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THE VISCOSITY OF LIQUID METALS

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AEC Contract No. W-7405-eng-48

THE VISCOSITY OF LIQUID METALS

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February 15, 1965

THE VISCOSITY OF LIQUID METALS

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ABSTRACT

An approximate form is suggested for the perturbation of the radial distribution function of a monatomic liquid by a nonuniform flow field. Substitution of this form into the microscopic expression for the pressure tensor yields an equation for liquid viscosity in terms of the equilibrium distribution function and the interatomic potential energy function. This equation establishes the basis for a corresponding states correlation of the viscosity of liquid metals based on atomic parameters.

The viscosity data for twenty-one molten metals are made to fall on a single curve by the adjustment of one microscopic parameter. It is found that this empirically-determined parameter apparently has the proper fundamental significance. Therefore it is possible to estimate it independently and to use the general correlation for estimating the viscosity of a metal for which data are not available. It is also suggested that the atomic parameters determined here might be used to correlate other properties of the liquid metals.

INTRODUCTION

In recent years a great deal of effort has been devoted to the measurement of the viscosity of molten metals. One reason for this is that the behavior of the viscosity of simple liquids is a vital key to understanding the nature of the liquid state. On the practical side these data are of interest because of the increasing application of liquid metal coolants in nuclear reactors. Also, the viscosities of many higher melting metals are required for design purposes in the metallurgical industries. For example, in the steel industry the recent development of continuous casting processes requires an accurate knowledge of the viscosity of the molten metal. It is obvious that the need for such data for less common metals and particularly for molten alloys will increase. Therefore, because of the experimental difficulties of measuring properties at such high temperatures, it would be desirable to have a reliable correlation from which the viscosity of a particular substance could be estimated. It is the purpose of this report to show how the existing theory of liquid structure can be treated to develop a useful correlation of the viscosities of liquid metals.

When seeking a meaningful correlation of a physical property such as viscosity, one desires to base his analysis upon a sound theory rather than to rely entirely

on empiricism or artificial models. In this way one hopes to obtain relationships and derived parameters which have real physical significance. With such a correlation one has confidence in extrapolating existing data over a range of conditions or in predicting values for substances where no experimental data are available. For example, the equations of statistical mechanics and kinetic theory have been applied very successfully to correlate the thermodynamic and transport properties of gases [1]. The liquid state however does not lend itself so readily to theoretical analysis. In general most theories of liquids have been based on models which generate parameters lacking fundamental significance and which impose a priori assumptions on the structure, interactions, or mechanisms of transport in the fluid. The theory of liquids based on the pair distribution function as developed by Kirkwood [2] and by Born and Green [3] escapes this criticism because of the generality of its formulation. In fact, all the thermodynamic and transport properties of a fluid can be expressed with general validity in terms of the intermolecular potential energy function and the appropriate distribution functions. The chief stumbling block in applying this theory lies in obtaining a valid representation of the pair distribution function.

The distribution functions which are of importance in the liquid must be determined from a system of coupled, nonlinear integral equations which have not been solved

very accurately for even the equilibrium case, not to mention the nonuniform situations which must be considered to treat irreversible phenomena. Since it appears that much more work is required before satisfactory results are to be obtained from the actual solution of these equations, a suggestion is made here for an approximate form of the solution for the pair distribution function under conditions of a nonuniform flow field. Substitution of this assumed form into the microscopic expression for the pressure tensor of a simple liquid leads to an equation for the coefficient of viscosity. Even though this equation still involves two unknown functions, the radial distribution function and the potential energy function, it can be made dimensionless to establish the basis for a corresponding states correlation of liquid viscosity based on molecular parameters. Such a correlation will be shown to be valid for the liquid metals.

