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Publication Date
1966-07-26

UCRL-17012 Preprint

University of California

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To be submitted to Physical Review

UCRL-17012
Preprint

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
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AEC Contract No. W-7405-eng-48

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Nuclear Magnetic Moment, Hyperfine Structure, and Hyperfine-Structure Anomaly of Ag^{110m}*

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ABSTRACT

The atomic-beam magnetic-resonance technique was used to measure the free-atom ground-state hyperfine structure separation $\Delta\nu$, the nuclear magnetic moment μ_I, and the hyperfine-structure anomaly ${}^{109}\Delta^{110m}$ of 260-day Ag^{110m} (I = 6). μ_I was obtained independently of $\Delta\nu$ by observing a $\Delta F = \pm 1$ doublet at its field-independent point. The results are $\Delta\nu(Ag^{110m}) = +30313.756(4)$ MHz, $\mu_I(Ag^{110m})$(uncorr) $= +3.587(4)$nm, and ${}^{109}\Delta^{110m} = -2.47(12)\%$. The results are shown to fit in well with existing theories.

I. INTRODUCTION

By measuring the difference in energy between various pairs of hyperfine-structure levels, one can use the unpaired s electron of the silver isotopes as a probe to determine the nuclear spin I, the hyperfine-structure interaction constant $\Delta\nu$, the nuclear magnetic moment μ_I, and the hyperfine-structure (hfs) anomaly Δ. The nuclear spin, magnetic moment, and hyperfine-structure anomaly all provide information about the nuclear state of the atom being studied. The $^2S_{1/2}$ electronic ground state of the silver isotopes makes them particularly well suited for atomic-beam magnetic-resonance (ABMR) studies. The spin of Ag^{110m} has already been measured in this Laboratory by the ABMR method.[1] This paper reports a further study of this isotope, including a measurement of $\Delta\nu$ and a direct measurement of μ_I.[2] The hfs anomaly with respect to Ag^{109} can then be determined from the relation

$$^1\Delta^2 = \frac{\mu_2}{\mu_1} \cdot \frac{\Delta\nu_1}{\Delta\nu_2} \cdot \frac{(2I_2+1)}{(2I_1+1)} \frac{I_1}{I_2} - 1, \qquad (1)$$

where subscript or superscript 1 stands for Ag^{109} and 2 for Ag^{110m}. Because the protons in Ag^{110m} are in a different configuration from those in Ag^{109}, the interpretation of the anomaly is somewhat different than in the case of two adjacent isotopes with similar proton configurations. For Ag^{109} and Ag^{110m} the Bohr-Weisskopf theory[3] of hfs anomalies predicts a very large anomaly, which was found in fact to exist.

II. APPARATUS AND PROCEDURE

The basic experimental technique used was the atomic-beam flop-in magnetic-resonance method of Zacharias. The beam was detected by collecting the atoms on sulfur-coated buttons which were then counted in low-background Geiger counters. For further details of the method, the reader is referred to the several review articles on the subject.[4]

The Ag^{110m} was produced in the Materials Test Reactor at Arco, Idaho, by an (n, γ) reaction on natural silver metal. About 2 g of silver foil was bombarded for 24 days at a flux of $5 \times 10^{14}/cm^2$-sec. Natural silver has two stable isotopes, Ag^{107} and Ag^{109}, in approximately equal abundance. In the reactor Ag^{108}, Ag^{108m}, Ag^{110}, and Ag^{110m} were all produced. Since the half-life of Ag^{108} is 2.4 min, and that of Ag^{110} is 24 sec, they had both decayed away by the time the sample was received. The half-life of Ag^{108m} is greater than 5 years and possibly as long as 100 years;[5] and it has a very low specific activity. This was verified by looking at a γ-ray spectrum of the sample taken with a Li-drifted Ge crystal and comparing it with the spectrum of another sample bombarded 7 years earlier. In the spectrum of the new sample none of the Ag^{108m} peaks stood out above the background, whereas they were the dominant peaks for the spectrum of the old sample in which most of the Ag^{110m} had decayed away. As a further check that the radioactivity used for detecting the beam arose from Ag^{110m}, two resonance buttons that were counted periodically, one for 8 months and one for 18 months, decayed with a half-life of about 260 days.

