking isospin into consideration.

The case in which the target is a π leads us to a discussion of the ρ meson in Section IV.

4. Mnemonic for quantum numbers of nonstrange baryons

For a given πp orbital angular momentum ℓ like the p-wave 1^+, we can make $J^P = \frac{1}{2}^+$ or $\frac{3}{2}^+$. Ispin $\frac{1}{2}$ chooses to form a particle (the $\frac{1}{2}^+$ nucleon) which has the lower value of J; I = 3/2 forms a resonance (the $\frac{3}{2}^+$ Δ) which has the higher value. This same association of lower I with lower J, higher I with higher J, seems to be general. Jerome A. Helland of Lawrence Radiation Laboratory illustrates it as shown in Table III.

III. SIXTEEN REGGE TRAJECTORIES

Figures 4 and 5 are Chew-Frautschi plots of all the particles as described by Drell elsewhere in this volume. The notation is that explained in the notes to Table II, and the optimistic assignments correspond to the "possible assignment" columns of that table. It should be pointed out that the Regge trajectories really are somewhat S-shaped curves about which little is known, so our straight lines represent only absence of information.

For the baryons (Fig. 4) there are three possible trajectories that may each join two established particles.

For the mesons (Fig. 5) the situation is still a very sorry one--no two known mesons lie on the same trajectory.

The slopes of all trajectories (baryon and meson) correspond to a range $R \approx \frac{1}{2} \frac{\hbar}{m_\pi c}$, and agree with current information on the slope of the vacuum trajectory near $m^2 = 0$.

IV. PERIPHERAL COLLISIONS, THE ρ MESON

A. One-Pion Exchange

Consider the reaction $\pi^+ p \rightarrow p\pi^+\pi^0$. It can be represented by the "one-pion-exchange" diagram of Fig. 6, where the target proton has a laboratory 3-momentum $\underline{P} = 0$, and recoils with momentum $\underline{P}'$. In 4-vector notation, where the energy is called P_0 and the 3-momentum $\underline{P}$ has components P_1, P_2, P_3, we can then write

$$P = \begin{pmatrix} M \\ 0 \end{pmatrix}, \quad P' = \begin{pmatrix} P_0' \\ \underline{P}' \end{pmatrix}, \quad P' - P \equiv \Delta = \begin{pmatrix} P_0' - M \\ \underline{P}' \end{pmatrix} \approx \begin{pmatrix} 0 \\ \underline{P}' \end{pmatrix}.$$

Chew and Low[16] pointed out that the 4-pion vertex would represent real $\pi\pi$ scattering if it were possible (it is not) to choose Δ^2 (which represents the exchange π) to have the value $+m_\pi^2$.

At that point ($\Delta^2 = m_\pi^{\ 2}$) Chew and Low tell us that the experimental cross section for production of a π^0 leading to a ρ with total energy ω is

$$\frac{d^2\sigma}{d\Delta^2 d\omega^2} = \frac{f^2 = 0.08}{2\pi}\left(\frac{\Delta/m_\pi^{\ 2}}{(\Delta^2 - m_\pi^{\ 2})^2}\right)\left(\begin{matrix}\text{Kinematic}\\ \text{Factors}\end{matrix}\right) \times \sigma_{\pi\pi}(\omega). \quad (34)$$

The factors $f^2(\Delta/m_\pi)^2$ represent the probability of creating a p-wave pion at the πp vertex in Fig. 6. The factor $\sigma_{\pi\pi}/(\Delta^2 - m_\pi^{\ 2})^2$ represents the probability of real $\pi\pi$ scattering at the second vertex, and $1/(\Delta^2 - m_\pi^{\ 2})$ is the pion propagator.

The experimental importance of (34) is that it sometimes holds in the physical region (i.e., $\Delta^2 < 0$) for small $|\Delta|$, say $<$ 400 MeV/c.

Figure 7 displays ρ production by 1.255-GeV/c $\pi^+ + p$, with the recoil momentum $\underline{P}'$ selected to be $<$ 400 MeV/c.

Both charged and neutral ρ have now been produced in many different reactions, so that it is clear that ρ has ispin = 1. Hence J = 1 or 3. The fact that $\sigma_{\pi\pi} \approx 12\ \pi\lambda^2$ as shown in Fig. 7 suggests strongly that J = 1.

B. Other Tests on the Peripheral Production of ρ

1. Angular Distribution Test

Consider the $\pi\pi$ vertex in Fig. 6, in the $\pi\pi(=\rho)$ rest frame. If we measure an angle θ with respect to the beam direction, the ρ wave function must be $Y_J^0(\cos\theta)$; and if ρ has a spin J = 1, then its amplitude must be $\cos\theta$. Hence the angular distribution of the decay of ρ in its own rest frame must be $\cos^2\theta$. For some experiments this test works beautifully;[17, 18] sometimes it does not work so well.[19] But $\cos^2\theta$ is always the dominant term, and this adds further to our conviction that ρ indeed has J = 1.

2. Treiman-Yang Test

Treiman and Yang[20] have pointed out that if the two vertices of Fig. 6 are connected only by a spinless pion, then the overall reaction be azimuthally symmetric about the direction of the exchanged pion. This means that there must be no correlation between the plane of the ρ produced at the $\pi\pi$ vertex and the plane made by the target and the recoil proton. In the laboratory reference frame the latter plane is not defined, so it is conventional to choose for a reference frame either that the rest frame of the ρ, or of the beam pion. We next want to give two examples of reactions which probably would introduce a correlation between these planes. First, suppose that there is a final-state interaction between the scattered π^+ and the recoil proton. This could be important if certain orientations of the $\pi\pi$ scattering permitted the formation of the Δ isobar. It is then easy to see that a correlation between planes would be introduced. Second suppose that only a single particle was exchanged, but that it had a nonzero spin; again a correlation could exist.

3. Results of Above Tests on ρ Production

Pickup et al.[21] have applied tests 1 and 2 to about 100 ρ produced by 1.4-BeV/c $\pi^- p \rightarrow p\pi^-\pi^0$. They report that if they choose $\Delta <$ 80 MeV/c ($\Delta/m_\pi < 1/2$) and select the dipion mass ω to lie inside the ρ band, the Treiman-Yang test is satisfied within statistics. But although the angular distribution of $\theta_{\pi\pi}$ is nearly $\cos^2\theta$, there is a definite linear term which seems to be related to a competing process, namely Δ formation, which cannot proceed via one-pion exchange ("OPE").

Carmony and Van de Walle[19] and Xuong[21] tested the 1500 ρ produced by 1.2-BeV/c $\pi^+ p \rightarrow p\pi^+\pi^0$ which are shown in Fig. 7. In contrast with Pickup et al., they can include data for Δ up to 400 MeV/c ($\Delta/m_\pi = 3$) and still get an impressive (but misleading) $\cos^2\theta_{\pi\pi}$ angular distribution. Then when but they apply the Treiman-Yang test it fails, and cannot be satisfied until the selection of events is further restricted to areas of the Dalitz plot far from the bands corresponding to Δ.

In conclusion, present experience with the ρ shows that the combination of Tests 1 and 2 is a quite sensitive measure of OPE.

V. MESONS (η, ω, K) THAT DECAY INTO 3π

We shall study the decay modes $M \to 3\pi$, where M stands for any of the mesons $\omega, \eta, K^+, K_1^0, K_2^0$. We limit our considerations to mesons with I and J both ≤ 1.

A. Dalitz-Fabri Variables

Following Dalitz, the kinematics of the 3π decay may be analyzed as shown in Fig. 8, where $\underline{q}$ denotes the relative π_2-π_3 momentum in the $(\pi_2\pi_3)$ rest frame, and $\underline{p}$ the momentum of π_1 in the rest frame of M.

In his lectures here at Varenna, H.K. Ticho discusses the general properties of a Dalitz plot and shows that unit area is proportional to Lorentz-invariant phase space and that collinear events lie on the boundary. Dalitz also invented the idea of displaying the symmetry of the pions by picking energy axes normal to the bases of an isosceles triangle (see Gell-Mann and Rosenfeld, Appendix C), as shown in Fig. 9. For every point P inside the triangle one has PL + PM + PN = constant = height. If the height is taken equal to the Q value of the reaction, then the geometrical relation that we have just written represents the energy conservation relation $T_1 + T_2 + T_3 = Q$. Every point P thus may represent a decay event, provided momentum is conserved.

We shall soon discuss the population of the events on a Dalitz plot and express the wave function in terms of powers of p and q: p^ℓ and q^L. It is worth noticing that one can qualitatively associate angular momenta $\underline{\ell}$ and $\underline{L}$ with these two exponents. To see this association, write down the ℓ-wave eigenfunction ψ_ℓ for a free pion as

$$\psi_\ell(\gamma, \theta) \approx j_\ell(pr)\, Y_\ell(\cos\theta),$$

where $j_\ell(pr)$ is a Bessel function and Y_ℓ is a spherical harmonic. In the small radius approximation ($pr \ll \ell$) the leading terms of the Bessel functions are proportional to p^ℓ (or q^L), so that the total wave function for the 3π system is proportional to the leading terms $p^\ell q^L$.

We conclude that if we see a Dalitz plot which behaves for small p like $|\psi|^2 \propto |p^\ell|^2 = p^{2\ell}$, then the angular momentum involved must be mainly ℓ, with perhaps some admixture of $\ell+2$, $\ell+4$, etc., whose presence we cannot detect.

To form the spin J of the meson we simply write

$$\underline{J} = \underline{\ell} + \underline{L} \tag{35}$$

(i.e., there are no complications with intrinsic spins, since we deal here with three spinless pions).

The "orbital" parity of $\psi(p^\ell, q^L)$ is $(-1)^{\ell+L}$. However, the parity P of our meson M has an extra factor of $[P(\text{pion})]^3 = (-1)^3 = -1$, so

$$P(\text{meson}) = -(-1)^{\ell+L}. \tag{36}$$

Note that for a J = 0 meson, $\underline{\ell}$ and $\underline{L}$ must be equal, so $\ell + L$ is even and $P(J = 0 \text{ meson}) = -1$, i.e. a 0^+ meson simply cannot decay into three pions.

To take advantage of the fact that neutral mesons are in an eigenstate of C, we find it convenient to choose L so that the dipion is neutral, i.e. $\underline{q} = \underline{p}_{\pi^+} - \underline{p}_{\pi^-}$, $\underline{p} = \underline{p}_{\pi^0}$. Then C is $(-1)^L$.

For charged mesons we get a simplification if we let L describe the $\pi^+\pi^+$ (or $\pi^-\pi^-$) dipion. Then L can only be even.