Theory

According to the Born and Green theory [3], the pressure tensor $\underline{\underline{P}}$ in a monatomic fluid can be expressed as

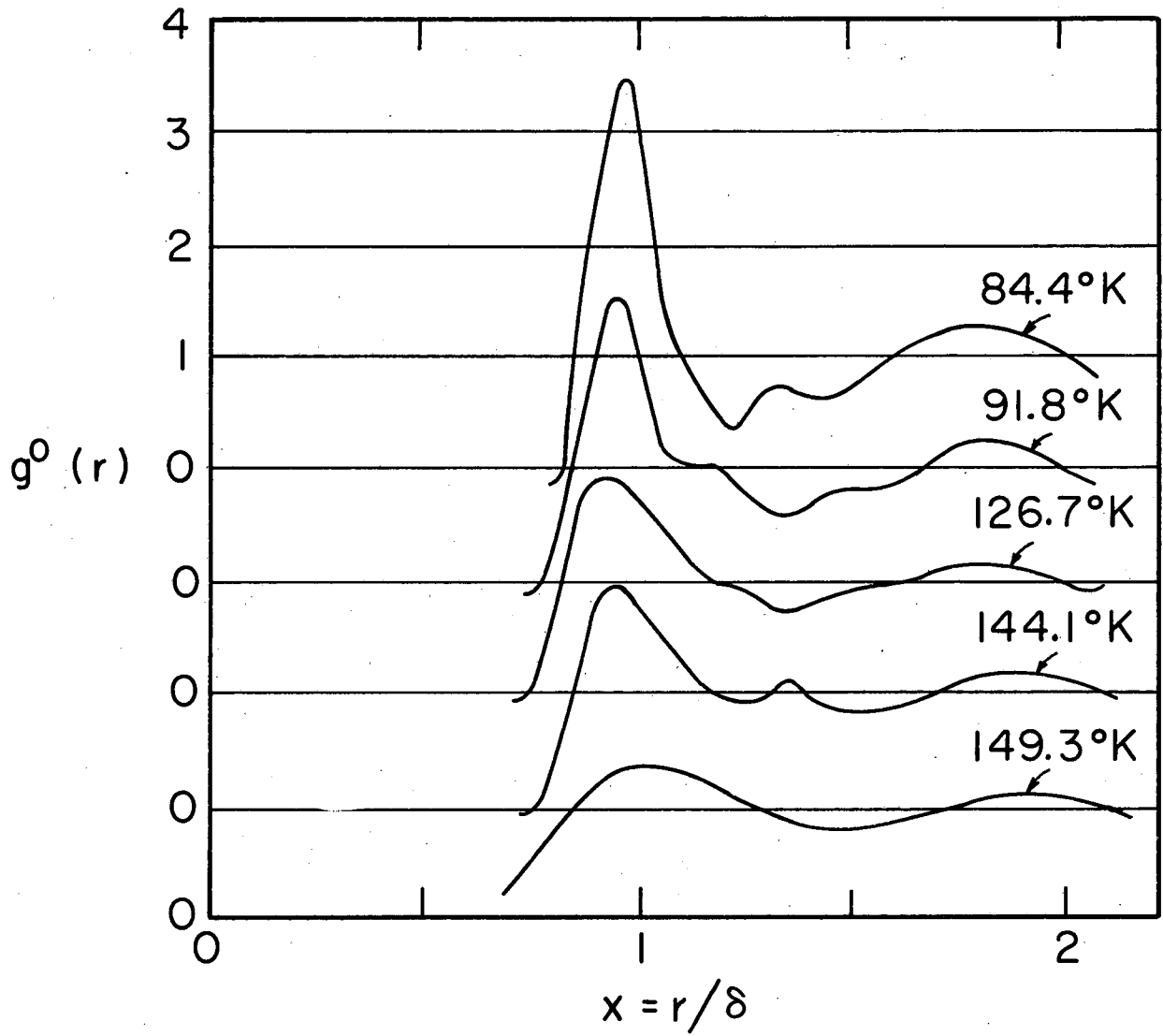
$$\underline{\underline{P}} = m \iiint \underline{\underline{\xi}} (\underline{\underline{\xi}} - \underline{\underline{u}}) \tilde{f}(\underline{\underline{\xi}}) d^3 \underline{\underline{\xi}} - \frac{n^2}{2} \iiint \underline{\underline{r}} \nabla \phi(r) g(r) d^3 \underline{\underline{r}} \quad (1)$$

