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# Clarifying the Role of Alignability in Similarity Comparisons

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**Abstract**

Structure-mapping theory has successfully predicted a number of empirical results concerning ordinary literal similarity processing. In particular, it predicts a distinction between alignable differences—those connected to the common structure derived in a comparison—and nonalignable differences, which are not so connected and which are held to be less salient than alignable differences (Markman & Gentner, 1993). Recently, Estes and Hasson (2004) have challenged the claim that alignable differences are more salient than nonalignable differences. In this paper, we address their criticisms and present data supporting an alternative interpretation of their results.

## Introduction

Similarity is an important construct in cognitive science. It plays a central role in research on categorization and classification (e.g., Medin & Schaffer, 1978; Nosofsky, 1984), induction (e.g., Osherson, et al., 1990; Sloman, 1993), decision-making (e.g., Medin, Goldstone, & Markman, 1995), and learning and transfer (Gentner, Ratterman, & Forbus, 1993). Structure-mapping theory treats similarity as the outcome of a comparison process that yields a maximal alignment between structured mental representations (e.g., Gentner, 1983; Gentner & Markman, 1997; Markman & Gentner, 1996).

The structure-mapping theory of similarity has its roots in analogy (Gentner, 1983). Similarity is claimed to involve the alignment and mapping of conceptual structure from one concept to another. The theory assumes that concepts are represented as structured descriptions comprising object attributes, objects, relations among objects, and higher order relations among relations. As in a feature-based theory of similarity (Tversky, 1977), structure-mapping assumes that similarity is a positive function of common properties and a negative function of distinctive properties.

However, a unique aspect of structure-mapping theory is its distinction between two kinds of differences, alignable differences (ADs) and nonalignable differences (NDs), and its claim that these differentially affect the similarity computation. ADs are related to the conceptual structure common to the compared items, and take the form of different values of a common predicate or along a common dimension. For example, a car has 4 wheels, a motorcycle 2; though this is a difference, the values derive from the common property of having wheels. NDs are not connected to the common system and generally take the form of an assertion about one term that is denied for the other: e.g., kittens have claws, but hoses do not. This difference is nonalignable—there is no clear common predicate along which these properties could be aligned.

Evidence that this distinction is psychologically real comes from several studies (Gentner & Gunn, 2001; Gentner & Markman, 1994; Markman & Gentner, 1993; Markman & Gentner, 1996; Medin, Goldstone, & Gentner, 1990). For example, people can list differences more quickly for similar pairs (e.g., hotel and motel) than between dissimilar pairs (e.g., hotel and banana)(Gentner & Markman, 1994). A further claim is that ADs are more salient than NDs, and therefore serve as better memory cues (Markman & Gentner, 1997).

Some important evidence for the claim that ADs count more against similarity than NDs (all else being equal)[1] comes from Markman and Gentner (1996). In one study, participants were presented with similarity triads comprising a standard scene plus two alternatives, from which one was to be chosen as most similar to the standard. In Figure 1, the standard consists of an archer aiming at a bull's-eye. There is an implied relational structure in the scene, namely, AIMS(archer, bull's-eye). In both alternative scenes a bird appears. In the right scene, the bird replaces the bull's-eye and thus becomes part of the scene's implied causal relational structure, namely, AIMS(archer, *bird*). In this case, the bird is an AD, because it is connected to the common predicate AIMS (relative to the standard). This is not the case in the left alternative where the bird is simply added to the scene, but does not participate in the common causal relational structure.

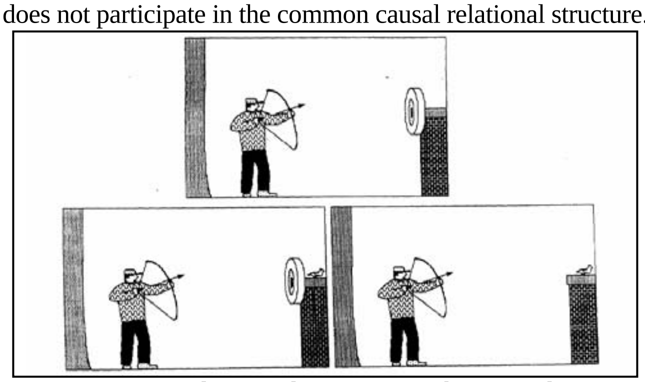

Figure 1: Sample stimulus item, Markman and Gentner (1996). The bird is a nonalignable difference on the left, and an alignable difference on the right.

