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The experimental study of Alfven-wave propagation makes possible a quantitative comparison between the predictions of hydromagnetic theory and experimental results. Our hydromagnetic wave guide for the generation and propagation of torsional Alfvén waves has been described previously.[1] It consists of a cylindrical hydrogenic plasma contained in a copper cylinder 86 cm long and 14.6 cm in diam which is immersed in an axial magnetic field of approximately 15 kgauss. A torsional hydromagnetic wave is induced at one end of the tube by an oscillating radial current, which is driven from an external circuit.

We have measured the radial distribution of the oscillating magnetic field that is associated with the wave, and find a good agreement with the results of an analysis of the modes of propagation to be expected. We have observed reflections of the hydromagnetic waves from the end of the cylindrical plasma[2] and have used this effect to measure the radial magnetic-field distribution of a wave that has made three transits of the hydromagnetic wave guide. Since the lowest-order mode has the least amount of attenuation, the wave that has made three transits of the tube would be expected to consist almost entirely of the lowest-order mode. This is verified experimentally.

Using Maxwell's equations, Ohm's law, and Newton's second law of motion one can show[3] that the radial distribution of the azimuthal magnetic field $b_\theta$ associated with the wave is described by a first-order Bessel function $J_1(k_{cn}r)$, where the various principal modes are designated by $n=1, 2, 3 \cdots$, and the $k_{cn}$ are evaluated from a boundary condition that was determined experimentally in the following manner. Radial current probes inserted into the wall of the wave guide failed to detect a current associated with the propagating wave. Therefore the radial current density [which is also proportional to $J_1(k_{cn}r)$] is zero at the wall, and the boundary condition is $J_1(k_{cn}a)=0$, where $a$ is the wave guide radius. The attenuation length for ohmic losses is proportional to $(k^2_{cn}+k^2)^{-1}$, where $k$ is the wave number of the wave. Since we have $k_{c1} < k_{c2} < k_{c3} \cdots$, the attenuation of the lowest-order mode is considerably less than the attenuation of the higher modes. Thus, although an arbitrary disturbance at the driving end can excite many modes, only the lowest-order modes survive for an appreciable distance.

The radial variation of $b_\theta$ has been measured with six magnetic probes inserted into the plasma at six radial positions near the receiving end of the tube. Results for the wave which has made one transit of the tube are shown in Fig. 1. The solid line represents the lowest mode of propagation plus 30% of the second mode. The amount of second mode present was determined by a best fit to the experimental data. The effective plasma radius for wave propagation appears to be about 4mm less than the radius of the copper cylinder. This may be a measure of the thickness of the sheath or layer of neutral particles that isolates the wave currents from the conducting boundary.

When the wave reaches the receiving end, it undergoes a reflection which corresponds to that of an open-ended transmission line. A similar reflection occurs at the driving end, and we can then observe a wave that has made three transits of the wave guide, as shown in Fig. 2. According to theory, a second mode which was present to the extent of 30% after one transit would be present after three transits to the extent of only 0.4%, i. e. it would be negligible. Thus the solid line in Fig. 2 represents the lowest mode only.

REFERENCES

1. T. K. Allen, W. R. Baker, R. V. Pyle, and J. M. Wilcox, Phys. Rev. Letters 2, 383 (1959); J. M. Wilcox, F. I. Boley, and A. W. DeSilva, Phys. Fluids 3, 15 (1960).