The atomic beam was produced by electron-bombardment heating of a tantalum oven containing about 80 mg of the silver foil. The rest of the apparatus used is very similar to that used by Vanden Bout in his measurements on gold isotopes, and is described in a recent paper.[6]

The Hamiltonian of an atom with J = 1/2 in the C-field region of an atomic-beam machine is

$$H = a\,\vec{I}\cdot\vec{J} - \mu_0 g_I \vec{I}\cdot\vec{H}_0 - \mu_0 g_J \vec{J}\cdot\vec{H}_0\,, \tag{2}$$

where H_0 is a uniform static magnetic field, μ_0 is the Bohr magneton, and g_I and g_J are the nuclear and electronic g factors.

This Hamiltonian can be diagonalized, and the energy levels $W(F, M_F)$ are given by the well-known Breit-Rabi formula,[7]

$$W(F, M_F) = -\frac{h\Delta\nu}{2(2I+1)} - g_I\mu_0 M_F H_0 \pm \frac{h\Delta\nu}{2}\left[1 + \frac{4M_F x}{(2I+1)} + x^2\right]^{1/2}, \tag{3}$$

where $x = \frac{(g_I - g_J)}{h\Delta\nu}\mu_0 H_0$, and the plus sign goes with the state $F = I + 1/2$ and the minus sign with $F = I - 1/2$. It has also been assumed that second-order corrections are negligibly small, so that $\Delta\nu = a(I + 1/2)$. This assumption is justified in Part III.

In Eq. (2), a and g_I are unknown parameters to be determined by experiment. The procedure is to perform a series of measurements of the energy difference between pairs of (F, M_F) levels, and after each experiment make a least-squares fit of the data to Eq. (2), varying a and g_I to obtain the best fit. The values of a and g_I giving the best fit are then taken as the starting point for the next measurement, which should reduce the uncertainty in a and g_I.

First, the so-called "standard transition" $(13/2, -11/2) \longleftrightarrow (13/2, -13/2)$ was observed, starting at low magnetic field (≈5 G), and then at increasingly higher fields. Each time, improved values of a and g_I

were determined and used to predict the transition frequency at a higher field. After this transition had been observed at fields up to 6000 G, a and g_I were known well enough that one could look for $\Delta F = \pm 1$ transitions, which would fix $\Delta\nu$ and g_I accurately enough to determine a hfs anomaly.

There are two reasons for looking at $\Delta F = \pm 1$ transitions. First one can get a value of $\Delta\nu$ accurate to seven or eight significant figures; and second, one can determine μ_I independently of $\Delta\nu$, as explained below.

In this atomic-beam apparatus the resonance line-width is relatively independent of the transition frequency. Provided field inhomogeneities are small or one is at a field-independent point, one can (by using microwave cavities with long transition regions) make the line-width as small at high frequencies ($\approx$ 24 GHz) as at low frequencies ($\approx$ 1 MHz). This means that by looking at high-frequency $\Delta F = \pm 1$ transitions one can get a measurement of $\Delta\nu$ accurate to many more significant figures than one can by observing only low-frequency transitions. It should also be mentioned that although it is desirable to measure $\Delta\nu$ directly at almost zero field, where the dependence on g_J and g_I is small, it was not possible to do so in this case because of a lack of frequency-generating equipment in the 30-GHz range. Instead $\Delta\nu$ was determined principally by the best fit of the three $\Delta F = \pm 1$ transitions listed in Table I. All three lines were measured at their field-independent points.

The second reason for looking at $\Delta F = \pm 1$ transitions is that, in addition to using the frequencies of the two lines of the hyperfine doublet $(13/2, -7/2) \longleftrightarrow (11/2, -9/2)$ and $(13/2, -9/2) \longleftrightarrow (11/2, -7/2)$

to find $\Delta\nu$, the difference in frequency of the two lines equals $\frac{2\mu_0 H g_I}{h}$. Observing such a doublet at its field-independent point enables one to get a direct measurement of μ_I independently of $\Delta\nu$ and thus to determine the hfs anomaly.