[1] Of course, alignable differences will not win out in every case. For example, suppose the ND were an elephant and the AD were a flea.

According to structure-mapping theory, the alternative scenes will be aligned with the standard, and differences connected to the common structure (i.e., ADs) will become more salient, and will reduce similarity more than differences unrelated to the common structure (i.e., NDs). As predicted by structure-mapping, the AD alternative was judged less similar to the standard.

Recently, Estes and Hasson (2004) have carried out studies that challenge the claim that ADs are more salient than NDs, and the corollary claim that ADs reduce subjective similarity more than NDs. They aimed to provide a fair test of the predictions of structure-mapping by using stimuli patterned after a figure that was used by Markman and Gentner (1996) to illustrate the distinction between ADs and NDs.

In their first experiment, Estes and Hasson addressed the question of whether ADs are more important than NDs in judgments of similarity. Figure 2 presents a set of sample stimuli. In the example, the standard comprises a shaded square above an unshaded circle. One alternative (the "Standard + ND" alternative) contains the standard and an added nonalignable object (i.e., the triangle). The other alternative is partly alignable with the standard but has an internal object (an attribute AD or a relation AD) that does not match the corresponding object in the standard. Note that in the AD alternatives, the internal structure changes from that of the standard. In the ND alternative, the standard's structure is preserved: the new object is simply added on the side.

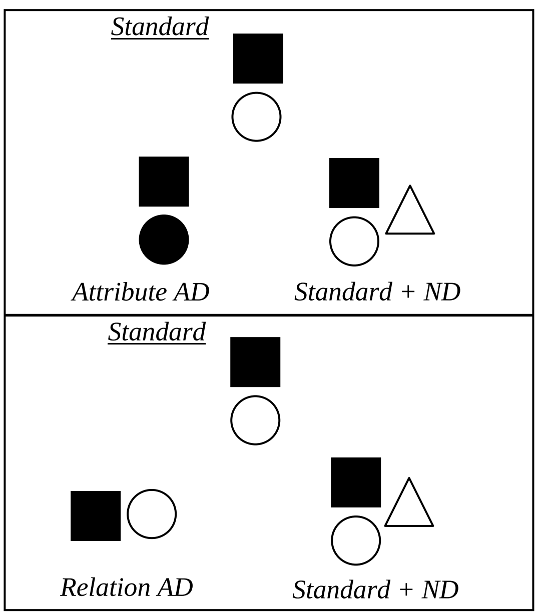

Figure 2: Sample comparisons, Estes and Hasson, Expt 1.

The results of interest for this paper involve those triads in which the *Standard + ND* alternative (S+ND) was paired with one of the AD alternatives. In these cases, the S+ND alternative was judged less similar to the standard than either the attribute AD item or the relation AD item. Estes and Hasson interpreted these results to suggest that the ND (the triangle) was more salient than the AD and that the ND detracted more from similarity than either the attribute AD or the relation AD. This runs contrary to the structure-mapping prediction that the S+ND alternative would be considered most similar to the standard (because its nonalignable difference—the extraneous object—should have mattered less than the alignable difference in the AD alternative—which, as noted above, leads to a difference in internal structure). Estes and Hasson concluded that these results support the distinction between alignable and nonalignable differences (consistent with structure mapping theory), but not that ADs are more "important" than NDs.

However, these conclusions hinge critically on the assumption that the number of objects (i.e., 2 vs. 3) is not being used as an alignable dimension. But if participants take number as an alignable dimension then they can process alignments (and notice ADs) based either on the *number* of objects or on the *internal structure* of the objects. If participants attend to internal structure, then they will choose the S+ND alternative in Fig. 2 as most similar to the standard. But if they align on the basis of number, then they will choose the alternative with the same *number* as most similar—the AD alternative. Estes and Hasson take the latter response as contradicting structure-mapping theory. We suggest it results from a different dimension of alignment. If so, the problem then becomes determining which kind of alignable difference will be chosen.