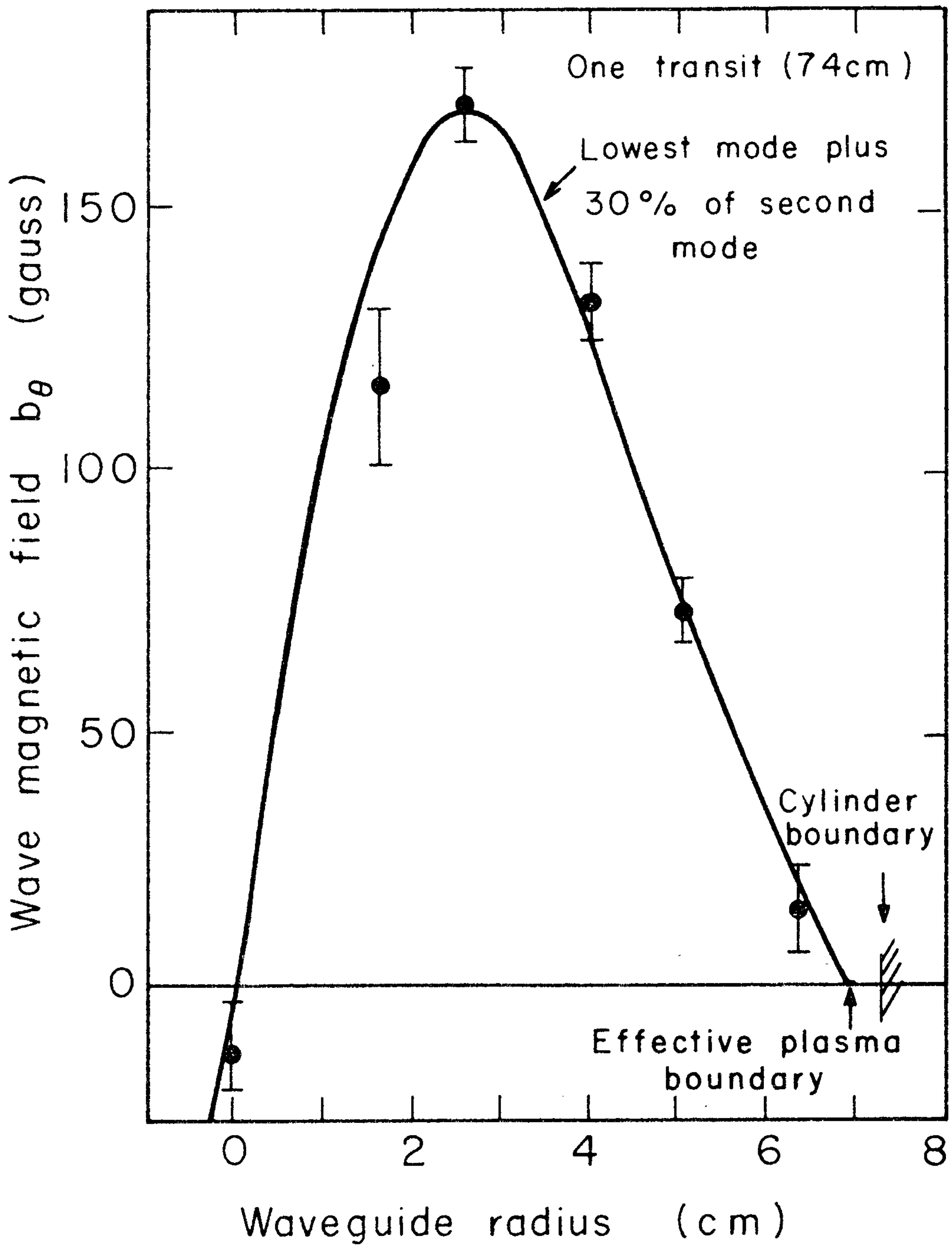
2. Reflection of plasma Alfvén waves has independently been observed by Shigeo Nagao and Teruyuki Sato, Tohoku University, Sendai, Japan (private communication).

3. W. A. Newcomb, in Magnetohydrodynamics (Stanford University Press, Stanford, California, 1957), p. 109.

## FIGURE LEGENDS

Fig. 1. Measured radial distribution of the wave magnetic field after one transit of the wave guide (74 cm). The solid line is proportional to $J_1(k_{c1}r) + 0.3\ J_1(k_{c2}r)$, i.e., the first mode plus 30% of the second mode. Vertical bars indicate the standard deviation of the mean for eight measurements plus an estimate of calibration uncertainties.

Fig. 2. Measured radial distribution of wave magnetic field after two reflections, i.e., after three transits of the wave guide (247 cm). The solid line is proportional to $J_1(k_{c1}r)$, i.e., the first mode only.