To observe the two lines of this doublet the microwave cavity hairpin shown in Fig. 1a was used. This cavity was operated in the TE_{011} mode having the rf field configuration shown in Fig. 1b. As can be seen from the drawing, this mode provides the necessary oscillating magnetic field perpendicular to the static C field for observing a $\Delta M_F = \pm 1$ transition. A typical resonance obtained by using this cavity at a field-independent point is shown in Fig. 2.

III. RESULTS

A least-squares fit of all the data shown in Table I gave the following results:

$$\Delta\nu(Ag^{110m}) = +\ 30313.756(4)\ \text{MHz},$$

$$\mu_I(Ag^{110m})\ (\text{uncorr}) = +\ 3.587(4),$$

$${}^{109}\Delta^{110m} = -\ 2.47(12)\%.$$

The errors in parentheses are twice the standard deviation given by the least-squares fit. With 20 data points and two independent variables, $\chi^2 = 2.03$. When the test of external consistency is used, the quoted errors give a confidence level greater than 99%. When g_J was allowed to vary as a third parameter the value of g_J obtained, $g_J = -2.002349(5)$, was consistent with the previously measured value,[8] $g_J = -\ 2.0023474(22)$. The value of μ_I agrees with the value of

Easley et al,[9] $\mu_I = + 3.55(4)$nm. The hyperfine-structure anomaly was computed by using

$$\Delta\nu = - 1976.932075(17)\,\text{MHz},^{8}$$

and

$$\mu_I(\text{Ag}^{109}) = - 0.129924(4)_{\text{nm}}.^{10}$$

The small χ^2 indicates that the errors assigned were very conservative, and that the Breit-Rabi formula gives a good description of the hyperfine levels of Ag^{110m}. Since the program fits a, not $\Delta\nu$, and since the Breit-Rabi formula is only a first-order perturbation-theory calculation, an estimate was made of any second-order corrections to $\Delta\nu$ which would cause a deviation from the interval rule. This estimate was made assuming that the electronic wave functions were the solutions of the Dirac equation for an electron with zero binding energy in a Coulomb field, and that the quadrupole moment of Ag^{110m} was 1 barn. The shift in $\Delta\nu$ caused by the mixing of the ${}^2S_{1/2}$ electronic ground state with the nearest ${}^2D_{3/2}$ excited state by a second-order quadrupole interaction was about 34 Hz, less than the experimental uncertainty. For these same wave functions the second-order dipole interaction matrix element between these states vanishes. Because of a parity selection rule the second-order dipole and quadrupole interaction matrix elements between the ${}^2S_{1/2}$ and the ${}^2P_{1/2}$ and ${}^2P_{3/2}$ levels also vanish. Thus, within the accuracy of this experiment, $\Delta\nu = a(I + 1/2)$.

IV. DISCUSSION

The spin I = 6 of Ag^{110m} is consistent with the shell model if one assumes the configuration $\pi(g_{9/2})^{-3}_{7/2}\ \nu(d_{5/2})^{-1}$. Justification for putting the protons in a configuration with seniority three comes from noting that both Ag^{109m} and Ag^{107m} have I = 7/2, and that the magnetic moment of Ag^{109m} is consistent with having three $g_{9/2}$ proton holes coupled to I = 7/2.[11] I = 6 for Ag^{110m} then follows from the weak Nordheim rule as modified by Brennan and Bernstein.[12] An estimate of the magnetic moment using free-nucleon g factors gives μ_{free} = + 3.39 nm. By taking empirical g factors from neighboring odd-a nuclei, Easly et al.[9] obtained μ_I = + 3.54 nm, in good agreement with the experimental value of 3.59 nm. We have repeated this calculation and get a similar answer.

