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Hot Gluon Plasma at Large Distances*

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Abstract:

We study some general features of the pure gluon plasma in equilibrium at high temperature and large distances, using both imaginary- and real-time formalisms. The plasma description in terms of a (confining) Yang-Mills theory in three spatial dimensions (YM_3) for magnetic gluons is outlined. Arguments are given that imply that, in such a regime, neither a direct quartic coupling of electric gluons nor a Higgs-like mechanism contribute. It is also shown that the only renormalization group equation valid in that regime is the trivial one, associated to the superrenormalizable YM_3 theory, and, on basis of this, it is argued that there are no free gluons at large distances. The real-time gluon Green function is given to one loop order and the instabilities it produces in the gluon plasma are displayed. These instabilities appear to reflect the long-range confining behaviour of YM_3.

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1 Introduction

The attempts at understanding Quantum Chromodynamics (QCD) at high temperature greatly motivated by the conjectured possibility of creating a quark-gluon plasma in ultrarelativistic heavy ion collisions, have originated a very active research along this decade [1],[2]. It has been argued that asymptotic freedom, via a renormalization group analysis of QCD, implies that, above certain critical temperature, quarks and gluons behave as weakly interacting (almost free) in general, that is, for short and large distances[3],[4],[5].

Based upon a general decoupling theorem[6] it was argued that continuum four-dimensional QCD would reduce at high temperature to a pure three-dimensional Yang-Mills theory for magnetic gluons coupled eventually to a massive coloured scalar (electric gluon) field[7],[8]. That simplification is named dimensional reduction. Further works with increasing degree of generality, have contributed to understand this effect[5],[9],[10],[11]. Dimensional reduction of QCD on the lattice has also been discussed[12].

At a rather early stage, this dimensional reduction already signaled certain surprises in the apparently weakly interacting plasma of quark and gluons (possibly colour confinement[8]). Then, it was conjectured that at sufficiently large distances, the high-temperature plasma would display features analogous to those of the low-temperature hadronic phase[12]. Thus, the question still arises whether the high-temperature plasma really behaves as weakly interacting at all distances.

From another line of research, some controversies about the linear response and instability properties of the hot gluon plasma have been brought up. Some authors have found a negative damping constant of the collective excitations[13]-[18], meaning an unstable plasma, while others have found a positive stabilizing damping constant[19]-[21]. Although this issue is not yet settled, one may well ask if the origin for these discrepancies lies in the physics itself, *i.e.* if the instability is real.

The previous discussions strongly indicate the convenience and the interest of carrying through further analysis of the hot gluon plasma, in order to clarify whether there is really almost-free behaviour at all distances and the possible origin of the instability. We shall concentrate on the pure gluon plasma (no quarks) in the regime of high temperature and large distances. The dimensional reduction is outlined and discussed in section 2. Further possible contributions (direct quartic coupling of electric gluons) and mechanisms (Higgs-like) are discussed critically in section 3. A new renormalization group analysis is carried out and discussed in section 4. In section 5, we discuss some useful properties of the real-time formalism. The linear response and stability properties of the gluon plasma are analyzed and discussed in section 6. Section 7 summarizes our findings.

2 The gluon plasma in imaginary time

2.1 Survey of general properties

We consider the quantized nonabelian gluon plasma in infinite three-dimensional space in thermodynamical equilibrium at finite temperature β^{-1} (> 0). In the imaginary-time ($-\beta/2 \leq \tau \leq \beta/2$) formalism, the partition function $Z[J]$ including an external source $J = J^a{}_\nu$ ($J^a_\nu(-\beta/2, \vec{x}) = J^a_\nu(+\beta/2, \vec{x})$) for gluons and the finite temperature connected and renormalized gluon correlation functions in momentum space ($\tilde{D}^{a_1 \cdots a_n}_{\nu_1 \cdots \nu_n}$'s) are given through

$$Z[J] = N \int [DA] \int [DcD\bar{c}] exp\{\int_{-\beta/2}^{+\beta/2} d\tau \int d^3\vec{x} J^a_\nu A^{a\nu} - S\}$$

$$= Nexp[\sum_{n=2}^{+\infty} (n!)^{-1} \beta^{-n+1} \sum_{r_1=+\infty}^{+\infty} \cdots \sum_{r_{n-1}=-\infty}^{+\infty}$$
$$\int \prod_{h=1}^{n-1} \frac{d^3\vec{q}_h}{(2\pi)^3} \tilde{J}^{a_h\nu_h}(Q_h) \tilde{J}^{a_n\nu_n}(Q_n) \tilde{D}^{a_1 \cdots a_n}_{\nu_1 \cdots \nu_n}(Q_1 \cdots Q_n; \mu)].$$

N will denote generically a normalization constant, $[DA] = \prod_{\tau,\vec{x},\nu,a} dA^a_\nu(\tau\vec{x})$ and A^a_ν is the renormalized gluon field ($\nu = 0,1,2,3$), periodic in τ. $[DcD\bar{c}] = \prod_{\tau,\vec{x},a} dc^a(\tau\vec{x}) d\bar{c}^a(\tau\vec{x})$ and c^a, $\bar{c}^a$ are the renormalized ghost fields, also periodic in τ. The superscript a denotes colour.

On the other hand

$$S = S_0 + \int_{-\beta/2}^{+\beta/2} d\tau \int d^3\vec{x} \mathcal{L}_{rc} , \quad (1)$$

where

$$S_0 = \int_{-\beta/2}^{+\beta/2} d\tau \int d^3\vec{x} [\frac{1}{4} F^a_{\rho\nu} F^{a\rho\nu} + \frac{1}{2\xi} (\partial_\rho A^{a\rho})(\partial_\nu A^{a\nu}) + (\partial^\mu \bar{c}^a)(D^{ab}_\mu c^b)] ,$$

with

$$F^a_{\rho\nu} = \partial_\rho A^a_\nu - \partial_\nu A^a_\rho + g f^{abc} A^b_\rho A^c_\nu, \quad D^{ab}_\mu = \partial^{ab} \partial_\mu - g f^{abc} A^c_\mu , \quad (2)$$

while $(\partial_\rho) = (i\frac{\partial}{\partial\tau}, \nabla_{\vec{x}})$, f^{abc} are the $SU(N)$ structure constants. The parameters g and ξ are, respectively, the renormalized coupling constant and the gauge parameter, all at zero temperature in the MS scheme[22]. The last term on the right-hand-side of eq. 1 contains all renormalization counterterms: the integrand $\mathcal{L}_{rc}$ is the same polynomial in A, c, $\bar{c}$ with the same renormalization constants as for zero temperature, in the MS scheme[22]. One has

$$Q_h = (i\omega(r_h), \vec{q}_h), \quad \omega(r_h) = \beta^{-1} 2\pi r_h , \quad (3)$$

$$r_h = 0, \pm 1 \pm 2, \cdots \ , \ \ \sum_{h=1}^{n} Q_h = 0,$$

$$\tilde{J}^a_\nu(Q) = \int_{-\beta/2}^{+\beta/2} d\tau \int d^3\vec{x} J^a_\nu(\tau\vec{x}) exp\{i[\vec{q}\cdot\vec{x} - \omega(r)\tau]\} \ ,$$

μ is the finite momentum scale introduced by the renormalization procedure (in the MS scheme).