B. Ispin Considerations

Next we introduce ispin, i.e. we write the overall ψ as the product of $\psi(\underline{p}, \underline{q})$ or $\psi(\ell, L)$, times $\psi_I(\text{ispin})$:

$$\psi_{3\pi} = \psi(\underline{p}, \underline{q})\, \psi(I_1, I_2, I_3). \tag{37}$$

Bose statistics requires that ψ be symmetric under interchange of any of the two pions. But since we have factored ψ in (37) each ψ factor separately must have identical symmetries under interchange.

ψ_I can represent I = 3, 2, 1, or 0; we discuss only the cases 1 and 0.

Case I. I = 0 (sextant symmetry).

We want to discuss the properties of $\psi_0(I_1, I_2, I_3)$. To simplify the notation, write $\underline{I}_1 = \underline{a}$ (when I_1 is the ispin vector of the first π), $I_2 = \underline{b}$, $I_3 = \underline{c}$.

Then there is only one way to combine $\underline{a}, \underline{b}, \underline{c}$ to make I = 0, namely,

$$\psi_{I=0} = \underline{a} \cdot \underline{b} \times \underline{c}, \qquad (38)$$

which is totally antisymmetric, so $M \to 3\pi$ is forbidden. Since the ispin factor is antisymmetric in the exchange of any two pions, the spatial factor $\psi(\underline{p}, \underline{q})$ must have the same property, and its square, which is the population of events on a Dalitz plot, must by symmetric. This means that the Dalitz-plot population must be symmetric about every median; i.e. it must have "sextant symmetry." It is then conventional in discussing I = 0 Dalitz plots to fold all the data until they fall into one sextant; this simplifies the statistical analysis.

This total antisymmetry can also be seen in a more familiar way. Take any of the three pions: it has I = 1, therefore the remaining dipion must be in an I = 1 state if the 3π final state has I = 0. A dipion state with I = 0 or 2 would be symmetric, but our I = 1 state is antisymmetric. (The ρ meson is a familiar example.) Since we have chosen an arbitrary pion to start with, we conclude that the I = 0 three-pion state is totally antisymmetric.

Case II. I = 1

We can have a totally symmetric isovector state

$$\psi_{I=1}^{symm}(\underline{a}, \underline{b}, \underline{c}) = \underline{a}(\underline{b}\cdot\underline{c}) + \underline{b}(\underline{c}\cdot\underline{a}) + \underline{c}(\underline{a}\cdot\underline{b}). \tag{39}$$

But we can also form states of the nonsymmetric (nsym) form:

$$\psi_{I=1}^{nsym} = \underline{a}\times(\underline{b}\times\underline{c}) \text{ or } \underline{b}\times(\underline{c}\times\underline{a}) \text{ or } \underline{c}\times(\underline{a}\times\underline{b}). \tag{40}$$

By $\underline{b}\cdot\underline{c}$ in (39) we mean the iscalar combination of two vectors. Using the Clebsch-Gordan table included in Crawford's lectures, we have

$$\underline{b}\cdot\underline{c} = \frac{1}{\sqrt{3}}\,(b^+c^- - b^0c^0 + b^-c^+). \tag{41}$$

Assuming $I_{3\pi} = 1$, and assuming the symmetrical form (39), we now show, with the help of Clebsch-Gordan coefficients, that the ratio $R_\eta = \frac{\pi^+\pi^-\pi^0}{\pi^0\pi^0\pi^0}$ is proportional to 2/3 whereas the ratio $R_{K^+} = \frac{\pi^+\pi^+\pi^-}{\pi^+\pi^0\pi^0} \propto 4$. We say "proportional to" rather than "equal to" because there is still a phase-space factor which is slightly larger for $\pi^0\pi^0$ than for $\pi^+\pi^-$.

Proof: Combining (39) and (41), we have

$$\psi_I = \sqrt{\frac{1}{3}}\left\{\underline{a}(b^+c^- - b^0c^0 + b^-c^+) + \underline{b}(a^+c^- - a^0c^0 + a^-c^+) + \underline{c}(a^+b^- - a^0b^0 + a^-b^+)\right\}. \tag{42}$$

For the decay of a neutral meson like η we are interested in the neutral component of (42), so that we have

$$\psi_I = \sqrt{\frac{1}{3}}\left\{a^0(b^+c^- - b^0c^0 + b^-c^+) + b^0(a^+c^- - a^0c^0 + a^-c^+) + c^0(a^+b^- - a^0b^0 + a^-b^+)\right\}. \tag{43}$$

The $\pi^+\pi^-\pi^0$ state appears six times, but with different (noninterfering) permutations of a, b, c, so that when one squares the amplitude one gets $(1/3)(6\times 1^2) = 6/3$. The $\pi^0\pi^0\pi^0$ state appears in three terms, and they clearly interfere, so squaring it, we get $(1/3)(3^2) = 9/3$. Therefore the ratio

$$R_\eta = \frac{\pi^{+-0}}{\pi^{000}} = \frac{6}{9} = \frac{2}{3}. \tag{44}$$

In the case of K^+ decay we show below that (39) again dominates over (40). For K^+ we are interested in the positive component of (39), i.e.,

$$\psi_I = \sqrt{\frac{1}{3}} \left\{a^+(b^+c^- - b^0c^0 + b^-c^+) + b^+(a^+c^- - a^0c^0 + a^-c^+) + c^+(a^+b^- - a^0b^0 + a^-b^+)\right\}. \quad (45)$$

The $\pi^+\pi^+\pi^-$ state appears twice as $a^+b^+c^-$, twice as $a^+b^-c^+$, and twice as $a^-b^+c^+$, so that, squaring the amplitudes, we get $(1/3)(3\times 2^2) = 4$. The $\pi^+\pi^0\pi^0$ state appears three times, always in a different permutation, so that $\pi^+\pi^0\pi^0 \propto \frac{1}{3}(3\times 1^2) = 1$. Hence, for K^+, we have

$$R_{K^+} = \frac{\tau}{\tau'} = \frac{\pi^{++-}}{\pi^{+00}} = 4/1. \qquad \text{Q.E.D.} (46)$$

C. Allowed vs G-Forbidden Decays

In sections D and E below we discuss the properties of 3π states $\psi(3\pi)$ with I = 0 and 1 respectively, but we do not want to imply that the meson producing these states necessarily has the same quantum numbers as $\psi(3\pi)$; after all, K mesons decay to 3π via the weak interaction, and we shall show that $\eta \to 3\pi$ only after emitting and reabsorbing a virtual photon.

This brings us to the topic of "G-forbidden" em decays. Consider the η, whose quantum numbers are $0(0^{-+})$. A 0^- meson cannot decay to 2π. It can decay to $\gamma\gamma$ and to $\gamma\pi^+\pi^-$, but these modes seem to be so slow that a 3π mode competes, even though it is G-forbidden. How can this be? We have already shown in (29) that a single photon changes C and hence must either change G or else change I by one unit. Suppose now that this photon is merely virtual, and is reabsorbed as sketched in Fig. 10. The first vertex changes C, and the second vertex changes it back again; i.e., this juggling of a virtual photon preserves C. Since there are two em vertices, $\Delta|I|$ can be 0, 1, or 2. Our rule $G = C(-1)^I$, Eq. (9), then says that if $\Delta|I| = 1$, G changes; if $\Delta|I| = 0$ or 2 then G cannot change and nothing has been accomplished in the way of producing an intermediate meson state that can then fall apart into 3π. However, we see that photon emission and reabsorption can change G and change I by one unit so as to permit the isoscalar η to decay into an isovector configuration of 3π. Of course each em vertex decreases the amplitude by a factor e (really $e/\sqrt{\hbar c}$), so that $\psi(3\pi)$ is down by e^2, and $|\psi|^2$ is down by e^4; i.e., the rate is down by approx 10^4 with respect to comparable G-allowed rates.

In summary, G-forbidden decays have the following properties, which have been pointed out by many authors:[22, 23, 24]

$$C = \text{No}, \quad G = \text{Yes}, \quad \Delta|I| = 1, \quad \Gamma \propto e^4 . \tag{47}$$

D. Isoscalar (C = -1) 3π States

1. General Properties

In this section we discuss the properties of I = 0 3π states, of which there are three: pseudoscalar, axial vector, and vector [Eq. (36) rules out the scalar possibility]. Combining (8) and (9), we have

$$G = -1 = C(-1)^I , \tag{48}$$

therefore

$$C = -(-1)^I . \tag{48a}$$

So for I = 0 all $\psi(3\pi)$ states have C = -1, and hence--by (15a)--L must be odd.

These states can arise from the allowed decay of isoscalar mesons with G = -1, or from the G-forbidden decay of isovector mesons with G = +1, which, according to (9), also have C = -1.

In Table IV we have listed in the first column the three possibilities for J and P. We now discuss each row in turn. Although the first row applies to a pseudoscalar <u>meson</u>, the $\psi(3\pi)$ must have scalar <u>spatial</u> transformation properties. Parity then forces ℓ to be odd also. The simplest possibility is then q^1 and p^1, and the simplest scalar quantity is $\underline{q} \cdot \underline{p} = qp\cos\theta$, which vanishes when $\cos\theta = 0$, i.e., when $p_+ = p_-$ or $E_+ = E_-$ (see Fig. 11). Thus ψ vanishes along the vertical median of the Dalitz triangle. The antisymmetrized form is the simplest scalar expression that vanishes along all three medians. $|\psi(\text{antisymm})|^2$ is drawn in isometric projection in Fig. 12(b) (taken from Reference 25).

The second row of Table IV is the axial meson (1^+) illustrated in Fig. 12(a); $\psi_{3\pi}$ must then be a spatial vector (1^-), i.e., ℓ is even, and $\underline{p}$ need not enter. Then $\psi = \underline{q}$ is the simplest vector, and ψ must vanish when the dipion has $q = 0$, i.e., when π^+ and π^- "touch" in momentum space. To antisymmetrize ψ we first tried $(\underline{p}_1 - \underline{p}_2) + (\underline{p}_2 - \underline{p}_3) + (\underline{p}_3 - \underline{p}_1)$, but because $\underline{p}_1 + \underline{p}_2 + \underline{p}_3 = 0$ this has the unfortunate property of vanishing. The energy factors w_i preserve the antisymmetry and make ψ nonzero. Note that ψ (antisymm) $\to 0$ when <u>any</u> dipion has $q = 0$. It also vanishes at the symmetry point where $w_1 = w_2 = w_3$. In Reference 1(b) Dalitz gives a more general proof that ψ must in fact vanish at the symmetry point for $J = 0^-$, 1^+ and 2^-.

For the third row of Table IV, ψ must be an axial vector (1^+), so ℓ is odd and $\psi \propto \underline{q} \times \underline{p}$, which vanishes for q and p collinear. Collinear decays correspond to the boundary of the Dalitz plot, as illustrated in Fig. 12(c). This correspondence can be seen by noting that both collinear decays and the boundary have one degree of freedom less than the normal configuration.

