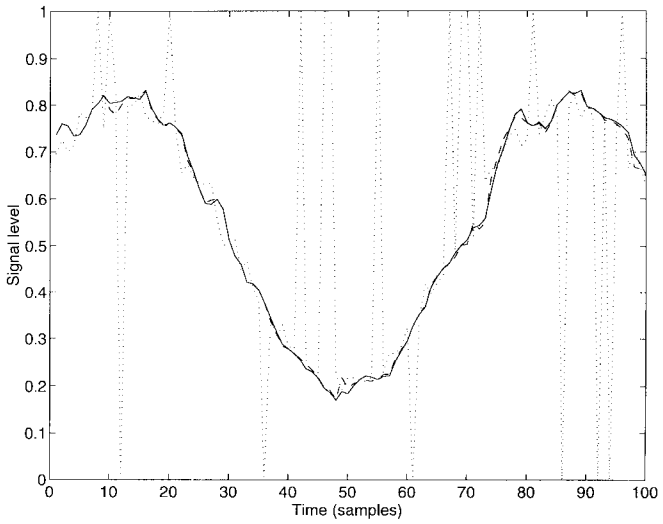

Fig. 2. Sinusoidal signal corrupted by impulsive and Gaussian noise (dotted), the actual polynomial prediction (solid), and the recursive prediction (dashed).

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REFERENCES

[1] P. Heinonen and Y. Neuvo, "FIR-median hybrid filters with predictive FIR substructures," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 36, pp. 892–899, June 1988.

[2] P. T. Harju and T. I. Laakso, "Polynomial predictors for complex-valued vector signals," *Electron. Lett.*, vol. 31, no. 19, pp. 1650–1652, Sept. 1995.

[3] S. J. Ovaska, "Improving the velocity sensing resolution of pulse encoders by FIR prediction," *IEEE Trans. Instrum. Meas.*, vol. 40, pp. 657–658, June 1991.

[4] S. J. Ovaska, O. Vainio, and T. I. Laakso, "Design of predictive IIR filters via feedback extension of FIR forward predictors," in *Proc. 38th Midwest Symp. Circuits Syst.*, Rio de Janeiro, Brazil, Aug. 1995, pp. 370–375.

Blind System Identification Using Minimum Noise Subspace

Yingbo Hua, Karim Abed-Meraim, and Mati Wax

***Abstract*—Developing fast and robust methods for identifying multiple FIR channels driven by an unknown common source is important for wireless communications, speech reverberation cancellation, and other applications. In this correspondence, we present a new method that exploits a minimum noise subspace (MNS). The MNS is computed from a set of channel output pairs that form a "tree." The "tree" exploits, with minimum redundancy, the diversity among all channels. The MNS method is much more efficient in computation than a standard subspace method. The noise robustness of the MNS method is illustrated by simulation.**

I. INTRODUCTION

Blind identification of multiple-channel FIR system driven by a common source has recently received much attention due to its potential applications in wireless communications. In contrast to the traditional cost-function based adaptive approaches and the more recent higher order statistics (HOS)-based methods, the second-order statistics (SOS)-based methods have become a popular topic in this community since [3], e.g., see [2] and the references therein. Among many SOS-based methods known so far is the subspace (SS) method shown in [1]. The SS method applies the MUSIC concept to a relation between the channel impulse responses and the noise subspace associated with a covariance matrix of the system output. In this correspondence, we present a new variation of the SS method. Instead of exploiting the full noise subspace, this new method exploits a minimum noise subspace (MNS). The MNS method represents a solid extension of an observation made in [1] that the full noise subspace of the system output covariance matrix is generally not necessary to asymptotically yield the unique (up to a constant scalar) estimate of channel responses. We will show that the minimum dimension of the noise subspace required for unique system identification is $M-1$, where M is the number of FIR channels. Although not any set of $M-1$ noise vectors yields unique identification, each vector in a *proper* set of $M-1$ noise vectors can be computed from one of $M-1$ covariance matrices that correspond to a *proper* set of $M-1$ (distinct) pairs of channel outputs. Any $M-1$ pairs of channel outputs that span a "tree" pattern as shown in Fig. 1 are a *proper* choice. The MNS method is much more efficient in computation than the SS method. Simulations have shown that the MNS method is slightly less robust to channel noise than the SS method.

