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Perceptual Learning in Mathematics: The Algebra-Geometry Connection

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Abstract

mportant component of expertise is the rapid pickup omplex, task-relevant pattern structure, yet such skills seldom trained explicitly. We report initial results ying principles of perceptual learning to the essing of structure in mathematics, specifically the ection between graphed functions and their symbolic essions. Subjects in two experiments viewed graphs nctions and made a speeded, forced choice match from ral equations. Training consisted of many short trials is active classification task involving examples of a tion (e.g., sine) subjected to various transformations . scaling, shifting, reflection). Experiment 1 used *rastive feedback* -- the graph for a trial was shown rimposed on the canonical function to accentuate sformations. Subjects showed substantial performance s from 45 minutes of training and transferred to new nces, new function families and a new task. In eriment 2, with contrastive feedback removed, subjects ved no transfer to new functions. The results indicate value of perceptual training in producing nematical expertise and the value of contrastive back in particular.

at does it mean to attain mathematical expertise? ction most often emphasizes declarative knowledge – nd concepts. A student may learn, for example, that nction y = Sin x can be generated by a certain uction involving a triangle. Having learned about the unction, the student may be able to answer certain l questions and work out problems on a test.

re is more to expertise, however, than facts, concepts ferences. Suppose we ask the student who is familiar / = Sin x what the graph of y = Sin (x–2) would look Chances are the student will not know immediately. g the answer may be an inference process requiring l steps: We can substitute 2 for x to find that the on now crosses the x axis at (2,0) instead of (0,0). If eck a few more points, perhaps the answer would e clear.

v would an expert respond to this query? At a glance, ntuitive that the "-2" in the function y = Sin (x-2) the whole function rightward on the x axis by two If it were y = Sin (x+2), the shift would be to the Likewise, y = 3 Sin x amplifies the function along the ; y = Sin (4x) compresses along the x; and y = 2–Sin x causes a reflection around the x axis and shifts the whole function upward by two units. The expert detects at a glance the structural relationships in each equation and intuitively knows their meaning in the spatial representation (the graph in Cartesian coordinates).

This kind of expertise is important, and in some tasks, decisive. To be able to look at a plot of data and recognize how it could be approximated by an equation, or to visualize the consequences of changes in an equation for the shape of the function, would seem to be basic to the use of mathematics in science. The student who can work out these connections through factual knowledge and reasoning lags behind one who intuits the relevant patterns at a glance. The scientist's ability to extract relevant structures in both equations and graphs allows her attention and effort to be allocated to the scientific problem at hand -- without having to pause to work through what e^{-x} looks like. These aspects of skilled performance are our concern in this paper.

How these skills arise, in mathematics or in other domains, may appear mysterious. In the examples of sine functions above, we could specifically state to the student, in a lecture or text, all of the transformations mentioned. We might find that even after the student learned to state the facts, classification of new examples would be slow and arduous. Attaining the fluent pattern classification skills of the expert may not come from this kind of instruction. We would say that the students "need experience" and that they will attain greater fluency with time. The same is said to beginning radiologists, instrument pilots, accountants and novices in other domains about the structures and patterns they work with.

The passage of time is not very satisfactory as an explanatory notion. A specific hypothesis about the development of such skills is that they involve perceptual learning (Gibson, 1969). Broadly, perceptual learning is defined as "an increase in the ability to extract information from the environment, as a result of experience and practice with stimulation coming from it" (Gibson, 1969). Perceptual learning is a cornerstone of advanced human performance. In some domains, it leads to competence that appears nearly magical. The magic comes from processes that allow for continuing improvement in the extraction of pattern structure with practice. For example, in 1996, Garry Kasparov, the world champion chess player, defeated Deep Blue, a chess-playing computer that examined 125 million possible moves per second. In 1997, Kasparov lost a close match to an improved Deep Blue that examined approximately 250 million moves per second. How can a human, who examines a smaller number of possible moves on each turn (about 4) even begin to compete with a machine that computes all the possibilities for exactly what the board will look like many moves later? The grandmaster has developed pattern pickup skills, specifically relevant to the game of chess (Chase & Simon, 1973). These allow efficient processing of the board structures that will be relevant to the outcome of the game. Much of this knowledge is not accessible to the player. If it could be clearly articulated, the grandmaster's strategy could be implemented in a computer chess-playing program, allowing computers that look at mere thousands or even hundreds of moves to defeat the best humans.