In their second study, Estes and Hasson adopted the logic of Markman and Gentner's (1996) Experiment 3 to test a more specific prediction of structure-mapping theory. Figure 3 shows sample stimuli.

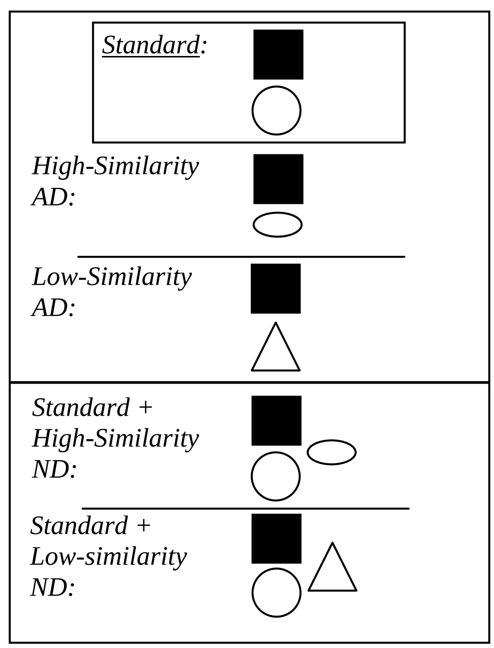

Figure 3: Sample stimulus set, Estes and Hasson, Expt. 2.

If ADs are more salient than NDs, then variations in ADs should matter more than variations in NDs in judging similarity to the standard. In their experiment, similarity ratings were obtained between pairs of figures formed by pairing a standard with one of 4 alternative items, as in Figure 3.

In Figure 3, the common relational structure is assumed to be the vertical spatial relation "square above circle." If (as predicted by structure-mapping) ADs matter greatly in judging similarity, then similarity-to-standard will vary greatly between the AD pairs. That is, replacing the circle in the standard with a triangle will lower similarity more than replacing the circle with an oval, a more similar object. If NDs are less important than ADs in judging similarity, then (1) similarity-to-standard will be relatively unaffected by *adding* an object on the side; and (2) similarity-to-standard will be relatively unaffected by *which* object is added—an oval or a triangle. In other words, the difference between the oval and the triangle should matter greatly when they occupy an alignable difference slot, and should matter very little when they occupy a nonalignable difference slot.

Consistent with structure-mapping, judged similarity-to-standard was greater for high-similarity AD items than for low-similarity AD items, and did not differ between the high- and low-similarity ND items. However, contrary to the structure-mapping account, the overall similarity ratings for the AD items were higher than those for the ND items. (Structure-mapping predicts lower similarity for AD items, because the differences matter.)

But these findings can be reconciled with structure-mapping if we make the assumption that participants were treating the number of objects as an alignable dimension. In this case, they would essentially be saying that two objects align better with two objects than with three objects. To clarify: the S+ND alternatives, whether high-similarity or low-similarity, both have three objects. Thus, if number is treated as an alignable dimension, structure-mapping would predict no difference in their similarity to the standard. However, because both of the AD items have two objects, for participants who aligned on the basis of number, these items would be considered more similar to the standard than the three-object S+ND alternatives.

Estes and Hasson assumed instead that participants aligned only along the common spatial configuration and not on the number of objects. They argued against the possibility that participants treated number as an alignable dimension but did not provide direct empirical evidence to support their claim. (We return to this point in the discussion).

What is needed, then, is a direct test of whether number of objects can act as an alignable dimension. To that end, we adopted the design and stimuli of Estes and Hasson's first experiment, but asked participants about the information that figured in their decisions. If people's choices of the same-number (AD) alternative are motivated by considering number as a salient alignable dimension, then when people choose the same-number alternative, they should mention sameness of number as the reason for their choice. In contrast, if they choose the S+ND alternative (with its nonalignable object) as most similar to the standard, then their comments should indicate a focus on the fact that it matches the standard, despite the extra object.

## Experiment 1

### Method

**Participants** Thirty-seven Northwestern undergraduates participated for either course credit or monetary compensation.