2. <u>The ω Meson</u>

The Columbia-Rutgers data on 1100 ω decays are presented in Fig. 13.[26] It is clear that the data are perfectly consistent with 1^{--} and not with the other tow $I = 0$ possibilities listed in Table IV. No charged mode of ω has ever been seen, so we conclude that ω has the quantum numbers $0(1^{--})$.

E. Isovector (C = +1) 3π States

1. General Properties

Table V summarizes the properties of the 3π states with I = 1. Equation (48a) requires that they all have C = +1. As discussed in obtaining Eq. (47), these 3π states can be reached in several different ways.

(a). Strong decay of a meson with any of the quantum numbers $1(0^{--})$, $1(1^{+-})$, $1(1^{--})$, ···. However, no such mesons are known, nor can the $\bar{N}N$ system have such quantum numbers. The simplest model would be a ηπ resonant state.

(b). G-forbidden decay of a G = +1 meson with I = 0 or 2. The η is an example of the I = 0 case.

(c). Weak decay of the K mesons, as we shall see below.

The reasoning used to construct Table V is identical with that used for Table IV; we take the first row as an example. Given J^P of the meson in Column 1, we get spatial parity by changing P by $(-1)^3 = -1$. We have just mentioned that (47a) requires all the neutral states to have C = +1 and hence L even. For the charged states we choose L to apply to the doubly charged diparticle, and then L must be even, by symmetry. We then choose ℓ so as to satisfy the spatial parity. The simplest spatial scalar quantity with L = ℓ = even is a constant, so that the Dalitz plot should tend to be uniformly populated. Of course the constant can be multiplied by any scalar quantity, e.g., $(1 + q^2 + p^2 + p^2q^2 + \cdots)$; but η decay and K decay both have low Q values, so that barrier penetration suppresses the higher values of L and ℓ, leaving q^0p^0 dominant. Pure q^0p^0 is of course symmetric, so ψ(ispin) is also symmetric and $\pi^0\pi^0\pi^0$ can be produced with $R_\eta = 2/3$, as proved in (44). Terms like $\underline{q} \times \underline{p} \times p^2$ are nonsymmetric and cannot yield $\pi^0\pi^0\pi^0$.

For the remaining rows it is not hard to see that the simplest spatial vector with L even is $\underline{p}$. The simplest spatial axial vector is $\underline{q} \times \underline{p}$, but this has L odd, so we choose $(\underline{q} \times \underline{p})(\underline{q} \cdot \underline{p})$. Note that the $\underline{q} \times \underline{p}$ factor makes ψ vanish at the boundary of the Dalitz plot, and $\underline{q} \cdot \underline{p}$ makes it vanish at the vertical median.

2. The η Meson

The η is a neutral meson with a mass of 548 MeV, $\Gamma < 10$ MeV (probably about 1 keV) which decays into several neutral modes 3/4 of the time and into $\pi^+\pi^-\pi^0$ 1/4 of the time. References to the data are given in Table II. Note that η has a mass of $4m_\pi + 8$ MeV, so that its Q value for 3π decay is limited.

The ispin of η is taken to be zero for two reasons: first, no evidence for any charged η has ever been observed; secondly, the η was discovered in the reaction

$$\pi^+\eta \rightarrow p\eta^0, \text{ with } \sigma^0 \approx 0.6 \text{ mb}.$$

If η has I = 1, then charge independence sets a lower limit on the cross sections $\sigma^\pm$ for the related reactions:

$$\pi^\pm p \rightarrow p\eta^\pm .$$

More precisely, there is a "triangle inequality" $\sqrt{\sigma^+} + \sqrt{\sigma} \geqslant \sqrt{2\sigma^0}$. But Carmony et al. find no evidence for these reactions and report that the inequality is badly violated.[27]

The Dalitz plot for 287 η published up to Aug. 1962 is shown in Fig. 14 (taken from Reference 26). It has two salient features:

First: It does not show sextant symmetry. If symmetric, the 200 events in the top and bottom sectors would be distributed 100 ± 7 in each ; but this is not true by 3.8 standard deviations. So we are not dealing with an I = 0 state.

Second: Of the three $I = 1$ J^P possibilities listed in Table V, none except 0^- comes anywhere near fitting the data. However, the population is not perfectly flat, but instead favors low T_0. A good fit is

$$|\psi|^2 \propto 1 - 0.8\,z, \text{ where } -1 \leqslant z \leqslant +1 \tag{49}$$

i.e., $z = 1/2 - (T_0/T_0 \text{ max})$. Therefore within the errors we can write

$$\psi \propto 1 - 0.4\,z.$$

The first term, 1, is of course totally symmetric; 0.4 z is not. As pointed out in Eq. (40) of Gell-Mann and Rosenfeld,[1(d)] the nonsymmetric term 0.4 z contributes only about 1/4 of $(0.4)^2 = 4\%$ to the η decay rate. Thus the symmetric state dominates η decay, and we expect (44) to hold, i.e., we expect

$$\frac{\Gamma(\pi^0\pi^0\pi^0)}{\Gamma(\pi^+\pi^-\pi^0)} \approx \frac{3}{2} \times \text{phase space} \approx 1.7\,. \tag{50}$$

Therefore we conclude that η is a $0(0^{-+})$ meson and that it decays slowly to 3π after emission and reabsorption of a virtual photon.

Table II tells us $\Gamma(\eta \rightarrow \text{neutrals})/\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0) = 3$. Combining this with Eq. (50), we see that another neutral mode, presumably $\gamma\gamma$, is important; i.e., we estimate

$$\frac{\Gamma(\gamma\gamma)}{\Gamma(\pi^+\pi^-\pi^0)} = 3 - 1.7 = 1.3 \tag{51}$$

This mode has recently been reported by the Cambridge bubble chamber group, with a branching fraction consistent with (51).

The remaining unsettled question about η is why the G-forbidden 3π mode ($\propto e^4$) seems to be enhanced so that it dominates the $\pi^+\pi^-\gamma$ mode ($\propto e^2 \times$ p-wave barrier penetration). Experimentally $\pi^+\pi^-\gamma$/(all modes) seems to be $\lesssim 5\%$.[23] Theoretically this has not been explained very satisfactorily, but it is intelligently discussed by Brown and Singer,[29] and by Gell-Mann, Sharp, and

Wagner.[30] Brown and Singer estimate a width Γ about 1/4 keV, giving a mean life of 3×10^{-18} sec.

3. K-Meson Decay into 3π

We wish to discuss very briefly the following decay modes of the spinless K meson:

τ and τ' (i.e. $K^+ \rightarrow \pi^+\pi^+\pi^-$ or $\pi^+\pi^0\pi^0$),

$K_2 \rightarrow \pi^+\pi^-\pi^0$ or $\pi^0\pi^0\pi^0$,

$K_1 \rightarrow \pi^+\pi^-\pi^0$.

These modes are discussed by Gell-Mann and Rosenfeld,[1(d)] but we want to show that all except K_1 are covered by Table V.

τ Decay. Let L refer to the $\pi^+\pi^+$ dipion. Then L is even. The $\Delta|I| = 1/2$ rule favors I = 1. This is then the 0^- row of Table V. The Dalitz plot should be almost uniformly populated. It is. The same remarks apply to τ'. Equation (46) relates τ and τ'.

K_2 Decay. Tables IV and V remind us that the J = 0 states of 3π have P = -1 (i.e., 0^-). Since $CP|K_2\rangle = -|K_2\rangle$, C must be +1. Then I = 1 is allowed, and is favored by $\Delta|I| = 1/2$, so again we expect (and find) the properties of the first row of Table V. Equation (44) again predicts $\pi^0\pi^0\pi^0/\pi^+\pi^-\pi^0 \approx 3/2$.

K_1 Decay. This time, since $CP|K_1\rangle = +|K_1\rangle$, C must be -1, so I = 0 or 2; I = 2 is suppressed by $\Delta|I| = 1/2$. I = 0 is suppressed by the complicated $(w_1-w_2)(w_2-w_3)(w_3-w_1)$ form of the antisymmetrized wave function shown in Table IV and Fig. 12(b). Since $3\pi^0$ can have only C = +1 it is forbidden.

I intend to take up related topics in UCRL-10492 (Rev.).

Acknowledgments

I want to thank I. Derado and G. Giacomelli for their help in writing up these notes, and to acknowledge helpful discussions with Robert W. Huff.

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Table I. Nonstrange mesons. The column labeled $\bar{N}N$ lists the $\bar{N}N$ states having same quantum numbers as the mesons. A single arrow indicates electromagnetic decay, a double arrow indicates strong decay. References are given in Table II.

Ispin	Pseudoscalar mesons				Vector mesons			
	particle	J^{PG}	C	$\bar{N}N$	particle	J^{PG}	C	$\bar{N}N$
0	η $\rightarrow \gamma\gamma, \pi^+\pi^-\pi^0$	0^{-+}	+	$^1S_0$	ω $\Rightarrow \pi^+\pi^-\pi^0$	1^{--}	-	$^3S_1, {}^3D_1$
1	π^0 $\rightarrow \gamma\gamma$	0^{--}	+	$^1S_0$	ρ $\Rightarrow \pi\pi$	1^{-+}	-	$^3S_1, {}^3D_1$

Table II.