where m is the mass of an atom, $\underline{\underline{\xi}}$ is the microscopic particle velocity vector, $\underline{\underline{u}}$ is macroscopic fluid velocity, and $f(\underline{\underline{\xi}})$ is the velocity distribution function. In the

second term n is the particle number density, $\phi(\underline{r})$ is the pair interatomic potential energy function, \underline{r} is the microscopic relative position vector, and $g(\underline{r})$ is the pair distribution function. The first term in this expression corresponds to the kinetic contribution to the flux of momentum, that is, the momentum which is transported by the actual movement of molecules through the fluid. It is this term which is considered in the kinetic theory gases as presented by Chapman and Cowling [1].

In a nonuniform liquid, however, most of the momentum is transported by intermolecular forces, and this contribution to the pressure tensor is given by the second term in equation (1). This integral in effect adds up the interactions between all pairs of atoms. The quantity $\frac{n^2 g(\underline{r})}{2}$ is the time-average number of pairs with a separation \underline{r} .

At equilibrium the pair distribution function $g(\underline{r})$ is symmetrical and is known as the radial distribution function $g^{\circ}(r)$. This quantity can be measured experimentally by X-ray or neutron diffraction techniques [4]. Shown in Figure 1 are experimental curves representing the radial distribution function g° of liquid argon at various temperatures which were measured by Eisenstein and Gingrich [5]. The curves have been vertically displaced to show the effect of temperature more clearly. In each



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Fig. 1

case $g^{\circ}(r)$ approaches unity at large separations which means that the long-range structure is random. The distance coordinate has been normalized by a length δ , the distance to the first peak which is also approximately the position of the minimum of the pair potential energy function ϕ .

A significant characteristic of the radial distribution function is the effect of temperature as demonstrated by Figure 1. It is seen that increasing temperature causes the function g° to tend toward unity at all positions along the curve. This means that the increased thermal motion of the particles tends to destroy the order in the liquid even at small separations.

Under conditions of thermal and mechanical equilibrium when the pair distribution function g is equal to the symmetrical function g° , the shear stresses in the pressure tensor vanish, and equation (1) reduces to the equation of state [6]

$$p = nkT - \frac{2\pi}{3} n \int_0^{\infty} r^3 \frac{d\phi(r)}{dr} g^{\circ}(r) dr \quad (2)$$

where p is the thermodynamic pressure.

When the fluid is not in equilibrium, the macroscopic pressure tensor includes certain off-diagonal elements as well as the pressure defined by equation (2), and it can be written as [7]

$$\underline{\underline{P}} = p \underline{\underline{I}} + \underline{\underline{\tau}} \quad (3)$$

where $\underline{\underline{I}}$ is the unit tensor and $\underline{\underline{\tau}}$ is the stress tensor. Substitution of equations (1) and (2) into equation (3) leads to a microscopic expression for the pressure tensor

$$\underline{\underline{\tau}} = \underline{\underline{\tau}}_k - \frac{n^2}{2} \iiint \underline{\underline{r}} \frac{d\phi(r)}{dr} g'(r) dV \quad (4)$$

where $\underline{\underline{\tau}}_k$ is the kinetic contribution to $\underline{\underline{\tau}}$ and the second term represents the contribution of intermolecular forces. Since the thermodynamic pressure p includes the equilibrium form of the pair distribution function, the function which appears in equation (4), $g'(r)$, is the difference between $g(r)$ and $g^0(r)$. That is, the function $g'(r)$ is the deviation of $g(r)$ from its equilibrium value due to nonuniformity in the liquid. The magnitude of the intermolecular force portion of the shear stresses depends directly on the extent of this deviation.

