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Demonstration of a Degenerate Band Edge in Periodically-Loaded Circular Waveguides

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Abstract—We demonstrate the existence of a special degeneracy condition, called degenerate band edge (DBE), between two Bloch modes in periodically-loaded circular all-metallic waveguides at microwave frequencies. The DBE condition has been associated with a dramatic reduction in group velocity and with some unique resonance properties, but it has not been shown in hollow waveguide structures yet. Hence, we show here its existence in two periodic waveguide examples. The unit cell of the first structure is composed of a circular waveguide loaded with two inner cylinders with elliptical irises with misaligned angles. The second structure is composed by loading the waveguide with elliptical rings. The demonstration of DBE in those waveguide is explained through a simple multi-transmission line approach where the conditions to obtain DBE are clarified, and suggests that the DBE can occur in several other analogous periodic waveguides. These structures can be potentially used to investigate unconventional gain schemes in traveling wave tubes or other kinds of distributed amplifiers, oscillators and novel pulse compressors.

Index Terms—Electromagnetic bandgap, periodic structures.

I. INTRODUCTION

LOW-WAVE structures (SWSs) such as coupled-cavity traveling wave tubes, are one of the main building block in high power microwave electronic devices such as amplifiers and oscillators [1]. Among the various implementations of periodic SWSs, here we are investigating those that provide significant reduction in group velocity of the electromagnetic propagating mode, which can result in new phenomenological improvements in resonator quality factors, reduction of size, and mitigation of losses [2], [3]. We focus here on a particular class of SWSs based on periodic structures with at least two Bloch modes that exhibit a degeneracy conditions, such as the degenerate band edge (DBE) condition. This condition has not been explored yet in waveguides like those in this letter. The DBE phenomena was first introduced in [2], [3] by observing frozen mode regimes in multilayer anisotropic dielectric structures. In general, the DBE condition causes a quartic power dependence at the band-edge of the Bloch wavenumber k versus frequency ω , i.e.

$$\omega - \omega_d = \alpha(k - k_d)^4 \quad (1)$$

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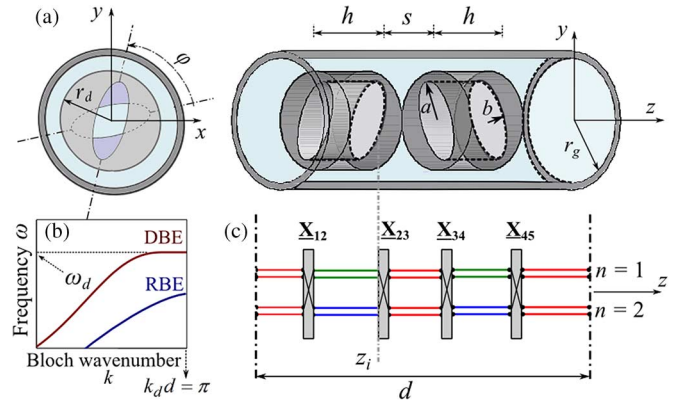


Fig. 1. (a) A unit cell composed of a circular air-filled waveguide loaded with two discs with elliptical irises misaligned by an angle φ . The periodic structure not only supports a regular band edge (RBE), but also DBE under certain conditions. (b) Typical dispersion relation of two modes one exhibiting an RBE and another exhibiting DBE. (c) Equivalent multi-TL that is adopted to understand the occurrence of the two modes in (b) and DBE in particular.