The hyperfine-structure anomaly, defined in Eq. (1), arises because the hyperfine-structure interaction of an s electron is sensitive to the detailed structure of the magnetic field inside the nucleus. The magnetism arising from the intrinsic spin and that from the orbital motion of the nucleons have different spatial dependencies. Thus, the relative contributions of intrinsic spin and orbital motion to the magnetic moment are different from their relative contributions to the hyperfine-structure interaction inside the nucleus. This means that a is not strictly proportional to g_I.

One can estimate Δ, using the theory of Bohr and Weisskopf[3] (B-W). Using $\Delta\nu = a(I + 1/2)$, $\mu_I = g_I\, I$, and rewriting Eq. (1) as

$$\frac{a_1}{a_2} = \frac{g_1}{g_2}\,(1 + {}^1\Delta^2) = \frac{g_1}{g_2}\;\frac{(1 + \epsilon_1)}{(1 + \epsilon_2)}$$

redefines

$$ {}^1\Delta^2 = \frac{1 + \epsilon_1}{1 + \epsilon_2} - 1 \,. \tag{4}$$

Here ϵ is the fractional reduction in the hyperfine structure under that for a point nucleus. One can then use the B-W theory to calculate ϵ for an odd-A nucleus. This was done for Ag^{109} as explained below. The B-W theory can also be adapted to the case of an odd-odd nucleus. In the spirit of the B-W theory, one can show that[13]

$$\epsilon_{o-o} = \beta_p \epsilon_p + \beta_n \epsilon_n, \tag{5}$$

where ϵ_p and ϵ_n are the ϵ's of the neighboring odd-A nuclei and β_p and β_n are the fractional contributions to the magnetic moment from protons and neutrons respectively.

Equation (5) was used to estimate $\epsilon(Ag^{110m})$. The β's were determined by assuming that the contribution to the magnetic moment from the protons was the same as that for Ag^{109m} and adjusting the neutron g factor to fit the Ag^{110m} magnetic moment; ϵ_p was taken to be the ϵ estimated for Ag^{109m} and ϵ_n was estimated for a $d_{5/2}$ neutron. Both $\epsilon(Ag^{110})$ and $\epsilon(Ag^{109})$ were calculated by using the uniform spherical nuclear charge distribution of Eisinger and Jaccarino[14] and the trapezoidal nuclear charge distribution of Stroke et al.[15] to determine the electron wave functions. The nuclear parameters were taken from the paper of Eisinger and Jaccarino. Table II lists the calculated values of the ϵ's and Δ's as well as the experimental value of ${}^{109}\Delta^{110m}$.

Since the errors in the calculated Δ's are probably ±1% or greater, both values include the experimental result.

If the magnetism in two nuclei arises in much different relative amounts from intrinsic spin and from orbital motion, as with Ag^{109} and Ag^{110m}, the B-W theory predicts a large anomaly. This prediction of a large anomaly is borne out. For two nuclei with different shell-model configurations, the B-W theory would predict an anomaly even for the extreme case in which both nuclei had magnetic moments equal to the Schmidt value. This is in contrast to the case of two nuclei with the same configuration for which the predicted anomaly would vanish if both moments were equal to the Schmidt limit--or, in fact, just equal to each other. That the nuclear magnetism of Ag^{109} is very different from that of Ag^{110m} is evident from the large difference in their magnetic moments. The large hfs anomaly confirms this difference, but the present state of nuclear theory makes it difficult to extract much additional information from this large hfs anomaly.

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*This work was supported in part by the National Science Foundation and the Atomic Energy Commission.

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Table I. Resonance data. $\chi^2 = 2.03$.