We do not yet have good process models of perceptual learning. Evidence indicates, however, that the performance of certain acts of information processing, not the passage of time, lead to advances in perceptual learning (Gibson, 1969; Hock, 1987; Karni & Sagi, 1993; Kellman & Kaiser, 1994; Lewicki, 1992; Pick 1965; for a review see Goldstone, 1998). Impressive changes in detection and pattern classification have often been obtained, even using relatively brief training procedures in laboratory experiments. Such experiments shed light on the conditions that lead to perceptual learning, and they raise the possibility of systematizing procedures that might accelerate the development of pattern extraction skills in educational and training settings. In short, even while we lack complete process models, we know quite a bit and can learn more about how to produce perceptual learning. Expert pattern processing skills represent a component of expertise that differs from declarative knowledge and must be trained differently. In the present research, we seek to apply perceptual learning principles to mathematics and investigate how they can be optimized.

What are the ingredients for obtaining perceptual learning? Our answer is tentative, but a number of ideas have received support. In the first place, information pickup is a skill that is not much exercised by hearing a recitation of facts. Training using many short trials requiring a speeded response may optimize perceptual learning. In these trials, the learner must be exposed to a range of variation in a stimulus set that contains the invariants that support some discrimination or classification (Gibson, 1969). Often it is suggested that the learner must be actively involved in a classification task, i.e., must attend and respond on a number of trials. Where discovery of differences is primary, such trials may allow learning without feedback (Gibson, 1969), whereas for other classifications feedback may be crucial.

The Algebra – Geometry Connection

In the present research, we developed a perceptual learning module (PLM) to advance subjects' abilities to relate graphs of mathematical functions to their symbolic expressions. Obviously, connecting graphs and equations is a complex task, one that no doubt has conceptual and perceptual components.[1] We do not attempt to separate these components here. We chose the task, however, because subjects appear to be quite poor at it initially, despite having satisfactorily completed relevant coursework in mathematics. It seemed plausible that their difficulties were due in part to the limitations of traditional instruction and might be overcome by perceptual learning.

We have three ultimate goals in this research. One is to test whether a brief period of perceptual training can improve subject's performance in interpreting graphs and equations. If so, a second goal is to determine what variables are important in producing and optimizing perceptual learning. These include the type of feedback and the role of active classification. Finally, we are interested in how acquired pattern processing skills transfer to new stimuli and more complex tasks. In this paper, we report some initial findings related to all three of these goals.

Experiment 1

In Experiment 1, we tested a perceptual learning procedure for developing pattern processing skills in matching graphs to equations. The procedure incorporated a number of ingredients that may be important in accelerating perceptual classification skills. First, subjects performed an active classification task. They performed many short trials, each requiring a speeded, perceptual classification response. On these trials, a graph appeared, followed two seconds later by 3 choices of equations, one of which matched the graph. Functions spanned a range of variation appropriate for subjects to extract the invariant patterns specifying various aspects. Finally, a particular kind of feedback display --