In the MS scheme one has

$$\tilde{D}^{a_1\cdots a_n}_{\nu_1\cdots\nu_n}(Q_1\cdots Q_n;\mu) = Z_D(\alpha,\epsilon)\tilde{D}^{a_1\cdots a_n}_{\nu_1\cdots\nu_n}(Q_1\cdots Q_n;\epsilon)_0 \ , \tag{4}$$

and

$$\alpha = \frac{g^2}{4\pi} \ = \ Z_\alpha^{-1}(\alpha,\epsilon)\alpha_0\mu^{2\epsilon}. \tag{5}$$

$Z_D(\alpha,\epsilon)$ and $Z_\alpha(\alpha,\epsilon)$ are suitable products of the zero temperature renormalization constants and they depend solely on α and ϵ. In turn, ϵ is the usual parameter characterizing dimensional regularization in a space time of dimension d ($d = 4+2\epsilon$). α_0 is the unrenormalized coupling constant. $\tilde{D}^{a_1\cdots a_n}_{\nu_1\cdots\nu_n}(Q_1\cdots Q_n;\epsilon)_0$ is the connected but unrenormalized correlation function. The latter does depend on β, α_0 as well but is independent on the finite momentum scale μ. Consequently at fixed β, since α is a function of μ, ϵ, α_0:

$$\mu\frac{d}{d\mu}|_{\epsilon,\ \alpha_0\ fixed}\tilde{D}^{a_1\cdots a_n}_{\nu_1\cdots\nu_n} \ = \ \mu\frac{d}{d\mu}|_{\epsilon,\ \alpha_0\ fixed}[Z_D\tilde{D}^{a_1\cdots a_n}_{\nu_1\cdots\nu_n,0}] \tag{6}$$

$$= \ [\frac{\mu\frac{d}{d\mu}Z_D}{Z_D}]_{\epsilon,\ \alpha_0\ fixed}\tilde{D}^{a_1\cdots a_n}_{\nu_1\cdots\nu_n} \ . \tag{7}$$

This last equation, which will be crucial in section 4 yields the following renormalization group equation at finite temperature (through the same arguments as for zero temperature[22]):

$$[\mu\frac{\partial}{\partial\mu} + \alpha\beta_{rg}\frac{\partial}{\partial\alpha} + \xi\delta_{rg}\frac{\partial}{\partial\xi} - \gamma_{rg}]\tilde{D}^{a_1\cdots a_n}_{\nu_1\cdots\nu_n}(Q_1\cdots Q_n;\mu) \ = \ 0. \tag{8}$$

Here β_{rg}, γ_{rg}, and δ_{rg} are the standard ultraviolet-finite renormalization group functions at zero temperature[22] (since they are determined by the zero-temperature renormalization constants). All dependences in the temperature are now contained in $\tilde{D}^{a_1\cdots a_n}_{\nu_1\cdots\nu_n}(Q_1\cdots Q_n;\mu)$ which also depends on α, and ξ.

2.2 The partition functions $Z_{ME}[j]_{HT}$ and $Z_M[j_M]_{HT}$

Below, latin indices will replace Lorentz indices when the latter are restricted to the values $1,2,3$. Let $j = j^a_\mu = j^a_\mu(\vec{x})$, $\mu = 0,1,2,3$, be an external τ-independent gluon source. We consider the new generating functional $Z[j]_{HT}$ and connected Green functions $\tilde{D}_{HT}$ at high temperature (HT) defined through

$$Z[j]_{HT} = N\int[dA(0)]_M\int[dc(0)d\bar{c}(0)][dA(0)]_E\ exp\{\beta\int d^3\vec{x}j^a_\nu A^{a\nu}(0) - S_{HT}\}$$

$$= N exp\{\beta\sum_{n=2}^{+\infty}(n!)^{-1}\int[\prod_{h=1}^{n-1}\frac{d^3\vec{q}_h}{(2\pi)^3}\tilde{j}^{a_h\nu_h}(\vec{q}_h)]\tilde{j}^{a_n\nu_n}(\vec{q}_n)\tilde{D}^{a_1\cdots a_n}_{\nu_1\cdots\nu_n}(\vec{q}_1\cdots\vec{q}_n)_{HT}\},$$

where

$$\sum_{j=1}^{n}\vec{q}_j = 0,\quad \tilde{j}^{a\nu}(\vec{q}) = \int d^3\vec{x}j^{a\nu}(\vec{x})exp\{i\vec{q}\vec{x}\}\ ,$$

M and E will stand for magnetic and electric respectively.

In turn, $[dA(0)]_M = \prod_{\vec{x}ja} dA^a_j(0,\vec{x})$, $A^a_j(0) = A^a_j(0,\vec{x})$, $j = 1,2,3$, $[dc(0)d\bar{c}(0)] = \prod_{\vec{x}a} dc^a(0,\vec{x})d\bar{c}^a(0,\vec{x})$. $[dA(0)]_E = \prod_{\vec{x}a} dA^{0a}(0,\vec{x})$, $A^{0a}(0) = A^{0a}(0,\vec{x})$. The new action is ($\vec{x} = (x_j)$, $j = 1,2,3$)

$$S_{HT} = S^M_{HT} + S^E_{HT}\ ,$$

where

$$S^M_{HT} = \sum_{j=1}^{3} S_{HTj}\ ,$$

and

$$S^E_{HT} = \sum_{j=4}^{5} S_{HTj}\ .$$

With

$$S_{HT1} = \beta\int d^3\vec{x}\frac{1}{4}\sum_{h,j=1}^{3} F^a_{hj}(0)F^a_{hj}(0),$$

$$S_{HT2} = \beta\int d^3\vec{x}\frac{1}{2\xi}[\sum_{h=1}^{3}\frac{\partial A^a_h(0)}{\partial x_h}][\sum_{j=1}^{3}\frac{\partial A^a_j(0)}{\partial x_j}]\ ,$$

$$S_{HT3} = \beta\int d^3\vec{x}\sum_{h=1}^{3}[\frac{\partial\bar{c}^a(0,\vec{x})}{\partial x_h}][D^{ab}_h(0)c^b(0,\vec{x})]\ ,$$

$$S_{HT4} = \beta\int d^3\vec{x}\frac{1}{2}\sum_{h=1}^{3}[\frac{\partial A^{a0}(0)}{\partial x_h} + gf^{abc}A^b_h(0)A^{c0}(0)][\frac{\partial A^{a0}(0)}{\partial x_h} + gf^{ade}A^d_h(0)A^{e0}(0)]\ ,$$

and

$$S_{HT5} = (m^2_{E,fin} - m^2_{E,div})\beta/2 \int d^3\vec{x} A^{a0}(0)A^{a0}(0) .$$

$F^a_{hj}(0)$ and $D^{ab}_h(0)$, $h,j = 1,2,3$ are given by eq. 2 with A's replaced by $A(0\vec{x})$'s. Notice that $\tilde{D}^{a_1\cdots a_n}_{\nu_1\cdots\nu_n}(\vec{q}_1\cdots\vec{q}_n)_{HT}$ depends on β.

