Title
THE LAW OF ONE PRICE, PURCHASING POWER PARITY AND EXCHANGE RATES

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ABSTRACT

Exchange-rate economics is filled with puzzles. The asset approach has failed. Purchasing Power Parity is useful at best in the long run. There is no clear link between exchange rates and fundamentals. With no empirically supported theory for exchange rates, open-economy macro models are built on sand. This paper shows for the first time how recognizing differences between retail, wholesale and auction markets helps solve the puzzles, provides a theory of exchange rates based on auction markets for assets *and* commodities, and suggests a link between fundamentals and exchange rates.

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JEL: D25, D4, E43, E44, F30, F31, F41, G14, G15, Q17, R41.

1. Introduction.

Classic articles by Officer (1976) and Rogoff (1996) describe a conventional view of Purchasing Power Parity (CPPP).[1] Although Rogoff explicitly recognizes transaction costs, CPPP appears to implicitly assume a world like the one in the Pure Theory of Trade where there are no information or transaction costs and transportation from one international location to another is instantaneous. For example, it assumes that, given exchange rates, arbitrage should equate spot retail commodity prices.[2]

CPPP tries to be more "realistic" by recognizing "non-traded" goods. But it fails to realize that the costs that make say haircuts non-traded, make all retail goods non-traded. If there were no information or transaction costs and travel from one location to another were instantaneous, New Yorkers would just as soon go to a barber in London as go to a local barber. Haircuts, like everything else, would be traded.

Information and transaction costs also divide markets into retail, wholesale and auction.[3] At retail, *all* goods are non-traded. No one buys shoes in a department store in New York and sells them to a department store in London or Paris. A Londoner in New York might buy a pair of shoes and take them home, but that is hardly "trade".

To be useful, theories must have semantic or correspondence rules that link at least some theoretical terms like "price" or "price index" to something we observe. Theory normally guides such rules. One would not measure a theoretical term like "domestic output" using a stock market measure like the DOW. Semantic rules separate the conventional view of the Law of one Price (CLOP) and CPPP from the auction approach to the LOP (ALOP) and PPP (APPP)

[1] Officer ignores the role of the Law of One Price while Rogoff sees it as the foundation for all versions of PPP.
[2] The conventional approach then goes on to reject effective arbitrage and the LOP because it does not equate those prices. See below.
[3] The role of information and transaction costs in dividing markets seems obvious, but, as far as we are aware, no one has explained *how* those costs divide markets.

developed below.[4] Implicit semantic rules for the CLOP and CPPP link the theoretical concept "price" to observed prices for things like Kleenex in retail or wholesale markets. Semantic rules for the ALOP and APPP link "price" to prices in auction markets.

Goods are widely traded at the wholesale level, but there is almost no arbitrage. It is only in auction markets where arbitrage is routine. This distinction between markets also applies to financial markets where research routinely uses data from auction markets, not retail or wholesale markets. The CLOP and CPPP mix auction and retail markets.[5]

The discussion of arbitrage and the Law of One Price in Pippenger (2016) distinguishes between these markets. It can be summed up as follows: After taking into account time in transit and differences in transaction costs, arbitrage is probably as effective in commodity auction markets as in financial auctions markets. Articles like Engel and Rogers (1996), Asplund and Friberg (2001), and Parsley and Wei (2001) claim, or appear to claim, that arbitrage is ineffective and that the LOP fails. But they use volatile exchange rates from auction markets and sticky commodity prices from retail markets where arbitrage is not possible.

2. The LOP.

The Law of One Price is relevant for both asset and commodity markets, but the details differ.

2.1. Asset versus commodity.

Whether markets are commodity or asset, arbitrage between retail markets is usually impossible and arbitrage between wholesale markets rare. Arbitrage takes place primarily between auction markets.

[4] Unlike most exchange-rate economics, Sarno (205, 686) recognizes the importance of semantic rules for relative price levels.
[5] The CLOP and CPPP literature, at times, recognizes that mixing retail and auction prices might be a problem, but pays little attention to that problem.

Arbitrage between auction markets for identical financial assets is relatively simple, partly because the transfer of ownership is effectively instantaneous. If, after converting to a common currency and accounting for transaction costs, prices differ between international locations, arbitragers buy low and sell high. This rule applies to spot financial markets because it is the ownership of financial assets, not their location, that is relevant.

Trade between auction commodity markets is more complex because location is important, transport takes time and transaction costs are higher for commodities. But after taking into account the special conditions in auction markets for commodities, trade in those markets should resemble the trade in auction financial markets described in Akram, Farooq and Sarno (2008).

Figure 1 describes trade in a world with retail, wholesale and auction markets.