TENTATIVE DATA ON STRONGLY INTERACTING PARTICLES Sept. 1962 A. H. Rosenfeld

Particle	Established Quantum No. $I(J^{PG})$	Possible Assignment: Quantum No. $I(J^{PG})$	Possible Assignment: Regge[1] Trajectory	Mass (MeV)	Γ[2] (MeV)	Mass^2 (BeV)^2	Dominant Decays: Mode	%	Q[4] (MeV)	p or p_{max} (MeV/c)
Vacuum ?	-	$0(2^{++})$	$+\omega_\alpha$	-	-	-	(even no. pions) $\bar{K}K(K_1K_1$ etc.) [5]			
η	$0(0^{-+})$		$+\omega_\beta$	548	< 10	.30	neutrals[3] $\pi^+\pi^-\pi^0$	75 25±4	- 136	- 175
ω	$0(1^{--})$		$-\omega_\gamma$	782	< 15	.62	$\pi^+\pi^-\pi^0$ [3, 5] $\pi^0\gamma$	86 14±4	368 647	326 379
$\pi\begin{cases}\pi^0\\ \pi^\pm\end{cases}$	$1(0^{--})$		$-\pi_\beta$	π^0 135 $\pi^\pm$ 140	0 0	.018 .02	$\pi^0\to\gamma\gamma$ [6] $\pi^\pm\to\mu\nu$	100 58	135 34	67 30
ρ	$1(1^{-+})$		$+\pi_\gamma$	750	100	.56	$\pi\pi$[3] (p-wave)	100	471	348
ζ (?)	1(?)	$1(0^{+-})$	$-\pi_\alpha$	560	< 15	.31	$\pi\pi$	?	290	245
$K\begin{cases}K^0\\ K^\pm\end{cases}$	$\frac{1}{2}(0^-)$		κ_β	K^0 498 $K^\pm$ 494	0 0	.24	$K_1^0\to\pi^+\pi^-$ [6] $K^\pm\to\mu\nu$	2/3K_1 58	219 388	206 236
$K^*_{1/2}(888)$	$\frac{1}{2}(1^-)$		κ_γ	888	50	.78	$K\pi$(p-wave)	100	251($K^0\pi^-$)	283
$K^*_{1/2}(730)$?	$\frac{1}{2}$(?)	?	?	730	< 20	.53	$K\pi$	?	101($K^-\pi^0$)	161
$N\begin{cases}n\\ p\end{cases}$	$\frac{1}{2}(\frac{1}{2}+)$		N_α	n 940 p 938	0	.88	$e^-\bar{\nu}p$[6] -	100 -	.78 -	1.2 -
$N^*_{1/2}(1688)$="900 MeV πp"		$\frac{1}{2}(\frac{5}{2}+)$	N_α	1688	~100	2.84	$N\pi$(f-wave)	?	610	572
$N^*_{1/2}(1512)$="600 MeV πp"		$\frac{1}{2}(\frac{3}{2}-)$	N_γ	1512	~150	2.28	$N\pi$(d-wave)	?	434(π^-p)	450
$N^*_{3/2}(1238)$= "Isobar"	$\frac{3}{2}(\frac{3}{2}+)$		Δ_δ	1238	100	1.53	$N\pi$(p-wave)	100	160(π^-p)	233
$N^*_{3/2}(1920)$	$\frac{3}{2}(J\geq\frac{5}{2})$	$\frac{3}{2}(\frac{7}{2}+)$	Δ_δ	1920	~200	3.69	$N\pi$ + other	?	842(π^-p)	722
Λ	$0(\frac{1}{2}+)$		Λ_α	1115	0	1.24	π^-p [6]	67	38	100
$Y^*_0(1815)$	$0(J\geq\frac{5}{2})$	$0(\frac{5}{2}+)$	Λ_α	1815	120	3.29	($\bar{K}N$+other)	?	383(K^-p)	541
$Y^*_0(1405)$	0(?)	$0(\frac{1}{2}-)$	Λ_β	1405	50[5]	1.97	$\Sigma\pi$ $\Lambda 2\pi$	{100}	69($\Sigma^-\pi^+$) 10($\Lambda\pi^+\pi^-$)	144 69
$Y^*_0(1520)$	$0(\frac{3}{2}-)$		Λ_γ	1520	15	2.31	$\Sigma\pi$(d-wave) $\bar{K}N$(d-wave) $\Lambda 2\pi$	60 30 10	194($\Sigma^0\pi^0$) 88(K^-p) 125($\Lambda\pi^+\pi^-$)	267 244 253
$\Sigma\begin{cases}\Sigma^+\\ \Sigma^0\\ \Sigma^-\end{cases}$	$1(\frac{1}{2}+)$		Σ_α	1189 1191 1196	0 0 0	1.42 1.42 1.42	$n\pi^+$ [6] $\Lambda\gamma$ $n\pi^-$	50 100 100	110 76 117	185 74 192
$Y^*_1(1385)$	$1(J\geq\frac{3}{2})$	$1(\frac{3}{2}+)$	Σ_δ	1385	50	1.92	$\Lambda\pi$ $\Sigma\pi$	98 2±2	135($\Lambda\pi^0$) 49($\Sigma^-\pi^+$)	210 119
$Y^*_1(1685)$?	1(?)	?	?	1685?	?	2.85?	($\Lambda\pi$+others)	?	435	459
$\Xi\begin{cases}\Xi^0\\ \Xi^-\end{cases}$	$\frac{1}{2}$(?)	$\frac{1}{2}(\frac{1}{2}+)$	Ξ_α	1311 1321	0	1.72	$\Lambda\pi^0$[6] $\Lambda\pi^-$	- -	61 66	131 138
$\Xi^*(1530)$	$\frac{1}{2}$(?)	?	?	1530	<7	2.34	$\Xi\pi$	100	74($\Xi^-\pi^0$)	148

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FOOTNOTES (Table II.)

? Means data that either I have not seen, or of which I am not yet convinced.

[1] The reader can use the data on p. 1 without reference to this shorthand notation. The first (and perhaps the only useful) contraction comes in choosing a single symbol to denote baryon number B, strangeness S, and I-spin I. Thus for the S = 0 meson with I = 0 (like ω) we chose ω. For the S = 0 meson with I = 1 (like π, ρ) we chose π. For K and $K^*_{1/2}$ we chose a Greek κ. Suggestive names (N, Λ, Σ, Ξ) existed for the baryons with I = 1/2, 0, and 1. For I = 3/2 [e.g., the $N^*_{3/2}$ ($3/2^+$, 1238) and $N^*_{3/2}$ (1922) isobars], we invent symbol Δ; if $\Xi^*_{3/2}$ shows up, we suggest O (omicron). One shock is that Λ (I = 0) now stands for something that can break up into Σπ, but is forbidden by conservation of I to break up into Λ and a single π.

The symbols above are useful independent of the idea of a Regge trajectory. In addition, the Regge conjecture suggests that particles (e.g., ω, N, Δ, etc.) having the same parity, but J-values differing by 2, can lie in the same trajectory. To emphasize this point, and to further condense the notation, we suggest the following subscripts to denote parity and a string of J's differing by 2:

Subscript	For mesons	For baryons
α	0^+, 2^+ ... (e.g., vacuum or ABC)	$\frac{1}{2}+$, $\frac{5}{2}+$, ... (thus p = N_α)
β	0^-, 2^- ... (e.g., π meson)	$\frac{1}{2}-$, $\frac{5}{2}-$, ...
γ	1^-, 3^- ... (γ for "vector")	$\frac{3}{2}-$, $\frac{7}{2}-$, ... [e.g., $D_{3/2}$Kp resonance Y_0^* (1520)]
δ	1^+, 3^+ ... (none known)	$\frac{3}{2}+$, $\frac{7}{2}+$ ... (e.g., the 3/2, 3/2 isobar Δ_δ)

G parity is written as a prescript (this avoids confusion with the charge of a particle). If two trajectories exist with the same symbol, the lower one can be given a "prime;" thus the vacuum is already $_+\omega_\alpha$. If the ABC meson exists, its trajectory will be $_+\omega'_\alpha$. One can use the trajectory notation for particles; thus Δ_δ (1238) means $N^*_{3/2}(\frac{3}{2}^+$ at 1238 MeV), and Δ (1920) means $N^*_{3/2}(\frac{7}{2}^+$ at 1920 MeV).

If one writes the mass, e.g., Δ_δ (1920), one does not have to write the * to indicate an excited state. (This notation was evolved in conversation with G. Chew and M. Gell-Mann.)

Where the properties of a particle are essentially unknown, it has been given the simplest possible assignment merely because it had to be listed somewhere.

[2] Γ = empirical full widths at half-max with background substracted.

[3] For analysis of possible neutral decay modes, see Tables 2 and 3 in G of R. Lynch, Proc. Phys. Soc. (London) 80, 46 (1962).

[4] Q values apply to decays to neutral particles (unless that mode is forbidden).

[5] See notes below on this particle.

[6] Common electromagnetic or weak decays are listed for convenience. The masses come from Table I, except for $m(\Xi^-)$ for which see note on Ξ^- below.

References and Notes on Individual Particles

In addition to the references and notes below, many new data were presented at the International Conference on High-Energy Physics, CERN, July 1962. (Proceedings of the 1962 International Conference on High-Energy Physics, Interscience, 1962). (For a complete bibliography to 11-7-61, see M. Lynn Stevenson, Bibliography on Pion-Pion Interaction, UCRL-9999).

ω_a - represents the vacuum trajectory, which crosses J = 1 at m = 0, and should cross J = 2 at about 1 BeV. Its $K\bar{K}$ decay mode should be K^-K^+, K_1K_1, K_2K_2 in the ratio 2:1:1. The data of Erwin et al., Bigi et al., and Alexander et al. (CERN, 1962), could be either a 0^{++} or 2^{++} interaction. These data should not be confused with the K_1K_2 data of Bertanza et al. (see notes on ω below).

ω_a' - could be the ABC meson [Abashian, Booth, and Crowe, Phys. Rev. Letters 7, 35 (1961)].

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ω - Maglić, Alvarez, Rosenfeld, and Stevenson, Phys. Rev. Letters 7, 178 (1961);

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Bertanza et al. (CERN 1962 and Phys. Rev. Letters 9, 180 (1962) have reported a low-energy K_1K_2 interaction at about 1000 MeV. Possible explanation for this effect is a second $\omega(-\omega_\gamma)$.

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ρ - See summary by Stevenson, UCRL-9999, and CERN (1962).

N^* - For reviews see Falk-Vairant and Valladas, Rev. Mod. Phys. 33, 362 (1961); B. J. Moyer, Rev. Mod. Phys. 33, 367 (1961). For recent data, see J. Helland, UCRL-10378, and CERN (1962). There is no evidence that $N^*_{1/2}$ (1512) is a pure resonance with a single phase shift going through 90 deg. At 1640 MeV in I = 3/2 there is another shoulder, probably not a pure resonance. The πp phase shift for $N^*_{3/2}$ (1238) goes through 90 deg at 1238 MeV, but because of a $\pi\lambda^2$ factor, the πp cross section reaches its maximum at 1225 MeV; see de Hoffman et al., Phys. Rev. 95, 1586 (1954); and Klepikov, Mescheryakov, and Sokolev, JINR-D-584, (1960).

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$\Xi^*_{1/2}$-(1520) at CERN, 1962. Bertanza et al., Phys. Rev. Letters 9, 180 (1962);
(BNL - Syracuse) reported m = 1535, $\Gamma \leq 35$ MeV.
Pjerrou et al., Phys. Rev. Letters 9, 114 (1962) reported m = 1529, $\Gamma < 7$ MeV.

Ξ^- -(1321). Mass from Bertanza et al., Phys. Rev. Letters 9, 229 (1962).

Table III. Nonstrange Baryons.