According to the Navier-Stokes stress argument for an incompressible Newtonian fluid [8] the coefficient of shear viscosity η is defined as the constant of proportionality between the stress tensor and the deformation of the velocity $[\underline{\nabla}u + (\underline{\nabla}u)^\dagger]$, where $(\underline{\nabla}u)^\dagger$ is the transpose of the gradient of the velocity $\underline{\nabla}u$,

$$\underline{\underline{\tau}} = -\eta [\underline{\nabla}u + (\underline{\nabla}u)^\dagger]. \quad (5)$$

Comparing this macroscopic definition of viscosity with the microscopic expression for the stress tensor, equation (4), and neglecting $\underline{\underline{\tau}}_k$ since it is very small in a liquid, one obtains a formal expression for the

viscosity of a monatomic liquid

$$\eta [\nabla \underline{u} + (\nabla \underline{u})^{\dagger}] = \frac{n^2}{2} \iiint \underline{r} \nabla \phi(r) g'(\underline{r}) dV. \quad (6)$$

This form is essentially equivalent to the expression given by Born and Green [9], except for the omission of the kinetic term. This approximation can be justified by calculating the kinetic contribution to viscosity from liquid self-diffusion data. For example, the self-diffusion coefficient of a liquid metal is on the order of 10^{-5} cm²/sec. According to Kirkwood [10] the kinetic portion of viscosity η_k is related to the self-diffusion coefficient D by

$$\eta_k = \frac{n m}{2} D. \quad (7)$$

For mercury at 300°K, $\eta = 1.55$ cp and $\eta_k = 0.02$ cp.

Thus it is seen that η_k is on the order of only a few percent of the total viscosity.

In order to proceed to the calculation of viscosity from equation (6), it is necessary to obtain the pair distribution function under nonequilibrium conditions and in particular to relate it to the velocity gradient in the fluid. This problem seems to be far from a rigorous solution since it is not yet possible to calculate the distribution function of a liquid in equilibrium very accurately. However, it is possible to derive a simple approximation to the exact function by taking recourse to physical arguments about the effect of a flow field on the liquid structure.

In general the exact solution for the pair distribution function must be determined from a complicated integro-differential equation [11] which is difficult to solve even in the simplest case [11a]. In order to obtain an approximate form for the pair distribution function we consider the equilibrium solution to be known and assume that the general solution to the complete equation can then be expressed as a perturbation series expansion about the equilibrium value, that is,

$$g(\underline{r}) = g^0(r) + F_1(r)s + F_2(r)s^2 + \dots \quad (8)$$

The function $g(\underline{r})$ is written as a power series in some perturbation parameter s which characterizes the disturbance of the liquid structure by the flow field. " s " should be small and vanish at equilibrium to assure convergence. The coefficients $F_i(r)$ are assumed to be independent of the disturbance and functions of r only. Then s must be a function of the polar angles θ and ϕ . This separation of variables seems reasonable since the disturbance must depend on the velocity gradient, and in the region where interatomic forces are appreciable the velocity gradient is very nearly linear. Therefore s should not depend on the magnitude of the position vector \underline{r} but only on its direction.

According to conservation of mass, the average value of $g(\underline{r})$ is unity as is the average value of $g^0(r)$. Therefore the integral of each higher term in equation (8)

over r must vanish. This condition will be satisfied if $F_i(r)$ is proportional to $[1 - g^0(r)]$. Thus it is assumed that

$$F_i(r) = A_i [1 - g^0(r)]. \quad (9)$$

The factors A_i are constants of integration which would have to be evaluated from the actual solution of the complete equation for $g(r)$. We shall not concern ourselves with their actual values at this time.