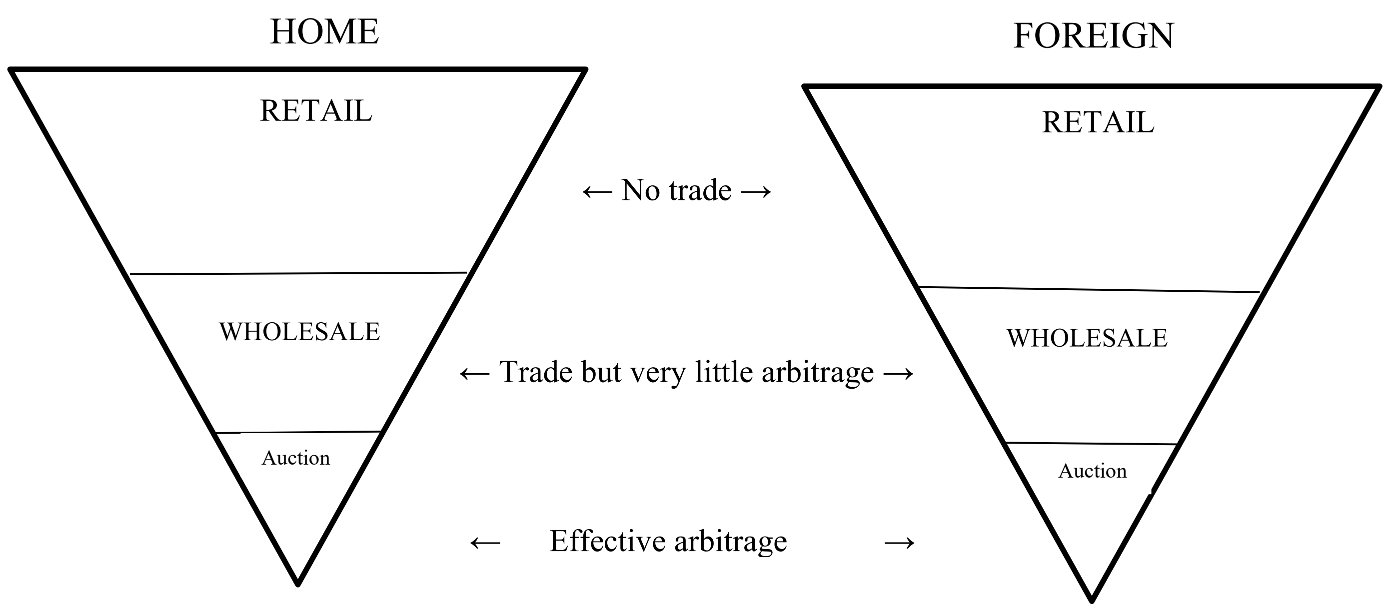

FIGURE 1
Arbitrage and Trade

Retail markets in different countries are isolated. There is no direct trade between them, but they are linked indirectly. A firm producing shoes in Milan sells those shoes to retailers in

London, Paris and New York. Kansas farms produce wheat that is traded in auction markets and that is milled into the flour that bakeries in London, Paris and New York use to bake bread. Retail markets in London, Paris and New York are linked, but the links are indirect and work slowly. As pointed out below, several of the common puzzles about exchange rates exist because they implicitly assume the kind of direct links that exist in the Pure Theory of Trade. Recognizing the differences between markets helps resolve those puzzles.

2.2. Commodities.

As pointed out above, arbitrage in spot commodities is usually impossible due to time in transit.[6] As a result, the Law of One Price normally does not apply to spot commodities. This subsection shows how the LOP can work for prices from auction markets. It illustrates the economic logic behind the ALOP. It does not "prove" that the ALOP holds. Only rigorous testing can confirm the ALOP.

The discussion starts with intertemporal arbitrage between near traded and more distantly traded goods in a given location. For simplicity, it assumes that there is no implicit return to holding a commodity. Firms like Bunge Ltd. and Carghill Inc. are as willing to hold a ton of a particular kind of wheat spot as to own a claim on that wheat in 90 days.

W is a particular kind of wheat with a specific protein content and specified values for all the other characteristics normally included in contracts to buy or sell W. The example uses spot and 90-day forward. It starts with domestic equilibrium.

2.2.1. Domestic equilibrium. For domestic equilibrium it must be impossible to make a risk-free profit by buying spot and selling forward or the opposite. Equilibrium also excludes losses. As the next Subsection shows, given reasonable assumptions, responding to losses or

[6] A few commodities like gold are exceptions.

risk-free profits can produce international equilibria involving *forward* prices and *forward* exchange rates.

The United States is the home country. $(\$/W)_0$ is the spot price of W in U.S. Gulf ports and $(\$/W)_{90}$ is the 90-day forward price of W in those ports. ${}_0CC\$_{90}(\$/W)_{90}$ is the cost in future dollars of carrying W forward 90 days in Gulf ports. ${}_0CC\$_{90}$ is exogenous because W is only one of a wide variety of grains that is carried forward. ${}_\$i_{90}$ is the 90-day interest rate in the U.S. It is exogenous because the borrowing and lending associated with trade in W is a miniscule part of the relevant capital market. For simplicity, the discussion ignores the difference between bid and ask prices, and borrowing and lending rates.[7]

Eq. (1) is one way to write the local equilibrium condition.

$$[(\$/W)_{90} - {}_0CC\$_{90}(\$/W)_{90}]/(1+ {}_\$i_{90}) = (\$/W)_0 \tag{1}$$

After deducting carrying costs, the present value of wheat carried forward equals the current value of that wheat. Buying W spot and selling it forward, or the opposite, does not produce a profit or loss.