where ω_d is the DBE angular frequency, $k_d = \pi/d$ represents the edge of the Brillouin zone, d is the length of the periodic structure's unit cell (Fig. 1), and α is a problem dependent constant. Consequently, group velocity $v_g = \partial_k \omega$, its first and second derivative are identically zero, but a non-zero third derivative exists, with $\alpha = \partial_k^3 v_g / 24$ (the partial derivatives are defined here as $\partial_k^n \omega \equiv \partial^n \omega / \partial k^n$). This leads to a gigantic increase in the density of electromagnetic modes (or density of states) and group index as we have shown in [3]. Due to the extremely low group velocity of modes close to the DBE condition inside the structure, large field enhancement occurs [2], [3], suggesting their use as novel SWSs for high power generation [4]. This is achieved by the manifestation of giant amplification mechanisms in Fabry-Pérot cavities operating with DBE modes compared to those in conventional oscillators operating based on a single mode [1], [3], [4]. At radio frequencies, DBE condition have been observed only in periodically-coupled microstrip lines [5] but not yet in metallic waveguides. In this letter we propose for the first time a realization of DBE in periodic metallic waveguide structures, such as the periodically loaded circular waveguide unit cells in Fig. 1.

II. DEGENERATE BAND EDGE IN LOADED WAVEGUIDES

Consider the perfect electric conductor (PEC) air-filled circular waveguide with a radius r_g and periodic unit cell of length d , as in Fig. 1(a). The waveguide is loaded with two discs of equal length h , with an elliptic iris, separated by a distance s . The two elliptical irises have identical dimensions with major and minor radii b and a , as shown in

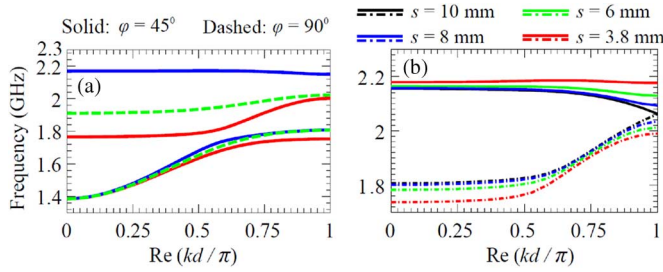


Fig. 2. (a) Dispersion diagram of the periodic structure in Fig. 1 for different misalignment angles of the elliptical irises for $h = 10$ mm and $s = 5$ mm. Each dashed line is split into two by changing φ from 90 to 45 degrees. (b) Same as (a), but for varying s for $\varphi = 45^\circ$, and we show only two higher order modes.