	Calibrating isotope data			Ag^{110m} Data		
Run No.	Isotope	Freq. (MHz)	H (gauss)	Transition $(F, m_F) \leftrightarrow (F', m_F')$	Freq. (MHz)	Residual (MHz)
34A	Cs^{133}	1.757(5)[a]	5.015(14)	(13/2, -11/2) ↔ (13/2, -13/2)	1.085(10)	0.0055
34B	Cs^{133}	17.762(5)	50.091(14)	(13/2, -11/2) ↔ (13/2, -13/2)	10.815(10)	-0.0087
34C1	Cs^{133}	36.362(5)	101.135(14)	(13/2, -11/2) ↔ (13/2, -13/2)	21.950(5)	0.0004
34C2	Cs^{133}	36.360(5)	101.130(14)	(13/2, -9/2) ↔ (13/2, -13/2)	43.860(20)	-0.0049
34D	Cs^{133}	54.670(5)	150.035(13)	(13/2, -11/2) ↔ (13/2, -13/2)	32.700(10)	-0.0002
34E	Cs^{133}	93.692(5)	250.083(12)	(13/2, -11/2) ↔ (13/2, -13/2)	54.980(10)	-0.0010
34F1	Cs^{133}	134.926(10)	350.101(24)	(13/2, -11/2) ↔ (13/2, -13/2)	77.647(10)	0.0014
34F2	Cs^{133}	134.930(4)	350.111(9)	(13/2, -11/2) ↔ (13/2, -13/2)	77.650(5)	0.0022
34H	Cs^{133}	646.583(4)	1264.793(6)	(13/2, -11/2) ↔ (13/2, -13/2)	304.762(5)	-0.0003
34J	Cs^{133}	3407.191(7)	3362.028(4)	(13/2, -11/2) ↔ (13/2, -13/2)	1002.145(10)	0.0005
34K	Cs^{133}	9629.936(17)	5999.865(6)	(13/2, -11/2) ↔ (13/2, -13/2)	2455.238(12)	0.0041
34L	Cs^{133}	11514.957(30)	6717.940(11)	(13/2, -7/2) ↔ (11/2, -7/2)	25666.650(60)	-0.0014
34M	Cs^{133}	23833.024(20)[b]	6717.998(8)	(13/2, -7/2) ↔ (11/2, -7/2)	25666.659(28)	-0.0084
34P	Cs^{133}	23189.056(20)[c]	5822.970(7)	(13/2, -7/2) ↔ (11/2, -7/2)	25543.879(14)	-0.0103
48C1	Cs^{133}	11522.751(150)	6720.880(56)	(13/2, -7/2) ↔ (11/2, -9/2)	23820.514(10)	-0.0013
48C2	Cs^{133}	11522.834(150)	6720.912(56)	(13/2, -9/2) ↔ (11/2, -7/2)	23826.640(10)	-0.0005
54A	Cs^{133}	11523.079(60)	6721.004(23)	(13/2, -7/2) ↔ (11/2, -9/2)	23820.516(5)	0.0007
54B	Cs^{133}	11523.035(50)	6720.988(19)	(13/2, -7/2) ↔ (11/2, -9/2)	23820.515(3)	0.0001
54C	Cs^{133}	11523.130(50)	6721.024(19)	(13/2, -9/2) ↔ (11/2, -7/2)	23826.641(4)	0.0008
54D	Cs^{133}	11523.102(50)	6721.013(19)	(13/2, -9/2) ↔ (11/2, -7/2)	23826.641(3)	0.0000

Constants for Cs^{133}: $\Delta\nu = 9192.631770$ MHz.

$g_I = 3.98994 \times 10^{-4}$ Bohr magnetons

$g_J = -2.0025417(24)$ Bohr magnetons

a. Unless otherwise specified the Cs^{133} transition was $(4, -3) \leftrightarrow (4, -4)$.

b. Cs^{133} transition was $(4, 2) \leftrightarrow (3, 1)$.

c. Cs^{133} transition was $(4, 3) \leftrightarrow (3, 1)$.

Table II. Values of the hfs anomalies.

Charge Distribution	$\epsilon(Ag^{109})$ (%)	$\epsilon(Ag^{110m})$ (%)	${}^{109}\Delta^{110m}$ (%)
Uniform	-3.3	-0.5	-2.8
Trapezoidal	-3.8	-0.6	-3.2
Experimental Result			-2.47

FIGURE CAPTIONS

Fig. 1a. Schematic drawing of microwave cavity hairpin, (b) rf fields in cavity

Fig. 2. $(13/2, -9/2) \leftrightarrow (11/2, -7/2)$ transition for Ag^{110m} at 6721 gauss.

