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Author

Carlip, S

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The Dynamics of Supertranslations and Superrotations in 2+1 Dimensions

S. CARLIP*

*Department of Physics
University of California
Davis, CA 95616
USA*

Abstract

Supertranslations, and at least in 2+1 dimensions superrotations, are asymptotic symmetries of the metric in asymptotically flat spacetimes. They are not, however, symmetries of the boundary term of the Einstein-Hilbert action, which therefore induces an action for the Goldstone-like fields that parametrize these symmetries. I show that in 2+1 dimensions, this action is closely related to a chiral Liouville action, as well as the “Schwarzian” action that appears in two-dimensional near-AdS physics.

*email: carlip@physics.ucdavis.edu

Asymptotically flat spacetime looks asymptotically like Minkowski space. One might therefore expect its asymptotic symmetries to be the symmetries of Minkowski space, the Poincaré group. Surprisingly, this is not the case at null infinity \mathcal{I}^\pm : it has been understood since the 1960s [1, 2] that the symmetries are described by the larger BMS group, which includes angle-dependent supertranslations. These symmetries have recently received renewed attention, thanks in part to their relationship to soft graviton theorems [3, 4], the gravitational memory effect [5], and perhaps the black hole information loss problem [6].

The goal of this paper is to show that the Goldstone-like excitations associated with these symmetries acquire a dynamics, induced from the boundary term in the action at \mathcal{I}^\pm . For simplicity, I focus here on the case of (2+1)-dimensional spacetimes, where the BMS group also includes superrotations [7].* The resulting boundary action includes a chiral Liouville theory, which is also closely related to the “Schwarzian action” that appears in nearly anti-de Sitter gravity in two dimensions [10–12], and it has intriguing connections to the coadjoint orbit quantization of the Virasoro group [13–16]. Chiral Liouville theory has previously been associated with BMS_3 [17, 18], but in a somewhat less direct way, via a Chern-Simons formulation whose generalization to higher dimensions seems problematic.

The basic idea is fairly simple. If one starts with a theory with an action I_{bulk} and places it on a manifold with boundary, one must usually add a boundary term I_{bdry} to the action. Classically, I_{bdry} is required for the existence of extrema: a variation of the bulk action gives boundary terms from integration by parts that must be cancelled off. Quantum mechanically, I_{bdry} is required for the proper “sewing” of path integrals, the path integral analog of a sum over intermediate states. The key observation [19] is that even if the bulk action is gauge invariant, the boundary action may not be. Hence field configurations that would normally be considered physically equivalent can become distinct at the boundary, giving rise to new degrees of freedom. The action for these “would-be gauge degrees of freedom” is induced from I_{bdry} , and can sometimes be calculated explicitly. In particular, for (2+1)-dimensional asymptotically anti-de Sitter gravity, it is a Liouville theory [20–22] with the proper central charge to match the Brown-Henneaux asymptotic symmetry [23]. We shall see here that a similar conclusion holds for the asymptotically flat case.

1. Metric and diffeomorphisms

To discuss the behavior of the metric near (future) null infinity, it is useful to choose coordinates in which the approach to \mathcal{I}^+ is easy to describe. Bondi coordinates [1] are defined by requiring that near infinity, spacetime is foliated by outgoing null cones, here labeled by a coordinate u (see figure 1). In 2+1 dimensions, the metric then takes the form

$$ds^2 = -2V dudr + g_{uu} du^2 + 2g_{u\phi} dud\phi + r^2 e^{2\omega} d\phi^2 \quad (1.1)$$

*The role of superrotations in (3+1)-dimensional spacetimes is not yet clear [8, 9].