[1] We should make clear what we mean by "perceptual." It is common to think of the senses as providing low-level, concrete, sensory information, such as color, and invoking higher cognitive processes, such as inference processes, to account for more abstract descriptions of reality. Our view of *perception* is a much more inclusive one: Perceptual mechanisms respond to abstract patterns of information and produce abstract and meaningful descriptions of reality (c.f., J. Gibson, 1966, 1979; Marr, 1982; for recent discussions see Kellman & Arterberry, 1998; Barsalou, in press). Although this is not the place to attempt to find a clear boundary between perception and conception, our working hypothesis is that any potentially detectable pattern in the stimulus is a candidate for perceptual learning. The shape of a sine function in a graph, and the difference in patterns between y = Sin x and y = Sin 2x are candidates for perceptual learning, as are aspects of the elements, positions and sequencing of symbols in the equations. On the other hand, knowledge that the sine function is derived from a certain construction involving a triangle is not potentially discernible from looking at the graph of y = Sin x; contributions of that knowledge to performance are therefore not perceptual. The reader who worries about the boundaries of perceptual learning may feel more at home with our occasional substitution of the more neutral phrase "pattern learning."

what we call *contrastive feedback* -- was used. This display showed the canonical function (e.g., y = Sin x) as a dotted line in the background, and the particular function for the trial (e.g., y 4 Sin (x+2)) in front. Contrastive feedback may highlight the particular pattern transformations relating the basic function to its variants. It is a form of augmented feedback (e.g., Lintern, 1980).

Method

Participants. Participants were 20 undergraduate students at the University of California, Los Angeles who received credit units for participation in the one-hour experiment.

Materials and Apparatus. Stimuli were designed with *Mathematica*, version 2.2.1, and consisted of graphs of four types of mathematical functions: Cosines, Logarithms, Sines, and Exponentials. They were displayed in a Power Macintosh 7100/66.

Design and Procedure. Subjects were randomly assigned to one of 2 training groups: Sines and Exponentials, or Cosines and Logarithms.

Training consisted of 8 blocks with 20 trials each. At the beginning of each block, the basic function on which the subject was to be trained was displayed on the screen (i. e., the graph and equation for y = Sin x). Then, for each trial, a graph with some transformation from the basic function was presented (i. e., Sin 3x) along with three equations.

Variations within function families were created using 6 transformations. These included:

1) X-shifting: Adding some integer to the variable x within the scope of the basic function (e.g., y = Sin (x+4))
2) Y-shifting: Adding some integer outside the scope of the basic function (e.g., y = 4+Sin x)
3) X-scaling (e.g., Sin x/4)
4) Y-scaling (e.g., y = 4 Sin x)
5) X-reflection (e.g., y = Sin (-x))
6) Y-reflection (e.g., x = - Sin x)

Many problems included combinations of these (e.g., y = 2-Sin(5x)). The materials given to the 2 training groups were matched regarding the types of transformations seen during training. However, matching of specific equations were not necessarily made. For example, if y = Sin 2x was given to one group, the other group would not necessarily be given y = Cos 2x. The second group may have received Cos 4x, for example.

The subject had to choose which of the possible answers corresponded to the graph. Responses were entered on a keyboard. A trial feedback screen reported whether the subject's answer was right or wrong. This screen also showed a display with both the tested graph (depicted with a thick blue line) and the basic one (depicted with a dotted line) superimposed, along with a label indicating the correct equation. This design for the feedback screen aimed to highlight the relevant transformations relating the tested expression and graph to the basic function type. At the end of each block, average accuracy and reaction time for the previous 20 trials was reported. The training phase of the experiment lasted 40-45 minutes.

Dependent Measures. Accuracy and reaction times for the first 10 trials of the first 2 blocks were taken as a pre-test measure, and scores for the last 10 trials of the last 2 blocks were taken as a measure of the subjects' end-of-training performance (EOT). Three kinds of posttests were administered to both groups:

1) A *Familiar Functions Posttest* (FFP) presented in the same format as training, composed of 10 *new* instances in the function families the subjects saw in training.

2) An *Unfamiliar Functions Posttest* (UFP) in the same format, consisting of 10 trials from the function families they had not seen (i. e., subjects trained on Sines and Exponentials were tested on Cosines and Logarithms).