One transit (74cm)
Lowest mode plus
30% of second
mode
Cylinder
boundary
Effective plasma
boundary
Wave magnetic field $b_\theta$ (gauss)
150
100
50
0
0
2
4
6
8
Waveguide radius (cm)


Fig. 1

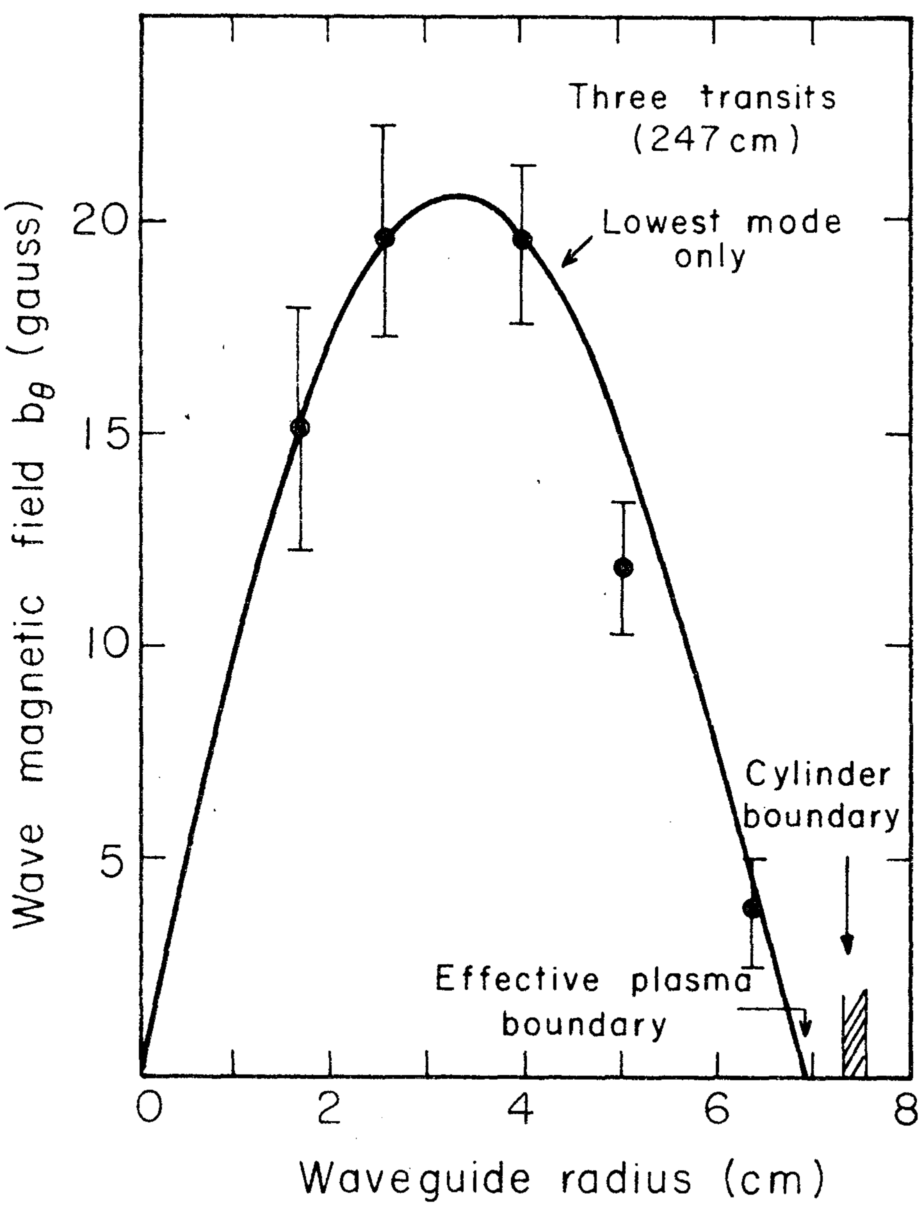

Three transits
(247 cm)
Lowest mode
only
Cylinder
boundary
Effective plasma
boundary
Wave magnetic field $b_\theta$ (gauss)
20
15
10
5
0
2
4
6
8
Waveguide radius (cm)

Fig. 2