Here $m^2_{E,fin}$ and $m^2_{E,div}$ are mass terms for electric gluons which are ultraviolet finite and ultraviolet divergent, respectively. They are due to one- and two-loop diagrams and it has been shown that the difference is free of infrared divergences[11].

We shall also introduce the purely magnetic (M) high temperature function ($j_M = j^a_h(0\vec{x})$, $h = 1,2,3$)

$$Z_M[j_M]_{HT} = N\int[dA(0)]_M\int[dc(0)d\bar{c}(0)]exp\{-\beta\int d^3\vec{x}\sum_{h=1}^{3} j^a_h(\vec{x})A^a_h(0) - S^M_{HT}\}$$

$$= Nexp[\beta\sum_{n=2}^{+\infty}(n!)^{-1}\int[\prod_{h=1}^{n-1}\frac{d^3\vec{q}_j}{(2\pi)^3}\tilde{j}^{a_h}_{i_h}(\vec{q}_h)]\tilde{j}^{a_n}_{i_n}(\vec{q}_n)\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots\vec{q}_n)^M_{HT}].$$

$Z[j]_{HT}$ is the generating functional for the euclidean Yang-Mills field (A^a_h, $h = 1,2,3$) interacting with a massive scalar coloured field A^{0a} in three spatial dimensions (in short, the $(YM+A^0)_3$ theory). Similarly, $Z_M[j_M]_{HT}$ is the generating functional for the euclidean Yang-Mills theory in three spatial dimensions (in short, the YM_3 theory). By recalling the properties of the euclidean YM_3 theory, it follows that $Z_M[j_M]_{HT}$ does not require any ultraviolet divergent renormalization constant. Hence all $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots\vec{q}_n)^M_{HT}$ contain no ultraviolet divergent piece and, hence, they do not depend on any momentum scale μ induced by any renormalization procedure (which is unnecessary for the euclidean YM_3 theory). Similar statements hold for the $(YM + A_0)_3$ theory, that is, for $Z[j]_{HT}$ and its correlation functions.

2.3 Dimensional reduction

Let β^{-1}_{crit} denote generically the critical temperature related to deconfinement. For fixed g, we shall consider the high temperature regime, where $\beta\mu \ll 1$, $\beta\beta^{-1}_{crit} \ll 1$, and, eventually, $\beta \to 0$. Then to all perturbative orders in g, the following properties can be justified. They amount to the statement that $Z[j]_{HT}$ describes the gluon plasma dimensionally reduced at high temperatures and large distances.

i) For fixed $\vec{q}_1\cdots\vec{q}_{n-1}$, with $\beta|\vec{q}_j| \ll 1$, $j = 1,\cdots n-1$, $\vec{q}_n = -\sum_{j=-1}^{n-1}\vec{q}_j$ and vanishing external frequencies ($r_j = 0$: recall eq. 3)

$$\tilde{D}^{a_1\cdots a_n}_{\mu_1\cdots\mu_n}((0,\vec{q}_1)\cdots(0,\vec{q}_n);\mu) \xrightarrow{\beta\to 0} \tilde{D}^{a_1\cdots a_n}_{\mu_1\cdots\mu_n}(\vec{q}_1\cdots\vec{q}_n)_{HT} .$$

ii) Let

$$J^a_\mu(\tau\vec{x}) \xrightarrow{\beta\to 0} j^a_\mu(\vec{x}) ,$$

where $\int d^3\vec{x} j^a_\mu(\vec{x}) exp(i\vec{q}\vec{x})$ takes on its largest value for $|\vec{q}| \ll \beta^{-1}$ while it is entirely negligible for both $|\vec{q}| \approx \beta^{-1}$ and $|\vec{q}| \gg \beta^{-1}$, by assumption. Then

$$Z[J] \stackrel{\beta \to 0}{\longrightarrow} Z[j]_{HT} \, . \tag{9}$$

2.4 Extreme dimensional reduction

It has been conjectured that *electric* gluons (those associated to A^a_0, with vanishing Lorentz indices) decouple safely from the magnetic ones in the regime of high temperatures and large distances. Arguments (with increasing details and generality) [7,5,11] have been provided which do support the statement that the contributions of electric gluons become subdominant compared to those of magnetic ones for all correlation functions $\tilde{D}^{a_1 \cdots a_n}_{i_1 \cdots i_n}((0\vec{q}_1) \cdots (0\vec{q}_n); \mu)$ in the above regime. This additional simplification will be called extreme dimensional reduction.

It can be formulated as follows. In addition to the assumptions contained in i) and ii) in the previous subsection, we shall suppose that all external three momenta in any $\tilde{D}^{a_1 \cdots a_n}_{i_1 \cdots i_n}(\vec{q}_1 \cdots \vec{q}_n)_{HT}$ fulfill $\beta|\vec{q}_j| \ll g$, $j = 1 \cdots n-1$, and that $j^{a0}(\vec{x}) = 0$. Then in such a regime one has (for fixed g and to all perturbative orders)

$$\tilde{D}^{a_1 \cdots a_n}_{\mu_1 \cdots \mu_n}(\vec{q}_1 \cdots \vec{q}_n)_{HT} \stackrel{\beta \to 0}{\longrightarrow} \tilde{D}^{a_1 \cdots a_n}_{i_1 \cdots i_n}(\vec{q}_1 \cdots \vec{q}_n)^M_{HT} \, ,$$

and

$$Z[j, j^{a0} = 0]_{HT} \stackrel{\beta \to 0}{\longrightarrow} Z_M[j_M]_{HT} \, . \tag{10}$$

2.5 Discussion

Let us go from the euclidean three-dimensional space to the Minkowski $(1+2)$ one (that is to one time dimension plus two spatial ones). Then the direct continuation of $Z_M[j_M]_{HT}$ so resulting describes the minkowskian Yang-Mills theory in $1+2$ dimensions (the YM_{1+2}, in short), with large coupling constant $g\beta^{-1/2}$. But the YM_{1+2} theory is a confining one, that is, all its physical states are colour singlets and there are no coloured states (like single gluons) in it [23]. Consequently $Z_M[j_M]_{HT}$ also describes a confining theory. It also follows that the plasma of magnetic gluons in the regime of high temperature and large distances cannot reduce to an ideal gas of free (or almost free) massless gluons (interactions at large distances among magnetic gluons are not small!).

This conclusion contradicts the statement appearing quite frequently in the literature that a very hot gluon plasma becomes identical to a gas of almost non-interacting massless gluons for short and large distances. However some authors have made different statements as we shall discuss in section 4.2.

A posteriori we shall justify the neglection of electric gluons in the regime under consideration through the following qualitative argument. Electric gluons acquire, due to Debye screening, masses of order $g\beta^{-1}$ and, hence they are concentrated in small regions of size $g^{-1}\beta$, which is much smaller than $|\vec{q}|^{-1}$, $\vec{q}$ being an external three momentum in any $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots\vec{q}_n)_{HT}$. On the other hand, in perturbation theory magnetic gluons are not screened and, so, they extend and oscillate over large distances with any wavelength of order $|\vec{q}|^{-1} \gg g^{-1}\beta,\ \beta$. Then it is reasonable to neglect the influence of very small objects, like the electric gluons, upon extended structures, like the magnetic ones. The latter interact among themselves and give rise, non-perturbatively, to colourless magnetic glueballs of the YM_3 theory, which are also small structures of size $g^{-1}\beta$. No contradiction of principle arises in this picture, since the small-size colour singlet magnetic glueballs are not expected to overlap spatially with the tiny electric gluons. Thus, in this picture, the electric gluons may behave as deconfined, while the magnetic ones are absolutely confined inside small colourless magnetic glueballs.