If, starting in equilibrium, $(\$/W)_0$ falls, $(\$/W)_{90}$ rises, carrying costs fall or interest rates fall, there is a "risk-free" profit.[8] $[(\$/W)_{90} - {}_0CC\$_{90}(\$/W)_{90}]/(1+ {}_\$i_{90})$ is greater than $(\$/W)_0$. Arbitragers buy low and sell high. They borrow $W(\$/W)_0$ spot dollars, which they repay with $W(\$/W)_0(1+ {}_\$i_{90})$ future dollars, and buy W spot. They sell W forward and carry it forward to meet their future commitment. $[(\$/W)_{90} - {}_0CC\$_{90}(\$/W)_{90}](1+ {}_\$i_{90}) - (\$/W)_0$ is the risk-free profit. As more W is purchased spot and carried forward, spot purchases raise $(\$/W)_0$ and forward sales lower $(\$/W)_{90}$ until arbitrage restores equilibrium.

[7] They would contribute to the thresholds discussed below.
[8] This profit is free of any risk associated with uncertain prices, but it is not completely free of risk. There is always the risk that some contracting agent will default. From this point on we will take this exception for granted and omit the "".

If, starting in equilibrium, $(\$/W)_0$ rises, *$(\$/W)_{90}$* falls, carrying costs rise or interest rates rise, carrying W forward produces losses. $[(\$/W)_{90} - {}_0CC\$_{90}(\$/W)_{90}]/(1+ {}_{\$}i_{90})$ is less than $(\$/W)_0$. Buying low and selling high is impossible because it is impossible to bring future wheat back to the present. Two mechanisms work to restore equilibrium.

The first involves arbitragers who sell high and buy low. They "borrow" spot W and sell it short, invest the proceeds and buy forward.[9] This arbitrage reduces the inequality written as $[(\$/W)_{90} - {}_0CC\$_{90}(\$/W)_{90}] < (\$/W)_0(1+ {}_{\$}i_{90})$ by raising $(\$/W)_{90}$ and lowering $(\$/W)_0$, but it does not restore equilibrium unless the cost of selling short is zero. Let ${}_{\$}\varepsilon_{90}W$ represent the cost of borrowing W for 90 days over and above the interest rate. If ${}_{\$}\varepsilon_{90}$ is zero, as long as $[(\$/W)_{90} - {}_0CC\$_{90}(\$/W)_{90}] < (\$/W)_0(1+ {}_{\$}i_{90})$, arbitragers make a risk-free profit by selling short and buying forward. If ${}_{\$}\varepsilon_{90}$ is positive, selling short produces a risk-free profit only as long as $[(\$/W)_{90} - {}_0CC\$_{90}(\$/W)_{90}] < (\$/W)_0(1+ {}_{\$}i_{90} - {}_{\$}\varepsilon_{90})$. Arbitrage lowers $(\$/W)_0$ and raises $(\$/W)_{90}$, but it does not fully restore equilibrium

A second mechanism is also at work. Firms stop carrying W forward because it produces a loss.[10] Not carrying W forward raises $(\$/W)_{90}$ and lowers $(\$/W)_0$ until, under normal circumstances, equilibrium is restored.

Together these two mechanisms work to restore equilibrium.[11] How well auction markets respond to such shocks is an empirical issue that needs to be addressed. What follows considers what happens when local equilibria hold.

Eq. (1) can be written in a more useful form as follows:

[9] See Wikipedia for the details of selling short.
[10] Firms that were carrying W forward and covered that commitment do not take a loss. If they did not cover, they do.
[11] It is possible that this process does not restore equilibrium.

$$(\$/W)_{90}[1 - {}_{0}CC\$_{90}] = (\$/W)_{0}(1+ {}_{\$}i_{90}) \tag{1'}$$

After accounting for the carrying costs, the future value of present wheat equals the future value of future wheat.

Similar transactions produce a similar equilibrium in Rotterdam. The notation for Rotterdam is as follows: $(€/W)_0$ is the spot euro price of W in Rotterdam and $(€/W)_{90}$ is the forward euro price of W in Rotterdam in 90 days. ${}_{0}CC€_{90}(€/W)_{90}$ is the cost in future euros of carrying W forward by 90 days in Rotterdam. ${}_{0}CC€_{90}$ is exogenous for the same reason ${}_{0}CC\$_{90}$ is exogenous. ${}_{€}i_{90}$ is the 90-day euro interest rate. It is exogenous for the same reason ${}_{\$}i_{90}$ is exogenous. Eq. (2) describes the relevant equilibrium condition in Rotterdam.

$$(€/W)_{90}[1 - {}_{0}CC€_{90}] = (€/W)_{0}(1+ {}_{€}i_{90}) \tag{2}$$

Full international equilibrium assumes local equilibrium.

2.2.2. International equilibrium. Comparative advantage drives trade. See Wikipedia for a discussion of comparative advantage. Here we consider just W where exchange rates are exogenous. In that context trade depends on where, in the absence of trade, W is cheapest in a common currency.

Where ever W is cheapest, direct trade between spot commodity markets in different locations is impossible, as is direct arbitrage between forward markets of the same maturity in different locations. But arbitrage is possible between t = x and t = y, as long as they are both positive and y is sufficiently greater than x to allow for time in transit.[12] In this example, x is zero and y is 90 days.