II. CHANNEL MODEL AND THE SS METHOD

We consider M parallel FIR channels driven by a common source. The output vector of the ith channel can be written as

$$\mathbf{y}_i(n) = \mathbf{H}_i\mathbf{s}(n) + \mathbf{w}_i(n)$$

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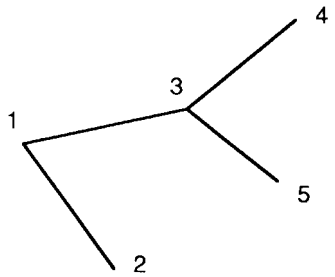

Fig. 1. "Tree" that connects $M = 5$ channel outputs as its "nodes." A tree must have no loop and connect all its nodes. Here, the nodes 2, 4, and 5 are "ending" nodes, and the nodes 1 and 3 are "branching" nodes. (The tree spanned by $M-1$ pairs of channel outputs is the same as the tree by $M-1$ pairs of the columns of $\overline{\mathbf{H}}(z)$ in the proof for Lemma 3.)

where

$$\mathbf{y}_i(n) = [y_i(n) \quad y_i(n+1) \quad \cdots \quad y_i(n+N-1)]^T$$
$$\mathbf{s}(n) = [s(n-L) \quad s(n-L+1) \quad \cdots \quad s(n+N-1)]^T$$
$$\mathbf{w}_i(n) = [w_i(n) \quad w_i(n+1) \quad \cdots \quad w_i(n+N-1)]^T$$
$$\mathbf{H}_i = \begin{bmatrix} h_i(L) & \cdots & h_i(0) & 0 & \cdots & 0 \\ 0 & h_i(L) & \cdots & h_i(0) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & h_i(L) & \cdots & h_i(0) \end{bmatrix},$$
$$N \times (N+L).$$

where

$y_i(n)$	output sequence of the ith channel
$s(n)$	input sequence
$w_i(n)$	noise sequence on the ith channel (uncorrelated with $s(n)$)
$h_i(k)$	impulse response of the ith channel.
L	maximum order of the M channels
N	window length on each channel output.

Then, we write

$$\mathbf{y}(n) = \mathbf{H}\mathbf{s}(n) + \mathbf{w}(n)$$

where

$$\mathbf{y}(n) = \begin{bmatrix} \mathbf{y}_1(n) \\ \vdots \\ \mathbf{y}_M(n) \end{bmatrix}, \quad \mathbf{w}(n) = \begin{bmatrix} \mathbf{w}_1(n) \\ \vdots \\ \mathbf{w}_M(n) \end{bmatrix}$$

and

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_M \end{bmatrix}.$$

The matrix $\mathbf{H}$ is known as the $MN \times (N+L)$ generalized Sylvester matrix [6], which has the full column rank $N+L$ under the following assumptions:

A1) The M channels do not share a common zero.
A2) $N \geq L+1$.

The blind identification problem here is to find $\mathbf{H}$ from the sequence $\{\mathbf{y}(n)$ for $n = 1, 2, \cdots, T\}$. The SS method [1] exploits the sample covariance matrix of all channel outputs:

$$\mathbf{R}_y = \frac{1}{T} \sum_{n=1}^{T} \mathbf{y}(n)\mathbf{y}(n)^H$$

where ^{H} denotes the conjugate transpose. As T becomes large, this matrix has the asymptotical structure

$$\mathbf{R}_y = \mathbf{H}\mathbf{R}_s\mathbf{H}^H + \mathbf{R}_w$$

with

$$\mathbf{R}_s = \frac{1}{T} \sum_{n=1}^{T} \mathbf{s}(n)\mathbf{s}(n)^H$$

and

$$\mathbf{R}_w = \frac{1}{T} \sum_{n=1}^{T} \mathbf{w}(n)\mathbf{w}(n)^H.$$

The SS method then computes the eigendecomposition of $\mathbf{R}_y$:

$$\mathbf{R}_y = [\mathbf{U}_s \quad \mathbf{U}_n] \begin{bmatrix} \Sigma_s & \\ & \Sigma_n \end{bmatrix} [\mathbf{U}_s \quad \mathbf{U}_n]^H$$

where the matrix $\mathbf{U}_n$ consists of the $MN - N - L$ less dominant eigenvectors of $\mathbf{R}_y$. In addition to assumptions **A1)** and **A2)** if

A3) the source covariance matrix $\mathbf{R}_s$ has the full rank $N+L$, and
A4) the noise covariance matrix $\mathbf{R}_w$ is proportional to the identity matrix (which is true when the noise is white and T is very large)

then it can be shown [1] that range $(\mathbf{U}_n)$ is the orthogonal complement of range $(\mathbf{H})$. Hence, range $(\mathbf{U}_n)$ is referred to as the noise subspace. The SS method yields an estimate $\mathbf{H}_e$ of $\mathbf{H}$ by solving the equation

$$\mathbf{U}_n^H \mathbf{H}_e = \mathbf{0}$$

in a least square sense (where $\mathbf{H}_e$ is subject to the same structure as $\mathbf{H}$). This estimate is uniquely (up to a constant scalar) equal to $\mathbf{H}$ under the assumptions **A1)**–**A4)** [1].

III. The MNS Method

In the MNS method, we first select $M-1$ distinct pairs from the M channel outputs $\{\mathbf{y}_i(n), i = 1, \cdots, M\}$. The $M-1$ pairs of channels (or channel outputs) must form a tree pattern, as shown in Fig. 1, where the channels are the "nodes" of the tree. Then, for each pair of channel outputs, we compute the covariance matrix

$$\mathbf{R}_y^{i,j} = \frac{1}{T} \sum_{n=1}^{T} \begin{bmatrix} \mathbf{y}_i(n) \\ \mathbf{y}_j(n) \end{bmatrix} \begin{bmatrix} \mathbf{y}_i(n) \\ \mathbf{y}_j(n) \end{bmatrix}^H$$

and its least dominant eigenvector $\tilde{\mathbf{v}}^{i,j}$. Let $\tilde{\mathbf{v}}^{i,j} = \begin{bmatrix} \tilde{\mathbf{v}}^{i,j}(i) \\ \tilde{\mathbf{v}}^{i,j}(j) \end{bmatrix}$, where each subvector has the dimension $N \times 1$. Then, define the "zero padded" vector

$$\mathbf{v}^{i,j} = \begin{bmatrix} \mathbf{v}^{i,j}(1) \\ \vdots \\ \mathbf{v}^{i,j}(M) \end{bmatrix}$$

where

$$\mathbf{v}^{i,j}(k) = \begin{cases} \tilde{\mathbf{v}}^{i,j}(i) & k = i \\ \tilde{\mathbf{v}}^{i,j}(j) & k = j \\ \mathbf{0} & \text{otherwise.} \end{cases}$$

Then, we form a $MN \times (M-1)$ matrix $\mathbf{V_n}$ consisting of the $M-1$ vectors $\{\mathbf{v}^{i,j}\}$. Similar to the SS method, the MNS method yields an estimate $\mathbf{H}_e$ of $\mathbf{H}$ by solving the equation $\mathbf{V}_n^H \mathbf{H}_e = \mathbf{0}$ in a least square sense (where $\mathbf{H}_e$ is subject to the same structure as $\mathbf{H}$). The significant computational advantage of the MNS method over the SS method is obvious. In particular, the SS method requires a full eigendecomposition of an $MN \times MN$ matrix, but the MNS method computes the single least dominant eigenvector of a $2N \times 2N$ matrix in parallel for each of $M-1$ pairs of channel outputs.

