A generative model for feedback networks

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Outline

1. Motivation
   - An example
2. Model
3. Results
   - Network properties
   - Simulations

Cycle formation in growing network

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- `business network` (seeking for not too similar business partners)

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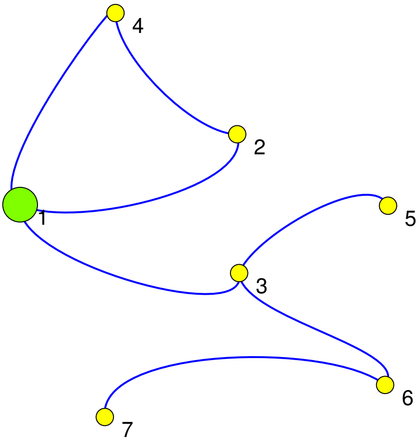
An example

Creating a strategic alliance in business 3 links away.



A company which wants to make a strategic alliance.

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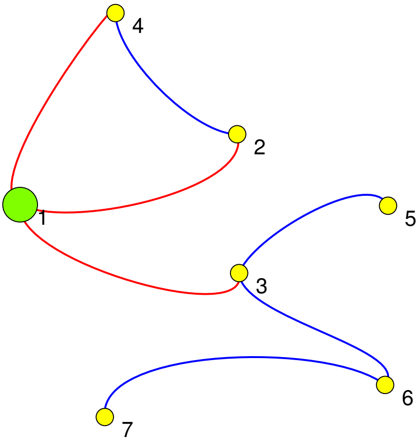

Possible paths on the way. First two from the top do not lead to a successful alliance. The company chooses the link to company 3.

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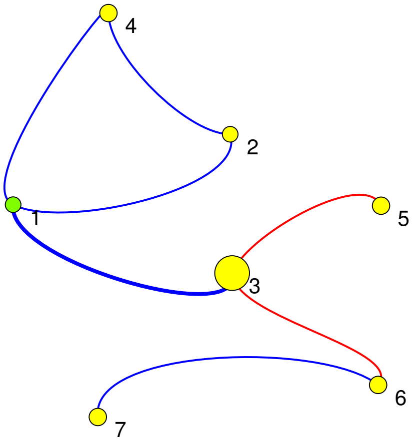

Step 1: $1 \rightarrow 3$

Company 3 can choose between two possible paths. The top one does not lead to a successful alliance. It chooses the link to company 6.

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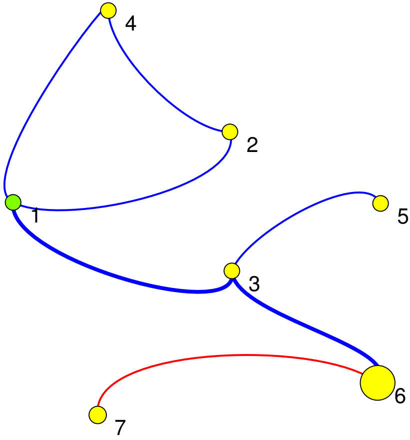

Step 2: $3 \rightarrow 6$

From company 6 there is only one way to choose the next company (company 7).

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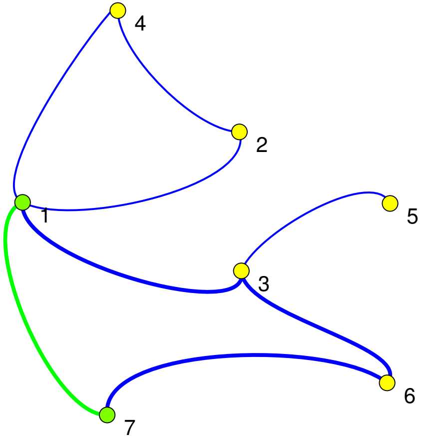

Step 3: 6 → 7

The path with 3 consecutive links was found. Alliance is created from company 1 to company 7.