Fig. 1(a). However, their major axes are misaligned by an angle φ . Let $\mathbf{E}_t(\boldsymbol{\rho}, z)$ and $\mathbf{H}_t(\boldsymbol{\rho}, z)$ be the transverse components of the electric and magnetic fields at a cross-section plane defined at $z = \text{constant}$. They can be represented as expansions of the normal modes inside the waveguide at any z as $\mathbf{E}_t(\boldsymbol{\rho}, z) = \sum_n \mathbf{e}_n(\boldsymbol{\rho}) V_n(z)$ and $\mathbf{H}_t(\boldsymbol{\rho}, z) = \sum_n \mathbf{h}_n(\boldsymbol{\rho}) I_n(z)$, where $\mathbf{e}_n(\boldsymbol{\rho})$ and $\mathbf{h}_n(\boldsymbol{\rho})$ are the n th electric and magnetic eigenvectors that depend on the local transverse cross-section [6]. Scalars V_n and I_n , interpreted as transmission line's (TL) voltage and current, are the amplitudes of those fields. To treat the DBE condition it has been convenient to define a state vector $\boldsymbol{\Psi}(z) = [\mathbf{V}^T(z) \mathbf{I}^T(z)]^T$, that describes the evolution of electromagnetic waves along the z -direction using a TL approach [6]. At an interface between two different waveguide cross-sections (for example, in Fig. 1, at z_i located just before the end of the first ring), we apply the boundary conditions matching the transverse fields in each of the two segments $\mathbf{E}_t(\boldsymbol{\rho}, z_i) = \mathbf{E}_t(\boldsymbol{\rho}, z_{i+1})$ and $\mathbf{H}_t(\boldsymbol{\rho}, z_i) = \mathbf{H}_t(\boldsymbol{\rho}, z_{i+1})$. Then we define an interface matrix $\mathbf{X}_{i,i+1}$ that is composed of the projections of the electric and magnetic eigenmode functions belonging to both waveguide segments at z_i and z_{i+1} (see [6] for detailed analysis). Hence the state-vector is transformed as $\boldsymbol{\Psi}(z_i) = \mathbf{X}_{i,i+1} \boldsymbol{\Psi}(z_{i+1})$ across a cross section discontinuity. The rotation matrix $\mathbf{X}_{i,i+1}$ is strongly dependent on the misalignment angle φ in Fig. 1, that mixes modes in the various segments and therefore has a significant impact on the dispersion of modes and the possibility of attaining different band edge condition, among which DBE is the main interest [4], [5], as to be shown later. A detailed formulation is developed in [4] using transfer matrices. We employ only two TLs in the MTL model that are necessary and sufficient to observe a DBE. The purpose of the TL procedure outline here is to (i) demonstrate the one-to-one equivalence of waves in the periodic waveguide and those in the equivalent 2TL model, (ii) develop a general framework whereby many interesting characteristics associated with a DBE can be observed, and (iii) highlight the importance of the misalignment angle to tune the band edge conditions. Here, by using full-wave simulations, we demonstrate the DBE is obtained in a realistic waveguide structure, confirming the prediction based on TL formulation, for the periodic structure in Fig. 1, with $r_g = 40$ mm, $r_d = 35$ mm, $b = 25$ mm, and $a = 10$ mm. In Fig. 2(a) we report the dispersion diagram of the Bloch modes supported by the periodic structure using the eigenmode solver by CST Microwave Studio for two different misalignment angles 45° and 90° . We observe that for $\varphi = 90^\circ$ (dashed lines in Fig. 2(a)), the structure supports

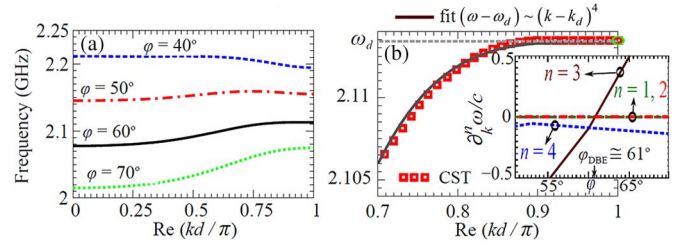


Fig. 3. (a) Dispersion diagram of the highest order mode in Fig. 2(b) obtained by varying the misalignment angle φ with $s = 3.8$ mm. (b) Dispersion for $\varphi = 61^\circ$ around 2.11 GHz and showing an accurate fit to equation (1), and the inset shows the normalized derivatives of group velocity at k_d varying as a function of φ proving the existence of a DBE at $\varphi_{\text{DBE}} \cong 61^\circ$.