We will now establish that under the assumptions **A1)**–**A4)**, a) the MNS method yields the unique estimate of $\mathbf{H}$, and b) $M-1$ is the smallest number of vectors from the noise subspace in order for an equation like $\mathbf{V}_n^H \mathbf{H}_e = \mathbf{0}$ to yield the unique estimate of $\mathbf{H}$.

Lemma 1 (Easy to Prove): For any equation $\mathbf{v}^H\mathbf{H} = \mathbf{0}$, where $\mathbf{v} = [\mathbf{v}(1)^T \ \cdots \ \mathbf{v}(M)^T]^T$ with $\mathbf{v}(i) = [\mathbf{v}_i(0) \ \cdots \ \mathbf{v}_i(N-1)]^T$ and $\mathbf{H}$ is a $MN \times (N+L)$ generalized Sylvester matrix, there uniquely corresponds a polynomial equation $\sum_{i=1}^{M} V_i(z)H_i(z) = 0$ of degree $N+L-1$, where $H_i(z) = \sum_{l=0}^{L} h_i(l)z^{-l}$ of degree L, and $V_i(z) = \sum_{l=0}^{N-1} v_i(l)z^{-l}$ of degree $N-1$. The converse is also true.

Lemma 2: If there are $q\mathbf{v}MN \times 1$ vectors $\{\mathbf{v}_i$ for $i = 1, \cdots, q\}$ such that $\{\mathbf{v}_i^H\mathbf{H} = \mathbf{0}$ for $i = 1, \cdots, q\}$, where $\mathbf{H}$ is a $MN \times (N+L)$ generalized Sylvester matrix, then $\mathbf{H}$ is (possibly) unique up to a constant scalar *only if* $q \geq M-1$.

Proof: Using Lemma 1, it is straightforward to show that $\{\mathbf{v}_i^H\mathbf{H} = \mathbf{0}$ for $i = 1, \cdots, q\}$ is equivalent to the polynomial matrix equation $\overline{\mathbf{V}}(z)\overline{\mathbf{h}}(z) = \overline{\mathbf{0}}$ of degree $N+L-1$, where $\overline{\mathbf{V}}(z)$ is a $q \times M$ polynomial matrix of degree $N-1$ uniquely corresponding to $\{\mathbf{v}_i$ for $i = 1, \cdots, q\}$, and $\overline{\mathbf{h}}(z)$ is an $M \times 1$ polynomial vector of degree L uniquely corresponding to $\mathbf{H}$. However, using the polynomial matrix theory [5], $\overline{\mathbf{h}}(z)$ is determined by the equation $\overline{\mathbf{V}}(z)\overline{\mathbf{h}}(z) = \overline{\mathbf{0}}$ uniquely up to a polynomial (or constant) scalar *only if* $q \geq M-1$.

It is easy to show that under the assumptions **A1**)–**A4**), the vector $\mathbf{v}^{i,j}$ satisfies $(\mathbf{v}^{i,j})^H\mathbf{H} = \mathbf{0}$. Since the MNS method only relies on $M-1$ noise vectors, Lemma 2 has now established that the MNS method exploits a "minimum" noise subspace.

Lemma 3: The MNS method yields the unique (up to a constant scalar) estimate of the channel responses under the assumptions **A1**)–**A4**).

Proof: From Lemma 1, the equation $(\mathbf{v}^{i,j})^H\mathbf{H} = \mathbf{0}$ is equivalent to a polynomial equation

$$V_j(z)H_i(z) + V_i(z)H_j(z) = 0$$

of degree $N+L-1$, where $V_i(z)$ and $V_j(z)$ are of degree $N-1$ and $H_i(z)$, and $H_j(z)$ are of degree L. Similarly, each subequation $(\mathbf{v}^{i,j})^H\mathbf{H}_e = \mathbf{0}$ in the "overall" MNS estimation equation $\mathbf{V}_n^H\mathbf{H}_e = \mathbf{0}$ is equivalent to a polynomial equation