**Materials** The materials comprised six picture triads, with each triad composed of a standard figure and a pair of alternative figures. These triads were the same as those in Estes and Hasson's first experiment. As in their study, we ran all possible pairs of alternatives against each standard. However, the triads of interest for the present discussion were of the form shown in Figure 3. Given a standard consisting of two objects in a configuration, the two-object alternative always had one change in internal structure (either in an attribute or in the spatial relation), and the three-object alternative comprised the standard and an added object on its side. Finally, the standard and the paired alternatives were presented in the same triad format as in Estes and Hasson's first experiment. A sample item is presented in Figure 4.

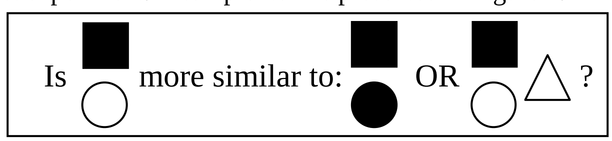


Figure 4: Sample item, Estes and Hasson, Expt. 1

**Procedure** The procedure was identical to that of Estes and Hasson's first experiment, with the addition of a response justification at the end of every item. Triads were presented on computer, and for each, participants were instructed to pick the alternative most similar to the standard by clicking on the chosen item. After each response, the question appeared, "What factors influenced your decision?" After the response was typed in, the participant was presented with the next triad. Left-right presentation was counterbalanced, and order of triad presentation was randomized.

**Response coding** The justification responses were sorted into three categories: (1) *number*, (2) *nonalignable difference*, and (3) *other*. Responses were coded as *number* if they referred to number: e.g., "2 items as opposed to 3." Responses were categorized as *nonalignable difference* if they referred to the extra object: e.g., "the same basic elements are present, just with the addition of an extra triangle." The category *other* was reserved for all responses that did not refer either to number or the extra object: e.g., "the shading trumps" or "same shapes with same shading, though in wrong positions."

**Results** We first looked at responses across triads: participants' similarity choices showed a strong advantage for the two-object alternative over the three-object alternative (*M*

= 0.76, $SD_{two\text{-}item}$ = 0.24), $t(36) = 6.4$, $p < .0001$. This closely replicates Estes and Hasson's main finding, which is important, because it suggests that providing justifications did not alter the similarity judgments.

The question is what best explains this pattern. Estes and Hasson interpreted participants' preference for the two- object alternative over the three-object alternative as due to the high salience of the nonalignable difference (the extra object). However, examination of participants' justifications undermines this conclusion in two ways. First, we found no significant difference between the likelihood of mentioning the alignable difference ($M$ = 0.40, $SD$ = 0.37) and of mentioning the number of objects ($M$ = 0.31, $SD$ = 0.33), $t(36)_{paired}$ = .87, $p$ = .40. If attention to the nonalignable difference were driving the similarity choices, we would have expected a large majority to mention the nonalignable difference in their justifications.

Second, and more importantly, there was a strong relationship between participants' justifications and their similarity preferences. Participants who gave number justifications were more likely to choose the AD alternative (which, like the standard, contains two objects), $r(36) = .43$, $p < .01$. In other words, when participants mentioned number, they treated it as an alignable dimension that mattered to similarity, and mostly chose the same-number alternative as most similar. In contrast, participants who mentioned the nonalignable difference (the added object) behaved as though it did not much matter: they tended to choose the ND alternative as most similar to the standard despite noting that difference, $r(36) = .38$, $p < .05$. Apparently these latter participants were aligning on the basis of common internal structure, while the former group (that mentioned number) was aligning on the basis of numerosity.

Note that this relationship is in the opposite direction to what Estes and Hasson would predict. Their claim is that nonalignable differences were more salient than alignable differences. However, our findings show that their pattern of results is consistent with alignable differences mattering more than nonalignable differences if number of objects is treated as an alignable dimension.

## Experiment 2

So far, Experiment 1 shows that number can serve as an alignable dimension, and can even win out over common spatial relational structure in some cases. Now let us step back and ask *when* we should expect the numerical dimension to be more salient than other bases for alignment. Two obvious possibilities come to mind. First, a given change in the number of objects should matter more for figures with few objects than for figures with many objects (by Weber-Fechner's law). The second factor is the degree of structure present. According to structure-mapping, people prefer alignments based on common systematic relational structure. The figures used in Experiment 1 represent a minimal level of structure. Perhaps greater levels of matching structure would compete more successfully against the numerical dimension. If so, then alignable differences in internal structure may outweigh differences in numerosity.