Symbol	State $J^P = (\ell - \frac{1}{2})^P$	Angular Momentum and Parity	State $J^P = (\ell + \frac{1}{2})^P$	Symbol
No particle		s-wave = 0^-	$\frac{1}{2}^-$	No particle
$N_\alpha(938)$	$\frac{1}{2}^+$	p-wave = 1^+	$\frac{3}{2}^+$	$\Delta_\delta(1238)$
$N_\gamma(1520)$	$\frac{3}{2}^-$	d-wave = 2^-	$\frac{5}{2}^-$	No particle
$N_\alpha(1588)$	$\frac{5}{2}^+$	f-wave = 3^+	$\frac{7}{2}^+$	$\Delta_\delta(1920)$

Table IV. I = 0 3π states, which necessarily have C = L = -1.

Meson Properties	Properties of $\psi_{3\pi}(\underline{p}, \underline{q}, \underline{L}, \underline{\ell})$				
Spin and Parity	Spatial Transformation	C and L	ℓ	Leading terms → Antisymmetrical form	Zeroes on Dalitz plot
$PS(0^{--})$	$S(0^{+})$	odd	odd	$\underline{q}\cdot\underline{p} \rightarrow (w_1 - w_2)(w_2 - w_3)(w_3 - w_1)$	medians
$A(1^{+-})$	$V(1^{-})$	odd	even	$\underline{q} \rightarrow (\underline{p}_1 - \underline{p}_2) w_3 + (\underline{p}_2 - \underline{p}_3) w_1 + (\underline{p}_3 - \underline{p}_1) w_2$	where $\underline{q} = 0$ and center
$V(1^{--})$	$A(1^{+})$	odd	odd	$\underline{q}\times\underline{p} \rightarrow (\underline{p}_1 \times \underline{p}_2) + (\underline{p}_2 \times \underline{p}_3) + (\underline{p}_3 \times \underline{p}_1)$	boundary

Table V. Isovector 3π states. For neutral states, C = +1, so L is even if it applies to the dipion. For charged states like $\pi^+\pi^+\pi^-$, L must again be even if it applies to $\pi^+\pi^+$.

Meson	$\psi_{3\pi}(\underline{p}, \underline{q}, \underline{L}, \underline{\ell})$					
J^P	Spatial Transformation	C and L	ℓ	Momentum dependence of leading terms	ψ(ispin)	Zeroes on Dalitz plot
PS(0^-)	S(0^+)	even	odd	$q^0 p^0$ = const,	mainly symmetric	none
A(1^+)	V(1^-)	even	odd	$\underline{p}$,	non-symm	p = 0
V(1^-)	A(1^+)	even	even	$(\underline{q}\times\underline{p})(\underline{q}\cdot\underline{p})$,	"	Boundary and vertical median

Figure Captions

Fig. 1. Particles stable against strong decay.

Fig. 2. Particles and resonances. The data and references correspond to Table II.

Fig. 3. Fictitious $K^*_{3/2}$. Here $K^*_{3/2}$ and $\bar{K}^*_{3/2}$ are sketched as two separate quartets, although they would actually be charge-conjugate, and would have equal masses.

(Note to the Nuovo Cimento editor--this Fig. 3 is unimportant, reproduce it much smaller than Figs. 2, 4, 5.)

Fig. 4. Chew-Frautschi plot of the baryons. The dots corresponding to the mesons are merely shown for orientation. The notation and assignments are explained in Table II and its references. The solid lines correspond to "signature 1/2"; the dashed lines to "signature 3/2": i.e., Regge trajectories of signature 1/2 can generate particles of spin 1/2 + 2n (namely, 1/2, 5/2, etc.). These values of J are shown as solid black lines, joined by solid colored trajectories; dashed lines indicate the signature-3/2 set.

Fig. 5. Chew-Frautschi plot of the mesons. After this plot was made the spin and parity of $K^*_{1/2}(888)$ was found to be 1^-, so both the point and the trajectory should be raised from J = 0 to J = 1. There is no longer any reason why the spin of $K^*_{1/2}(730)$, labeled M, should be unity, if this meson exists.

Fig. 6. Production of a ρ meson by a peripheral collision.

Fig. 7. The total $\pi\pi$ cross section $\sigma_{\pi\pi}$ as a function of the dipion total energy ω squared, as determined in the physical region from the reaction 1.255 GeV/c $\pi^+ + p \Rightarrow p\,\pi^+\pi^0$. The line labeled $12\pi\lambda^2$ is the cross section given by Eq. (32) with J = 1 and $\delta = 90^{\circ}$. From D. D. Carmony, Reference 17.

Fig. 8. Dalitz-Fabri coordinates for three-body systems.

Fig. 9. Triangular Dalitz plot. Conservation of momentum forces events to lie inside either the circle or the triangle a-c-e in the nonrelativistic or relativistic limits respectively.

Fig. 10. G-forbidden decay via emission and reabsorption of a virtual photon.

Fig. 11. Configuration that lies on the vertical median of the Dalitz plot in Fig. 9.

Fig. 12. (a, b, c). Isometric projections of $|\psi|^2$ from Table IV. (d, e). Dalitz plot of ω decays and background ("control region") events: (d) is 241 control events, (e) is 270 "ω-region" events. The contour lines are the projections of the contours of (c) for a 1^- meson. From Stevenson et al, Reference 25.

Fig. 13. Columbia-Rutgers ω decays from Ref. 26.

Fig. 14. Dalitz plot for η decays.