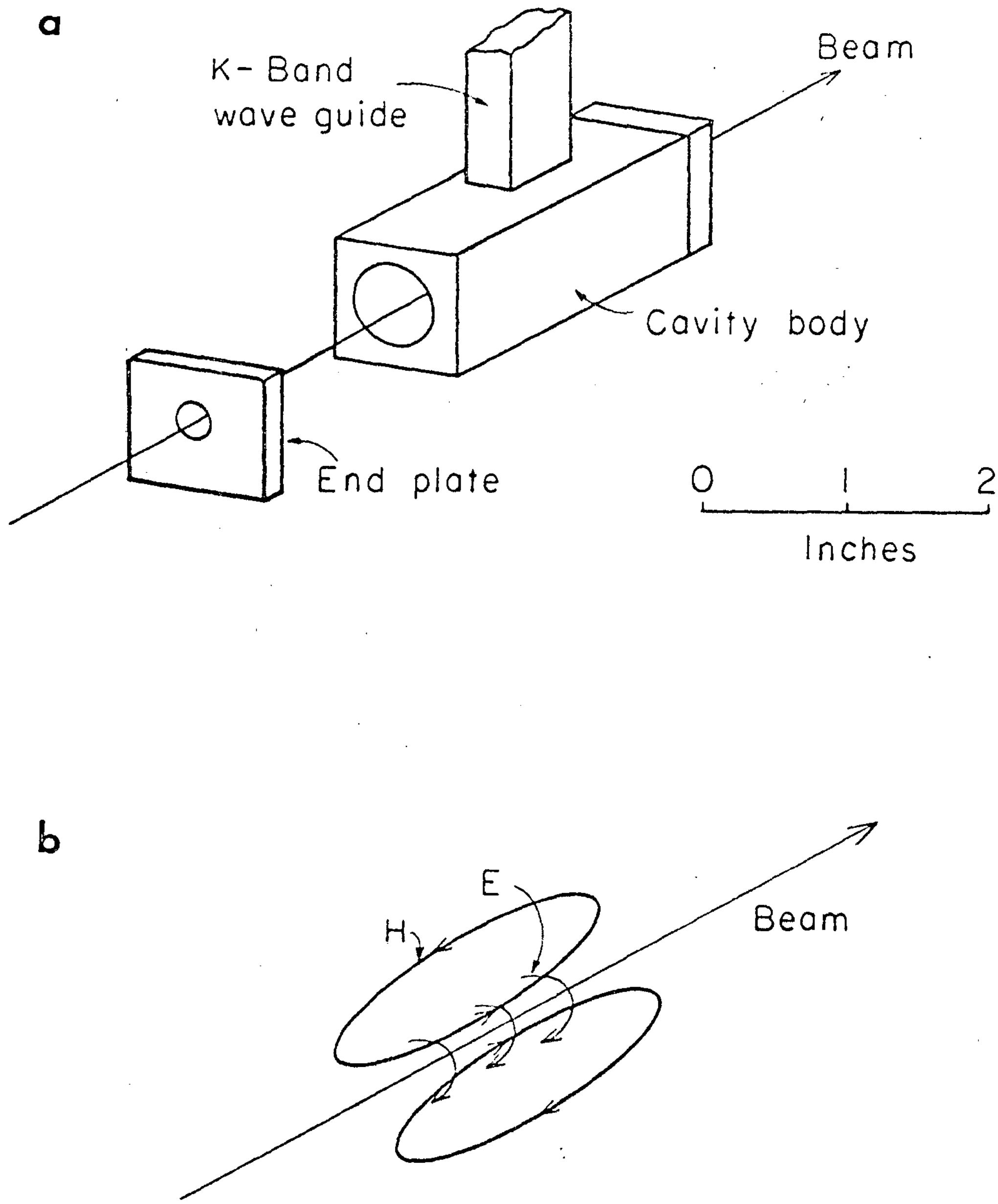

Fig. 1 MUB-12095

