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Causal Contrasts Promote Algebra Problem Solving

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Abstract

The causal-contrast approach is a new teaching method that recruits learners' implicit causal discovery process to improve math learning by juxtaposing contrasting information critical to discovering the goal of each solution step. Students often blindly memorize mathematical procedures and have difficulty transferring their knowledge to novel problems. By enabling learners to infer the goal of each step, the causal-contrast approach substantially improved high-school algebra problem solving compared to a traditional instructional control (Walker, Cheng & Stigler, 2014). The present study developed Walker et al.'s instructional materials into a computer-based teaching program and tested the new approach on community-college students, a population for whom the traditional approach is often ineffective. The study added two new conditions: a baseline that received no instruction and a condition using a teaching video from Khan Academy, a well-regarded online educational website representative of the traditional approach. A delayed post-test indicated that the causal-contrast condition produced dramatically greater success in solving transfer problems than the other three conditions.

Keywords: causal contrasts; causal induction; implicit learning; knowledge transfer; mathematics education

Introduction

Compared to previous years, U.S. students' ranking in science, technology, engineering, and math (STEM) subjects has been improving gradually; however, students' performance on the international mathematics assessments continue to fall below the international average (PISA, 2003, 2006, 2009, 2012). A massive amount of research has studied this issue, with the goal of improving students' mathematical learning and understanding.

One of the most common yet ineffective strategies that students use to learn mathematics is to blindly memorize the solution steps without truly understanding the reason behind each step (e.g., Stigler, Givvin, & Thompson, 2010). Without connecting the procedures to goals or concepts, repeatedly solving a large number of conventional problems may not be helpful for fostering the flexible use of mathematical knowledge (Cooper & Sweller, 1987; Schwartz et al., 2011). Previous research have suggested various ways to enhance mathematics education, such as

emphasizing worked examples to support schema acquisition (Carroll, 1994; Sweller & Cooper, 1985), self-generating explanations during learning to improve understanding and knowledge integration (e.g., Chi et al., 1989; Chi, 2000), labeling subgoals to illustrate the reason why certain solution steps should be applied to promote learning and transfer (e.g., Catrambone, 1998) and using comparisons and contrasting examples in the instruction to improve the learning of concepts and procedures (e.g., Hattikudur & Alibali, 2010; Rittle-Johnson & Star, 2007; Richland & McDonough, 2010).

Fewer studies have examined the use of causal learning to support mathematical learning and problem solving. Whereas mathematics is traditionally taught using explicit instruction to convey analytic knowledge, the causal contrast approach is an instructional method that recruits an implicit empirical-learning process to help students discover the reasons underlying mathematical procedures (Walker et al., 2014). Humans have a natural capacity for learning cause-and-effect relations (Cheng, 1997; Gopnik et al., 2004; Leslie & Keeble, 1987). The causal-contrast approach hypothesizes that operation-outcome relations in mathematics can be thought of as causal. The approach 1) induces students to formulate the goal of an operation in a mathematical procedure by allowing them to fail to solve a challenging problem using their prior mathematical knowledge and 2) promptly provides the relevant *conditional contingency* information (Cheng & Holyoak, 1995) by adding a *contrasting problem*, one that controls for confounding factors by being as similar as possible to the challenging problem but with the features causing the difficulty removed. "No confounding" is a pre-requisite for causal learning. The goal is for students to readily discover the goal of each step in the solution. It is not the use of comparison *per se* that characterizes the causal-contrast approach, but rather the targeting of critical concepts by juxtaposing contrasting information designed to enable discovering the causes of outcomes. The causal-contrast approach is also distinct from subgoal-learning in that it does not explicitly mention what the subgoals are. Instead, it recruits the student's natural causal reasoning to construct

the goals and subgoals within a causal structure that supports the application of solution steps.

Traditional instructional approaches teach students analytically and explicitly the rules and steps for solving specific types of problems. Figure 1 illustrates how this approach teaches how to solve quadratic equations.

