## Title

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# Response induction coil magnetometers to perturbations in orientation

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## Summary

We explore the data collected by a 3- component induction coil magnetometer system with respect to motion of the instruments in earths static magnetic field. The sensitivtiy of the inductiuon coil magnetometer leads to unprecediented accuracy on tilt measurements. We model the signals observed during seismic events as being perturbations in coil orientation. In theory, these perturbations can include ground roll, ocean motion, nearby cultural seismicity, or any other field with a tilting effect. Using data from a magnetic observatory near Parkfield CA we invert several time series of coil data during different levels of seismic activity in an attempt to determine the magnitudes of rotation at which our model accurately describes the coil data. Finally, we explore the transfer function between the coils and nearby seismic instruments (accelerometers, tiltmeters, and velocity seismometers).

## Introduction

The induction coil magnetometer (ICM) is in principle, sufficiently sensitive to detect μ-radian changes in coil orientation. We combine rigid body mechanics, Faraday's law, magnetotelluric (MT) processing techniques, and field data to determine the extent to which the instrument is able to deliver estimates of ground tilt.

The ICM is basically a wire wound around a ferromagnetic core material. A variation in ambient magnetic induction oriented along the core results in an EMF being induced at the output wires.

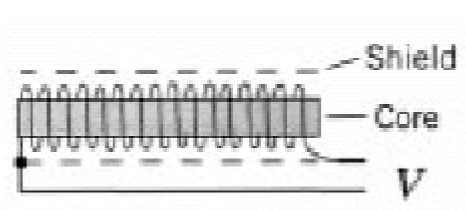


**Figure 1:** A magnetic induction coil (feedback loop for phase stability not shown)

The V response is directly proportional to the change in magnetic flux Φ which threads the coil, as per Faraday's Law of magnetic induction.

$$emf = -N \frac{d\Phi}{dt} \qquad \text{(Equation 1).}$$

Where N (number of turns of wire) is a property of the instrument, and the negative term is due to Lenz's Law, which states the induced *emf* will act in such a way as to reduce the net flux (Φ).

The ICM has been used with much success to observe small time varying magnetic fields, such as those generated by ionosphere-magnetosphere coupling currents, or cultural noise sources. The ICM is insensitive to the background field of the earth which is many orders larger than these detectable fields, but does not vary on a short enough timescale to induce detectable *emf* in the coil. The coils which collect the data we present are EMI BF-4 type, which are ICM's with a feedback loop for phase stability. A BF-4 is a circular cylindrical of diameter 6cm, and length 142cm. The BF-4 instrument response function and noise levels are plotted in figure 2.

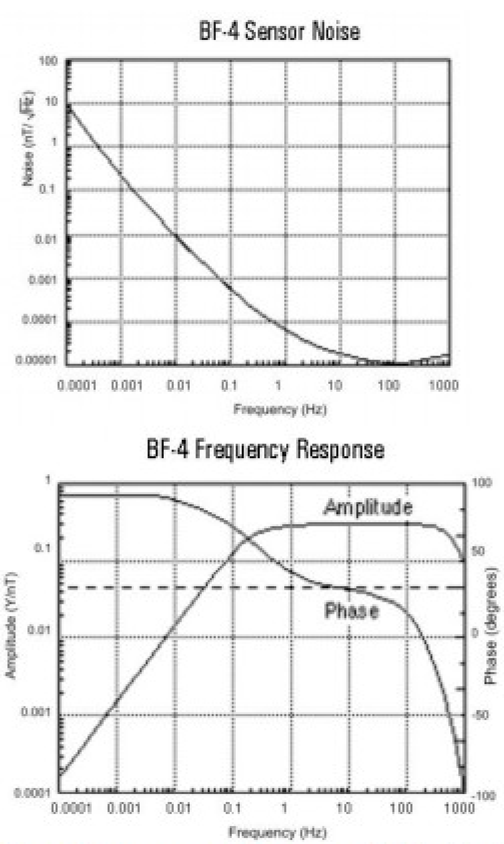


**Figure 2:** Frequency Response and Noise for BF-4 induction coils.

Strong signal output is observed during ground motion [Johnston et. al. 1981, Mueller et al 1998] but is seldom sampled at a high enough rate to resolve coherence between magnetometer and seismometer. Since very large magnetic fields are observed during strong ground motion, and the response of an induction coil should not vary as a result of small translations in position, we suppose that much of the coseismic observed signal is due to rotation of the coils.

Our intents in this presentation are:

1. Characterize an array of instruments (including remote reference array) which functioned during events which caused small angle motion of the coils.

2 a) Derive analytic expressions for induced *emf* as a function of angle, and an inverse expression for system orientation in terms of *emf* b) Offer a linear approximation to these equations

3. Apply our equations to the field-data of magnetic field variations. Solve them for the three Euler angles. This essentially maps the tilt of the site where the coils are installed.