3) A Remote Transfer Posttest (RTP) assessing transfer to a different task. In this task, subjects tried to make sense of complicated "combination" functions (such as y = - Cos (2x) * Log (-x) or y = Exp (-x/3)–5 Sin (5-x)). Eight such combination functions were generated for both studied and not studied pairs of function families. This yielded two written forms with 8 graphs and 8 functions. Subjects' task was to perform a matching test, indicating the correct equation for each graph. Total time and accuracy to complete each form was measured. Half of the subjects did one form first, and the other half did the other form first.

A control group of 20 subjects received no training and was given the same tests as in the experimental condition: the post-test with both sets of basic functions and the RTP with combination functions. In the regular training, the pretest (initial learning trials) formed a within-subjects baseline. The control group served as an additional baseline group for assessing effects in FFP, UFP, and RTP. Because subjects were not given any feedback in the control group, their performance allowed a check on possible rapid learning effects that might have elevated performance in the pretest.

Results

Training, FFP and UFP Results. Training had clear and highly reliable effects on accuracy (shown in Figure 1) and RT. Subjects improved from about 50% correct in the pretest to about 70% in the final block of training. Response times decreased about 40% during training. Accuracy at EOT and in the FFP were both higher than in the pretest and did not differ from each other. In the UFP, in which subjects saw new instances, accuracy was slightly higher than the pretest but not as high as in FFP. Accuracy in the control group was similar to the pretest and worse than the EOT and FFP. Reaction times were negatively correlated with accuracy.

These patterns were confirmed by the analyses. Accuracy and RTs were analyzed using a 2 (functions trained: sines & exponentials or cosines & logarithmics) X 4 (phase: pretest, EOT, FFP, UFP) analysis of variance (ANOVA), with repeated measures on the latter factor. There was a significant main effect of phase, for accuracy $F(3,54) =$

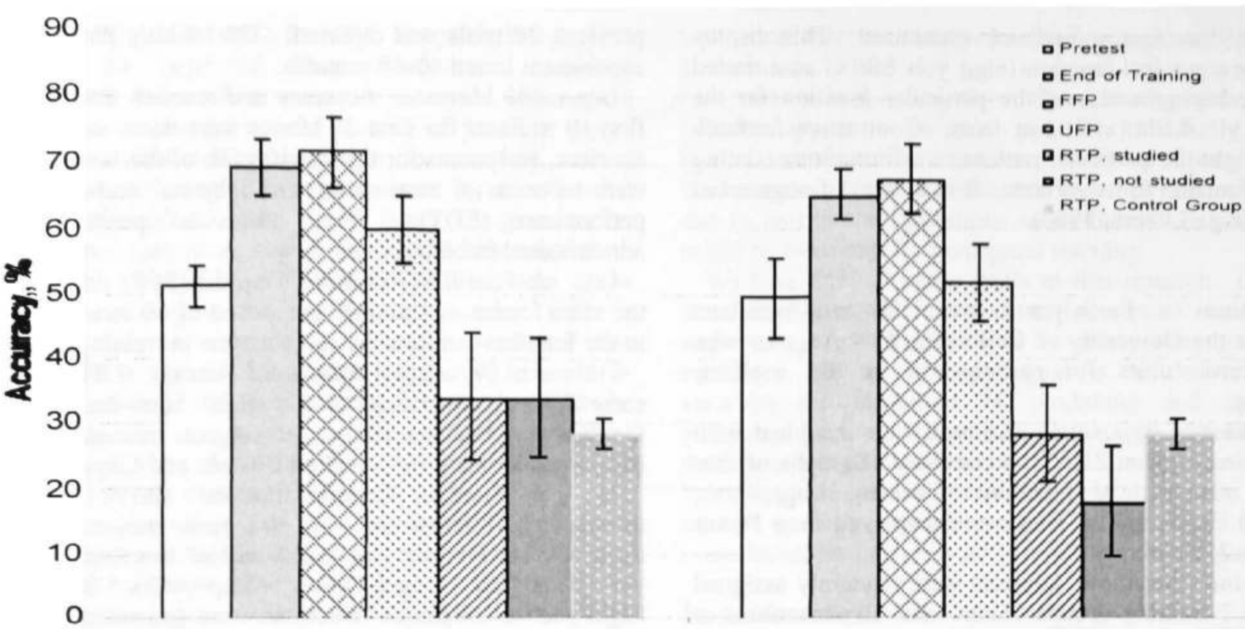

Figure 1. Results for Experiments 1 and 2. Mean accuracy for the different stages of training are shown, including pre-test, end of training, familiar functions post-test (FFP), unfamiliar functions post-test (UFP), remote transfer post-test (RTP) for studied and non-studied functions, and remote transfer post-test for the control group.