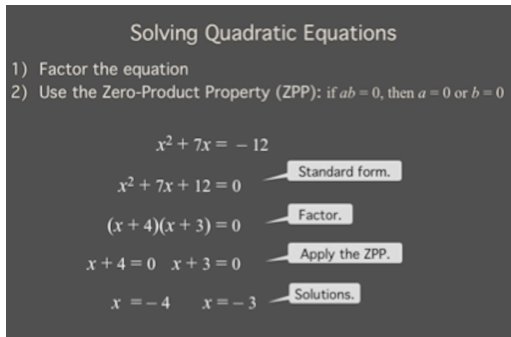


Figure 1. A cumulative static screen shot of sequentially presented and narrated traditional instructions.

For example, a student presented with the top equation in the figure is shown how to re-arrange the equation into standard form (second equation in the figure), factor the expression on the left-hand side, then determine the possible values of x , the desired unknown, using the zero-product property (i.e., if $a \cdot b = 0$, then $a = 0$ or $b = 0$). For a substantial fraction of students, as evidenced by their failure to flexibly generalize their learning to novel problems, this approach does not lead to an understanding of the causal structure of the solution, the reason behind each step in the procedure and how the steps work together. For example, what is the purpose of factoring the expression on the left-hand side of the equation? And what is the relationship between factoring and the zero-product property?

Opportunities to compare worked examples (Rittle-Johnson & Star, 2007) to explain solutions (e.g., Chi et al., 1989; Chi, 2000) or to isolate subgoals (e.g., Catrambone, 1998) may enable understanding of the causal structure of a problem. However, even then a student may not identify all the concepts in the causal structure, or have all the requisite information to infer the causal relations when the concepts are identified. While Sfard (2007) emphasizes the use of teacher-student discourse (students compare their own solution to teacher’s and explain the differences) to help students identify and correct misconceptions, the students may not discover the purposes of the concepts critical to transfer their success to novel problems.

The causal-contrast approach more directly targets specific links in the causal structure that make use of critical mathematical concepts essential to the solution of relevant types of problems. For example, students are first asked to try solving a quadratic equation (see top equation in Figure 2). If they fail to solve it, they are presented with the next two equations in the figure and asked to solve them. After students solve these, they are asked why these problems are easier for them to solve than the top equation. This comparison enables students to readily discover a cause of

their failure: unlike the second and third equations, the top equation has both an x and an x^2 term, preventing one from isolating x by simply rearranging the equation. The newly formulated cause – namely, having both x and an x^2 terms – in turn becomes an “effect” for a subsequent operation, namely, factoring, to remove. Without their initial attempt to isolate x , learners would have no “effect” for which to discover its cause. This effect – failure to isolate x by rearranging the equation – is often not explicitly noted in traditional instruction. Similarly, without comparing the top equation with the next two equations, a learner who is asked to explain why the top equation is difficult may mention the effect alone, “I’m unable to rearrange the equation to isolate x ”, omitting to identify its “cause”: having both x and x^2 terms in the same equation. Thus, each comparison is designed to direct attention to the accurate formulation of an essential causal relation in the structure of the solution.

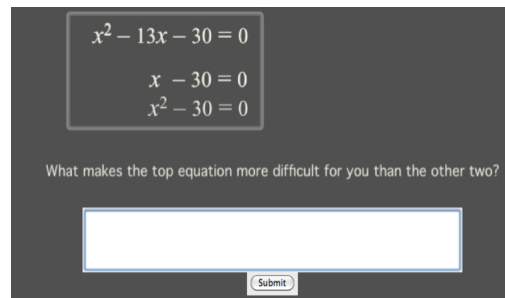


Figure 2. A screen shot of the causal-contrast intervention.

The process repeats as the student proceeds through the solution. In Figure 3, the pair of equations enclosed by the light gray rectangle illustrates the use of causal contrast for inferring a perceptual feature that causes a difficulty. Students are asked to solve the factored form of the quadratic equation (see top equation inside rectangle). If they fail to solve the problem, they are asked to solve a variant of the problem that has the difficulty removed (bottom equation inside rectangle), and to answer the question, “What values of x would make $x \cdot y = 0$ true, regardless of the value of y ?” In this simple form, most students have no difficulty. They are then asked to compare the two equations. The comparison may allow them to discover why they initially failed, for example, that the perceptual complexity of the product prevented them from recognizing that the expression on the left-hand side is a product, that inside each pair of parentheses is “just a number”. At this point, the perceptual complexity can switch causal roles, taking on the role of an effect that the student can prevent or remove in the future, for example, by pausing to take note of the perceptual complexity and to register the perceptual cues indicating when the zero-product property applies.