4. Investigate the transfer function (TF) between the coils and nearby accelerometers, attempting to differentiate the accelerometer's response into translational and rotational components.

Theory

The core-sourced component of magnetic field at our location is specified by:

$\mathbf{B}_{earth} = \langle B_x, B_y, B_z \rangle = \mathbf{B}$.

We begin by considering a single ICM whose position is fixed with respect to the earth's magnetic field, with orientation vector $\mathbf{C} = \langle C_x, C_y, C_z \rangle$, where **C** is a unit vector running along the cylindrical axis.

Clearly, the core-generated flux threading the coil, and hence the $B(t_k)$ at time $t_k$ is given by:

$$\Phi(t_k) = aN\mathbf{B} \bullet \mathbf{C}(t_k) \qquad \text{(Equation 2)}$$

where a is the cross sectional area of the coil, and N is the number of wire turns. We make the following assumptions about the set up:

1. The core-sourced component of the magnetic field in the region of the place of data collection does not vary spatially, nor temporally.

2. Initially we disregard the time dependant component of B. A remote reference (RR) site is available, should RR MT processing seem appropriate.

3. Each ICM is treated as a rigid body, and hence obeys Euler's theorem. The motion of the coil from times $t_k$ to $t_{k+1}$ can be expressed as the sum of a single translation and a single rotation about some axis $\boldsymbol{\omega}$.

4. The group of three orthogonal magnetometers is treated as a rigid body. Which is to say that the relationship: $\mathbf{C}_i \bullet \mathbf{C}_j = \delta_{ij}$ holds. Since the spacing between the sensors is a few meters, this assumption can only hold when the perturbing wavelengths are sufficiently long.

5. We assume (at first) that all coils are identical. There is no variation in cross-sectional area a, nor in number of turns N. We proceed to normalize **C** to have length aN for convenience of notation. By normalizing C to have length equal aN we find that B•C gives the the field read by the coil.

By assumptions (1) and (3) we see that any seismically induced *emf* at the ICM must be the result of coil rotation. The relationship between the core-generated flux threading the coil, at $t_k$ and $t_{k+1}$ is then given by

$$\Delta\Phi(t_{k+1}) = \mathbf{B}\bullet\mathbf{C}(t_{k+1}) - \mathbf{B}\bullet\mathbf{C}(t_k)$$
$$= \mathbf{B}\bullet(\mathbf{C}(t_{k+1}) - \mathbf{C}(t_k)) \text{ (Equation 2)}$$

In the domain where the frequency response is flat the $i^{th}$ BF-4 acts as a "BField" meter, measuring the time changing part of the magnetic field aligned with the coil ($B_i(t)$) directly. Labeling our three coils as $\mathbf{C}_1(t)$, $\mathbf{C}_2(t)$, $\mathbf{C}_3(t)$ then it is clear that the observed $\Delta Bi$ (i=1,2,3) of each coil is given by :

$$\Delta B_i(t_{k+1}) = \mathbf{B}\bullet(\mathbf{C}_i(t_{k+1}) - \mathbf{C}_i(t_k)) \text{ (Equation 3)}$$

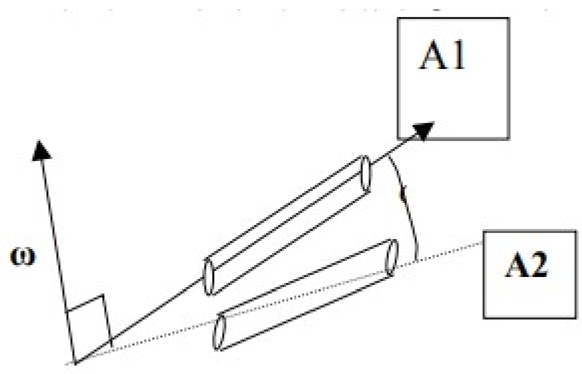


**Figure 3**
A1:Orientatin of $\mathbf{B}_{earth}$ (**C** coil and **B** are aligned at t=0)
A2:**C** at some later time

The forward problem here is determining the sensor observations as a function of the rotation vector **ω**. It is clear from equation 3 that it would be sufficient to know $\Delta \mathbf{C}_I = \mathbf{C}_i(t_{k+1}) - \mathbf{C}_i(t_k)$. Given that we know the system orientation at $t_k$ and the rotation vector **ω** there is an easy way to do this. By Euler's rotation theorem we know $\mathbf{C}_i(t_{k+1}) = \mathbf{A}\mathbf{C}_i(t_k)$, or:

$$\Delta \mathbf{C}_i(t_{k+1}) = (\mathbf{A} - \mathbf{I})\mathbf{C}_I(t_k) \quad \text{(Equation 4)}$$

