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SUMMARY

In this work the construction of the configuration space equivalent potential in Born approximation is considered for scattering by particles of unequal mass. The potential agrees in essence with the one obtained using the Chew-Frautschi prescription in S-matrix theory, i.e., the range of the exchange force arising from the exchange of a particle of mass M_p is

$$R_0 = \left[M_p^2 - \frac{(M_A^2 - M_B^2)^2}{s} \right]^{-\frac{1}{2}}$$

where (M_A, M_B) are the masses of the particles involved in the scattering and $s = (\text{CM energy of } (A,B))^2$. This clarifies the confusion in the recent literature regarding this point and sets the stage for determining the importance of the energy dependence of the range parameter in studying the dynamics of strongly interacting

particles. The examples considered are $K^*(890)$-exchange in πK scattering and N-exchange in πN scattering. It is shown that the energy dependence of the range parameter is not important in the former and may be crucial in the latter.

INTRODUCTION

Recently Balázs[1] has presented a new method for studying the dynamics of strongly interacting particles. The essential point is to construct an equivalent potential which when fed into the Schrödinger equation generates a unitary relativistic scattering amplitude (satisfying an unsubtracted Mandelstam Representation) when the nonrelativistic CM momentum (k^2) is set equal to the relativistic CM momentum (q^2). The construction of the equivalent potential has been carried out by Balázs[1] for the equal mass case when a single pole in momentum transfer is assumed to dominate the Born approximation. The resulting configuration space potential agrees in essence with the potential calculated using the Chew[2]-Frautschi prescription, i.e., the range of the force due to the exchange of a particle of mass m_p is $\frac{1}{m_p}$ and is energy independent. There has recently been a display of confusion in the literature regarding the construction of the equivalent potential for the unequal mass case. In particular consider πN scattering and the general features of the equivalent potential that should result from nucleon exchange in the crossed u-channel. Following the Chew-Frautschi prescription one finds the range of the force due to N-exchange is energy dependent and has the value

$$R_0 = \left[M^2 - \frac{(M^2 - m_\pi^{\ 2})^2}{S} \right]^{-\frac{1}{2}} .$$

One would certainly expect this feature to persist in the equivalent potential. The purpose of this paper is to show it does and thereby set straight a point which may be crucial in understanding the relevance of the equivalent potential prescription for studying the dynamics of strongly interacting particles.

II. CONSTRUCTION OF THE EQUIVALENT POTENTIAL FOR THE UNEQUAL MASS CASE

For illustrative purposes we shall consider πN and πK scattering without worrying about spin or isotopic spin complications, which only serve to obscure the essential point.

Consider the following Schrödinger equation

$$\nabla^2\psi + [k^2 - U(r,q^2)]\,\psi = 0 \tag{1}$$

where

$U(r,q^2) = 2\mu\, V(r,q^2)$

$\mu = \dfrac{m_A m_B}{m_A + m_B}$ = the reduced mass of the 2 particles

$V(r,q^2)$ = the interaction part of the Hamiltonian

k = magnitude of the non-relativistic CM momentum

q = magnitude of the Relativistic CM momentum

The quantity $V(r,q^2)$ is the potential that is needed to produce the relativistic amplitude for $k^2 = q^2$. It is constructed, as suggested by Balazs,[3] by requiring the Born approximation of

$V(r, q^2)$ to reproduce the Born approximation to the relativistic amplitude. For $\pi N(\pi K)$ scattering this corresponds to keeping the mesons that communicate with the t-channel $[\pi\pi \to N\bar{N}(\pi\pi \to K\bar{K})]$ and the baryons and mesons that communicate with the u-channel $[\pi N \to \pi N(\pi K \to \pi K)]$. Let us consider this prescription for both Direct (t-channel singularities) and exchange (u-channel singularities) forces, keeping in mind the following notation for the invariant variables for the s-channel:

<u>For πN scattering:</u> ch s $\pi N \to \pi N$

ch t $\pi\pi \to N\bar{N}$

ch u $\pi N \to \pi N$

The invariant variables for the s-channel are

$$s = (\text{CM energy})^2$$
$$t = -2q^2(1 - \cos\theta) \qquad (2)$$
$$u = 2(M^2 + m_\pi^2) - s - t$$

where M = nucleon mass, m_π = pion mass, θ = CM scattering angle and

$$q^2 = \frac{[s - (M + m_\pi)^2]\,[s - (M - m_\pi)^2]}{4s} \qquad (3)$$

For πK scattering: There is a complete analogy with the relevant formulas given above for πN scattering. Just replace M by M_K = mass of the Kaon.