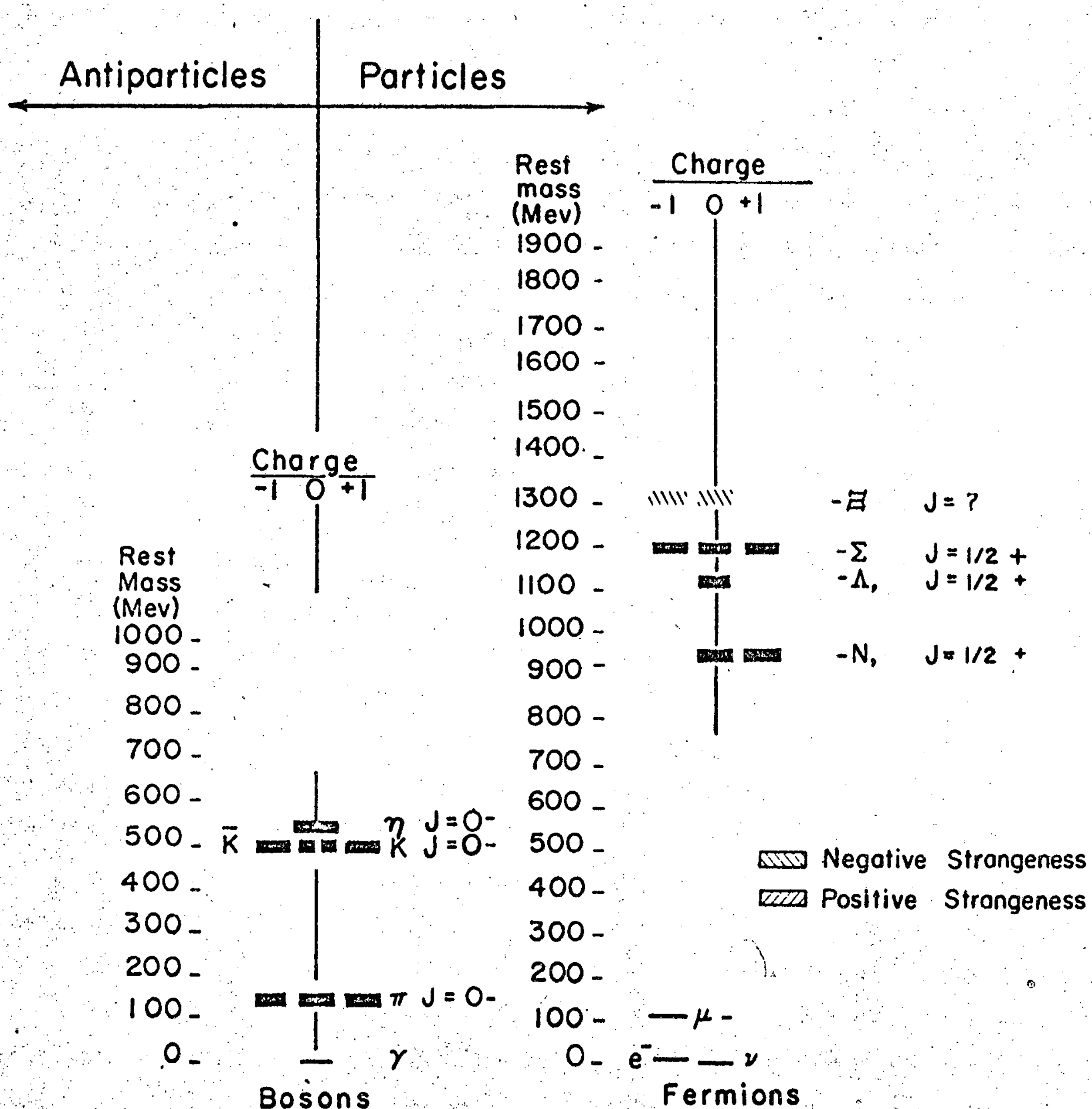

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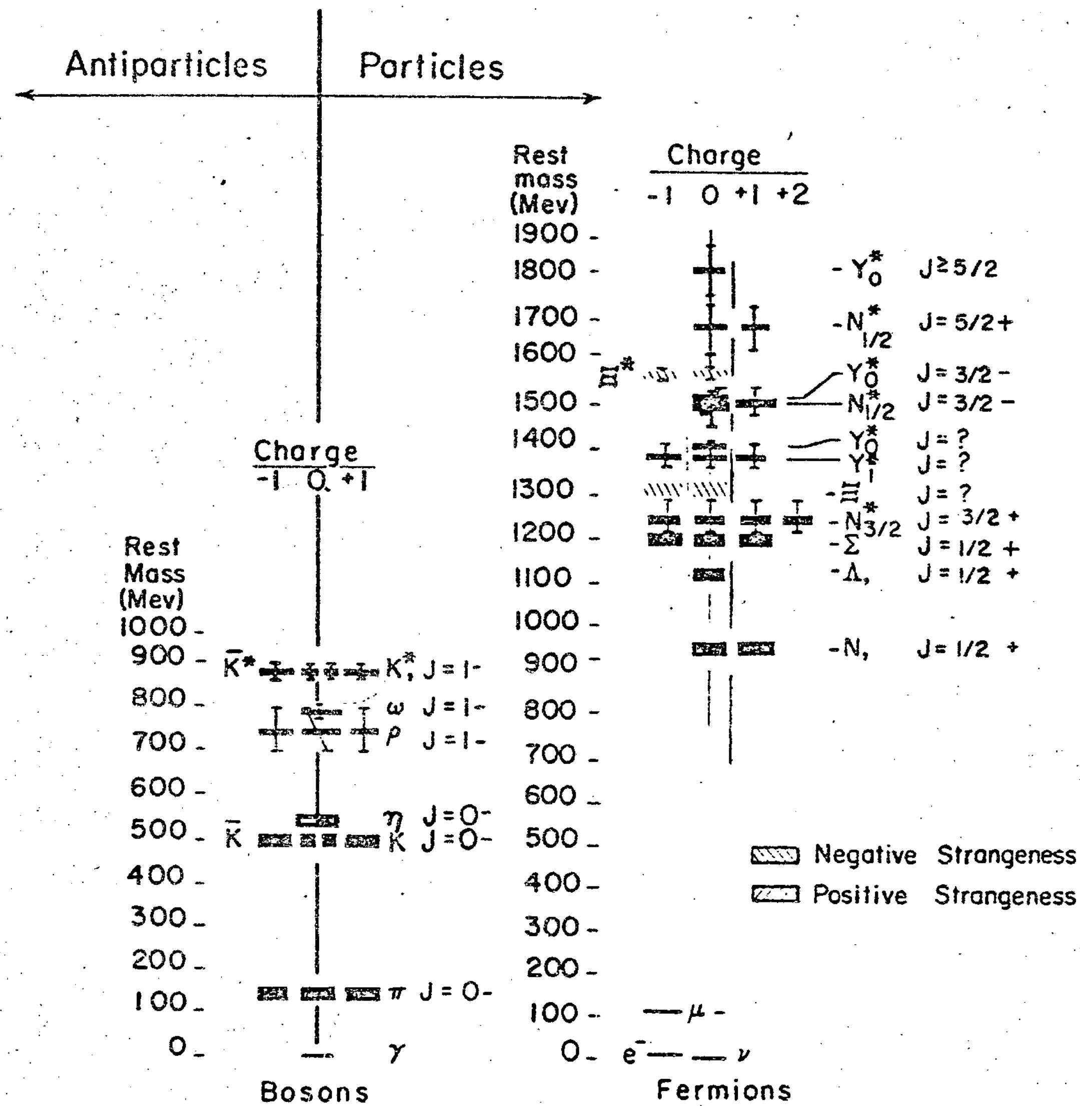
Fig. 1

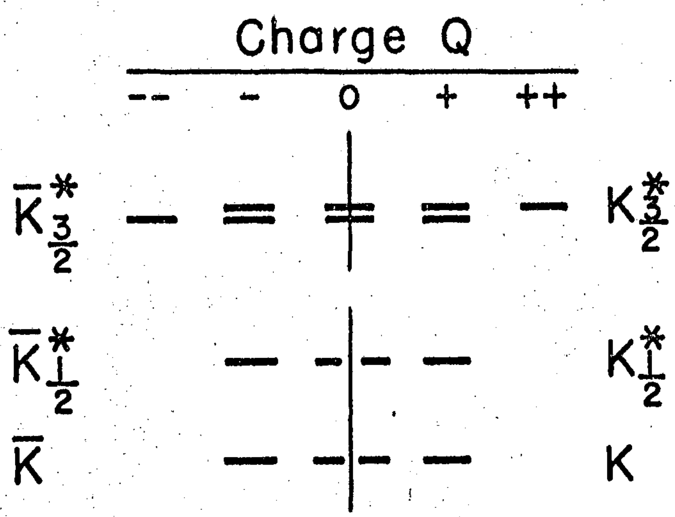


Strongly Interacting Particles are colored as follows: Red for Strangeness S = 0; Blue for S = ±1; Green for S = -2. To display antiparticles, one reflects the whole diagram about the heavy central line labeled "antiparticles | particles".

A useful empirical rule for mass differences _within_ mass multiplets: "With the exception of the π, masses increase towards the left (i.e. towards negative charge)."

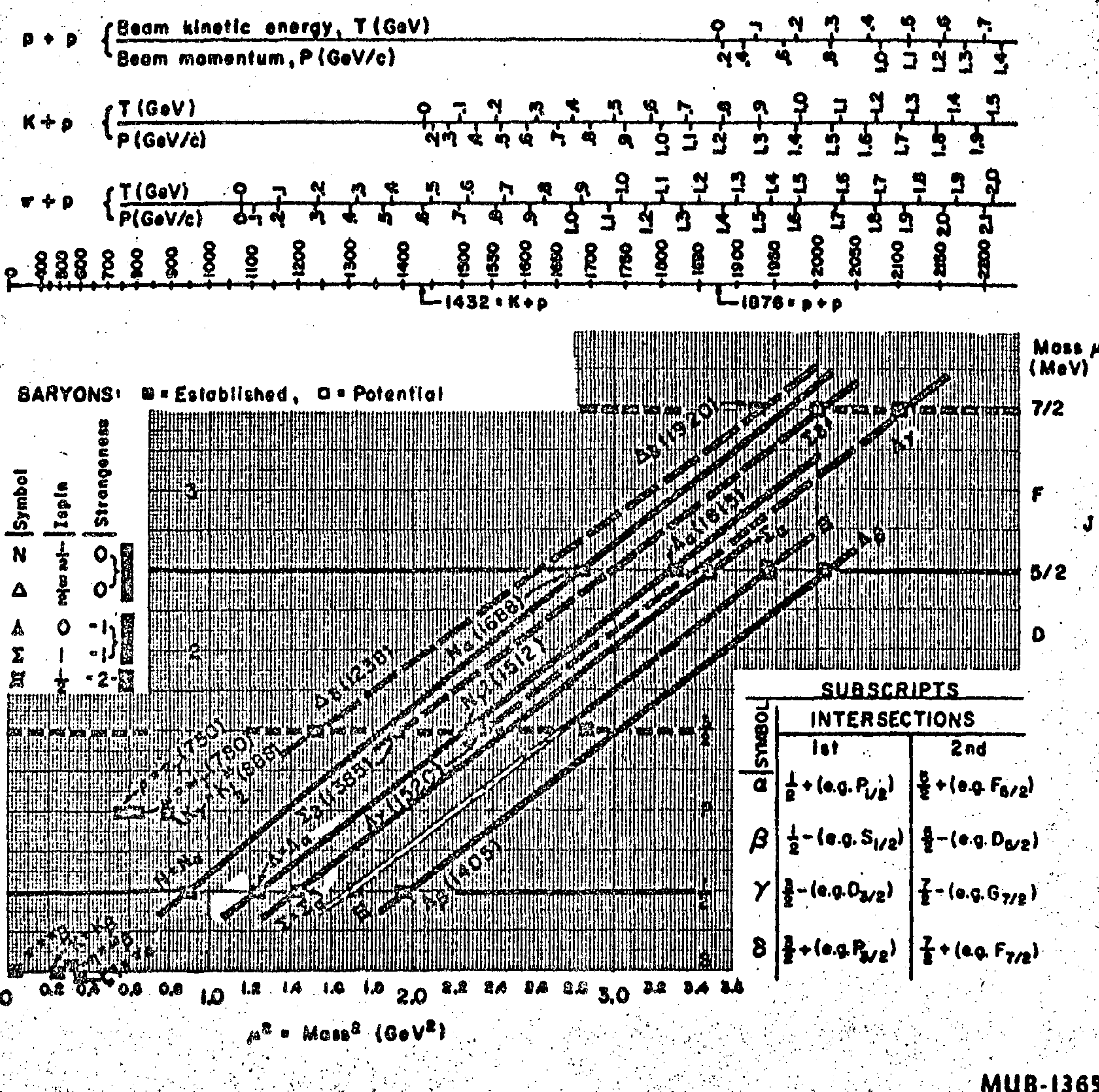
MUB-1026

Fig. 2



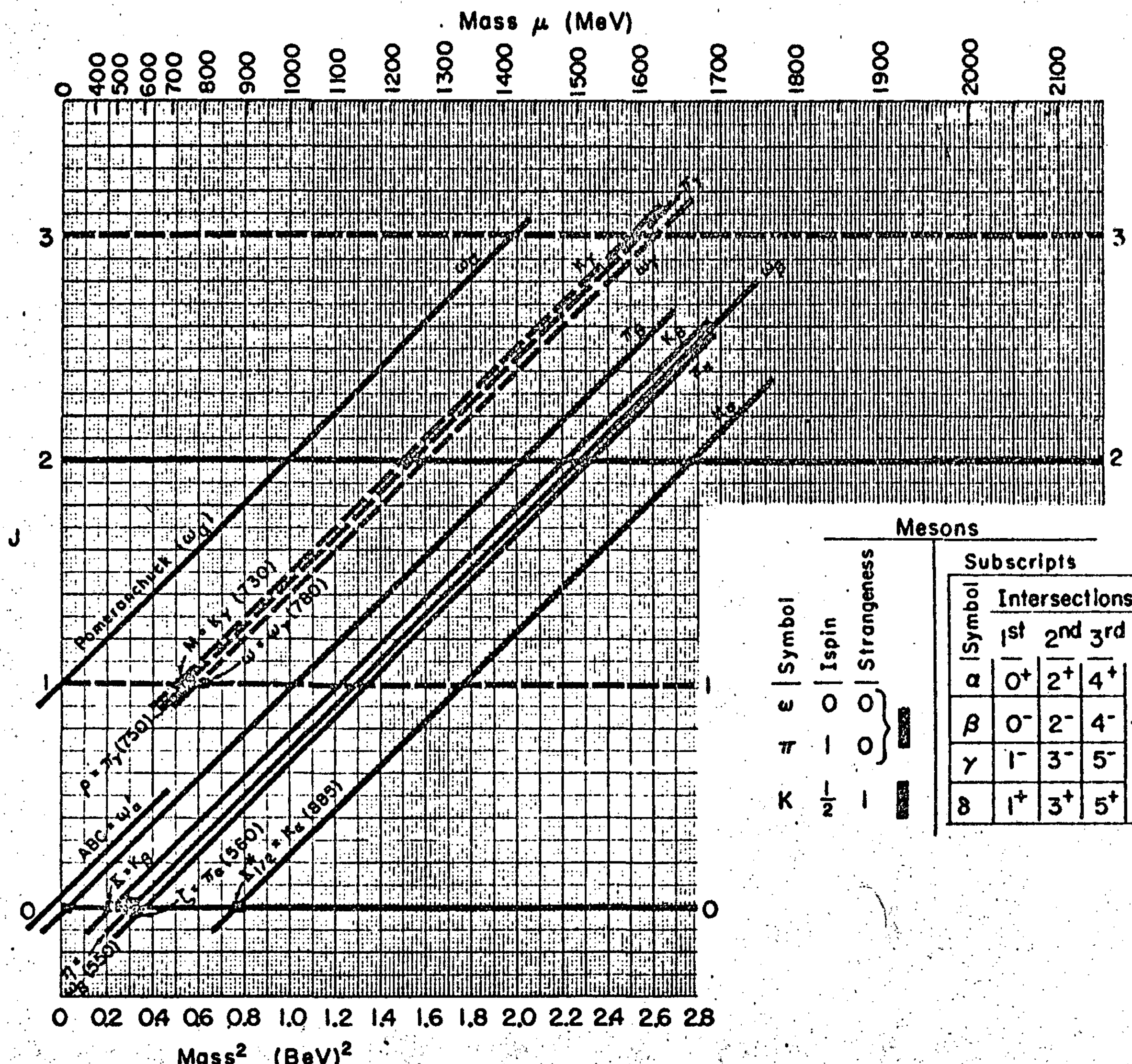
MU-28227

Fig. 3



p + p
Beam kinetic energy, T (GeV)
Beam momentum, P (GeV/c)
K + p
T (GeV)
P (GeV/c)
π + p
T (GeV)
P(GeV/c)
1432 = K+p
1876 = p+p
Mass μ (MeV)
BARYONS: ■ = Established, □ = Potential
Symbol
Isospin
Strangeness
J
7/2
F
5/2
D
SUBSCRIPTS
INTERSECTIONS
SYMBOL
1st
2nd
μ² = Mass² (GeV²)
MUB-1365