We tested this idea in Experiment 2. The design was similar to that of Experiment 1, but the stimulus items were composed of many geometrical shapes arranged in a rich, well-structured configuration (see Figure 5). We predict that number will become less salient as a potential alignable dimension than in Experiment 1, both because the change in number is less noticeable, and because the more systematic relational match between the standard and the alternatives will compete more strongly with the numerical match. Thus, similarity-to-standard should be higher for the S+ND alternative (which preserves internal structure but differs in number) than for the AD alternative (which changes internal structure but matches in number).

### Method

**Participants** Fifty-six Northwestern undergraduates participated for course credit.

**Materials** There were four picture triads, each consisting of a standard, an AD alternative that had the same number of objects as the standard, and a Standard + ND alternative that had one more object than the standard (see Figure 5 for a sample set). The triads were arranged so that the two alternatives were presented below the standard, one to the left of the standard, the other to the right of it. The standards consisted of simple geometrical shapes arranged regularly in a 3 X 3 grid. The AD alternative was similar to the standard, except that one shape within the grid was replaced by a different shape. The Standard + ND alternative was identical to the standard (and therefore matched the configuration of the standard), but had an extra shape outside the grid, making a mismatch in the number of elements. The AD alternative matched the standard in number but mismatched its configurational structure (since the different object *replaced* an original object within the grid).

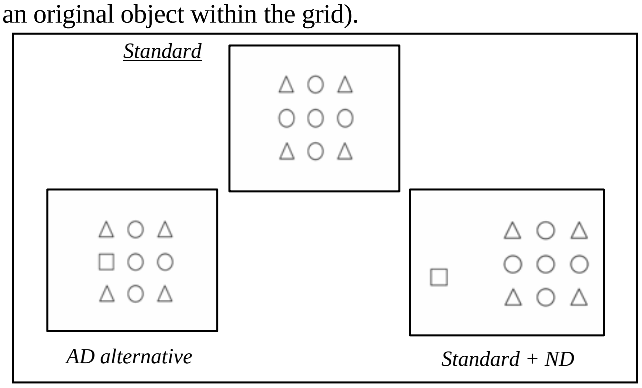


Figure 5: Sample stimulus set, Expt. 2

**Procedure** Participants were shown triads with two alternatives labeled A and B, and were instructed to indicate which alternative was most similar to the standard. They then provided a justification for their response.

**Results** We computed the proportion of times each participant chose the Standard + ND alternative as most similar to the standard and compared these proportions to chance (0.5). Consistent with our prediction, participants showed a significant preference for the S+ND alternative ($M = 0.81$, $SD = 0.56$), $t(55) = 6.55$, $p < .0001$. This finding contrasts strongly with the pattern in Experiment 1. In that study, 76% of the participants preferred the same-number alternative, whereas in this study, 81% preferred the different-number (but same configuration) alternative. Further, across all participant justifications, only one participant mentioned number as influencing the similarity comparison. This pattern of results suggests that number plays a less prominent role in determining similarity when the stimuli have richer, more systematic relational structure.

## Discussion

In recent years, structure-mapping theory has been gaining ground as a theory of similarity; and more generally, as a theory of the comparison process. With its emphasis on treating the process of computing similarity as one that seeks maximal alignment of conceptual structure, structure-mapping theory has met with a great deal of empirical success (for a review, see Gentner and Markman, 1997).

Estes and Hasson (2004) challenged structure-mapping's predicted role of kinds of differences in similarity comparisons. Estes and Hasson argued that nonalignable differences can affect similarity judgment as much or more than alignable differences, contrary to the predictions of structure-mapping theory. They argued further that their data provide an important counterexample to a key prediction of structure-mapping because the stimuli they used were simple and were designed to be good candidates for testing structure-mapping's key predictions. Indeed, the standard figures in their studies were taken from Markman and Gentner's (1996) paper, so it seemed reasonable that they should qualify as a fair stimulus choice.