Previous work

- lots of work on generative models for graphs (preferential attachment model of Albert and Barabási (1999), copying model of Kumar et al. (2000)); do not create cyclic networks
- social networks model of Newman (2003); not an evolving network model
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Growth of a model (1)

with 3 parameters: α, β, γ

At each time step

- select a `starting node` *i* according to probability

$$P_\alpha(i) = \frac{[deg(i)]^\alpha}{\sum_{m=1}^{N}[deg(m)]^\alpha}$$

- assign of search `distance` *d* according to probability

$$P_\beta(d) = \frac{d^{-\beta}}{\sum_{m=1}^{\infty} m^{-\beta}}$$

- generate a search path (selection of the `following nodes` (*l*s) on the path)

$$P_\gamma(l) = \frac{\left[1 + u(l)^\gamma\right]}{\sum_{m=1}^{M}\left[1 + u(m)^\gamma\right]}$$

$u(x) \equiv$ unused degree of x

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If the search path

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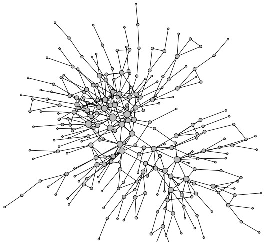
Initial condition (asympototically not important): 1 node.

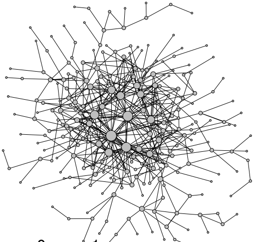
Outline

Representations of network models

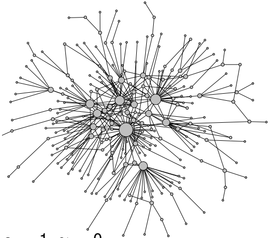
with 250 nodes, $\beta = 1.3$



$\alpha = 0, \gamma = 0$



$\alpha = 0, \gamma = 1$



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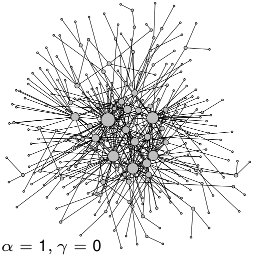

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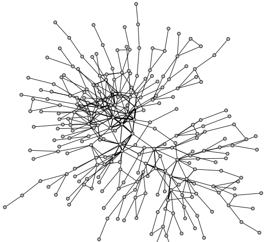

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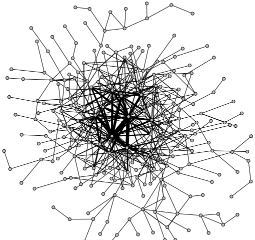

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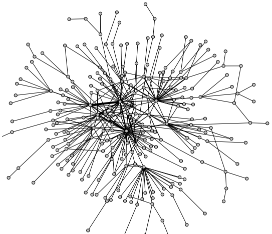

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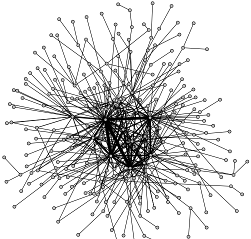

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Role of parameters in network evolution.

- $\alpha \ldots$ the `attachment` parameter describes forming hubs (highly connected nodes)
- $\beta \ldots$ the `distance decay` parameter accounts for density of the network
- $\gamma \ldots$ the `routing` parameter increases search – more cycle formations, it accounts for more interconnected network

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Network evolution depends on local information, but cycle formation depends on global properties of the network:

- successful search `decreases` mean distance of a node to other nodes
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Simulations

The assumption

Successful searches and adding nodes influence the frequency of one another $\longrightarrow$ long-range interactions among nodes.
We simulated the networks to check whether the degree (k) distributions can be described of the form (generalized q-exponential function)

$$p(k) = p_0 k^{\delta} e_q^{-k/\kappa}$$

where the q-exponential (Tsallis, 1988) function e_q^x is defined as

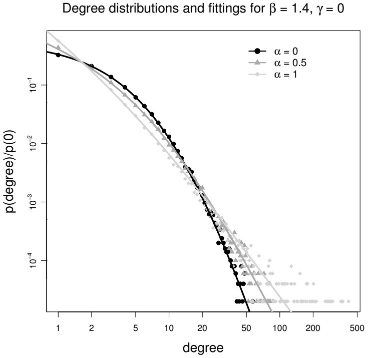
$$e_q^x \equiv \left[1 + (1-q)x\right]^{1/(1-q)} \qquad (e_1^x = e^x)$$