four modes (with positive k). In fact, each dashed curve in Fig. 2(a) corresponds to two curves on top of each other, and forms a couple of modes orthogonally-polarized, with a 90-degree rotation symmetry in the $x - y$ plane obeying the same dispersion relation. All four modes exhibit regular band edges (RBE), just below the cutoff frequency of the hollow circular waveguide (~ 2.197 GHz). Indeed, if we consider the equivalent periodic 2TL model in Fig. 1(b), with at least two uncoupled TL segments and mode coupling matrices \mathbf{X} at each cross section discontinuity that introduces mixing between TL modes, the mixing associated to 90° misalignment is not sufficient to develop a DBE. However, when $\varphi = 45^\circ$, we report that each dashed line Fig. 2(a) splits into two different curves, which means that now \mathbf{X} introduces significant mixing between the two TL modes that breaks symmetry between the two polarizations. For smaller separation such as $s = 3.8$ mm, a DBE condition may be qualitatively observed because of the flatness of the dispersion curve [solid red curve in Fig. 2(b)] relative to other cases with larger s . However the mathematical degeneracy condition is yet to be precisely found; that dictates vanishing group velocity, its first and second derivatives at k_d as seen from (1). For that purpose, we vary the rotation angle φ and plot the dispersion diagram in Fig. 3(a) for the case with $s = 3.8$ mm shown in Fig. 2(b). We can see that the band edge feature for the highest order mode (dispersion near k_d) is prone to the variation of φ whereas we have observed that the lower order mode in Fig. 2(b) is not sensitive to the same variation (not shown here). This indicates the possibility of acquiring DBE for the mode in Fig. 3(a) by optimizing φ . We plot the group velocity and its three derivatives and we observe that for $\varphi \cong 61^\circ$ the two derivatives of group velocity vanish while the third derivative is non zero and negative; nonetheless group velocity and its first derivative are still vanishing for wide range indicating that the dispersion can still be maintained relatively flat near k_d for angles between 50 and 65 degrees. For the case with $\varphi = 61^\circ$, we confirm that the dispersion develops a DBE by observing how full wave dispersion well approximates (1) near the band edge by means of numerical fitting. The parameters obtained from numerical fitting are $\omega_d \cong 2\pi$ (2.114 GHz) and $\alpha \cong -4.52 \times 10^3$ (units of $\text{m}^4 \text{s}^{-1}$) obtained also from the inset of Fig. 3(b) as $\alpha = \partial_k^4 \omega / 24 \cong -4.53 \times 10^3$ ($\text{m}^4 \text{s}^{-1}$). The fitting parameters in this model are found by setting a root mean squared error (RMSE) in the order of $\sim 10^{-5}$, which indicates a very good fitting. In Fig. 4 we show the normalized electric field distribution of the eigenmode that exhibits a DBE at ω_d . The three components of the modal electric field are plotted in Fig. 4(a) in the $y - z$ plane at $x = 0$,

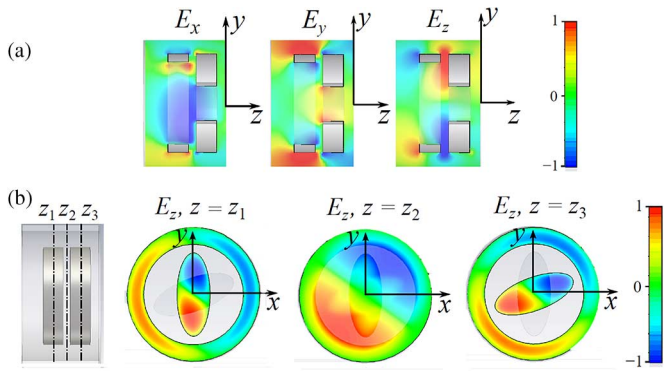


Fig. 4. (a) Normalized electric field distribution (three components, each normalized to its maximum) in the longitudinal plane of the mode that exhibit a DBE for the structure in Fig. 1 (b) The normalized z -directed electric field along the $+z$ -direction in the transverse plane at three different locations inside the unit cell.