$$V_j(z)H_{e_i}(z) + V_i(z)H_{e_j}(z) = 0$$

where the degrees of all polynomials are the same as in the previous polynomial equation. Combining these two polynomial equations yields

$$H_j(z)H_{e_i}(z) - H_i(z)H_{e_j}(z) = 0$$

. Using this equation for each of the $M-1$ pairs of channels, it follows that the solution to $\mathbf{V}_n^H\mathbf{H}_e = \mathbf{0}$ is equivalent to that of the polynomial matrix equation

$$\overline{\mathbf{H}}(z)\overline{\mathbf{h}}_e(z) = \overline{\mathbf{0}}$$

of degree $2L$, where $\overline{\mathbf{H}}(z)$ is an $(M-1) \times M$ polynomial matrix of degree L uniquely corresponding to $\{H_i(z)$ for $i = 1, \cdots, M\}$, and $\overline{\mathbf{h}}_e(z)$ is an $M \times 1$ polynomial vector of degree L uniquely corresponding to $\{H_{e_i}(z)$ for $i = 1, \cdots, M\}$ (or, equivalently, $\mathbf{H}_e$). Note that each row of $\overline{\mathbf{H}}(z)$ only has two nonzero entries and, hence, defines a pair of columns. The $M-1$ pairs of columns defined by the $M-1$ rows of $\overline{\mathbf{H}}(z)$ also span a "tree" that connects all M columns of $\overline{\mathbf{H}}(z)$ as its "nodes." This tree is identical to the tree spanned by the pairs of channel outputs (Fig. 1). Note that removing a column and a row of $\overline{\mathbf{H}}(z)$ associated with an "ending node" decreases the rank of $\overline{\mathbf{H}}(z)$ by one, and when $\overline{\mathbf{H}}(z)$ is 1×2, its rank is one. It follows by induction that $\overline{\mathbf{H}}(z)$ has the full row rank $M-1$. Therefore, the solution for the $M \times 1$ vector $\overline{\mathbf{h}}_e(z)$ to the equation $\overline{\mathbf{H}}(z)\overline{\mathbf{h}}_e(z) = \overline{\mathbf{0}}$ must be unique up to a polynomial scalar [5]. Furthermore, since $\overline{\mathbf{h}}(z)$ is a solution of degree L to $\overline{\mathbf{H}}(z)\overline{\mathbf{h}}_e(z) = \overline{\mathbf{0}}$ and there is no common zero among all channels (see **A1**)), $\overline{\mathbf{h}}(z)$ must be the unique solution up to a constant scalar.

Lemma 3 has established that the MNS method yields asymptotically the unique estimate of $\mathbf{H}$. This section has provided a much stronger result than a discussion in [4] on the MNS method.

It is worth noting that the concept behind the MNS method would become much simpler if assumption **A1**) was replaced by the stronger assumption that "no pair in the set of channel pairs that span a tree has a common zero." Under the latter assumption, it is easy to show that the least eigenvector associated with each pair of channels yields the unique estimation for that pair of channels, and hence, the $M-1$ least eigenvectors associated with $M-1$ properly chosen pairs of channels uniquely determine all channels. However, this correspondence has presented a much stronger result than the above observation.

TABLE I
IMPULSE RESPONSES OF THE M-CHANNEL SYSTEM. OTHER PARAMETERS ARE $M = 4$, $L = 5$, $N = 6$, $T = 245$

	$h_1(k)$	$h_2(k)$	$h_3(k)$	$h_4(k)$
k=0	0.4972-1.2784i	1.3516-0.2333i	0.8970+1.0377i	-0.4264+1.3037i
k=1	-0.0370+0.7256i	-0.5251+0.5021i	0.7265+0.0046i	-0.5314-0.4955i
k=2	1.4158+0.2768i	0.8012+1.1996i	-0.2867+1.4138i	-1.2052+0.7927i
k=3	0.6417+0.4440i	0.2181+0.7492i	-0.3031+0.7190i	-0.6886+0.3669i
k=4	-1.2418+0.5984i	-1.2837+0.5023i	-1.3182+0.4032i	-1.3450+0.3018i
k=5	0.0235+1.1777i	-0.7049+0.9438i	-1.1360+0.3118i	-1.0879-0.4518i