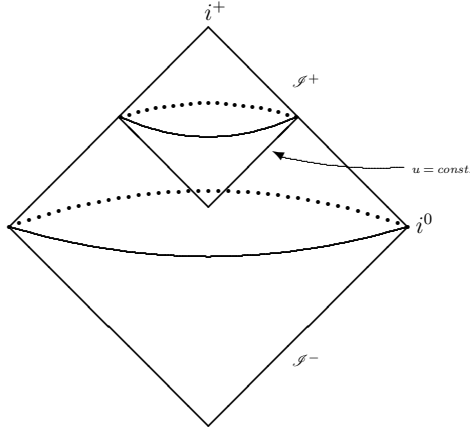


Figure 1: A (2+1)-dimensional spacetime foliated by null cones $u = \text{const.}$

where

$$V = \mathcal{O}(1), \quad g_{uu} = \mathcal{O}(r), \quad g_{u\phi} = \mathcal{O}(1), \quad \omega = \omega_0 + \frac{\omega_1}{r} + \dots \quad (1.2)$$

With the additional choice gauge $V = 1$, the radial coordinate r is an affine parameter for the geodesics generating the null cones. For convenience, I will make this choice here; it does not affect the final conclusions.

Barnich and Troessaert have analyzed the vacuum field equations for such a metric [24]. With the additional restriction $\omega_1 = 0$, they find that

$$\begin{aligned} g_{uu} &\sim -2r\partial_u\omega + e^{-2\omega} [-(\partial_\phi\omega)^2 + 2\partial_\phi^2\omega + \Theta] \\ g_{u\phi} &\sim e^{-\omega} \left[\Xi + \int^u d\tilde{u} \left\{ \frac{1}{2}\partial_\phi\Theta - \partial_\phi\omega [\Theta - (\partial_\phi\omega)^2 + 3\partial_\phi^2\omega] + \partial_\phi^3\omega \right\} \right] \\ &\text{with } \partial_u\Theta = \partial_u\Xi = 0 \end{aligned} \quad (1.3)$$

near \mathcal{S}^+ . The metric thus depends on the conformal factor ω and two functions Θ and Ξ of the angular coordinate ϕ , which can be shown to be the charges associated with supertranslations and superrotations.

We next evaluate the action of diffeomorphisms on this metric. Let us start with the standard flat metric

$$ds^2 = -2d\bar{u}d\bar{r} - d\bar{u}^2 + \bar{r}^2d\bar{\phi}^2 \quad (1.4)$$

and consider the diffeomorphisms

$$\bar{u} = u_0 + \frac{u_1}{r} + \dots, \quad \bar{\phi} = \phi_0 + \frac{\phi_1}{r} + \frac{\phi_2}{r^2} + \dots, \quad \bar{r} = ar + b_0 + \dots \quad (1.5)$$

where the coefficients are functions of new coordinates ϕ and u . For transformations that preserve the asymptotic form (1.1) of the metric, the functions ϕ_0 and u_0 —the asymptotic

reparametrizations of the circle and ϕ -dependent translations of u —are the superrotations and supertranslations.

The requirement that the new metric be of the form (1.1) with $V = 1$ and $\omega_1 = 0$ leads, after a straightforward calculation, to the conditions[†]

$$\begin{aligned} a\partial_u u_0 &= 1, & \partial_u \phi_0 &= 0, & e^{-\omega_0} a \partial_\phi \left(\frac{\partial_\phi u_0}{\partial_\phi \phi_0} \right) &= b_0 \\ u_1 &= -\frac{a}{2} \phi_1^2, & \phi_1 &= -\frac{1}{a} \frac{\partial_\phi u_0}{\partial_\phi \phi_0}, \\ \phi_2 &= -\frac{b_0}{a} \phi_1 \end{aligned} \tag{1.6}$$

The components of the metric are then

$$\begin{aligned} e^{\omega_0} &= \frac{\partial_\phi \phi_0}{\partial_u u_0}, \\ g_{uu} &= -2r\partial_u \omega_0 + e^{-2\omega_0} [-(\partial_\phi \omega_0)^2 + 2\partial_\phi^2 \omega_0 - (\partial_\phi \phi_0)^2 - 2\{\phi_0; \phi\}] \\ g_{u\phi} &= -e^{-\omega_0} \left[\partial_\phi^2 \left(\frac{\partial_\phi u_0}{\partial_\phi \phi_0} \right) - \frac{\partial_\phi^2 \phi_0}{\partial_\phi \phi_0} \partial_\phi \left(\frac{\partial_\phi u_0}{\partial_\phi \phi_0} \right) + (\partial_\phi \phi_0)^2 \left(\frac{\partial_\phi u_0}{\partial_\phi \phi_0} \right)^2 \right] \end{aligned} \tag{1.7}$$

where the Schwarzian derivative $\{\phi_0; \phi\}$ in g_{uu} is defined by

$$\{f; z\} = \frac{f'''}{f'} - \frac{3}{2} \left(\frac{f''}{f'} \right)^2 \tag{1.8}$$