A) Direct Force

Consider the attractive potential

$$V = - \frac{\lambda e^{-\beta r}}{r} \quad ; \tag{4}$$

the Born approximation to (4) is

$$f^B = - \frac{1}{4\pi} \int \underline{dr}\, e^{-i\vec{q}\,'\cdot\vec{r}}\, V(r,q^2)\, e^{+i\vec{q}\cdot\vec{r}} \quad , \tag{5}$$

where $(\vec{q},\vec{q}\,')$ are the initial and final CM momenta, respectively, for the pion. Substituting (4) into (5) and doing the 3 dimensional integration over configuration space gives

$$f^B = \frac{\lambda}{\beta^2 + \Delta_t^2} \quad , \tag{6}$$

where $\Delta_t^2 = + 2q^2(1 - \cos\theta)$. Keeping just the ρ-meson contribution to the t-channel, the Born approximation to the relativistic amplitude is just

$$f^B = \frac{\lambda}{m_\rho^2 - t} = \frac{\lambda}{m_\rho^2 + \Delta_t^2} \quad , \tag{7}$$

where m_ρ = mass of the ρ-meson. For (6) and (7) to be identical it is necessary to write $\beta = m_\rho$ and this is just the familiar result for the equal mass case, i.e., the range of the force is $\frac{1}{m_\rho}$ and is energy independent.

B) Exchange Force

Consider the attractive potential

$$V = - \frac{\lambda\, e^{-\beta r}}{r} P_{EX} , \qquad (8)$$

where P_{EX} is the exchange operator and

$$P_{EX}\, e^{i\vec{q}\cdot\vec{r}} = e^{-i\vec{q}\cdot\vec{r}} . \qquad (9)$$

Using (5) and (8)-(9) the Born approximation has the form

$$f^B = \frac{\lambda}{\beta^2 + \Delta_u^2} , \qquad (10)$$

where $\Delta_u^2 = 2q^2(1 + \cos\theta)$. Keeping just the nucleon contribution to the u-channel in πN scattering [or the K^* (890) contribution for πK scattering] the Born approximation to the relativistic amplitude is just

$$f^B = \frac{\lambda}{M_p^2 - u} , \qquad (11)$$

where M_p = mass of the appropriate particle

$$u = 2(M_A^2 + m_\pi^2) - s + 2q^2(1 - \cos\theta)$$

M_A = mass of the nucleon or Kaon.

For (10) and (11) to be equal note that

$$M_p^2 - u \equiv \beta^2 + \Delta_u^2, \text{ where } \beta^2 = M_p^2 - 2(M_A^2 + m_\pi^2) + s - 4q^2 .$$

If (3) is inserted into the expression for β^2 one finds

$$\beta^2 = M_p^2 - \frac{(M_A^2 - m_\pi^2)^2}{s} . \tag{12}$$

The range of the exchange force is therefore energy dependent and given by

$$R_0 = \beta^{-1} = \left[M_p^2 - \frac{(M_A^2 - m_\pi^2)^2}{s} \right]^{-\frac{1}{2}} . \tag{13}$$

This is just what one expects from the Chew-Frautschi prescription for constructing an exchange potential in configuration space. Finally, if one takes proper account of spin and isotopic spin complications the equivalent potential acting in the $J = I = 3/2$ state of the πN system for nucleon exchange[3] is

$$V^N(r,q^2) = -\frac{g^2/4\pi}{W}\,\frac{e^{-\beta r}}{r}\left\{(E+M)(W-M) + \frac{(E-M)(W+M)}{q^2}\times\left[\left(q^2+\frac{\beta^2}{2}\right)+\left(\frac{1+\beta r}{r^2}\right)\right]\right\} , \tag{14}$$

where $\beta^2 = M^2 - \frac{(M^2 - m_\pi^{\ 2})^2}{s}$ and $g^2/4\pi = 14.4$.

The equivalent potential appropriate for the $\ell = 1$, $I = \frac{1}{2}$ state of the πK system for $K^*(890)$ exchange is

$$V^{K^*}(r,q^2) = -8\,\Gamma_{in}^{\ K^*}\left(\frac{s+12}{\sqrt{s}}\right)\frac{e^{-\beta r}}{r} , \tag{15}$$

where $\beta^2 = M_{K^*}^{\ 2} - \frac{(M_K^{\ 2} - m_\pi^{\ 2})^2}{s}$ and $\Gamma_{in}^{\ K^*}$ is the input reduced width of the $K^*(890)$ resonance.