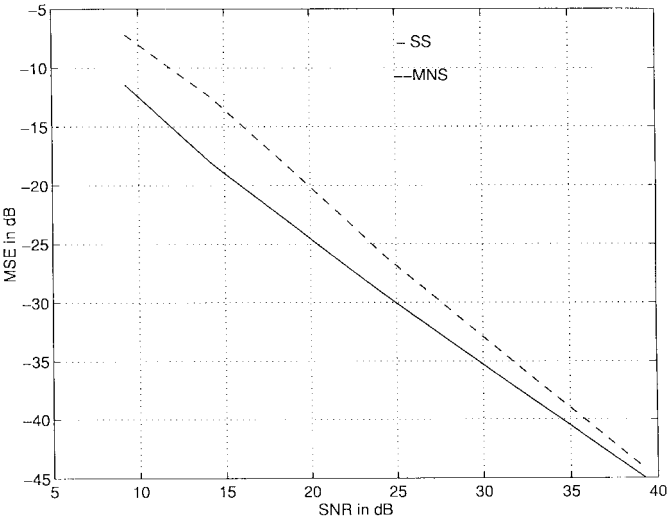

Fig. 2. Performance comparison of the SS method and the MNS method. MSE versus SNR.

IV. PERFORMANCE OF THE MNS METHOD

In our simulation, we used a system of four ($M = 4$) parallel FIR channels. The first channel is given by the GSM test channel [7] with six ($L = 5$) delayed paths. The other three channels are generated by assuming a plane propagation model for each path with corresponding electric angles uniformly distributed in $[0, \pi/3]$. A realization of the channel impulse responses is given in Table I. The output observation noise is an i.i.d. sequence of zero-mean Gaussian variables. The input signal is an i.i.d. sequence of zero-mean, unit-variance QAM-4 variables independent from the noise. The performance is measured by

$$\text{MSE (dB)} = 10 \log_{10}\left\{\frac{1}{N_r}\sum_{r=1}^{N_r}\|\mathbf{h}_r - \mathbf{h}\|^2\right\}$$

where N_r is the number of independent runs $(N_r = 100)$, $\mathbf{h}$ is the true (unit-norm) vector of the impulse responses $\{h_i(k)$ for $i = 1, \cdots, M$ and $k = 0, \cdots, L\}$, $\mathbf{h}_r$ is the estimated (unit-norm) vector of impulse responses at the rth run. Note that the equation $\mathbf{V}_n^H \mathbf{H}_e = \mathbf{0}$ was solved subject to $\|\mathbf{h}_e\| = 1$, and for each run, we computed $\mathbf{h}_r = \alpha \mathbf{h}_e$, where $\alpha = \mathbf{h}_e^H \mathbf{h}$ is a phase adjuster. The signal-to-noise ratio is defined as

$$\text{SNR (dB)} = 20 \log_{10} \left(\frac{\|\mathbf{h}\| \sigma_s}{\sqrt{M} \sigma_w} \right)$$

where σ_s and σ_w denote the deviations of the input and the noise, respectively. Fig. 2 compares the performances of the SS and MNS methods. This figure (which is associated with the case defined by the table) is quite typical among all the cases that we considered in our simulation. In the region where the MSE is relatively small (say -35 dB), the MNS method required an SNR of no more than 3 dB higher than the SS method, to yield a given value of MSE.

REFERENCES

[1] E. Moulines, P. Duhamel, J. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," *IEEE Trans. Signal Processing*, vol. 43, pp. 516–525, Feb. 1995.

[2] Y. Hua, "Fast maximum likelihood for blind identification of multiple FIR channels," *IEEE Trans. Signal Processing*, vol. 44, pp. 661–672, Mar. 1996.