It is not hard to check that these results match (1.3), but with coefficients that are now explicit functions of the parameters that label superrotations and supertranslations. In particular,

$$\Theta = -(\partial_\phi \phi_0)^2 - 2\{\phi_0; \phi\} \tag{1.9}$$

(Ξ becomes a complicated function of the u -independent part of $\partial_\phi u_0/\partial_\phi \phi_0$; we will not need its explicit form.)

2. Boundary terms, corner terms, and the induced action

To proceed further, we shall need the boundary term for the action. For a spacelike or timelike boundary with a fixed induced metric, the proper choice is the Gibbons-Hawking term [25], an integral of the extrinsic curvature of the boundary. For a lightlike boundary like

[†]I show in the appendix that the conclusions are essentially unchanged if $\omega_1 \neq 0$.

\mathcal{I}^\pm , the choice is less clear: the null normal to the boundary has no preferred normalization, so the analog of the extrinsic curvature, the expansion, is not unique. I will therefore take the less elegant approach of directly computing the boundary terms in the variation of the action and finding a boundary action to cancel them.

We start with the standard Einstein-Hilbert action

$$I = \frac{1}{\kappa^2} \int_M d^3x \sqrt{-g} R \quad (\text{with } \kappa^2 = 16\pi G_N) \quad (2.1)$$

on a manifold M with metric (1.1) and a boundary at $r = \bar{r}$; we will take the limit $\bar{r} \rightarrow \infty$ at the end. A standard calculation gives

$$\begin{aligned} \delta I &= e.o.m. + \frac{1}{\kappa^2} \int_{r=\bar{r}} d^2x \sqrt{-g} [g^{ab} \delta \Gamma_{ab}^r - g^{ar} \delta \Gamma_{ab}^b] \\ &= \dots + \frac{1}{\kappa^2} \int_{r=\bar{r}} d^2x \sqrt{-g} g^{ab} g^{rc} [\nabla_a \delta g_{bc} - \nabla_c \delta g_{ab}] \\ &= \dots - \frac{1}{\kappa^2} \int_{r=\bar{r}} d^2x [\partial_a (\sqrt{-g} \delta g^{ra}) + \sqrt{-g} \Gamma_{ab}^r \delta g^{ab} - \sqrt{-g} g^{rc} \partial_c (g_{ab} \delta g^{ab})] \end{aligned} \quad (2.2)$$

As in [24], let us set $V = 1$ and $\omega_1 = 0$ in (1.1). A short computation then gives

$$\delta I = \dots + \frac{1}{\kappa^2} \int_{r=\bar{r}} d^2x [2r \partial_u (e^\omega \delta \omega) + \partial_r (r e^\omega \delta g_{uu}) + 2g_{uu} \partial_r (r e^\omega) \delta \omega] + \mathcal{O}(\bar{r}^{-1}) \quad (2.3)$$

Using the asymptotic form (1.3) of the metric, this expression simplifies to

$$\delta I = \dots + \frac{1}{\kappa^2} \int_{r=\bar{r}} d^2x [-2r \partial_u (e^{\omega_0} \delta \omega_0) + \delta (e^{\omega_0} g_{uu}^{(0)}) + g_{uu}^{(0)} \delta (e^{\omega_0})] + \mathcal{O}(\bar{r}^{-1}) \quad (2.4)$$

where $g_{uu}^{(0)}$ means the $\mathcal{O}(1)$ part of g_{uu} .