3 Other possible contributions and mechanisms

3.1 On the possible quartic coupling of electric gluons

It has been proposed[10],[24] that a direct quartic coupling of electric gluons of the form $\frac{\lambda\beta}{2}\int d^3\vec{x}(A^{a0}(0)A^{a0}(0))^2 \equiv S_{HT,6}$ should be added to S_{HT}. Here λ is a dimensionless, finite and β-independent number of order g^4. We state that $S_{HT,6}$ should not be added to S_{HT} for the sake of consistency. In fact, such a direct coupling of electric gluons does not appear at any perturbative order in the systematic analysis which establishes eqs. 9 and 10 (*cf.* ref.[11]). Actually, $Z[j]_{HT}$ keeps track of all dominant contributions to all diagrams generated by $Z[J]$, but the former neglects all subdominant terms, consistently.

An independent reason for neglecting $S_{HT,6}$ completely, is that it gives rise, in any perturbative order, to contributions which are subdominant compared to those generated by S^M_{HT} (and their coupling to electric gluons) in the regime of high temperature and very large distances ($\beta|\vec{q}| \ll g$ and $\beta|\vec{q}| \ll 1$). For instance to order g^4, the four electric gluon amplitude due to $S_{HT,6}$ (*cf.* fig.1a) is $\lambda(\approx g^4)$ while that due to S^M_{HT} and $S_{HT,4}$ (with one loop of magnetic gluons (see fig. 1b) is seen to be of order $g^4/\beta|\vec{q}|$, $\vec{q}$ being a typical momentum of one of the four external electric gluons. Notice that $g^4/\beta|\vec{q}| \gg \lambda$ since $\beta|\vec{q}| \ll 1$.

Similarly, we compare the two diagrams of order g^8 with external electric gluons having three momenta of order $|\vec{q}|$ ($\ll \beta^{-1}$) displayed in figs 2a and 2b. First we consider fig. 2a: The two internal electric gluons in the loop can either be massless or have mass $m_{E,2} \approx g\beta^{-1}$. Then the magnitude of such a diagram is of order $\lambda^2/\beta|\vec{q}| \approx g^8/\beta|\vec{q}|$ or $\lambda^2/\beta m_{E,2} \approx g^8/g$, respectively. On the other hand, the diagram in fig. 2b is of order $g^8/(\beta|\vec{q}|)^3$ and, hence, it clearly dominates over fig. 2a with either massless or massive external electric gluons when $\beta|\vec{q}| \ll 1$ and $\beta|\vec{q}| \ll g$. Similar conclusions are obtained if the two external gluons in figs. 2a and 2b are both magnetic instead of electric and so on for other diagrams. Thus $S_{HT,6}$ should be discarded as it produces subdominant contributions and when $j^{a0} = 0$, we are left back at $Z_M[j_M]_{HT}$, that is, with extreme dimensional reduction.

3.2 On a possible Higgs mechanism

It has also been argued[24] that a Higgs mechanism could be operating for the new generating functional $Z'[j]_{HT}$ which is obtained when $S_{HT,6}$ is added to S_{HT}. Our analysis leads to discard this possibility as rather unlikely for the following independent reasons:
i) $S_{HT,6}$ should not be added to S_{HT} as explained above, and
ii) $m^2_{E,fin} - m^2_{E,div}$ can be decomposed into a completely finite part plus an (infrared-finite) ultraviolet divergent mass renormalization counterterm: the finite part is positive, at least for suitable small g (as in such a case it is dominated by the one loop contribution which is positive).

4 Dimensionally reduced theories

4.1 Renormalization group analysis

We shall present what we believe is a correct renormalization group analysis of the gluon plasma at high temperature and large distances, by incorporating dimensional reduction. First we shall consider the case of extreme dimensional reduction.

As commented before, a key property of $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots\vec{q}_n)^M_{HT}$ is that it is independent on any (finite) momentum scale μ induced by renormalization. Then

$$\frac{\partial}{\partial\mu}\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(q_1\cdots q_n)^M_{HT} \;=\; 0. \tag{11}$$

This fact follows directly by looking at $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots\vec{q}_n)^M_{HT}$ from the standpoint of the superrenormalizable YM_3 theory. On the other hand since there is a proper renormalization group equation at finite temperature for $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(Q_1\cdots Q_n;\mu)$ (*i.e.* eq. 8), one may ask whether there is also a correct and non-trivial one (leaving aside eq. 11), which holds in the regime of high temperature and large distances, that is, for $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots\vec{q}_n)^M_{HT}$.

In other words, do the operations yielding eq. 8 commute with taking the limit of high temperature and large distances?. We shall anticipate the answer: there are no non-trivial renormalization group equations for $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots\vec{q}_n)^M_{HT}$ and the only renormalization group equation is eq. 11, which is trivial. This also follows from the fact that the reduced YM_3 is superrenormalizable and has no non-trivial renormalization constants, (that is, in YM_3 all of them are equal to unity). For convenience, we shall now give independent short justification of both eq. 11 and the absence of non-trivial renormalization group equations, succesively.

First we shall apply some useful techniques[11] in order to arrive at eq. 11. For a generic loop contributing to $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}((0\vec{q}_1)\cdots(0\vec{q}_n);\epsilon)_0$ one has a typical boson series like

$$\frac{1}{\beta}\sum_{n=-\infty}^{+\infty} f(\omega = i\omega(n)) ,$$

where f is some suitable function associated to the loop and $\omega(n) = \beta^{-1}2\pi n,\ n = 0, \pm 1, \pm 2, \cdots$. As it is well known [1], (33) can be replaced by ($\epsilon \to 0^+$)

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi i} f(\omega) + [\int_{-i\infty+\epsilon}^{+i\infty+\epsilon} \frac{d\omega}{2\pi i} \frac{f(\omega)}{exp(\beta\omega) - 1} + \int_{-i\infty-\epsilon}^{+i\infty-\epsilon} \frac{d\omega}{2\pi i} \frac{f(\omega)}{exp(-\beta\omega) - 1}] . \qquad (12)$$

We make systematic use of this replacement in the loop contributions to $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}((0\vec{q}_1)\cdots(0\vec{q}_n);\epsilon)_0$ and using eq. 4 we consider $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}((0\vec{q}_1)\cdots(0\vec{q}_n);\mu)$. We carry out the elimination of ultraviolet divergences at fixed $\beta^{-1} \neq 0$ by using the zero-temperature renormalization constant Z_D in eq. 4.