$(\$/€)_0$* is the spot dollar price of the euro and *$(\$/€)_{90}$* is the 90-day forward price of the euro. *$(€/\$)_0$* is the spot euro price of the dollar and *$(€/\$)_{90}$ is the 90-day forward price of the dollar. For

[12] Time in transit depends on transportation costs. Fast ships are more expensive per ton than slow ships. Airplanes are faster and more expensive than fast ships. The greater the profit, the smaller the difference between x and y.

simplicity, the discussion ignores bid-ask spreads, $(\$/€)_0 = 1/(€/\$)_0$ and $(\$/€)_{90} = 1/(€/\$)_{90}$. Exchange rates are exogenous because the foreign exchange involved in trading W is only a minuscule part of the foreign exchange market. The 90-day interest rate in the euro area, ${}_€i_{90}$, is exogenous for the same reason ${}_\$i_{90}$ is exogenous.

${}_0TC\$_{90}(€/W)_{90}(\$/€)_{90}$ is the cost in future dollars of shipping W from a Gulf port to Rotterdam while ${}_0TC€_{90}(\$/W)_{90}(€/\$)_{90}$ is the cost in future euros of shipping W from Rotterdam to a Gulf port. ${}_0TC\$_{90}$ and ${}_0TC€_{90}$ are exogenous because W is only one of many grains traded between Gulf ports and Rotterdam.

Ignoring transport costs, carrying costs and interest rates for simplicity, W flows from Gulf ports to Rotterdam when, *in the absence of trade*, W is cheaper in Gulf ports, *e.g.*, when $(\$/W)_{90}$ is less than $(\$/€)_{90}(€/W)_{90}$.[13] W flows from Rotterdam to Gulf ports when $(€/W)_{90}$ is less than $(€/\$)_{90}(\$/W)_{90}$, *i.e.*, when $(\$/W)_{90}$ is greater than $(\$/€)_0(€/W)_{90}$. The effects of transport costs, carrying costs and interest rates are discussed below.

When Gulf ports have the price advantage, if W moves, it moves from Gulf ports to Rotterdam. In that case, one way to express equilibrium is that $(\$/€)_{90}(€/W)_{90}[1 - {}_0TC\$_{90}] = (\$/W)_0(1+ {}_\$i_{90})$. The future dollar value of spot W in a Gulf port equals the future dollar value of shipping W to Rotterdam, selling it forward there and selling those future euros forward at $(\$/€)_{90}$. Note that trade can continue from day to day in this equilibrium without any risk-free profits or avoidable losses. They become relevant when equilibria are violated.

If, starting in equilibrium yesterday, today $(\$/€)_{90}$ rises, $(€/W)_{90}$ rises, $(\$/W)_0$ falls, ${}_\$i_{90}$ falls or ${}_0TC\$_{90}$ falls, today there is an arbitrage profit because $(\$/€)_{90}(€/W)_{90}[1 - {}_0TC\$_{90}] >$

[13] Trade equates *observed* prices. In the absence of all impediments to trade, observed $(\$/W)_{90}$ would equal observed $(\$/€)_0(€/W)_{90}$ which-ever way W is moving. Observed price differentials are the result of impediments. Larger observed differentials do not necessarily increase the volume of trade, they can reduce trade. Other things equal, larger impediments increase *observed* differentials and *reduce* trade.

$(\$/W)_0(1+ {}_{\$}i_{90})$. Arbitragers borrow $W(\$/W)_0$ spot dollars which they repay with $W(\$/W)_0(1+ {}_{\$}i_{90})$ future dollars, buy W spot in a Gulf port, ship it to Rotterdam where they sell it forward for $W(€/W)_{90}$ and sell those forward euros for forward dollars. They do all this as closely to simultaneously as possible. Purchases raise $(\$/W)_0$ and sales reduce $(€/W)_{90}$, restoring equilibrium.

If, starting in equilibrium yesterday, today $(\$/€)_{90}$ falls, $(€/W)_{90}$ falls, $(\$/W)_0$ rises, ${}_{\$}i_{90}$ rises, or ${}_0TC\$_{90}$ rises, then $(\$/€)_{90}(€/W)_{90}[1 - {}_0TC\$_{90}] < (\$/W)_0(1+ {}_{\$}i_{90})$. If these changes are large enough, Gulf ports lose their advantage and W moves from Rotterdam to Gulf ports, lowering $(\$/W)_0$ by lowering $(\$/W)_{90}$ and raising $(€/W)_{90}$.

If the shock does not shift the advantage to Rotterdam, but reduces the Gulf port advantage so that it no longer covers the net transaction costs, trade stops. $(€/W)_{90}$ rises as selling stops and $(\$/W)_0$ falls as exports stop, but this absence of trade does not necessarily restore equilibrium.

The subsection on Thresholds below discusses what happens in those two cases. If Gulf ports continue to ship to Rotterdam, firms earn more future dollars by selling W spot and investing the proceeds than by shipping W to Rotterdam. Reduced exports lower $(\$/W)_0$ and raise $(€/W)_{90}$, which works to restore equilibrium.