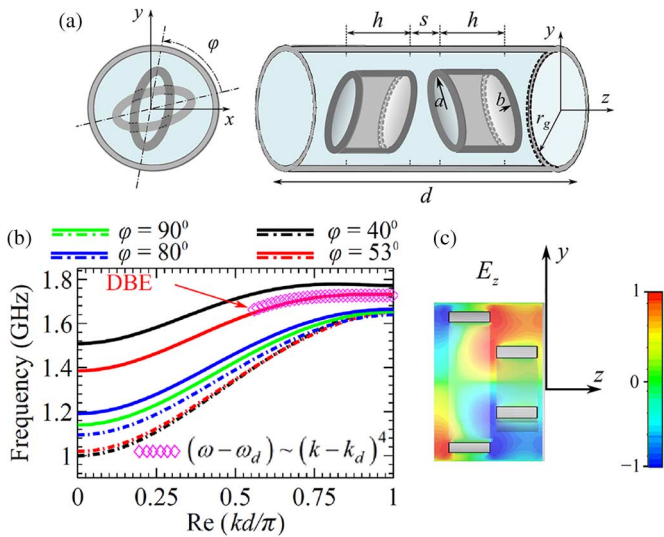


Fig. 5. Ring-loaded waveguide unit cell. (a) Dispersion diagram of the periodic structure for different misalignment angles of the elliptical rings, with $h = 15$ mm and $s = 2.5$ mm, $b = 25$ mm, $a = 10$ mm, and ring thickness is 5 mm. For $\varphi = 53^\circ$, the dispersion develops a DBE showing a fitting with the DBE dispersion relation indicated by the symbols. (c) Normalized z -directed electric field in the $y - z$ plane ($x = 0$) of the DBE mode.

each component is normalized to its own maximum. We report the DBE mode electric field distribution with a strong axial component (simulation data shows negligible longitudinal magnetic field H_z everywhere inside the waveguide, not shown here for brevity, indicating a transverse-magnetic (TM) -like mode at DBE). Also we plot the longitudinal electric field at different $x - y$ transverse cut-planes inside the structure in Fig. 4(b) showing the field inside irises as well as between the discs. Analogous optimized TM structures, may be utilized for electron beam devices in high power generation [1], since DBE mode in this structure is a slow-wave mode with a phase velocity of $\omega/k_d \sim 0.5c$, where c is the speed of light, that can be synchronized to an electron beam with optimized shape. Following the same procedure outlined above, in Fig. 5(a) we show a somehow simpler geometry with two hollow elliptical rings per unit cell. The rings' major axes are misaligned by an angle φ . For $\varphi = 90$ degrees, we report the lowest order modes in the structure and we observe two degenerate but orthogonal polarization modes represented in the dispersion relation in

Fig. 5(b) by dotted-dashed lines, exhibiting RBE. However, for different misalignment angles, mode splitting is evident and DBE is tuned at $\varphi = 53^\circ$, where we show the fitting to (1) with $\omega_d \cong 2\pi(1.731 \text{ GHz})$, $\alpha \cong -1.7 \times 10^4 \text{ (m}^4\text{s}^{-1}\text{)}$ and an RMSE of $\sim 3 \times 10^{-5}$. The axial electric field distribution of the DBE at k_d is shown in Fig. 5(c). We have also seen that severing each ring in Fig. 5 into thinner rings with careful design the periodic structure still enables the occurrence of DBE. An experimental implementation of DBE in circular waveguides should be carried out paying attention to the characteristics of a finite number of unit cells, which generally create mismatch conditions, such as transmission coefficients and quality factors, and their respective scaling with length. It is important to keep an all metallic realization, including ring support fixtures, in order to handle high power electron beam applications. Since DBE here is realized with a slow-wave waveguide, it can be used in high power oscillators, where very low starting oscillation current can be observed, contrary to conventional backward wave oscillators [1], [4]. The structure can also be used in pulse compression devices using the unique energy distribution in Fabry-Pèrot cavities with DBE.

III. CONCLUSION

We have demonstrated the existence of a special degeneracy condition, called degenerate band edge (DBE), between two modes in metallic circular waveguides. This is achieved by loading the waveguide with elliptical irises or elliptical rings with carefully designed misalignment angle. Full-wave eigenmode results confirmed the existence of DBE modes with an axial electric field component. The peculiar resonance condition associated to this special degeneracy condition and its unique advantages have been discussed previously [2]–[5] and they can be obtained now also in metallic waveguide technology. This allows us to explore potential applications such as the enhancement of wave-electron beam interaction in slow wave structures. In a similar context we have already shown a framework for such degeneracy condition that provides superior gain conditions compared to other types of uniform or periodic structure gain media [3], thus our proposed structures promise a great impact on high power amplifier and oscillator designs.

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