Ommiting indices and arguments the connected and renormalized correlation function 4 reads

$$\tilde{D} = \tilde{D}_1 + \tilde{D}_2 ,$$

where $\tilde{D}_1$ contains the largest possible number of square brackets like the one in eq. 12, while $\tilde{D}_2$ is the remainder. Moreover, all dependences in μ induced by renormalization are contained in $\tilde{D}_2$ ($\tilde{D}_2$ contains all remnants of renormalization). Next, one goes to the regime of high temperature and large distancs. There one finds that:
i)$\tilde{D}_2$ is subdominant and, hence, fully negligible compared to $\tilde{D}_1$ by powers of $\beta|\vec{q}_j| \ll 1$ ($\vec{q}_j$ being a generic external three momenta).
ii)$\tilde{D}_1$, which is μ-independent, approaches $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots\vec{q}_n)^M_{HT}$. This provides the alternative justification of eq. 11.

To justify the absence of non-trivial renormalization group equations, we consider the left-most hand side of eq. 6 and go to the regime under consideration (*ld* meaning *large distances*)

$$\lim_{\beta_{ld}\to 0} \mu\frac{\partial}{\partial\mu}|_{\alpha_0,\epsilon fixed} \tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}((0\vec{q}_1)\cdots(0\vec{q}_n);\mu) =$$

$$= \mu\frac{\partial}{\partial\mu}|_{\alpha_0,\epsilon fixed} \lim_{\beta_{ld}\to 0} \tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}((0\vec{q}_1)\cdots(0\vec{q}_n);\mu)$$

$$= \mu\frac{\partial}{\partial\mu}|_{\alpha_0,\epsilon fixed} \tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots\vec{q}_n)^M_{HT} .$$

In order to proceed further we would need an expression for $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots q_n)^M_{HT}$ in terms of an unrenormalized correlation function in the actual regime, say, $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots q_n;\epsilon)^M_{HT,0}$ which would be certainly μ-independent but ϵ-dependent times some non-trivial renormalization constant $Z[\alpha,\epsilon]_{HT}$ (which needs not coincide with $Z_D[\alpha,\epsilon]$). This would enable to carry through the actual analogue of the middle and right hand side steps in eqs. 6- 7 and, so, to derive a non-trivial renormalization group equation. But, in the actual regime, $Z[\alpha,\epsilon]_{HT}=1$ and $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots\vec{q}_n,\epsilon)^M_{HT,0}$ is ϵ-independent and coincides with $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots q_n)^M_{HT}$ (which is μ-independent and there is no other possibility!).

In fact, our previous justification of eq. 11, also displays that $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots\vec{q}_n)^M_{HT}$, which is the limit of the ultraviolet finite function $\tilde{D}_1$, is also ultraviolet finite and requires no renormalization counterterms. Consequently any conceivable renormalization constant $Z[\alpha,\epsilon]_{HT}$ for $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots\vec{q}_n)^M_{HT}$ has to reduce to unity.

Since the actual analogue of eq. 4 does not exist in a non-trivial way, the analogue of the steps in the middle and right-hand-side of eqs. 6- 7 cannot be carried out and the attempted derivation of a non-trivial renormalization group equation in the actual regime fails. The only correct possibility is the trivial one, namely, eq. 11. Our previous renormalization group analysis can be readily extended to $\tilde{D}^{a_1\cdots a_n}_{\nu_1\cdots\nu_n}(\vec{q}_1\cdots\vec{q}_n)_{HT}$. The conclusion is similar: The only renormalization group equation is the trivial one, namely

$$\frac{\partial}{\partial\mu}\tilde{D}^{a_1\cdots a_n}_{\nu_1\cdots\nu_n}(\vec{q}_1\cdots\vec{q}_n)_{HT} \;=\; 0\;.$$

4.2 Discussion

A)

The previous arguments imply that the equation which would result from replacing $\tilde{D}^{a_1\cdots a_n}_{\mu_1\cdots\mu_n}(Q_1\cdots Q_n;\mu)$ by $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots\vec{q}_n)_{HT}$ in eq. 8 and using eq. 11 is incorrect. Such an equation should also be discarded on the basis of the following arguments.
i)Would it hold, then the $\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots q_n)^M_{HT}$ extracted from the three-dimensional theory (namely YM_3) would provide, via such an (would-be) equation, information on the renormalization group functions β_{rg}, γ_{rg}, δ_{rg}, pertaining to the four-dimensional theory, which is unacceptable.
ii)It would imply, through the use of all steps in eqs. 6- 7, that
$Z_D^{-1}[\alpha,\epsilon]\tilde{D}^{a_1\cdots a_n}_{i_1\cdots i_n}(\vec{q}_1\cdots q_n)^M_{HT}$ is independent on μ when eq. 5 is used. The simultaneous validity of such an statement and eq. 4, with the same Z_D, cannot be expected to hold. This new unacceptable conclusion also shows *a posteriori* that not all steps in the actual analogue of eqs. 6- 7 are valid.

B)

One may ask whether eq. 8 at finite temperature and the known low order perturbative results for β_{rg}, *etc.* can be used reliably in order to obtain further information. The answer should not be expected to be affirmative in general. In fact let us integrate eq. 8 at fixed finite temperature using the standard method[22], which yields

$$\tilde{D}^{a_1\cdots a_n}_{\nu_1\cdots\nu_n}[exp(tQ_1)\cdots exp(tQ_n);\mu\alpha,\xi] =$$
$$\tilde{D}^{a_1\cdots a_n}_{\nu_1\cdots\nu_n}[Q_1\cdots Q_n;\mu\alpha(t),\xi(t)]exp\{-\textstyle\int_0^t dt'\gamma_{rg}[\alpha(t'),\xi(t')]\}\;,$$

where $\alpha(t)$ and $\xi(t)$ are the usual running coupling constants and gauge parameter ($\alpha(t=0)=\alpha$, *etc.*).

By using the lowest order perturbative results for β_{rg}[22], one finds ($\beta_1 = -\frac{11}{6}N$):

$$t \simeq \frac{\pi}{-\beta_1}[\frac{1}{\alpha(t)} - \frac{1}{\alpha}] \; , \tag{13}$$

with the understanding that both α and $\alpha(t)$ are small and positive and, moreover, $\alpha(t) \ll \alpha$. If the approximation 13 is to be reliable in the regime of high temperature and large distances, then for a typical three momentum $\vec{q}$ both

$$|\vec{q}| \ll \beta^{-1} \; , \tag{14}$$

and

$$|\vec{q}| exp\{\frac{\pi}{-\beta_1}[\frac{1}{\alpha(t)} - \frac{1}{\alpha}]\} \ll \beta^{-1} \; , \tag{15}$$

should hold simultaneously. Here lies the rub! For fixed small β and any small $|\vec{q}|$ fulfilling eq. 14, eq. 15 is to be violated as $\alpha(t) \to 0$ in general (except, perhaps, for some finite range, at best) precisely due to asymptotic freedom ($\beta_1 < 0$). Notice that the inclusion of $|\vec{q}| \ll \beta^{-1}g = \beta^{-1}(4\pi\alpha)^{1/2}$ in the above analysis does not improve the situation.

C)

In order to support our claims regarding the correct renormalization group analysis we shall collect here various statements by different authors, which appear scatterered in the literature.