Now, (2.4) is not, in general, a variation of any boundary action. This is to be expected: for a variational principle to make sense, we need to impose boundary conditions on some of the phase space variables. In fact, (2.4) tells us, roughly, that g_{uu} and e^ω are canonically conjugate variables, a fact that can be confirmed by looking at radial evolution in the Hamiltonian formalism. To choose boundary conditions, note that the conformally compactified metric near \mathcal{I}^+ is

$$d\bar{s}^2 = \rho^2 ds^2 \sim 2dud\rho + e^{2\omega} d\phi^2 \quad (2.5)$$

where $\rho = 1/r$. The only non-gauge-fixed component of the metric at \mathcal{I}^+ is e^ω , so it makes sense to hold this quantity fixed. If $\delta \omega_0 = 0$ at \mathcal{I}^+ , the boundary variation (2.4) will be cancelled by the variation of a boundary action

$$I_{\text{bdry}} = -\frac{1}{\kappa^2} \int_{\mathcal{I}^+} d^2x e^{\omega_0} g_{uu}^{(0)} \quad (2.6)$$

where ω_0 is now an arbitrary but fixed function of u and ϕ .

At finite \bar{r} , the action (2.6) is not quite the Gibbons-Hawking term: that term is obtained by fixing the full boundary metric, while we are only fixing $g_{\phi\phi}$. But (2.6) can be written in an invariant form resembling the Gibbons-Hawking term. Let n_a be the unit normal to the surface $r = \bar{r}$, and let ℓ_a be the null normal to the surfaces of constant u , normalized so that $\ell_a n^a = -1$. The projector

$$q^{ab} = g^{ab} + \ell^a n^b + \ell^b n^a + \ell^a \ell^b \quad (2.7)$$

projects onto circles of constant u and r . It may then be checked that the boundary action takes the geometric form

$$I_{bdry} = -\frac{1}{\kappa^2} \int_{\mathcal{S}^+} d^2x \sqrt{-^{(2)}g} q^{ab} \nabla_a n_b \quad (2.8)$$

(In [26], Detournay et al. construct a boundary action for three-dimensional Euclidean gravity, with the added restriction that $\omega_0 = 0$. In Euclidean signature, the extrinsic curvature is defined even in the limit $\bar{r} \rightarrow \infty$, and Detournay et al. argue that the proper boundary action is one-half of the usual Gibbons-Hawking term. The factor of one-half matches (2.8)—the standard Gibbons-Hawking term has a prefactor of $2/\kappa^2$ —and while (2.8) is not identical to the boundary action of [26], the differences vanish when $\omega_0 = 0$.)

Let us now restrict our attention to metrics of the form (1.7), that is, metrics obtained at least asymptotically from the standard flat metric by superrotations and supertranslations. The action (2.6) then becomes

$$I_{bdry} = \frac{1}{\kappa^2} \int_{\mathcal{S}^+} d^2x e^{-\omega_0} [(\partial_\phi \omega_0)^2 - 2\partial_\phi^2 \omega_0 + (\partial_\phi \phi_0)^2 + 2\{\phi_0; \phi\}] \quad (2.9)$$

There is one subtlety, though. From (1.6) and (1.7), we have

$$e^{-\omega_0} = \partial_u F \quad \text{with} \quad F = \frac{u_0}{\partial_\phi \phi_0} \quad (2.10)$$

Since, moreover, $\partial_u \phi_0 = 0$, the last two terms in (2.9) are total derivatives, which reduce to “corner” terms at the ends of \mathcal{S}^+ . We thus have

$$I_{bdry} = \frac{1}{\kappa^2} \int_{\mathcal{S}^+} d^2x e^{-\omega_0} [(\partial_\phi \omega_0)^2 - 2\partial_\phi^2 \omega_0] + \frac{1}{\kappa^2} \int_{\partial \mathcal{S}^+} d\phi F [(\partial_\phi \phi_0)^2 + 2\{\phi_0; \phi\}] \quad (2.11)$$

This is our induced boundary action for the superrotations and supertranslations.

3. Dynamics on \mathcal{S}^+

Let us begin by considering the first integral in the boundary action (2.11). If ω_0 is fixed, as we required to obtain our boundary action, this term is merely a fixed constant.

This is as it should be. The boundary term was chosen so that the action as a whole had no boundary variation. For a variation that is a pure diffeomorphism, the bulk action (2.1) is already invariant, so the boundary term should be as well.