In more general terms, the comparisons we construct provide *conditional contingency* information (Cheng & Holyoak, 1995) consisting of the state of an effect (e.g., succeeding to solve a problem or not) in the presence of a candidate cause (a feature of the math problem, e.g., an equation having both an x and an x^2 term) and in its absence

(an equation not having both an x and an x^2 term), with alternative causes held constant.

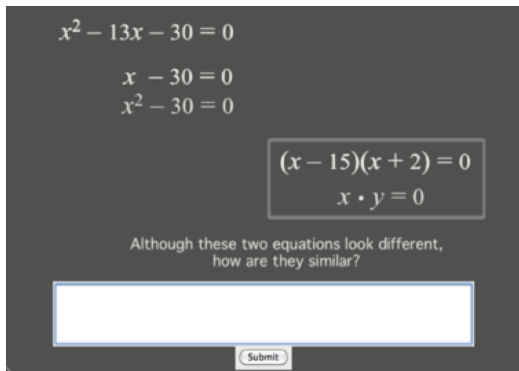


Figure 3. A later causal-contrast screen shot.

Comparing Causal-Contrast and Traditional Instruction

We conducted the present study on community-college students using a pretest/ intervention/posttest design to test a computer-based version of the Causal-Contrast instructional method (CC) against two computer-based traditional instructional methods and a baseline condition that received no instruction. The computer programs both animated and narrated the teaching materials for clearer and less attention-demanding instructions, reducing learners' cognitive load (Mayer & Moreno, 2003). By equating the instructions and feedback across conditions in the programs, we eliminated potential bias due to the human experimenter interacting with participants in Walker et al. (2014).

Although the causal approach was found to benefit both university students (at UCLA) and community-college students (Walker et al., 2014), the latter are less likely to benefit from traditional mathematics education. Causal induction is an implicit and evolutionarily old process (Hollis, 1997) and should be more uniformly available across the population than explicit reasoning processes.

One of the *Traditional* conditions (*T*) used materials identical to the causal contrast condition except for the instructional approach. The other traditional condition (*Khan Academy*; *K*) used an algebra teaching video from Khan Academy, a popular and well-regarded online educational website representative of traditional teaching. Khan Academy has become the largest school in the world; over 10 million students have watched its teaching videos online (Noer, 2012). Additionally, a *Baseline* condition (*B*) that did not receive any instruction was used as a control. All participants received information on the same mathematical concepts, solved the exact same set of problems with the same feedback.

The causal-contrast hypothesis predicts that students receiving the causal-contrast instructional intervention would have better performance in the post-test compared to students in the two traditional intervention conditions and in the baseline condition.

Participants

Sixty-eight community-college students recruited in Southern California and Downstate New York participated in our study. Participants (N=68) were randomly assigned into one of the three experimental conditions: Causal-Contrast (N=17), Traditional (N=18), Khan Academy (N=18), or the Baseline (N=15) control condition.

Study Design

This study consisted of two sessions, which were scheduled one to three weeks apart. In the first session, all participants were given a pretest measure assessing their knowledge of algebraic notation and of solving quadratic equations. Participants in the three experimental conditions received a lesson on solving quadratic equations on a computer followed by practice with an identical set of problems with identical feedback. For the baseline control group, immediately after the pretest, without receiving any instruction, participants were randomly assigned to receive one of two highly similar problem sets (Set A and Set B), which were the post-tests given to the experimental groups. The two sets consisted of analogous problems, and both assessed students' algebra problem solving mostly on quadratic equations. There was no time limit on the pretest and post-test, and the intervention lasted 25 minutes on average across the three experimental conditions. The groups did not differ in intervention time: $M_{\text{Causal-Contrast}} = 24.4$, $sd = 8.76$, $M_{\text{Traditional}} = 23.3$, $sd = 7.55$, and $M_{\text{Khan Academy}} = 29.5$, $sd = 12.3$, $F(2, 50) = 2.05$, $p = .14$. After a one- to three-week delay, participants in the experimental conditions were given one or the other post-test set alternately in the second session, and those in the *Baseline* condition received their second post-test set.