1) Kislinger and Morley[25] deduced that asymptotic freedom holds for the gluon plasma provided that the temperature and the momenta be large.

2) Polyakov[26] provided, through a qualitative argument, the following formula:

$$g^2(T) = [\; c\ln(\frac{2\pi T}{m_H})\;]^{-1}, \;\; T = \beta^{-1} \; , \tag{16}$$

c and m_H being proportional to the constants $-\beta_1$ (characterizing asymptotic freedom) and a typical hadronic mass, respectively. He also pointed out that this formula is correct only if $T \to m_H$; otherwise, $g^2(T)$ becomes independent of T and equal to the strong interaction coupling constant. Other authors do use formulae essentially identical to eq. 16 but they do not justify them.

For the reasons presented in this work, we pose serious objections to the use of such formula at high temperature in the large distance regime.

D)

One may naturally ask whether the renormalization group functions β_{rg}, *etc.* could be made β-dependent somehow, and about the consistency of such new variations with extreme dimensional reduction to YM_3, discussed previously.

It is possible to consider, instead of eq. 4, the renormalized correlation function

$$\tilde{D}'^{a_1\cdots a_n}_{\nu_1\cdots\nu_n}(Q_1\cdots Q_n;\mu) = Z'_D[\alpha,\epsilon,\beta]\tilde{D}^{a_1\cdots a_n}_{\nu_1\cdots\nu_n}(Q_1\cdots Q_n;\epsilon)_0 \; ,$$

with

$$Z'_D[\alpha, \epsilon, \beta] = Z_D[\alpha, \epsilon] + \delta Z'_D[\mu, \beta] ,$$

and

$$\alpha = \alpha_0 \mu^{2\epsilon}[Z_\alpha(\alpha, \epsilon) + \delta Z_\alpha(\mu, \beta)]^{-1} .$$

Here $\tilde{D}^{a_1 \cdots a_n}_{\nu_1 \cdots \nu_n}(Q_1 \cdots Q_n; \epsilon)_0$ and $Z_D[\alpha, \epsilon]$, $Z_\alpha[\alpha, \epsilon]$ are the same as in eq. 4, while the δZ's are some ultraviolet finite (ϵ-independent!) and temperature-dependent contributions, to be chosen at will, in principle. Consequently, $\tilde{D}'^{a_1 \cdots a_n}_{\nu_1 \cdots \nu_n}(Q_1 \cdots Q_n; \mu)$ satisfies a new renormalization group equation at finite temperature analogous to eq. 8 with new functions β'_{rg}, *etc.*

However, since the difference between $\tilde{D}'$ and $\tilde{D}$ are induced by the finite functions, they would amount to modify $\tilde{D}_2$. Then, in the high temperature and the large distance regime (with $\beta\mu \ll 1$), one should have that $\tilde{D}'$ and $\tilde{D}$ coincide and, for $\nu_h = i_h$, $h = 1, \cdots n$, they become equal to $\tilde{D}^{a_1 \cdots a_n}_{i_1 \cdots i_n}(\vec{q}_1 \cdots \vec{q}_n)^M_{HT}$ with the trivial eq. 11.

5 Connection to real-time formalism

In general the connection between the real and imaginary formalisms proceeds through the well known analytic continuation[27]. It turns out that such continuation also provides, in the regime of high temperature and large distances, direct relationships. In what follows, we shall consider only the real-time formalism presented in ref. [28], which will also be used in the next section.

Let $\tilde{G}^{ab}_{hj}(\omega, \vec{q})_{HT}$, $h, j = 1, 2, 3$, be the corresponding real-time Green function in the regime of high temperature and large distances ($\beta|\vec{q}| \ll 1$, $\beta|\vec{q}| \ll g$, $\beta\mu \ll 1$) for any real frequency ω, in Feynman gauge ($\xi = 1$). Then we claim that in such regime one has

$$\tilde{D}^{ab}_{hj}(\vec{q})^M_{HT} \sim -\beta \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \tilde{G}^{ab}_{hj}(\omega, \vec{q})_{HT} , \tag{17}$$

which will be used in a qualitative way in the next section.

We shall outline several arguments in order to justify eq. 17.

a) For the Schwinger model at finite temperature one can show that a relation similar to eq. 17 holds for the imaginary-time correlation function and the real-time Green function of the massive photon at high temperature[29].

b) For the pure and interacting spinless boson (ϕ^4, say) theory at finite temperature, one may consider the general representations for the imaginary-time two-point correlation function $\tilde{D}(\omega(r), \vec{q})$ and the real-time Green function $\tilde{G}(\omega, \vec{q})$ as given in appendix $A.1$ of ref. [28], namely

$$\tilde{D}(\omega(r), \vec{q}) = \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \frac{\rho(\omega', \vec{q})}{[i\omega(r) - \omega']} , \tag{18}$$

and

$$\tilde{G}(\omega',\vec{q}) = i\int_{-\infty}^{+\infty}\frac{d\omega''}{2\pi}\frac{\rho(\omega'',\vec{q})}{[\omega'-\omega''+i\epsilon]} + \frac{\rho(\omega',\vec{q})}{exp(\beta\omega')-1} \ ,$$

$\rho(\omega',\vec{q})$ being a suitable spectral function. In the regime of high temperature and large distances, one sees that the second term on the right hand side of eq. 18 dominates and then

$$-\beta\int_{-\infty}^{+\infty}\frac{d\omega'}{2\pi}\tilde{G}(\omega',\vec{q}) \sim -\int_{-\infty}^{+\infty}\frac{d\omega'}{2\pi}\frac{\rho(\omega',\vec{q})}{\omega'} \ ,$$

which, in turn, equals $\tilde{D}(\omega(r)=0,\vec{q})$. In particular, it is easy to check this relationship for the free case, where $\rho(\omega,\vec{q}) = 2\pi\epsilon(\omega)\delta(\omega^2-\vec{q}^2-m^2)$ (ϵ being the sign function).

c) Ref. [30] has established the existence of a well defined density matrix, namely, $exp(-\pi Q_c)exp(-\beta H)$ for the quantized non-abelian gluon plasma in the thermodynamical equilibrium at temperature β^{-1}. Here, Q_c is the ghost charge and H is the hamiltonian and it can be shown that both commute. The above density matrix plays for the Yang-Mills field a similar role as $exp(-\beta H)$ does for the spinless boson case. On the other hand, we recall that the free propagator of a free coloured vector boson in Feynman gauge is just $\delta^{ab}g_{\mu\nu}(\omega^2-\vec{q}^2)$. Then the analysis given in appendix $A.1$ of Dolan and Jackiw[28] and the previous arguments yielding eq. 17 for the spinless boson case can be carried over without difficulty to the actual Yang-Mills situation, these lead to eq. 17.

6 Stability of the gluon plasma in real time

We now turn to a study of the stability of the gluon plasma in the high temperature and large distance limit using the real-time approach of Dolan and Jackiw[28]. The study of the collective behaviour of the gluon plasma involves the use of linear response theory. Characterizing the medium by a response function $\tilde{\chi}^{\mu\nu}_{ab}(\omega,\vec{q})$ one can find the plasma's response (induced current δj^{μ}_{a}) to an external disturbance (applied field $A^{b}_{\nu E}$) by

$$\delta j^{\mu}_{a}(x) = \int d^4x'\chi^{\mu\nu}_{ab}(x-x')A^{b}_{\nu E}(x') \ .$$

The real part of $\tilde{\chi}(\omega,\vec{q})$ contains information about the resonant frequencies of the plasma and the imaginary part is related to the attenuation or amplification of waves propagating through the medium.