Fig. 4



Mesons

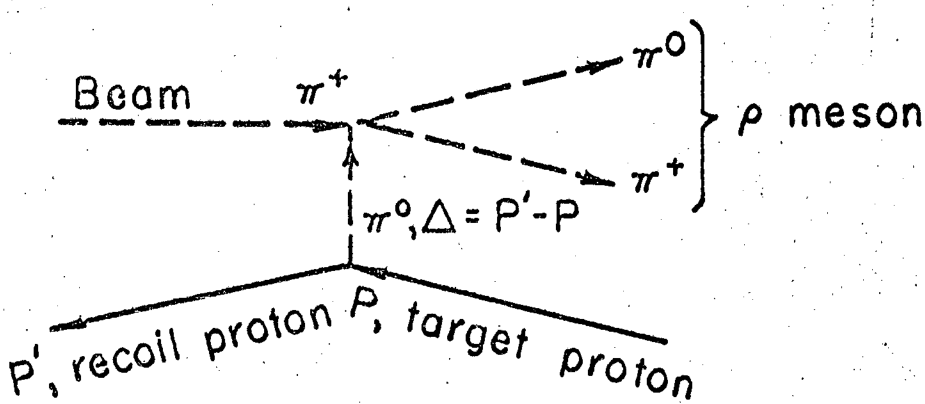
Symbol	Ispin	Strangeness
ε	0	0
π	1	0
K	$\frac{1}{2}$	1

Subscripts

Symbol	Intersections 1st	2nd	3rd	
α	0^+	2^+	4^+	---
β	0^-	2^-	4^-	---
γ	1^-	3^-	5^-	---
δ	1^+	3^+	5^+	---

MUB-1366

Fig. 5



MU-28228

Fig. 6

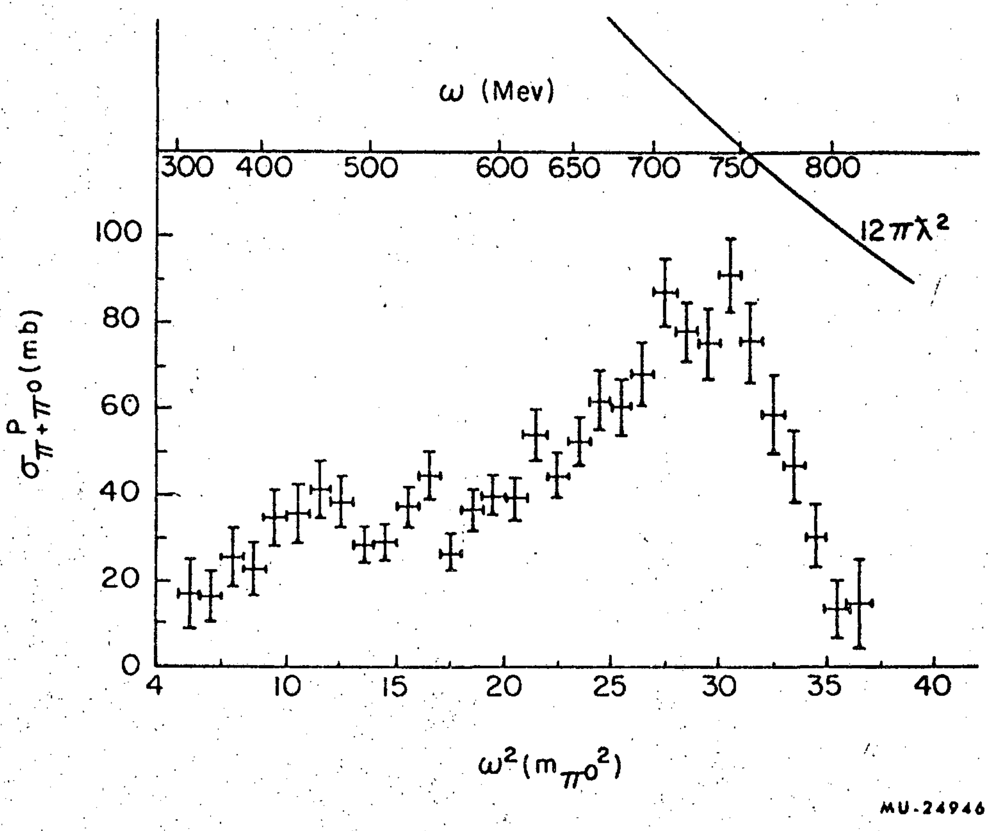
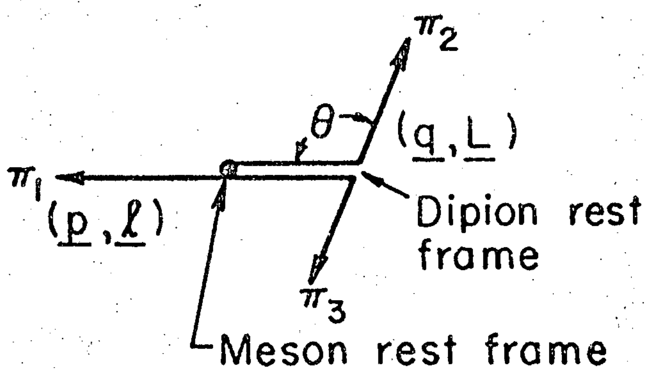

ω (Mev)
300
400
500
600
650
700
750
800
12πƛ²
100
80
60
40
20
0
4
10
15
20
25
30
35
40
σ^p_{π+π0} (mb)
ω²(m_{π0}²)
MU-24946

Fig. 7



MU-28229

Fig. 8

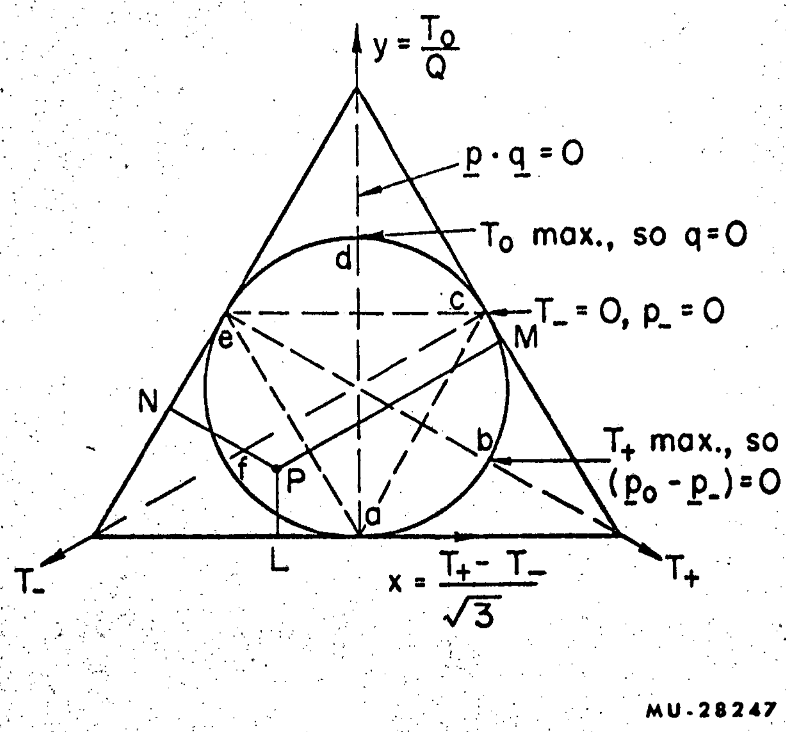
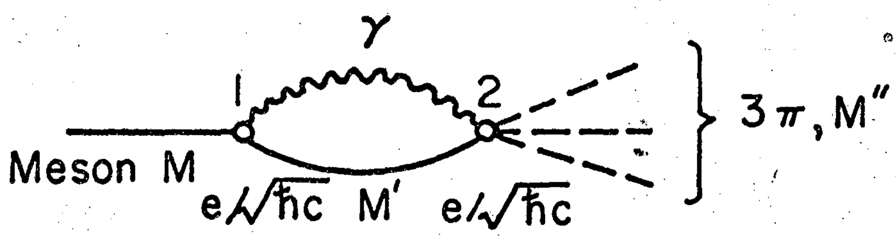
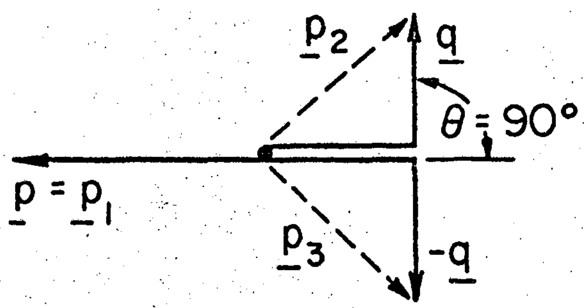

y = $\frac{T_0}{Q}$
p · q = 0
T_0 max., so q=0
T_- = 0, p_- = 0
T_+ max., so
(p_0 - p_-)=0
d
c
e
M
N
b
f
P
a
L
T_-
T_+
x = $\frac{T_+ - T_-}{\sqrt{3}}$
MU-28247