Instructional Materials

Causal-Contrast Approach. The causal-contrast instructional materials are as explained earlier. Participants in the causal-contrast approach were given three challenging problems that students tend to fail to solve. If the participants failed to solve a problem, a feedback slide informed them that they solved the problem incorrectly. A branching function was used to identify the nature of the failure (e.g., lack of understanding vs. careless mistakes) to enable the appropriate feedback. Instead of showing the correct solution, a *contrasting problem* was then presented. If the student solved a challenging problem successfully, the next challenging problem was presented.

Traditional Instructional Method. The instructional materials used in this condition were designed to make use of techniques that are representative of traditional instruction and were based on a popular textbook (Sullivan & Sullivan, 2007). Participants in this condition were shown step-by-step procedures through *worked examples*, written solutions of example problems that provide justifications for each procedural step (see Figure 1). The examples were followed by practice problems. Combining worked

examples and problem solving has been proved to facilitate learning (Sweller & Cooper, 1985). The instructions stated the subgoals of the critical steps in the procedure; for example, subjects were told that a quadratic equation is rearranged to standard form (having a zero on one side and a polynomial on the other) *so that* it could be factored and solved using the zero-product property. Emphasizing subgoals in problem solving has been shown to promote learning and transfer (Eiriksdottir & Catrambone, 2011).

Khan Academy. The instructional material for this condition was an online video from Khan Academy’s website: https://www.youtube.com/watch?v=uktzcTg_N7U. It makes use of animated digital technology designs. The video covers the necessary techniques for solving a quadratic equation, including factoring techniques and use of the zero-product property.

Measures

Post-test. The post-test included two types of problems: *instructed* and *transfer*. Instructed problems could be solved using the same solution procedures as the study problems. Transfer problems required generalization of concepts learned in the intervention. These problems included factorable quadratics in non-standard form. Table 1 lists the transfer problems:

Table 1. Transfer Problems

Q1	$4x^2 = 16x^4$
Q2	$(x - 3)(x^2 - 4) = 0$
Q3	$\sqrt{x - 3} + 5 = 9$
Q4	$(x - 1)^2 = 4(x - 1)$
Q5	$(x + 3)(x - 2) = 6$
Q6	$(2x + 1)(2 + x)x^2 = 0$
Q7	$\frac{4}{3}x^2 = 2x$
Q8	$(x - 3)(x - 7) + (x - 3) = 0$

Results

Pretest

A one-way between-subjects ANOVA performed on participants’ pretest performance shows that the four groups did not differ in their pretest performance: $M_{\text{Causal-Contrast}} = 73.2$ ($sd = 19.4$), $M_{\text{Traditional}} = 76.1$ ($sd = 16.0$), $M_{\text{Khan Academy}} = 68.6$ ($sd = 18.1$), and $M_{\text{Baseline}} = 70.3$ ($sd = 25.0$), $F(3, 64) = .497, p = .686$.

Delay Interval

A one-way between-subjects ANOVA performed on time between instruction and post-test (in days) shows that the four conditions did not differ in their average delay time: $M_{\text{Causal-Contrast}} = 14.5, sd = 5.37$, $M_{\text{Traditional}} = 13.1, sd = 5.61$, $M_{\text{Khan Academy}} = 12.9, sd = 4.83$, and $M_{\text{Baseline}} = 13.9, sd = 5.85, F(3, 64) = .318, p = .812$.

Post-test

Explanation of Analyses. Before evaluating the effects of instruction type on participants’ post-test performance, a correlation analysis was conducted to test whether the post-test performance was correlated with the pretest score. Pearson’s correlation confirmed that the pretest and post-test scores were strongly correlated, $r(68) = .711, p < .01$, suggesting that the participants’ post-test performance was in part due to their prior mathematical knowledge as indicated by their pretest scores.