[3] L. Tong, G. Xu, and T. Kailath, "A new approach to blind identification and equalization of multipath channels," in *Proc. 25th Asilomar Conf.*, Nov. 1991, pp. 856–860.

[4] Y. Hua, H. Yang, M. Wax, and M. Zhou, "Blind system identification using multiple sensors," in *Proc. IEEE ICASSP'95*, May 1995, pp. 3171–3174.

[5] G. Forney, "Minimal bases of rational vector spaces with applications to multivariable linear systems," *SIAM J. Contr.*, vol. 13, pp. 493–520, May 1975.

[6] R. R. Bitmead, S.-Y. Kung, B. Anderson, and T. Kailath, "Greatest common division via generalized Sylvester and Bezout matrices," *IEEE Trans. Automat. Contr.*, vol. AC-23, pp. 1043–1047, Dec. 1978.

[7] J. Proakis, *Digital Communications.* New York: McGraw-Hill, 1989.

[8] Y. Hua, K. Abed-Meraim, and M. Wax, "Blind system identification using minimum noise subspace," in *IEEE Workshop SSAP*, Corfu, Greece, June 24–26, 1996, pp. 308–311.

Optimum Block Adaptive Filtering Algorithms Using the Preconditioning Technique

Jae Sung Lim and Chong Kwan Un

***Abstract*— We propose three block adaptive algorithms using the preconditioning technique. The Toeplitz-preconditioned optimum block adaptive (TOBA) algorithm employs a preconditioner assumed to be Toeplitz, the SSOR-preconditioned optimum block adaptive (SOBA) algorithm uses a product of triangular matrices as a preconditioner, and the circulant-preconditioned OBA (COBA) algorithm is based on a circulant preconditioner. It is also shown that their tracking properties and convergence rates are superior to those of the OBA algorithm, the self-orthogonalizing block adaptive filter (SOBAF), and the normalized frequency-domain OBA (NFOBA) algorithm.**

I. INTRODUCTION

Among the block adaptive algorithms [1]–[7], the block LMS (BLMS) algorithm [1] is based on the block mean-square error (BMSE) and employs a fixed step size called the convergence factor that controls its convergence rate as well as its steady-state behavior. Unlike the BLMS algorithm, the optimum block adaptive (OBA) algorithm [2] employs a time-varying step size called the time-varying convergence factor, which is optimized in a least-square (LS) sense and updated at each block iteration. However, their convergence rate greatly slows down when the eigenvalue spread of the input autocorrelation matrix becomes large because they are inherently gradient algorithms.

The self-orthogonalizing block adaptive filter (SOBAF) algorithm [6] proposed by Panda *et al*. is based on the assumption that the autocorrelation estimate is a Toeplitz matrix. They showed that the algorithm converges under any input conditions at the same rate as it would if the input was white. The same concept was studied in [5] and [7] as slightly different approaches. In these approaches, the autocorrelation estimate is assumed to be a circulant matrix so that it can be diagonalized by the Fourier matrix. Particularly, Yon and Un developed the normalized frequency-domain OBA (NFOBA) algorithm by utilizing a relative normalization technique [7].

In this correspondence, we propose several adaptive algorithms using the preconditioning technique that can be regarded as a self-orthogonalization. The Toeplitz-preconditioned OBA (TOBA) is derived by employing the Toeplitz preconditioner, the symmetric successive overrelaxation (SSOR) preconditioned OBA (SOBA) is proposed by using the SSOR preconditioner assumed to be a product of triangular matrices, and the circulant-preconditioned OBA (COBA) algorithm is formulated by using the circulant preconditioner. In the algorithms, the filter tap weights are updated along the direction vector instead of the gradient vector. Additionally, the time-varying step size is optimized in the direction of the direction vector. The convergence rates of the proposed algorithms are fast, as compared with those of the SOBAF and NFOBA algorithms as well as the OBA algorithm. Moreover, the proposed algorithms do not have the initial instability problem existing in the SOBAF and NFOBA algorithms.

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