Assuming that adjustment restores $(\$/W)_{90}(€/W)_{90}[1 - {}_0TC\$_{90}] = (\$/W)_0(1+ {}_{\$}i_{90})$, full international equilibrium requires local equilibrium. Using the equilibrium condition in Gulf ports that $(\$/W)_{90}[1 - {}_0CC\$_{90}] = (\$/W)_0(1+ {}_{\$}i_{90})$, international equilibrium with trade from Gulf ports to Rotterdam can be written as follows: $(\$/€)_{90}(€/W)_{90}[1 - {}_0TC\$_{90}] = (\$/W)_{90}[1 - {}_0CC\$_{90}]$. Solving that equation for $[(\$/W)_{90}/(€/W)_{90}]$ yields eq. (3).

$$[(\$/W)_{90}/(€/W)_{90}] = [(\$/€)_{90}(1 - {}_0TC\$_{90})]/[1 - {}_0CC\$_{90}] \quad (3)$$

Exogenous exchange rates, transport costs and carrying costs determine relative prices in equilibrium.

Using the approximation that *log(1+a)* equals *a* when *a* is small, eq. (3) can be written in logarithmic form as eq. (3').

$$\log[(\$/W)_{90}/(€/W)_{90}] = \log(\$/€)_{90} + [{}_0TC\$_{90} - {}_0CC\$_{90}] \quad (3')$$

Transactions similar to those discussed above produce a similar equilibrium condition for buying in Rotterdam and selling in Gulf ports. It is $(€/\$)_{90}(\$/W)_{90}[1 - {}_0TC€_{90}] = (€/W)_0(1+ {}_€i_{90})$.

Using the local equilibrium that $(€/W)_{90}[1 - {}_0CC€_{90}] = (€/W)_0(1+ {}_€i_{90})$, this equilibrium can be written as follows: $(€/\$)_{90}(\$/W)_{90}[1 - {}_0TC€_{90}] = (€/W)_{90}[1 - {}_0CC€_{90}]$, which implies eq. (4).

$$(\$/W)_{90}/(€/W)_{90}= [1-{}_0CC€_{90}]/\{(€/\$)_{90}[1-{}_0TC€_{90}]\} = \{(\$/€)_{90}[1-{}_0CC€_{90}]\}/[1-{}_0TC€_{90}] \quad (4)$$

Exogenous exchange rates, transport costs and carrying costs determine $(\$/W)_{90}/(€/W)_{90}$ in equilibrium.

Using logarithms, eq. (4) can be written as eq. (4').[14]

$$\log[(\$/W)_{90}/(€/W)_{90}] = \log(\$/€)_{90} + {}_0TC€_{90} - {}_0CC€_{90} \quad (4')$$

Eqs. (3') and (4') differ. That difference reflects thresholds.

2.2.3. Thresholds. To see how transaction costs create thresholds, consider first a world without transport or carrying costs, but with a given exchange rate. Let $(€/\$)_{90}$ be that rate. $(€/\$)_{90}(\$/W)_{90}/(€/W)_{90}$ converts $(\$/W)_{90}/(€/W)_{90}$ to a common currency, the euro. For "low" $(\$/W)_{90}/(€/W)_{90}$ in the absence of trade, Gulf ports export to Rotterdam because in euros W is cheaper in Gulf ports.

[14] Note that $\log(\$/€)_{90}$ in eq. (4') equals $-\log(€/\$)_{90}$ where $(€/\$)_{90}$ is from eq. (4).

As $(€/\$)_{90}(\$/W)_{90}/(€/W)_{90}$ in the absence of trade rises because $(\$/W)_{90}/(€/W)_{90}$ rises, that advantage declines until we reach a point where $(€/\$)_{90}(\$/W)_{90}/(€/W)_{90}$ equals one and the euro price of W in the absence of trade is the same in Gulf ports and Rotterdam. Trade stops. Call that $(€/\$)_{90}(\$/W)_{90}/(€/W)_{90}$ tipping point T where T equals one. As $(\$/W)_{90}/(€/W)_{90}$ in the absence of trade rises, $(€/\$)_{90}(\$/W)_{90}/(€/W)_{90}$ rises beyond T, and $(€/W)_{90}/(\$/W)_{90}$ in the absence of trade falls, the advantage switches to Rotterdam because the euro price of W in the absence of trade is now lower in Rotterdam than in Gulf ports.

Now consider the effect of transport costs. For a range of $(€/\$)_{90}(\$/W)_{90}/(€/W)_{90}$ below T, transport costs prevent Gulf ports from exporting to Rotterdam. Call the minimum $(€/\$)_{90}(\$/W)_{90}/(€/W)_{90}$ tipping point L. $L = 1 - {}_0TC\$_{90}$. For Rotterdam to export to Gulf ports, Rotterdam's advantage must cover its transport costs. Call that higher $(€/\$)_{90}(\$/W)_{90}/(€/W)_{90}$ tipping point U. $U = 1 + {}_0TC€_{90}$.