Baseline Group’s Post-test Sets A and B. To examine whether our two post-test sets in the *Baseline* group produced different performance, a one-way repeated measures ANCOVA with pretest as a covariate was conducted on the post-test scores, separately for the instructed problems and for the transfer problems. For neither type of problems was there a significant difference between the scores of Post-test Sets A and B, $F(1,13) = .090, p = .768, \eta p^2 = .007$ and $F(1,13) = 1.15, p = 0.303, \eta p^2 = .081$ for the instructed and transfer problems respectively. Because the two sets produced highly similar performance, our analyses below collapsed across both sets.

Comparison across Conditions. Because the effect of mathematical instructions is strongly influenced by learners’ prior knowledge (Clarke, Ayres & Sweller, 2005; Rittle-Johnson et al., 2009), to show a clearer picture of the effect of the interventions, we separated the participants into three groups based on their pretest scores: *low-pretest* (i.e., pretest $<$ or $=$ 50%), *medium-pretest* (i.e., 50% $<$ pretest $<$ 90%), and *high-pretest* (i.e., pretest $>$ or $=$ 90%). Tables 2, 3, and 4 show the respective means of pretest scores, instructed problems scores, and transfer problems scores in each subgroup for each intervention condition.

Table 2. Descriptive Statistics for Pretest Performance

		Causal-Contrast	Traditional	Khan Academy	Baseline
Low Pretest	Mean	37.5	50.0	43.8	31.7
	SD	17.7	0.00	2.50	20.2
	N	2	3	4	3
Medium Pretest	Mean	70.5	76.8	70.0	70.7
	SD	10.9	10.3	8.40	12.1
	N	10	11	11	7
High Pretest	Mean	93.0	93.8	96.7	93.0
	SD	4.47	2.50	2.89	2.74
	N	5	4	3	5

As shown in Table 2, there were relatively few low-pretest participants. Relative to other groups, these participants’ pre-test scores varied substantially across conditions, making the assessment of the effectiveness of training in this group problematic. In view of the nature our pretest problems, these participants’ pretest performance suggests that they lack sufficient arithmetic or algebra knowledge to benefit from the interventions. In contrast, the high-pretest participants can potentially still benefit because the transfer problems are considerably harder than the pretest problems.

In the following analyses, to better assess the instructional interventions in their effective range, we excluded the low-pretest participants. We combined the rest

of the participants because of our small sample sizes. Figure 4 shows these participants' post-test performance results.

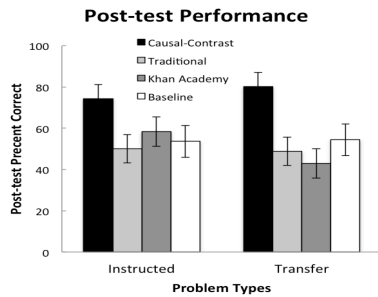


Figure 4. Estimated marginal means for medium- or high-pretest participants' Post-test scores.

Instructed Problems. A one-way ANCOVA on the post-test scores using pretest score as a covariate shows no significant difference in scores among the four conditions, $F(3, 51) = 2.37, p = .081, \eta^2 = .122$, perhaps due to our small sample size.

Table 3. Descriptive Statistics for Instructed Problems

		Causal-Contrast	Traditional	Khan Academy	Baseline
Low Pretest	Mean	12.5	4.17	25.0	4.17
	SD	17.7	7.22	35.4	7.22
	N	2	3	4	3
Medium Pretest	Mean	62.5	48.8	44.3	46.4
	SD	22.8	35.1	33.2	35.9
	N	10	11	11	7
High Pretest	Mean	95.0	65.6	91.7	67.5
	SD	11.2	25.8	14.4	24.4
	N	5	4	3	5

Transfer Problems. A one-way ANCOVA with pretest score as a covariate conducted on the transfer problems shows a significant difference among the intervention conditions, $F(3, 51) = 8.22, p < .001, \eta^2 = .326$. Follow-up pairwise comparisons using the Bonferroni correction indicate that the CC group outperformed the T group ($p = .002$), the K group ($p < .001$), and the B group ($p = .023$). The CC group transferred their knowledge about as well as UCLA students given similar training in our previous studies (Walker et al., 2014), with mean scores ranging from 80% to 82%. There was no statistically significant difference between the other three groups, $p > .50$.