**A** can be thought of as the product of three matrices, $\mathbf{A} = \mathbf{T}^{-1}\mathbf{\Omega}\mathbf{T}$, where **T** transforms our coordinate frame such that **ω** lies along a principal axis, **Ω** performs the rotation, and $\mathbf{T}^{-1}$ returns to the original frame. Formulae for **T**, and **Ω** can be found in [1], and will be presented in more detail. Qualitatively, to make **T**, we can rotate by θ about the **z-axis** (**k**) until the projection of **ω** onto the x-y plane lies along x (whose unit vector is denoted *i*) only. The angle θ obeys

$$(\boldsymbol{\omega} - \boldsymbol{\omega} \bullet \mathbf{k}) \bullet \mathbf{i} = |\boldsymbol{\omega}|(\cos(\theta)) \quad \text{(Equation 5)}$$

Similarly, rotate about the y axis by the angle φ defined by $(\omega \bullet k) = |\omega|(\cos(\varphi))$. Applying these two rotations in order results in **T**. **Ω** is now just a rotation by **|ω|** about the *z*-axis, and $\mathbf{T}^{-1}$ can be computed from **T**.

The Forward problem has the following form:

$$\begin{bmatrix} \Delta B_{Xobs} \\ \Delta B_{Yobs} \\ \Delta B_{Zobs} \end{bmatrix} = \begin{bmatrix} \Delta C_{1X} & \Delta C_{1Y} & \Delta C_{1Z} \\ \Delta C_{2X} & \Delta C_{2Y} & \Delta C_{2Z} \\ \Delta C_{3X} & \Delta C_{3Y} & \Delta C_{3Z} \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$

(Equation 6)

Where $B_{Qobs}$ refers to the change in the B field read by the $Q^{th}$ sensor

The rows of the $\Delta\mathbf{C}_i$ matrix are the change in the coil orientations (coils are assumed to hold one end fixed at origin as per assumptions 1 and 3). And the **B** vector is $\mathbf{B}_{earth}$.

Now that the forward problem is formulated we consider the inverse problem. Here we are given a multivarite time series of B field variations at three magnetometers. A sample of the data time series are shown below, together with seismometer output during a moderate earthquake. We have a time series of coil outputs, sampled at 40Hz, and knowledge of the value of $\mathbf{B}_{earth}$ at our site. The inverse problem is non-linear and is solved numerically.

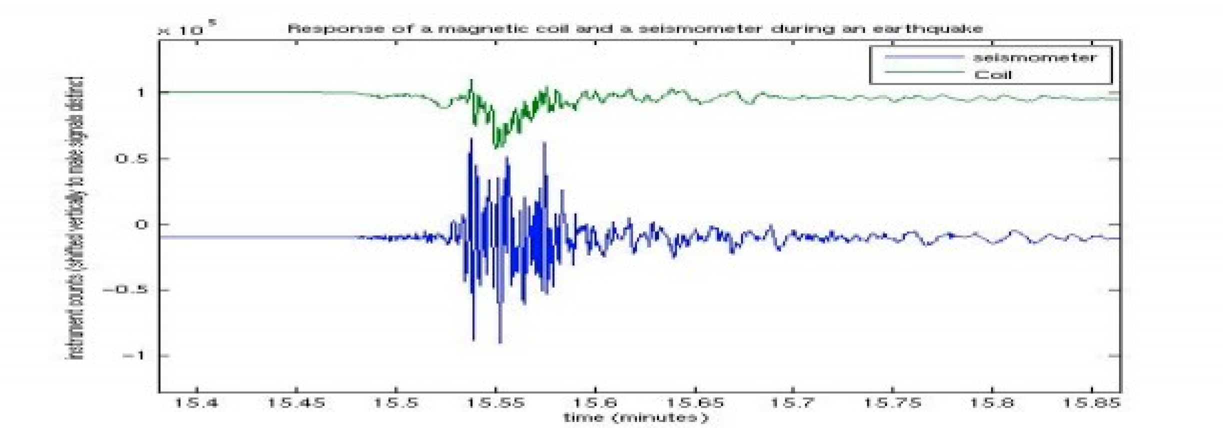


**Figure 4:** Responses observed during strong ground motion at Parkfield.

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[1] Fowles & Cassiday, Analytical Mechanics 6th Edn , 1999, Harcourt Brace and Company. [2] Johnston M.J.S., Mueller R.J., Keller V., Preseismic and Coseismic Magnetic Field Measurements Near the Coyote Lake, California, Earthquake of August 6, 1979 [3] Mueller R.J., Johnston M.J.S., Review of magnetic field monitoring near active faults and volcanic calderas in California: 1974–1995 Physics of the Earth and Planetary Interiors 105 1998 131–144 [4] Nichols E.A., Morrison H.F., Signals and Noise in Measurements of LowFrequency Geomagnetic Fields, Journal of Geophys. Res. Vol 93. No B11, 13743- 13754, 1988 [5]Sabaka T.J., Olsen N., Langrel R.A., A Comprehensive Model of the quiet-time, near-Earth magnetic field: phase 3, Geophys. J. Int. (2002), 151 32-68