Ignoring carrying costs, U is the upper threshold and L is the lower threshold. Between those thresholds the equilibrium conditions developed above do not hold. As a result, $(\$/W)_{90}/(€/W)_{90}$ can move more or less freely between U and L. Including carrying costs changes U and L, but it does not change the logic behind thresholds.[15]

2.2.4. Summary. Under some hopefully reasonable assumptions, after accounting for thresholds, the ALOP holds for individual commodities in auction markets. That top down reasoning does not tell us whether or not the ALOP actually holds. That question *must* be answered empirically. Firms like Bunge Ltd. and Carghill Inc. together with the auction markets in Rotterdam and Gulf ports should have the information necessary for such tests. But that

[15] When there are carrying costs, in the logarithmic version, $L = -[{}_0TC\$_{90} - {}_0CC\$_{90}]$ and $U = [{}_0TC€_{90} - {}_0CC€_{90}]$. Under normal circumstances, both $[{}_0TC\$_{90} - {}_0CC\$_{90}]$ and $[{}_0TC€_{90} - {}_0CC€_{90}]$ should be positive.

information is not easily available. Collecting it and performing the appropriate tests will be expensive and time consuming. Is the game worth the candle?

To help determine whether or not the game is worth the candle, the rest of this paper assumes that the ALOP holds for commodities in auction markets and examines some of its implications beginning with its implications for APPP. It would be strange if, after accounting for time in transit, transaction costs and thresholds, the LOP held in auction markets for financial assets but not in auction markets for commodities.

3. APPP.

The logic of the shift from LOP to PPP is the same for APPP as for CPPP. For CLOP, ignoring transaction costs, a ratio of prices like $(\$/W)_0/(€/W)_0$ denoted $q_i(t)$ equals the domestic price of foreign exchange S(t). That equality also holds for any bundle of $q_i(t)$ denoted $Q(t)$ so that S(t) = Q(t). The shift to PPP involves a reinterpretation of this relationship when i is large. For S(t) = q_i(t), $q_i(t)$ adjusts to $S(t)$. For PPP with large i the usual interpretation is that $S(t)$ adjusts to $Q(t)$ where $Q(t)$ is the ratio of a domestic price level $P_D(t)$ relative to a foreign price level $P_F(t)$, *i.e.*, $P_D(t)/P_F(t)$ or just $P(t)$.[16]

For CPPP, $P(t)$ is made up of current consumer or wholesale price indexes, which can have different weights, and $S(t)$ is the current spot price of foreign exchange in an auction market. That is S(t) = P(t) where it is understood that $S(t)$ moves more or less freely within thresholds around $P(t)$.

For APPP, "$P(t)$" is a ratio of price indexes using auction prices for y days forward where weights are identical denoted $P_A(t+y)$ and "$S(t)$" is the current domestic price of foreign

[16] As part of this shift, costs that were exogenous like transport and carrying costs become endogenous.

exchange y days forward denoted $F(t+y)$. $F(t+y) = P_A(t+y)$ where now $F(t+y)$ moves more or less freely within thresholds around $P_A(t+y)$.[17]

Using auction markets, the next section develops an asset and commodity theory of exchange rates, *i.e.*, ACTFX. For ACTFX it is convenient to express APPP logarithmically as $f(t + x) = p_A(t + x)$ where $f(t + x)$ is the log of $F(t + x)$ and $p_A(t + x)$ is the log of $P_A(t + x)$.

4. ACTFX

No one claims that the asset approach to exchange rates is a success. We know no more about the short-run behavior of exchange rates now than we did before the switch to an asset approach in the 1970s. The widely recognized puzzle that there is no link between economic fundamentals and exchange rates is an implicit recognition of that fact. But casual observation suggests that asset markets affect exchange rates. We see exchange rates apparently responding to interest rates.

The objective of this section is to develop a theory of exchange rate determination based on *auction* markets for both assets *and* commodities. It begins with Covered Interest Parity.

4.1. CIP

There is substantial empirical support for Covered Interest Parity (CIP). See for example Akram, Farooq and Sarno (2008). CIP says that $F(t+y)/S(t) = (1+{}_{us}i_y)/(1+{}_{€}i_y)$. This equality is usually expressed in a logarithmic approximation as $f(t+y) – s(t) = {}_{us}i_y – {}_{€}i_y$.

CIP is an example of the ALOP in financial markets. Suppose $f(t+y)$ equals $s(t)$, but ${}_{€}i_y$ is less than ${}_{us}i_y$. There is a risk-free profit. A large money market bank can borrow a million euro at ${}_{€}i_y$, use that million euro to buy a million dollars, invest that million dollars at the higher ${}_{us}i_y$ and sell those dollars forward for euros, earning an almost instantaneous risk-free profit of

[17] As in the example above, x =0.

€1,000,000.00(1 + ${}_{us}i_x$) minus €1,000,000.00(1 + ${}_{€}i_x$). As Akram, Farooq and Sarno (2008) point out, opportunities for such profits do not last much longer than a few minutes.

The usual interpretation of CIP is that ${}_{us}i_y - {}_{€}i_y + s(t)$ determine $f(t+y)$. That interpretation is reasonable because the volume of transactions in spot foreign exchange markets is greater than in any individual forward market. But that interpretation is less convincing when we compare the combined volume of transactions in *all* forward markets to the volume in the spot market.

Eq. (5) is an aggregate version of CIP where each maturity is weighted by the relative volume of transactions, w_x. As far as we are aware, no one has ever expressed CIP in this way before.

$$s(t) = \sum_{y=1}^{N} w_x[f(t+y) + {}_{€}i_y - {}_{us}i_y] \tag{5}$$

Eq. (5) is an aggregate restatement of CIP where ${}_{€}i_y - {}_{us}i_y$ captures the role of capital markets in determining spot exchange rates. APPP captures the role of commodity markets.