The interesting boundary dynamics comes when one allows ω_0 to vary. Different choices of ω_0 correspond to different vacua, and diffeomorphisms that change ω_0 are analogous to Goldstone modes.[‡] At first sight, the boundary action (2.11) is not very interesting dynamically, since it seems to involve only angular derivatives. But recall the ω_0 itself contains time derivatives. In terms of the function F defined in (2.10), the action along \mathcal{I}^+ is

$$I_{\mathcal{I}^+} = -\frac{1}{\kappa^2} \int_{\mathcal{I}^+} d^2x \frac{(\partial_u \partial_\phi F)^2}{\partial_u F} = -\frac{1}{\kappa^2} \int_{\mathcal{I}^+} d^2x \frac{(\partial_\phi \partial_u u_0)^2}{(\partial_\phi \phi_0)(\partial_u u_0)} + \text{corner terms} \quad (3.1)$$

(The first expression here is the “right” one, since F , rather than u_0 or ϕ_0 , is specified by boundary conditions, but the second exhibits the pattern of derivatives more clearly.) Although it differs in detail, this action is similar in structure to the action of Alexeev and Shatashvili for the coadjoint orbits of the Virasoro group. It would be interesting to see if it has a closer relationship to the corresponding action for coadjoint orbits of BMS_3 [28].

4. Liouville theory, the Schwarzian action, and Virasoro orbits

We now turn to the second integral in (2.11),

$$I_{\text{corner}} = \frac{1}{\kappa^2} \int_{\partial \mathcal{I}^+} d\phi F [(\partial_\phi \phi_0)^2 + 2\{\phi_0; \phi\}] \quad (4.1)$$

This is a “corner term,” appearing at the boundaries of \mathcal{I}^+ , that is, at spacelike and future timelike infinity i^0 and i^+ . The existence of such a term should not surprise us: the leading supertranslations and superrotations are time-independent, so their action should reduce to one at a fixed time. (In higher dimensions, a supertranslation can originate at a finite time from a pulse of gravitational radiation arriving at \mathcal{I}^+ —this is a form of the gravitational memory effect [5]—but in 2+1 dimensions there are no gravitational waves.)

The action (4.1) is essentially identical to the “Schwarzian action” found by several authors [10–12] in the rather different setting of two-dimensional asymptotically nearly anti-de Sitter spacetime. The physical interpretations differ, but in both cases the action is related to deformations of circles (here at i^0 and i^+). It seems likely that the results of [12] on the quantum theory can be translated directly to this context.

[‡]I believe the comparison to Goldstone modes was first made by Kaloper and Terning, as cited in [27]. These modes are closely related to the “soft modes” of Strominger et al. [3].

The corner term is also intimately related to chiral Liouville theory. To see this, note that for (1.5) to be a diffeomorphism, ϕ_0 must be a monotonic function of ϕ , so we can write

$$\partial_\phi \phi_0 = \sqrt{\frac{\mu}{4}} e^{\frac{\gamma}{2} x} \quad (4.2)$$

where γ and μ are constants and, from (1.6), $\partial_u \chi = 0$. The corner term then becomes

$$I_{corner} = \frac{1}{\kappa^2} \int_{\partial \mathcal{S}_+} d\phi F \left[-\frac{\gamma^2}{4} (\partial_\phi \chi)^2 + \gamma \partial_\phi^2 \chi + \frac{\mu}{4} e^{\gamma x} \right] \quad (4.3)$$

To put this into a standard Liouville form, we lift back to two dimensions,

$$I_{corner} = \frac{1}{\kappa^2} \int_{\mathcal{S}_+} d^2 x e^{-\omega_0} \left[-\frac{\gamma^2}{4} (\partial_\phi \chi)^2 - \gamma \chi (\partial_\phi^2 \omega_0 - (\partial_\phi \omega_0)^2) + \frac{\mu}{4} e^{\gamma x} \right] \quad (4.4)$$

and introduce an auxiliary two-dimensional metric

$$d\tilde{s}^2 = e^{-2\omega_0} du^2 - d\phi^2 \quad (4.5)$$

with a scalar curvature

$$\tilde{R} = -2 (\partial_\phi^2 \omega_0 - (\partial_\phi \omega_0)^2) \quad (4.6)$$