III. DISCUSSION AND NUMERICAL RESULTS

It is of importance to ask what role the energy dependence of β plays in determining the dynamics. If the energy dependence of β does not matter we can use β for any value of s. Suppose we take $s \to \infty$; then $\beta^N = M$ (for nucleon exchange) and $\beta^{K^*} = M_{K^*}$ (for K^* exchange). If the energy dependence of β does matter the strongest effect (in the physical region) would be at threshold and it would be more appropriate to evaluate β there. Then $\beta_0^N \approx (2m_\pi M)^{\frac{1}{2}}$ (for N exchange)and $\beta_0^{K^*} \approx \left[(M_{K^*}^2 - M_K^2) + 2m_\pi M_K\right]^{\frac{1}{2}}$. Let us consider these examples in detail.

A) πK scattering : K^* Exchange Force

From (15) calculate the quantity

$$R^{K^*}(r) \equiv \frac{V^{K^*}(r,q^2,\beta^{K^*})}{V^{K^*}(r,q^2,\beta_0^{K^*})} = \exp\left\{-\left[M_{K^*} - \left\{(M_{K^*}^2 - M_K^2) + 2m_\pi M_K\right\}^{\frac{1}{2}}\right] r\right\}. \tag{16}$$

Table I shows $R^{K^*}(r)$ vs r for $M_{K^*} = 6.35$, $M_K = 3.54$ and $m_\pi = 1$. Since V^{K^*} is singular at $r = 0$ it will have its strongest effect for "small" r. For "large" r the potential goes to 0 like $e^{-\beta r}$. For all practical purposes $V^{K^*} = 0$ for $r \approx 0.4$. Table I therefore shows that the energy dependence of β is not important in discussing πK scattering and Finkelstein's work (Ref. 3) is essentially correct despite the error in writing out V^{K^*}.

B) πN Scattering : N Exchange Force

From (14) calculate the quantity

$$R^N(r) = \frac{V^N(r, q^2 = 0, \beta^N)}{V^N(r, q^2 = 0, \beta_0^N)} \approx \exp\left\{-\left[M - \sqrt{2m_\pi M}\right] r\right\} \times \left\{\frac{2M + M^2/2 + \left(\frac{1 + Mr}{r^2}\right)}{3M + \left(\frac{1 + \sqrt{2m_\pi M}\, r}{r^2}\right)}\right\} . \tag{17}$$

Table I shows $R^N(r)$ vs r for $M = 6.7$, $m_\pi = 1$. It is clear from Table I that the energy dependence of β is crucial for $r \approx 0.3$, which is a range of r where the potential can not be considered negligible. Hence the energy dependence of β for the N exchange force in πN scattering in the (3,3) state may very well have effects which would not be exhibited if this energy dependence were ignored. To check this point in detail requires integrating the Schrödinger equation numerically using the potential given by (14) with a cut-off.[4] This work is now in progress and will be discussed in a future publication.

IV. CONCLUSION

It is concluded that the energy dependence of the range parameter of the equivalent potential is not important for the K^* exchange potential in πK scattering but may very well be crucial for the N exchange potential in πN scattering in the (3,3) state.

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TABLE I

r	$R^N(r, q^2 = 0)$	$R^{K^*}(r)$
0	1	1
0.2	0.82	0.92
0.4	0.50	0.85
0.6	0.29	0.70
1.0	0.09	0.67
2.0	~ 0	0.45

FOOTNOTES AND REFERENCES

* Visiting Scientist. This work was performed under the auspices of the United States Atomic Energy Commission.

1. L. A. P. Balázs, Phys. Rev. 137, B1510 (1965).

2. G. F. Chew and S. Frautschi, Phys. Rev. Letters 5, 580 (1960).

3. L. A. P. Balázs, Phys. Rev. 139, B1646 (1965).
The equivalent potential for nucleon exchange is wrong in this paper. The correct equation is reported in the text of this work.
J. Finkelstein UCRL-16537 (unpublished). The equivalent potential reported for K^* exchange is wrong in this work. The correct form is given in the text of this work.

4. The potential given by equation (14) has a strong $\frac{1}{r^3}$ singularity at $r = 0$ and the solution to the Schrödinger equation does not exist unless the potential is parameterized. One way to parameterize the potential is to use a straight cut-off (giving one free parameter); another way is to introduce a repulsive core (giving 2 free parameters).