Since the response function is connected to the polarization of the medium $\tilde{\Pi}$ (*cf.* next equation below), it is also ultimately connected to the two-point gluon Green function (since $\tilde{\Pi}^{ab}_{\mu\nu} = (\tilde{G}^{-1})^{ab}_{\mu\nu} - (\tilde{G}_0^{-1})^{ab}_{\mu\nu}$, where $\tilde{G}^{ab}_{0\mu\nu}$ is the bare gluon propagator). Therefore, any possible confining feature contained in $\tilde{G}^{ab}_{\mu\nu}$ should also, presumably, be present in $\tilde{\chi}^{ab}_{\mu\nu}$ in the appropriate limits. To test this statement we study the stability of the gluon plasma in the high temperature and large distances limit. And in view of the connection between real and imaginary-time formalisms, eq. 17, we choose to do it in the real-time approach to provide an alternative way to study the gluon plasma in that regime.

$\tilde{\chi}$ and $\tilde{\Pi}$ are connected via the fluctuation-dissipation theorem[21]. To study the stability of the plasma we need only to consider the imaginary part of $\tilde{\chi}$. The longitudinal and transverse components of the imaginary part of the response function are given by

$$Im\tilde{\chi}_L^{ab} = -\frac{1}{\omega^2 - \vec{q}^2} \tanh(\beta\omega/2) Im\tilde{\Pi}_{00}^{ab} \ ,$$

and

$$Im\tilde{\chi}_T^{ab} = -\frac{1}{2}\frac{1}{\omega^2 - \vec{q}^2} \tanh(\beta\omega/2)(\frac{\omega^2}{\vec{q}^2} Im\tilde{\Pi}_{00}^{ab} - Im\tilde{\Pi}_{ii}^{ab}).$$

Since at large distances ($|\vec{q}| \to 0$) $\tilde{\Pi}_{00}^{ab}$ and $\tilde{\Pi}_{ii}^{ab}$ are related by

$$\tilde{\Pi}_{00}^{ab}(\omega, |\vec{q}| \to 0) = a\frac{|\vec{q}|^2}{\omega^2} \tilde{\Pi}_{ii}^{ab}(\omega, |\vec{q}| \to 0) \ ,$$

with $0 < a < 1$ [21], we have for the transverse component in this limit

$$Im\tilde{\chi}_T^{ab} = \frac{1}{2}\frac{1}{\omega^2 - \vec{q}^2} \tanh(\beta\omega/2)(\frac{\omega^2}{\vec{q}^2}\frac{1-a}{a} Im\tilde{\Pi}_{00}^{ab}) \ ,$$

i.e. both $Im\tilde{\chi}_L^{ab}$ and $Im\tilde{\chi}_T^{ab}$ are determined in this limit by $Im\tilde{\Pi}_{00}^{ab}$.

In the real-time approach[28] $\tilde{\Pi}$ can be separated into a temperature dependent and independent parts. In Feynman gauge, a detailed calculation of $\tilde{\Pi}_{00}^{ab}$ to order g^2 (corresponding to the diagrams of fig. 3) has been carried out[13],[14]. The temperature dependent part of $Im\tilde{\Pi}_{00}^{ab}$ was obtained to be

$$\begin{aligned} Im\tilde{\Pi}_{00}^{ab}(\omega, \vec{q}) = \ & \delta^{ab}\tfrac{Ng^2}{16\pi|\vec{q}|}\textstyle\int_0^\infty dp[(4p^2 + 4\omega p + \omega^2 - \vec{q}^2)(n_p + n_{p+\omega} + 2n_p n_{p+\omega})(\Theta_+ + \Theta_-)] \\ & + [\omega \to -\omega] \ , \end{aligned}$$

where N denotes the number of colours, $n_p = (e^{\beta p} - 1)^{-1}$ and the step functions are $\Theta_\pm = \Theta(-1 \pm 2|\vec{q}|p/(\omega^2 - \vec{q}^2 + 2\omega p))$. Upon taking the limit $|\vec{q}| \to 0$, we find ($\beta|\vec{q}| \ll 1$) that

$$\begin{aligned} Im\tilde{\Pi}_{00}^{ab}(\omega, |\vec{q}| \to 0) \ &> \ 0 \ for \ \omega < |\vec{q}| \\ &< \ 0 \ for \ \omega > |\vec{q}| \ , \end{aligned}$$

and consequently, as $|\vec{q}| \to 0$, $Im\tilde{\chi}_L^{ab}(\omega, \vec{q}) > 0$ in both regions and $Im\tilde{\chi}_T^{ab}(\omega, \vec{q}) < 0$ also in both regions. Inclusion of the temperature independent contribution does not, in this limit, affect these results as the temperature dependent part grows with larger values of the temperature and becomes dominant. As the sign of the response function is usually connected with either attenuation of gluonic waves (for positive values) or instabilities (for negative values), we see that in the limit of high temperature and large distances transverse coloured disturbances appear to be unstable and then to have the effect of making the gluon plasma collapse into another (perhaps confining) state.

A similar instability has also been seen in studies of the quark-gluon plasma and in some limits of the pure gluon plasma. Performing a numerical evaluation of an iterated sum of the ring diagrams of fig. 3 plus the quark loop, Authors of ref. [14] found a negative sign for $Im\tilde{\chi}_T$ and $Im\tilde{\chi}_L$ in several regions of the $\omega - |\vec{q}|$ plane. In a subsequent, and more refined, work[15][16] a gauge independent plasma damping constant was calculated, to lowest order in the long-wavelenght limit, and was determined to be negative definite. A later work[17] modified that result quantitatively but left the sign unchanged. A further confirmation of the sign, at the lowest order, came from the elegant gauge invariant linear response calculation presented in ref. [18].

The general connection 17 between the real- and imaginary-time formalisms suggests an, at least qualitative, interpretation of the above transverse instability. In fact if gluons do not behave as free in the regime of high temperature and large distances their real-time Green functions ($\tilde{G}$'s) should display some sort of instability which would match somehow the confining behaviour of the static imaginary-time correlation functions ($\tilde{D}^M_{HT}$), through eq. 17.

7 Conclusions

General properties of the pure gluon plasma at high temperatures and large distances have been investigated. It appears to be described, to all perturbative orders, by the pure Yang-Mills (YM_3) theory for magnetic gluons in three spatial dimensions, which is confining. Arguments are given which lead to discard a direct quartic coupling of electric gluons and related Higgs-like mechanisms. In such regime: i) there are no free gluons (but possible hadronic-like excitations[12]), and ii) there is no non-trivial renormalization group equation (and, hence, asymptotic freedom cannot be invoked to derive almost free behaviour at large distances). Results for the real-time gluon propagator in the plasma to one loop order are given, which imply the existence of instabilities of the gluon plasma (in agreement with other calculations). It is argued that such instabilities just reflect the confining behaviour of YM_3.