If we choose $\gamma^2 = \frac{\kappa^2}{2\pi}$, the action (4.4) becomes

$$I_{corner} = \frac{1}{4\pi} \int_{\mathcal{S}_+} d^2 x \sqrt{-\tilde{g}} \left[\frac{1}{2} \tilde{g}^{ab} \partial_a \chi \partial_b \chi + \frac{1}{\gamma} \chi \tilde{R} + \frac{\mu}{2\gamma^2} e^{\gamma x} \right] \quad (4.7)$$

with the restriction $\partial_u \chi = 0$. This is precisely the Liouville action for χ [29], with a classical central charge

$$c = \frac{12}{\gamma^2} = \frac{24\pi}{\kappa^2} = \frac{3}{2G} \quad (4.8)$$

A similar chiral Liouville action was found in [17, 18], by means of a Chern-Simons formulation, but here the meaning of the Liouville field is clear: it is precisely the parameter that characterizes superrotations.

To obtain this Liouville action, we incorporated ω_0 into the ‘‘background metric’’ (4.5), implicitly treating it, and the related function F , as fixed quantities at the corners. But it is clear from (2.10) that for a fixed F , the supertranslations and superrotations are not independent, so it should be possible to reexpress the corner action as an action for the supertranslation parameter u_0 . This is indeed the case: starting with (4.1) and setting

$$\partial_\phi \phi_0 = \frac{u_0}{F}, \quad u_0 = e^\sigma \quad (4.9)$$

a simple calculation yields

$$I_{corner} = -\frac{1}{\kappa^2} \int_{\partial \mathcal{I}^+} d\phi F \left[(\partial_\phi \sigma)^2 - \frac{1}{F^2} e^{2\sigma} - \left(\frac{\partial_\phi F}{F} \right)^2 \right] \quad (4.10)$$

which again has the general form of a Liouville action.

Our boundary action also has an intriguing relationship to the quantization of the coadjoint orbits of the Virasoro group [13, 14]. It is not quite the Alekseev-Shatashvili action of [13], which is not chiral, but it is the integral of the chiral stress-energy tensor of that theory,

$$T = b_0 (\partial_\phi \phi_0)^2 - \frac{c}{24\pi} \{\phi_0, \phi\} \quad \text{with } b_0 = -\frac{c}{48\pi} \quad (4.11)$$

where this value of b_0 corresponds to the orbit of $L_0 = 0$ [14]. This connection is further strengthened if we rewrite the boundary action (2.9) as

$$I_{bdry} = \frac{1}{\kappa^2} \int_{\mathcal{I}^+} d^2x e^{-\omega_0} \left[(\partial_\phi \omega_0)^2 - 2\partial_\phi^2 \omega_0 - \frac{48\pi}{c} T \right] \quad (4.12)$$

and allow ω_0 to vary while holding T fixed—that is, allowing the boundary supertranslations to vary while fixing the superrotations. Setting $\psi = e^{-\omega_0/2}$, we find that the equation of motion for ψ is Hill’s equation,

$$\psi'' - \frac{12\pi}{c} T \psi = 0 \quad (4.13)$$

where a prime means a ϕ derivative. This equation has two solutions, say ψ_1 and ψ_2 . If we set $\varepsilon = \psi_1^2$, $\psi_1\psi_2$, or ψ_2^2 , it is then easy to check [15, 16] that[§]

$$\delta_\varepsilon T = \frac{c}{24\pi} \varepsilon''' - 2T\varepsilon' - T'\varepsilon = 0 \quad (4.14)$$

But $\delta_\varepsilon T$ is just the variation of T under an infinitesimal conformal transformation, that is, an action of the Virasoro group, and its vanishing determines the coadjoint orbits of the Virasoro group.