4.2. APPP.

The next step to ACTFX adds the role of auction commodity markets where, ignoring thresholds, $f(t+y) = p_A(t+y)$. Using APPP, replace $f(t+y)$ in eq. (5) with $p_A(t+y)$. That replacement converts eq. (5) into eq. (6), ACTFX.

$$s(t) = \sum_{y=1}^{N} w_x[p_A(t+y) + {}_{€}i_y - {}_{us}i_y] \tag{6}$$

Eq. (6) describes how the interaction between auction markets for financial assets and auction markets for commodities affects spot exchange rates. One advantages of eq. (6) is that the data needed for testing it should be available on a daily basis.

Eq. (6) is directly relevant only for those countries with appropriate auction markets. That requirement restricts it to developed countries and not to all developed countries. But the economics behind eq. (6) applies to all countries. At the retail level all goods are non-traded. Arbitrage is rare at the wholesale level and the only routine arbitrage is in auction markets.

5. Puzzles.

In Rogoff (1996) the PPP puzzle is the very high short-run volatility of real exchange rates combined with the very slow rate at which the half-lives for deviations from PPP die out. His explanation is that, in spite of progress, international commodity markets remain highly segmented. When Rogoff refers to international markets being highly segmented, he is referring to retail commodity markets.[18]

We believe that the distinction between retail, wholesale and auction markets provides a better explanation. International auction markets are highly integrated. International retail markets are highly segmented and always will be.

In the years since 1996 PPP puzzles have increased and been refined. Rogoff's puzzle has become three related puzzles: "excessive" exchange rate volatility, short-run PP versus long run PPP and long half lives for deviations from PPP. Two additional puzzles are that PPP works during inflation, but not in normal times, and the lack of any fundamentals that explain the behavior of exchange rates.

We take up these puzzles in the following order: (1) PPP works when there is inflation, but not in normal times. (2) PPP may work in the long run, but not in the short run. (3) Long half-lives for real PPP differentials. (4) Exchange rate volatility is excessive. and (5) A lack of fundamentals.

[18] At one point, p. 650, Rogoff indirectly refers to auction markets. The prices for gold in his Table 2 appear to be from auction markets. But he appears to quickly dismiss the role of such prices for PPP.

5.1. Inflation versus normal.

Frenkel (1981) is a seminal source of the idea that PPP works during inflation but fails in normal times. Using wholesale and cost of living price indexes, he compares the performance of relative PPP during the inflationary 1920s to its performance during the 1970s. Based on relative $\bar{R}^2$s, Frenkel concludes that PPP worked during the inflationary 1920s, but not during the more normal 1970s.

Davutyan and Pippenger (1985) point out that this conclusion is a statistical illusion, a result of misinterpreting $\bar{R}^2$ in the context of thresholds. A simple example makes the point about using $\bar{R}^2$ to compare relative performance. Suppose CPPP is essentially constant and exchange rates never exceed the thresholds. CPPP always holds. Now consider the case where CPPP and exchange rates both rise due to inflation and exchange rates often exceed the thresholds. CPPP often fails. CPPP clearly works better in the first than the second case. But if you test relative CPPP during "normal" times the $\bar{R}^2$ is zero while it is positive with inflation.

Most of the problem is the result of comparing retail prices to auction prices. In normal times CPPP volatility is small due to sticky retail prices while exchange rates are volatile because they are auction prices. As inflation increases, retail prices become more flexible. In hyperinflation retail prices change by the hour or even by the minute.

The difference between inflationary and normal times should largely disappear with APPP. Using commodity prices from auction markets, thresholds are much narrower and the volatility of relative price levels much lager. The problem with $\bar{R}^2$ largely disappears and with it the apparent distinction between inflationary and normal times.[19]

5.2. Long run versus short run.

The evidence clearly rejects CPPP as a theory for the short-run behavior of exchange rates. But there is support for it as a long-run theory. See for example Sarno and Taylor (2002).

APPP also solves this puzzle. As above, the source of the puzzle is comparing sticky retail prices to volatile auction exchange rates.

CPPP fails in the short run because there is no direct link between international retail markets. At the retail level, all goods are non-traded. As a result, short-run thresholds for CPPP are very wide. Sticky retail prices reduce the short-run volatility of CPPP. In the short run, wide thresholds and sticky prices disconnect exchange rates from CPPP. The disconnect for wholesale prices is smaller, but still large.

In the long run, CPPP approaches APPP. Long run links between retail markets are stronger because the indirect links through wholesale and auction markets strengthen in the long run. These stronger links reduce long-run thresholds. Retail prices also are more flexible in the long run. Narrower threshold and more flexible retail prices reduce the disconnect. CPPP works better in the long run than short run.

APPP should substantially reduce the difference between long run and short run. With APPP short-run thresholds are narrower and prices more flexible. The difference between long run and short run should be much smaller with APPP and potentially disappear.

[19] Note that with APPP one regresses changes in forward rates against changes in a ratio of forward commodity prices.