In fact, Markman and Gentner used those geometrical stimuli only as a simple way to demonstrate the distinction between alignable and nonalignable differences, and never used them as stimuli in an experimental context. But at the very least, Estes and Hasson have shown that Markman and Gentner should have chosen better figures to demonstrate their point.

That said, however, the results of the present studies undermine Estes and Hasson's conclusions. First, against their claim that number did not play a role as an alignable dimension in their studies, we gathered direct empirical evidence that participants did indeed consider number as a salient alignable dimension. In Experiment 1, participants who mentioned number were highly likely to choose the same-number item as most similar to the standard. This is what would be expected if these participants treated number as an alignable dimension. Second, against their claim that nonalignable differences are highly salient in similarity judgments, we found that even when participants mentioned the nonalignable difference (the extra object in the three-object alternative), they were highly likely to choose that alternative as most similar to the standard. In other words, the nonalignable difference did not matter enough to determine the output of the comparison process.

Finally, in Experiment 2 we found that attention to the numerical dimension diminished sharply when the materials had more internal objects and more systematic internal structure. We suggest that this kind of richness is characteristic of most real-life comparisons.

### Numerical and spatial alignment in the Estes and Hasson studies

Estes and Hasson considered the possibility that participants treated number as an alignable dimension, but argued that the results of their Experiment 2 ruled out this possibility. (Figure 3 in this paper provides an example of one of their items.) They granted that the AS and AD alternatives differed in similarity (as predicted by structure-mapping), but went on to state that "[i]f the NS and ND alternatives were also conceived as alignable, then they should have exhibited an effect of variation. However they didn't; the alignable and nonalignable alternatives exhibited qualitatively different patterns of results" (p. 1091).

But the NS and ND alternatives have the *same* number of objects (namely, three) relative to the standard (which has two), so a lack of difference between them is exactly what would be predicted if participants were aligning on the basis of number. Because the numerical mismatch relative to the standard is the *same* for both alternatives, number provides no basis for preferring one alternative to the other.

The AS and AD alternatives also don't differ in *numerical* alignability with the standard (both have two objects). But unlike the NS and ND alternatives, the AS and AD alternatives *do* differ in the similarity of their structurally aligned objects (following the logic of Markman and Gentner's (1996) study); and, as predicted, they differ in similarity-to-standard. In contrast, because the NS and ND pairs are equally alignable (or nonalignable) with the standard both on the basis of internal spatial structure and on the basis of numerical match, there is no reason for structure-mapping to predict a difference in similarity.

In summary, our evidence suggests that contrary to Estes and Hasson's assumption, number was treated as an alignable dimension in these triads. We have provided converging evidence from protocols that number was often seen as alignable by participants; and that when it was, then participants chose the same-number alternative, just as structure-mapping would predict.

### When is number alignable?

These findings lead to the question of exactly *when* number will be treated as alignable. One answer would be that number will matter in the comparison process when participants notice number, and otherwise not. After all, structure-mapping is a model of the comparison *process*. Its input representations depend on people's prior knowledge, goals, and other factors over which the analogy process itself need have no sway. But this is unsatisfying. We can go further in laying out some principles as to when the numerical dimension will be important in alignment.

According to structure-mapping, the comparison process acts to find a maximal alignment of conceptual structure. If the common structure is relatively impoverished, numerosity should play a more prominent role in the similarity comparison. When the common structure is highly systematic, number should play a less prominent role as an alignable dimension. In this case, alignable differences that arise within the aligned relational structure will dominate. This is indeed what we found in Experiment 2. When we used figures that comprised many geometrical shapes arranged in a richer, more systematic relational structure, number did play a less salient role.

Another factor that may influence the relative importance of number versus common configurational structure is the degree of variation in the number of objects. The difference between two and three objects looms much larger than the difference between nine and ten objects. In Experiment 2 both numerical variation and configurational structure acted to diminish the importance of number as the salient alignable dimension. A third factor is the sheer number of objects in the figures: if the number is too large to subitize, then changes in number might not matter even if the change itself is rather large. We are currently conducting studies to tease apart these three factors.

As this discussion reveals, although we disagree with Estes and Hasson's conclusion, their challenge has had the positive effect of helping to illuminate the role of alignability in similarity comparisons.

## Acknowledgments

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