5. Next steps

I have shown that the supertranslations and superrotations in asymptotically flat (2+1)-dimensional gravity become genuine physical degrees of freedom at null infinity. Much as in the asymptotically anti-de Sitter case, Goldstone-like “boundary gravitons” are dynamical along \mathcal{I}^+ , while additional corner terms at spacelike and future timelike infinity induce a

[§]I thank Shahin Sheikh-Jabbari for explaining this point to me.

chiral conformal action for the superrotations. A good deal is known about this conformal field theory, though the “nonnormalizable sector” of Liouville theory is not fully understood. It should be possible to translate the field theoretical results into statements about quantum gravity, although one must presumably first understand the identification of the past and future theories as in [3].

Somewhat mysteriously, the superrotation charge Ξ occurring in (1.3) does not appear in our boundary action. This charge, which depends on the u -independent part of $\partial_\phi u_0$, occurs only at subleading order. Further investigation is needed to see whether we are losing part of the dynamics. In particular, it is not clear whether the chiral Liouville theory captures the full algebra of the BMS_3 group.

It would also be useful to reexpress these results more invariantly in terms of the conformally compactified spacetime, using the methods of [7]. For instance, it should be possible to express the boundary term (2.8) in terms of quantities on the compactified spacetime.

The main question, of course, is whether these results can be extended to a realistic (3+1)-dimensional spacetime. In contrast to the Chern-Simons approach of [18], the basic approach of this paper should generalize to arbitrary dimensions, but the details may well be quite different.

Appendix. Putting back ω_1

The calculations presented above have assumed that $\omega_1 = 0$ in eqn. (1.2), as in [24]. Here I will describe the (minimal) changes that occur if ω_1 is allowed to be nonzero. Note that this can be accomplished by a coordinate transformation $r \rightarrow r + f(u, \phi)$.

The first change is that b_0 is no longer determined in eqn. (1.6). Instead,

$$\omega_1 = -e^{-\omega_0} \partial_\phi \left(\frac{\partial_\phi u_0}{\partial_\phi \phi_0} \right) + b_0 \partial_u u_0 \quad (\text{A.1})$$

This shifts the metric components in (1.7) to

$$\begin{aligned} g_{uu} &= -2r \partial_u \omega_0 - 2e^{-\omega_0} \partial_u (e^{\omega_0} \omega_1) + e^{-2\omega_0} [-(\partial_\phi \omega_0)^2 + 2\partial_\phi^2 \omega_0 - (\partial_\phi \phi_0)^2 - 2\{\phi_0; \phi\}] \\ g_{u\phi} &= -\partial_\phi \omega_1 - e^{-\omega_0} \left[\partial_\phi^2 \left(\frac{\partial_\phi u_0}{\partial_\phi \phi_0} \right) - \frac{\partial_\phi^2 \phi_0}{\partial_\phi \phi_0} \partial_\phi \left(\frac{\partial_\phi u_0}{\partial_\phi \phi_0} \right) + (\partial_\phi \phi_0)^2 \left(\frac{\partial_\phi u_0}{\partial_\phi \phi_0} \right)^2 \right] \end{aligned} \quad (\text{A.2})$$

The variation (2.4) of the action also acquires an extra term:

$$\delta I = \dots + \frac{1}{\kappa^2} \int_{r=\bar{r}} d^2x [\delta(e^{\omega_0} g_{uu}^{(0)}) + g_{uu}^{(0)} \delta e^{\omega_0} - 2\omega_1 e^{\omega_0} \partial_u \delta \omega_0] \quad (\text{A.3})$$

However, if we define

$$\tilde{g}_{uu}^{(0)} = g_{uu}^{(0)} + 2e^{-\omega_0} \partial_u (e^{\omega_0} \omega_1) = g_{uu}^{(0)} \Big|_{\omega_1=0} \quad (\text{A.4})$$

it is easy to check that

$$\delta I = \dots + \frac{1}{\kappa^2} \int_{r=\bar{r}} d^2x \left[\delta(e^{\omega_0} \tilde{g}_{uu}^{(0)}) + \tilde{g}_{uu}^{(0)} \delta e^{\omega_0} \right] \quad (\text{A.5})$$

exactly as in (2.4). The addition on ω_1 thus has no effect on the induced boundary action.

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