5.3. Long half-lives for real CPPP differentials.

Obstfeld and Rogoff (2000) list long half-lives for real CPPP differentials as one of the six major puzzles in international macroeconomics. Again, the source of the problem is comparing sticky retail prices to volatile auction exchange rates.

Half-life differentials using CPPP are long because the tests use prices in retail markets where all goods are non-traded. It should not be a surprise that real price differentials between non-traded goods have half-lives measured in years.

APPP indexes do not yet exist. But comparing CLOP and ALOP provides some insight into what we can expect. As pointed out above, the evidence rejects CLOP. But the evidence supports ALOP. As Pippenger (2016) reports, real half-life differentials between spot commodity auction prices are measured in just a few weeks. For monthly intervals, spot half lives would essentially disappear despite the fact that ALOP does not hold for spot auction markets.

5.4. Excessive volatility.

As is well known, the volatility of exchange rates is much larger than the volatility of corresponding CPPP. This difference in volatility is the primary evidence behind the belief that exchange-rate volatility is "excessive". Again, the source of the problem is comparing sticky retail prices to volatile auction prices.

Exchanges rate between the U.S. and Canada have been floating for over 25 years. As an example of "excessive" volatility with CPPP, using monthly data from 1975 through 2020, the variance of the change in the log of the Canadian price of U.S. dollars is 0.000226 while the variance in the change in the log of the corresponding CPPP is only 0.000018, a ratio of over 12 to 1.[20] Exchange rate volatility is 12 times greater than CPPP volatility.

The explanation for this puzzle is similar to the one for the three previous puzzles. Exchange rates are from auction markets while commodity prices are from retail markets. No one would be surprised to find that the volatility of the price of a common variety of wheat on the Chicago Board of Trade, whose price changes from minute to minute, is 12 times greater than the volatility of the price of bread in Chicago grocery stores, whose price often does not change for days. Why are we surprised by a ratio of 12 to 1 when we compare auction exchange rates to retail price levels?

We do not yet have data for APPP, but we do have data for individual auction commodity markets, which can give us some insight into APPP. At least it compares auction to auction. Using weekly data from spot auction markets, Bui and Pippenger (1990) find that the volatility of spot exchange rates implied by spot relative prices, *e.g.* $[(\$/W)_0 \backslash (€/W)_0]$, is *greater* than the volatility of actual spot exchange rates. Instead of a ratio of 12 to 1, the ratio is less than 1.

Of course, their results apply to spot auction markets, not forward auction markets. In addition, they use individual auction prices, not indexes. But their results suggest that using APPP rather than CPPP would greatly reduce, if not eliminate, the primary evidence for excessive volatility.

5.5. Exchange-rate disconnect.

The exchange-rate disconnect refers to the lack of any clear link between exchange rates and economic fundamentals. It is another one of the six major puzzles in Obstfeld and Rogoff (2000). ACTFX has the potential to solve this puzzle.

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[20] All data are from FRED.

Casual observation suggests that relative price levels and financial markets are two important fundamentals. CPPP fails for the reasons discussed above. Why the asset approach to exchange rates fails is not yet obvious, possibly it is because an asset approach ignores APPP.

Using auction markets for assets *and* commodities, ACTFX combines relative price levels and financial markets. It has the potential to resolve the exchange-rate disconnect by linking exchange rates to financial and commodity markets. Only careful research can determine whether or not that potential is realized. Even if it is realized, ACTFX will only be a bridge to a deeper understanding of the links between fundamentals and exchange rates.

5.6. Summary.

The important role of transaction costs in exchange-rate economics is widely recognized. Transaction costs play a key role here, but not because they cause imperfect competition, sticky wages and sticky prices, which they do.[21] This paper stresses the role of information and transaction costs in dividing markets into retail, wholesale and auction. The position taken here is that to understand the effects of things like imperfect competition, sticky prices and sticky wages we must first recognize the more important effects of the distinction between retail, wholesale and auction markets. The ability of APPP and/or ACTFX to explain so many puzzles suggests that the game is worth the candle.

6. Summary and conclusions.

Information and transaction costs play an important role in open-economy macroeconomics. They are the source of market imperfections, sticky prices and non-traded goods. But exchange-rate economics has ignored another effect of such costs: the division of

[21] Without restrictions, appealing to transaction costs can explain everything, which means they explain nothing. Our position is simple. We assume that transaction costs behave like other costs. More precisely, they behave like the postulates on costs in Alchian (1959).

markets into retail, wholesale and auction. Ignoring that division, exchange-rate economics has routinely compared the behavior of sticky retail prices to the behavior of flexible auction exchange rates, which compares apples to oranges. Recognizing that division, we suggest a new way of thinking about the LOP and PPP based on auction prices that we call ALOP and APPP. ALOP and APPP solve several of the puzzles associated with exchange-rate economics. The distinction between retail, wholesale and auction markets also suggests a theory of exchange rates that we develop here for the first time using auction markets for commodities *and* assets. We call it ACTFX. ACTFX provides a potential link between exchange rates and fundamentals, and a potential solid foundation for open-economy macro models.

Testing the relative merits of APPP versus conventional PPP and comparing the relative merits of the asset approach to spot exchange rates versus ACTFX creates many opportunities for future research.

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