12.669, $p<0.001$, and RT, $F(3,54)=18.596$, $p<0.001$. Individual comparisons showed that participants were significantly better and faster at the end of the training than at the beginning (accuracy: $t(19)=7.241$, $p<0.001$; reaction time: $t(19)=7.486$, $p<0.001$), and in the UFP (accuracy: $t(19)=2.582$, $p<0.02$; reaction time: $t(19)=4.174$, $p<0.001$). Accuracy was higher in FFP than in both the pretest, $t(19)=4.547$, $p<0.001$, and the UFP, $t(19)=2.239$, $p<0.05$. RTs in the FFP were faster than in the pretest, $t(19)=4.098$, $p<0.001$, but not reliably different from UFP. Subjects' UFP performance was superior to pretest performance (accuracy: $t(19)=-2.281$, $p<0.05$; reaction time: $t(19)=2.666$, $p<0.015$). Accuracy did not differ between EOT and FFP, but the former phase showed better reaction time, $t(19)=4.198$, $p<.001$. There was a reliable interaction between function trained and condition, $F(3,54)=3.226$, $p<0.03$. For reasons that are unclear, a somewhat larger training effect was found with sines and exponentials than with cosines and logarithmic functions.

The control group did not differ from the pretest group in accuracy, $t(38)=-0.453$, n.s. UFP was faster than the control group ($t(38)=-3.551$, $p<.001$) but not more accurate ($t(38)=1.425$, n.s.). Reaction times in the control group were reliably slower than in the pretest, mean difference = 4.6 sec, $t(38)=-2.661$, $p<0.011$. This result suggests some rapid improvements in response time during the early training trials. (Note that in order to get 10 pretest trials with each function type studied, the pretest comprised the first 10 trials of each of the first two trial blocks, i.e., trials 1-10 and trials 21-30).

Remote Transfer Post-test Results. Performance on the two forms containing combination functions did not differ depending on the function families seen during training. This observation was verified by the analyses. Two 2 X 2 (functions trained by functions tested) ANOVAs, one for accuracy and another for reaction time, yielded no significant main effects or interactions. Comparisons with the control group revealed no reliable differences in terms of accuracy. Response times were quite long for the RTP, on the order of three minutes per form. Control subjects were reliably slower than Experiment 1 subjects for non-studied functions, $t(38)=-2.19$, $p<0.035$, and marginally slower for the studied functions post-test, $t(38)=-1.802$, $P<0.05$ (one tailed).

Discussion

The results of Experiment 1 support several conclusions. First, training designed specifically to foster perceptual learning can improve subjects' performance in relating graphs and equations. Although our subjects had previously learned about the relevant functions in mathematics classes, they had not become skilled in classifying patterns and recognizing transformations. A relatively brief intervention substantially improved both accuracy and speed. A second finding was that training generalized to new instances in the same function families and also to similar transformations deployed in new function families. Training in this experiment did not lead to reliable effects on our remote transfer (combination function) test, which proved quite difficult for subjects.

It does not appear that the 45 minutes of training were sufficient to achieve automatic pattern recognition. At the end of training, accuracy had not reached ceiling. Still, response times had fallen to about 5.5 sec on average. Response times on this order suggest that subjects were becoming more automatic in their pattern classification rather than using an elaborate reasoning process. Further training might lead to greater automaticity in classification.