Table 4. Descriptive Statistics for Transfer Problems

		Causal-Contrast	Traditional	Khan Academy	Baseline
Low Pretest	Mean	2.78	3.71	18.1	1.95
	SD	3.93	3.21	28.8	3.21
	N	2	3	4	3
Medium Pretest	Mean	75.0	44.4	32.3	38.9
	SD	14.1	32.3	29.30	34.8
	N	10	11	11	7
High Pretest	Mean	87.8	73.6	63.0	80.0
	SD	8.24	24.2	8.48	14.4
	N	5	4	3	5

Figure 5 shows performance on individual transfer problems for the four intervention groups. The CC group consistently outperformed the other three groups.

Comparing the frequency of participants who solved a problem correctly versus incorrectly, we see that the CC

group greatly outperformed the other three groups on Transfer Problem 6, $\chi^2(1, N=56) = 8.13, p = .004$. This superiority is notable in that the CC instructions could have misled the participants into formulating a simplistic rule regarding the joint presence of x and x^2 , one that ignores whether they are terms or factors. They could have been stymied, for example, because the factored equation still contains both x and x^2 terms, as they saw in the pair of contrasting problems in Figure 2. Instead, the intervention enhanced their correct flexible use of the relevant concepts and procedures.

Performance on Transfer Problem 3 is also notable in that this problem does not involve a quadratic equation. And yet the CC group outperformed the other three groups, $\chi^2(1, N=56) = 9.71, p = .002$. The CC group's deeper understanding of the causal structure of solving quadratic equations might have allowed them to reason more flexibly on another type of algebra problem. An intriguing possibility, which of course requires further research, is that the causal-contrast approach may awaken students' natural causal inference processes so that they create and test their own causal contrasts as they encounter new mathematical domains.

This possibility was collaborated by the CC group's superior performance on Transfer Problem 7, $\chi^2(1, N=56) = 6.21, p = .013$. This problem involved an x^2 term with a fractional coefficient, and there was no training on fractional coefficients in any of the interventions.

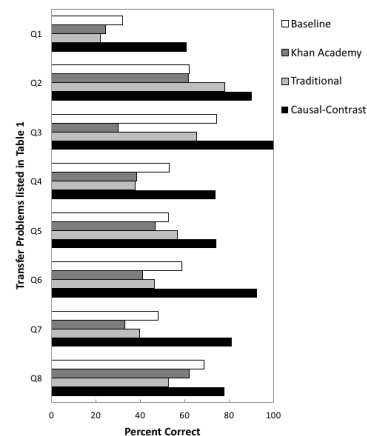


Figure 5. Performance on transfer problems in the four conditions.

Discussion

The causal contrast instructional approach invokes learners' natural causal induction process to identify the cause-and-effect relationships in a mathematical procedure. This approach decomposes the causal structure in a mathematical problem and accordingly caters the learning materials to allow learners to formulate the purposes of mathematical operations. These purposes are often not explicitly mentioned in traditional mathematical training.

Our current findings testing community-college students replicated and extended the results of previous studies

conducted on university students and community-college students (Walker et al., 2014). These studies show large improvements in solving algebra problems due to causal-contrast training. The improvements were especially dramatic in the community-college students.

Our comparison conditions now included a baseline condition and instructions from the Khan Academy as a benchmark. We eliminated the potential for experimenter-bias by incorporating all materials as animated computer programs, with no experimenter-participant interaction during the interventions.

Our results show that even when explicit analytic instruction focuses on teaching the reasons for mathematical procedures, students still often fail to learn in a way that promotes generalization to novel problems after a one-to-three week delay. In contrast, by allowing students to use an implicit, empirical learning process to discover the causal structure of solutions, students are able to fill in the missing links of their causal structure, need not rely on their rote memory of procedures, and become able to flexibly use their mathematical knowledge.

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