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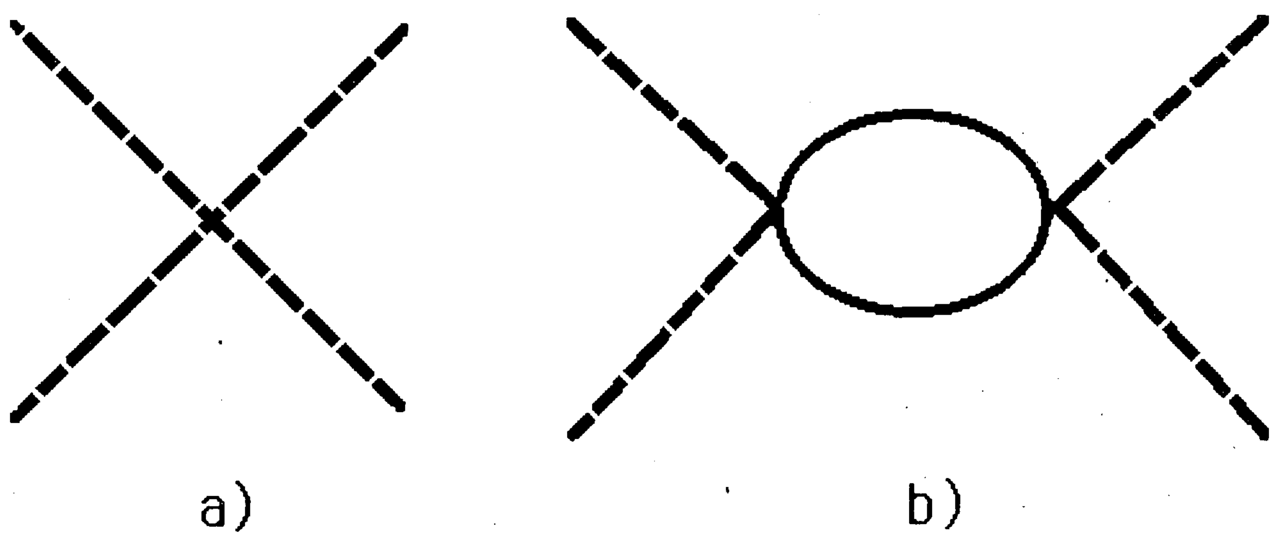

Figure 1

Dotted and continous lines represent electric and magnetic gluons, respectively, in the imaginary time formalism.

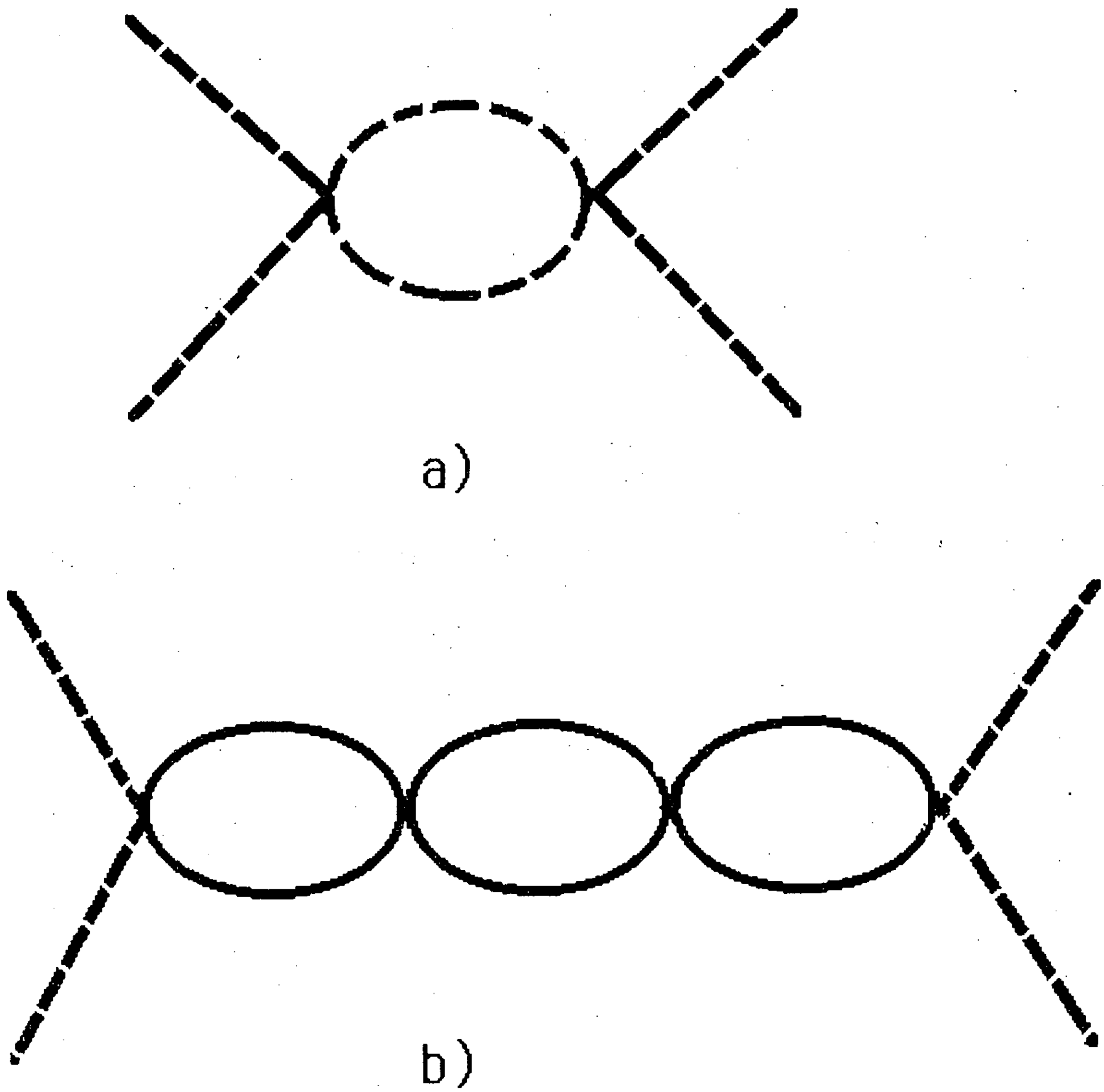

Figure 2

The conventions are the same as in fig. 1.

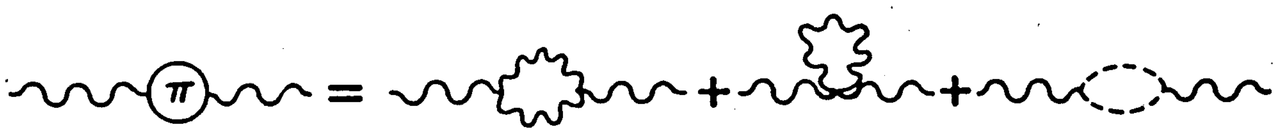

Figure 3:

In the real-time formalism, wavy lines and dotted lines represent gluons (electric and magnetic) and ghosts, respectively.

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