What did subjects learn? The transfer results suggest that the training effects were not merely about the trained functions, such as sines. Transformations such as compression, scaling and shifting have the same sorts of effects across function classes. Initial performance levels of subjects suggested that these generalities have not been well learned from conventional mathematics instruction. Our results showed enhancement of performance after training even with new function families, suggesting that the symbolic and graphical "meanings" of basic transformation patterns were learned to some degree. The results suggest that PLMs may have great promise for developing fluent pattern processing in mathematics.

Experiment 2: Contrastive Feedback

Experiment 1 indicated the efficacy of perceptual classification training, using several ingredients suggested by earlier research and by intuition. Which ingredients are crucial to the usefulness of this kind of training? These questions have hardly begun to be addressed, especially in the application of perceptual learning to complex skills, such as doing mathematics. A goal of our research is to determine systematically the effects of particular aspects of training in order to optimize PLMs in education.

In Experiment 2, we examined whether the particular kind of contrastive feedback used in Experiment 1 had important effects on learning. Recall that after each problem, participants viewed the problem function superimposed on the canonical function. If this particular type of feedback facilitated discovery of relevant pattern transformations, then eliminating it might reduce the success of training. In particular, we hypothesized that this type of feedback might have been especially helpful in producing transfer to new function families. Eliminating it might therefore be expected to reduce or eliminate transfer of learning to new functions.

Method

Participants. Twenty undergraduate students at the University of California, Los Angeles, received credit units for participation.

All aspects of the method were identical to Experiment 1 except that contrastive feedback was eliminated. Instead, feedback screens indicated whether the response on the trial had been correct and displayed the graph and correct equation for that problem.

Results

Training, FFP and UFP Results. As in experiment 1, training produced highly reliable effects on accuracy (shown in Figure 1) and RT. Subjects improved from about 50% correct in the pretest to about 65% in the final block of training. Response times decreased about 40% during training. Both the accuracy and speed at the end of training were maintained in the FFP. In contrast to the results of Experiment 1, there was little or no transfer of training to the UFP; training effects were largely confined to the familiar (studied) function types. The control group, pretest and UFP performance of the training group were very similar.

These observations were confirmed by the analyses. Accuracy and RTs were each analyzed using a 2 (functions trained) X 4 (phase: Pretest, EOT, FFP, UFP) ANOVA, with repeated measures on the latter factor. There were reliable main effects of phase, for accuracy $F(3,54) = 8.892$, $p<0.001$, and for RT, $F(3,54) = 8.405$, $p<0.001$. Individual comparisons showed that participants were significantly better and faster at EOT than at the beginning (for accuracy: $t(19) = 3.507$, $p<0.002$; for reaction time: $t(19) = 3.816$, $p<0.001$). Accuracy and reaction time were better in FFP than in the pre-test (accuracy: $t(19) = 3.789$, $p<0.001$; reaction time: $t(19) = 3.165$, $p<0.005$) and also better than in the UFP (accuracy: $t(19) = 3.47$, $p<0.003$; reaction time: $t(19) = 2.687$, $p<0.015$). Accuracy did not reliably differ between EOT and FFP, $t(19) = 0.76$, n.s. For reaction time, there was a marginal advantage for the EOT problems over FFP, $t(19) = 1.96$, $.05<p<0.10$. Subjects' UFP performance did not differ significantly from the pretest (accuracy: $t(19) = 0.453$, n. s.; reaction time: $t(19) = 0.805$, n. s.).

Accuracy in the control group did not differ from either the pretest ($t(38) = -0.59$, n.s.) or the UFP ($t(38) = -0.192$, n.s). Reaction times in the control group were significantly slower than in both the pretest, $t(38) = -2.706$, $p<0.01$, and UFP, $t(38) = -3.359$, $p<.002$.

Remote Transfer Post-test Results. As in Experiment 1, performance on the two forms containing combination functions did not differ depending on the trained function families. Two 2 X 2 (functions trained by functions tested) ANOVAs, one for accuracy the other for reaction time, yielded no significant main effects or interactions.