Fig. 9



MU-28248

Fig. 10



MU-28249

Fig. 11

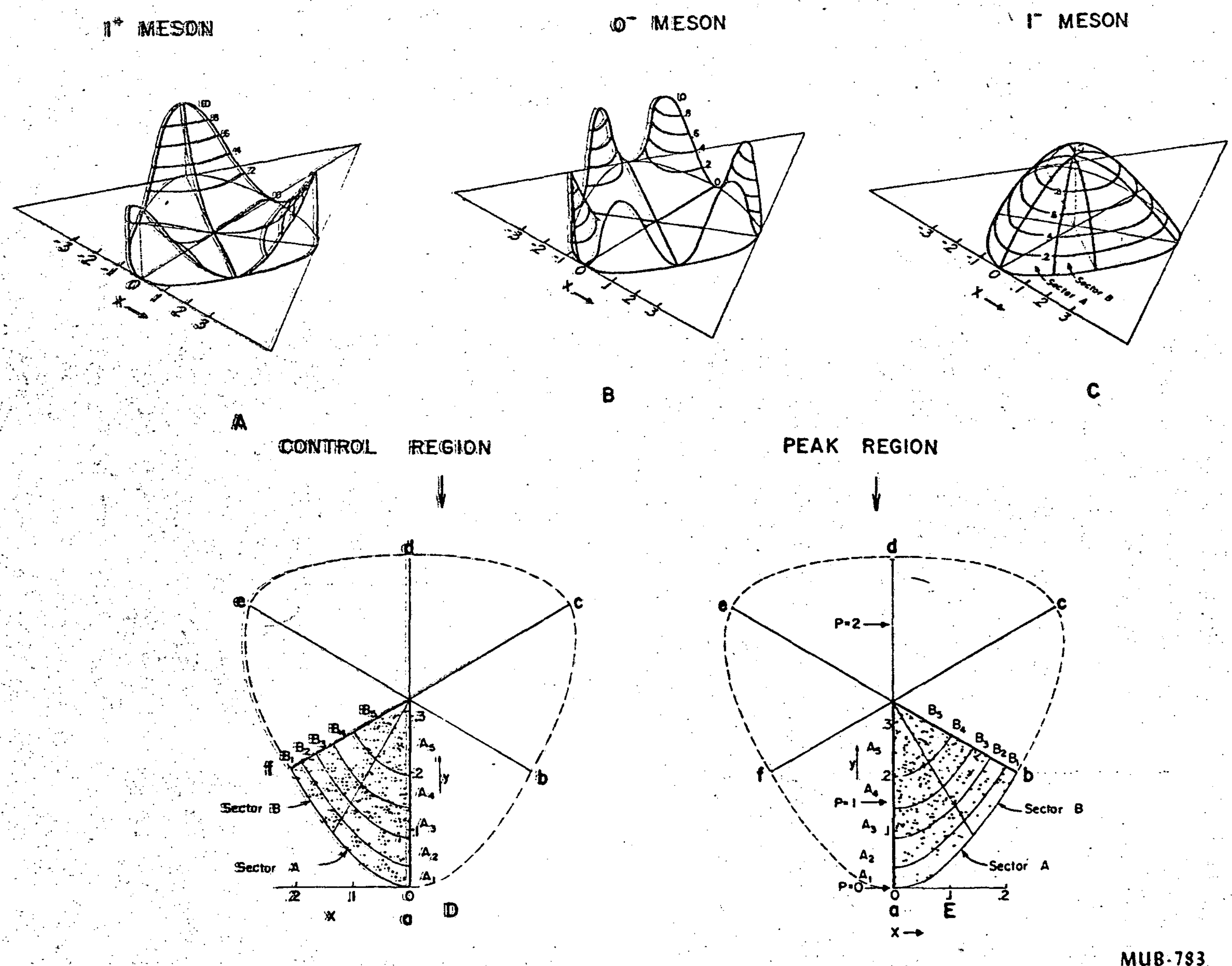
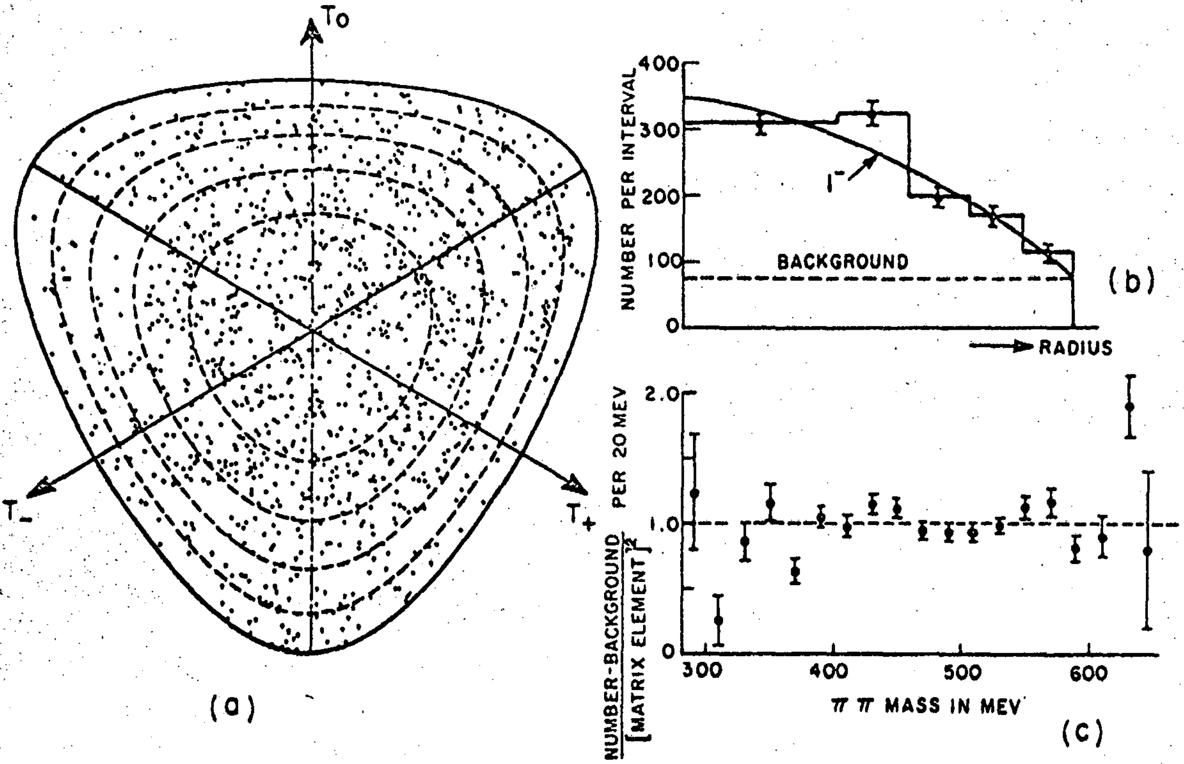

1+ MESON
0- MESON
1- MESON
A
B
C
CONTROL REGION
PEAK REGION
Sector B
Sector A
D
E
P=2
P=1
P=0
MUB-783

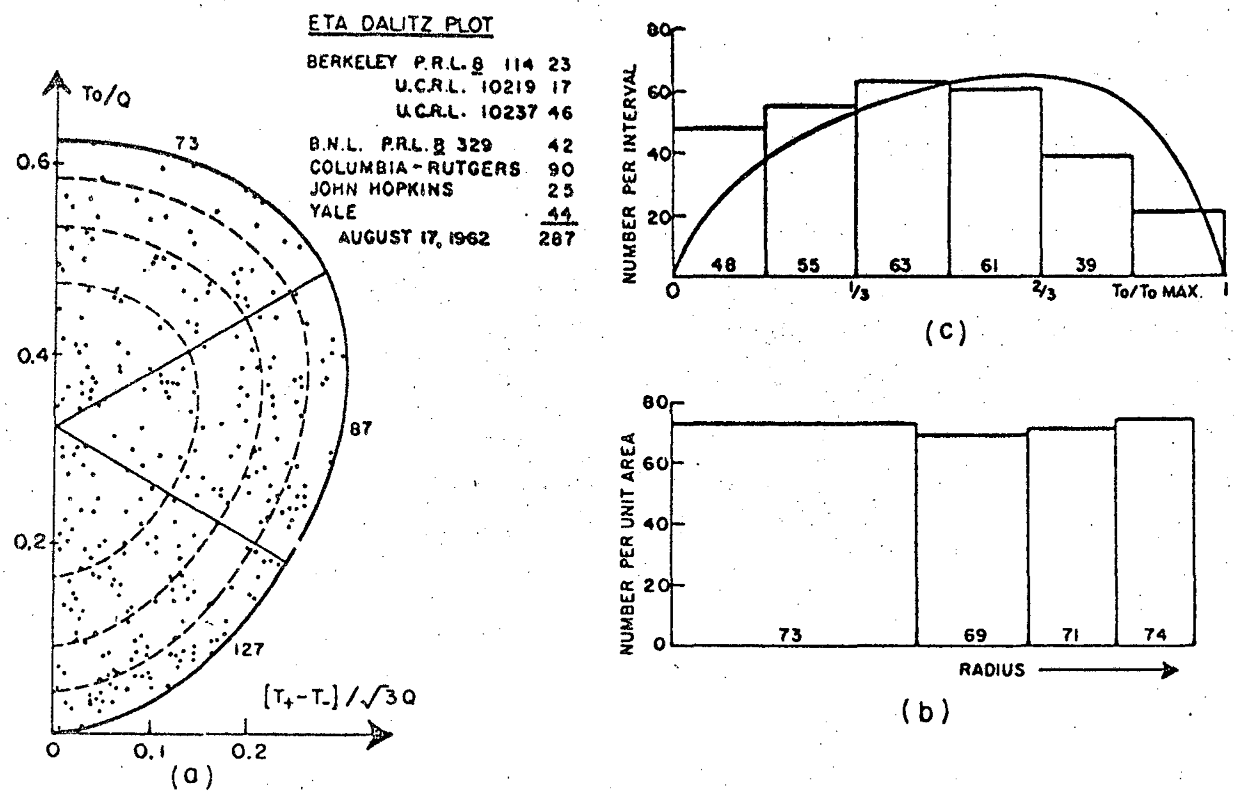
Fig. 12



(a) The Dalitz plot for 1100 omegas (including a background of 375 nonresonant triplets). (b) The density of points on the Dalitz plot compared to the expected density for a 1^- ω plus a uniformly distributed background. (c) The dependence of the $\pi\pi$ interaction in the $T=1$ $J=1$ state as a function of energy. This was obtained by summing the $\pi^+\pi^-$, $\pi^+\pi^0$, and $\pi^-\pi^0$ mass spectra for pion pairs from the ω decays, subtracting a background, and dividing by the distribution expected for 1^- decay into $\pi^+\pi^-\pi^0$. Since two of the three mass combinations are independent, an error corresponding to $(\frac{2}{3}N)^{1/2}$, where N is the number of pairs per interval before background subtraction, was assigned to each piont.

MUB-1361

Fig. 13



The Dalitz plot and projections for all published η decays into $\pi^+\pi^-\pi^0$. (a) shows the distribution of points, (b) the radial density, and (c) the projection of the points on the π^0 axis.

MUB-1362

Fig. 14