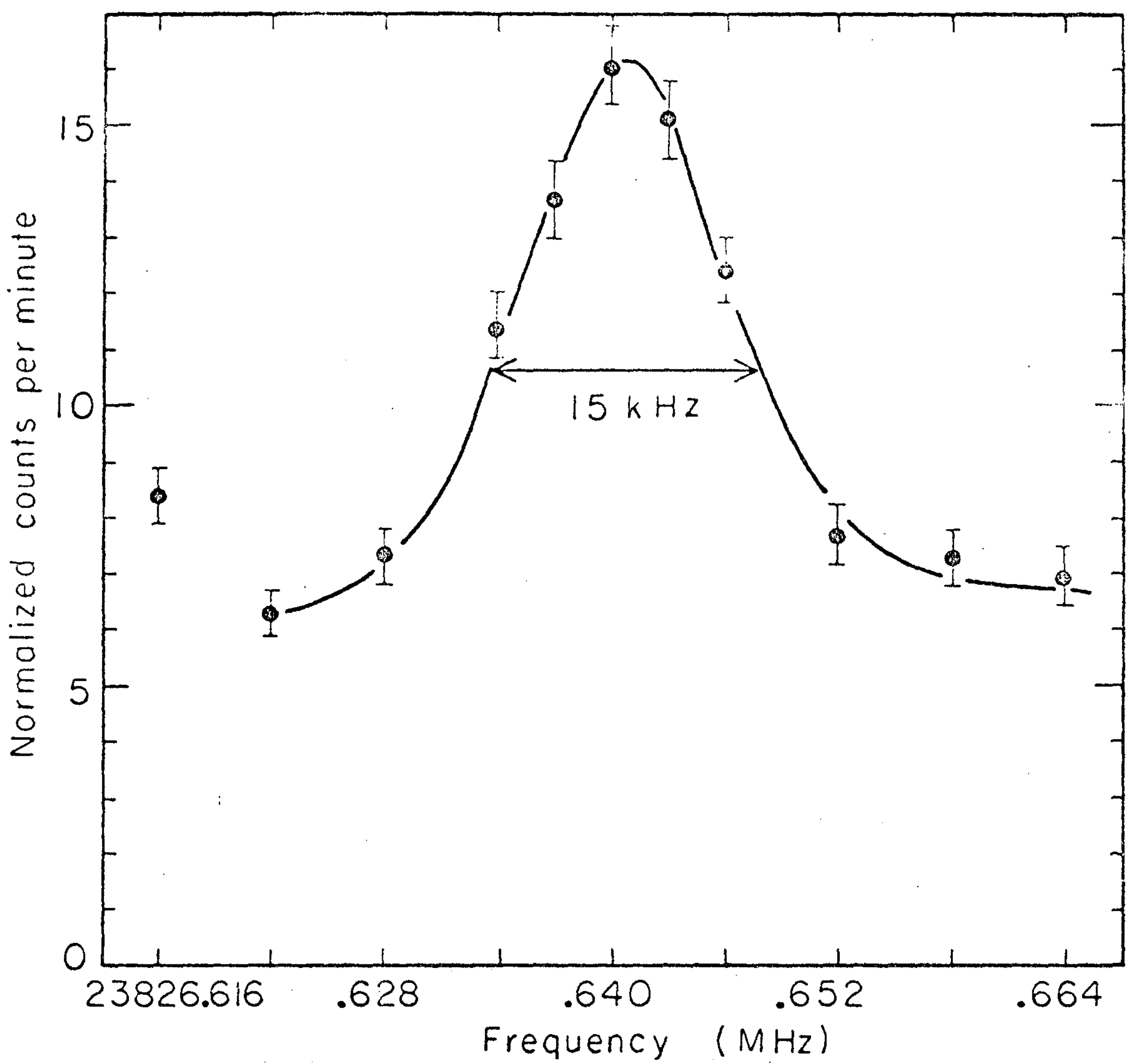

Fig. 2

MUB-12094