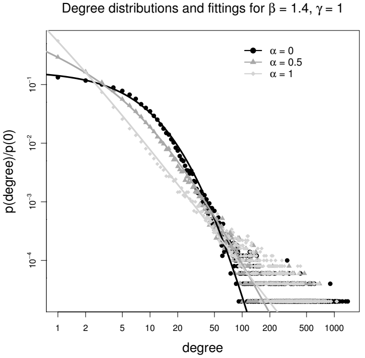
if $1 + (1-q)x > 0$, and zero otherwise.

Simulations
The procedure

- simulate 10 realizations of networks with 5000 nodes
- different parameters α, β and γ
- fit generalized *q*-exponential function to simulated distributions using `Gauss-Newton algorithm for nonlinear least-squares estimates` (some tail regions had to be manually corrected)
- get the fitted the parameters (q, κ and δ)

Simulations
Some results





Simulations
Goodness of fit tests

In order to test the q-exponential fits we used two nonparametric statistical tests

- `Kolmogorov-Smirnov test` (since q-exponential is defined on $[0, \inf)$ only, we used two sample test): null hypotesis was never rejected
- `Wilcoxon rank sum test`: null hypotesis rejected in 1/12 examples

Since data are very sparse in the tail, we excluded datapoints with probability $< 10^{-4}$.

Model parameters and q-exponential

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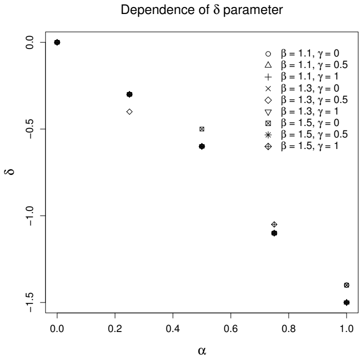

δ depends `only` on parameter α.

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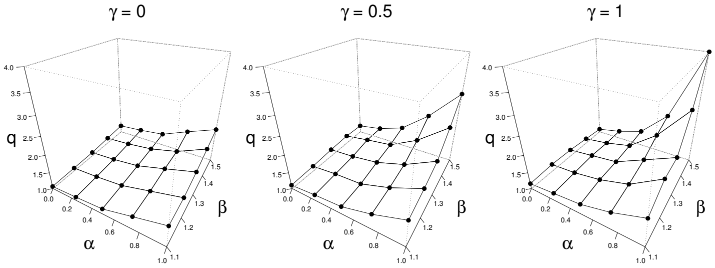
Dependence of parameter q.



Parameter q `grows rapidly` as each of the 3 model parameters increase.

Model parameters and q-exponential

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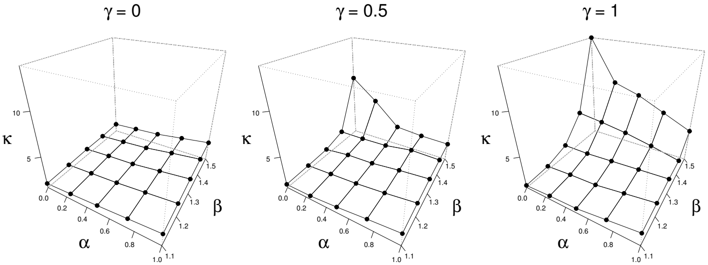
Dependence of parameter κ.



Parameter κ `diverges` when β and γ grow large and $\alpha = 0$.

Conclusion

- A generative model for creating graphs representing feedback networks was presented. Algorithm uses only local properties of the nodes.
- The simulated networks confirmed the assumption of long-range interactions in such a network (generalized q-exponential functions were fitted to empirical degree distributions).
- The competition between creating cycles (stronger feedback) and adding new nodes (growth in size).
- In the future
  - Apply the present model to real networks (biotech intercorporate networks).
  - Analyze more network model topological properties (e.g. mean distance of a node to other nodes).

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