Accuracy and reaction times on both forms (studied and unstudied functions) of the RTP did not differ reliably from a control group that received no training.

Cross-Experiment Comparisons

To more carefully assess the effect of contrastive feedback, we performed several analyses including the data from both Experiments 1 and 2.

Training, FFP and UFP Results. ANOVAs were performed on accuracy and reaction time data. Each was a 2 X 2 X 4 (experiment by functions trained by condition) design with repeated measures on the condition factor. There was a strong main effect of condition (accuracy: $F(3, 108) = 20.56$, $p <.001$; RT: $F(3,108) = 21.99$, $p <.001$), indicating the effects of training in both experiments. There was also a condition by functions trained interaction for accuracy, $F(3, 108) = 4.188$, $p <.008$, indicating that in both experiments training effects were greater for subjects trained on sines and exponentials. No other main effects or interactions reached significance in the accuracy or RT analyses.

The lack of a reliable experiment by condition interaction for accuracy contrasts somewhat with a difference between

Experiment 1 and 2 observed above. Specifically, in Experiment 1, participants showed better accuracy on unfamiliar functions (UFP) than in the pretest, $t(19) = -2.281$, $p < 0.05$, whereas no such difference appeared in Experiment 2, $t(19) = 0.453$, n.s.

This difference suggests that contrastive feedback produced better learning of transformations that could be applied to new function families. The lack of direct support for this interaction in the combined analysis is probably due to the greater variability present in between-subjects comparisons. Nevertheless, the mixed results suggest caution. Fortunately, the cross-experiment comparisons of RTP performance (below) provide some confirmation of the difference in training effects between the two experiments.

Remote Transfer Post-test Results. Remote transfer effects of training appeared to be somewhat larger in Experiment 1, which used contrastive feedback, than in Experiment 2, which did not. Even though no reliable differences were found between the studied and non-studied functions within each experimental condition, a difference was found in terms of accuracy when comparing experiments 1 and 2. More specifically, subjects in experiment 1 performed better than subjects in experiment 2 for the non-studied functions ($t(38)=1.906$, $p<.05$, one tailed). The superior performance of experiment 1 subjects' in the non-studied functions attests for the effect of the contrastive feedback in producing better learning of transformations that can be applied to new function families. No other comparisons reached significance.

General Discussion

Taken together, the results of experiments 1 and 2 indicate a clear training effect. Subjects in both experiments substantially improved their accuracy and speed in relating graphs and equations, as compared to a control group. Subjects' improvement extended beyond the specific examples on which they were trained. They showed equally good performance on new examples from the function families on which they had trained, indicating that learning did not depend merely on memorizing specific instances.

Subjects in Experiment 1 also transferred their learning to *new* function families. This result did not appear in Experiment 2, however. These outcomes suggest that the *contrastive feedback* used in Experiment 1 may have laid the foundation for transfer by directing attention to the relevant *transformations* in the stimulus patterns. The results are consistent with the idea that perceptual learning might be accelerated by augmented feedback that helps to direct attention to relevant features and dimensions.

Our remote transfer test was difficult and showed modest effects. Here again, however, a comparison between the two experiments revealed that Experiment 1 produced more gains in accuracy for the non-studied functions than Experiment 2, which did not have contrastive feedback.

The contributions of other procedural ingredients remain to be assessed. For example, it is often asserted that perceptual learning depends on an active classification task (e.g., Karni & Sagi, 1993), but there have been few careful tests of this idea. In the present work, both groups were given active classification tasks. Currently we are studying whether mere exposure in short episodes to corresponding graphs and equations produces learning and transfer.

In summary, our results indicate that a brief period of perceptual training can substantially improve subjects' performance in processing mathematical structures expressed in graphs and equations. Contrastive feedback – highlighting the dimensions of difference between a basic function and transformations of it -- enhances the learning of relationships that transfer to novel function families. Perceptual learning modules may have great promise in accelerating the development of components of expertise that do not arise easily from traditional instruction.

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