An appropriate form of the perturbation parameter s is assumed to be the ratio of two velocities: the ratio of a characteristic disturbing velocity u_δ to a characteristic restoring velocity u^*

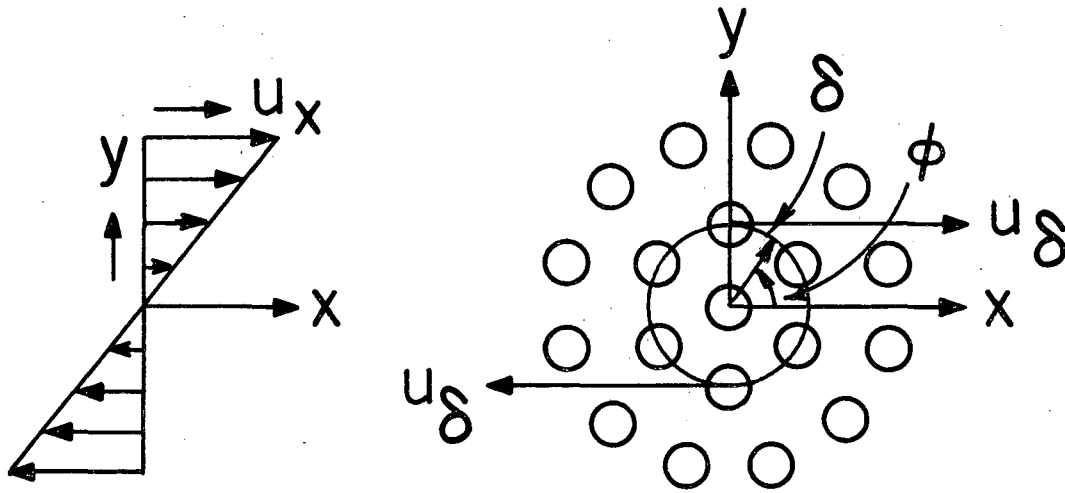
$$s = \frac{u_\delta}{u^*}. \quad (10)$$

The velocity which acts to restore the equilibrium liquid structure is assumed to be the average thermal speed of the atoms

$$u^* = \sqrt{3kT/m}. \quad (11)$$

" u_δ " is chosen to be the average velocity of the atoms in the first peak of the radial distribution function relative to that of the central atom. Figure 2 shows the significance of u_δ for the simple case of a linear velocity profile.

In this situation the deformation of the velocity contains only two nonzero elements, each of which is the velocity gradient $\frac{du_x}{dy}$:



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Fig. 2

$$[\underline{\nabla} \underline{u} + (\underline{\nabla} \underline{u})^\dagger] = \begin{pmatrix} 0 & \frac{du_x}{dy} & 0 \\ \frac{du_x}{dy} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (12)$$

The macroscopic picture of this flow field is shown on the left in the figure. The velocity u_x is drawn relative to that of the plane $y=0$. On the right the same situation is shown microscopically. Since we are considering a time-averaged picture u_δ is zero for the atoms in the $y=0$ plane. However, all other atoms on the spherical surface $r=\delta$ are on the average moving relative to the central atom because of the imposed u_x field. If one imagines an atom on $r=\delta$ to move away from its equilibrium position with velocity u_δ , one sees that there will be a density deficiency in that region for a short period of time, thus diminishing the value of g . However, because of the random motion of the atoms, as one atom is forced to move away from an energetically favorable region another will shortly replace it with the relaxation time depending on the thermal motion characterized by u^* . Thus the ratio assumed in equation (11) can be considered to be the function which characterizes the steady state disturbance of the short-range order in the liquid by the macroscopic velocity field. For this simple geometry u_δ can be expressed as

$$u_\delta = \delta \sin \phi \left(\frac{du_x}{dy} \right) \quad (13)$$

where ϕ is the angle measured from the plane $y=0$.

Since s turns out to be on the order of 10^{-12} one would expect terms higher than linear in s in equation (8) to be negligible under ordinary circumstances. If such is the case, the function $g(\underline{r})$ can then be approximated by

$$g(\underline{r}) = g^{\circ}(r) + A_1 [1 - g^{\circ}(r)] s . \quad (14)$$

This approximate form seems physically reasonable. Since the correction to $g^{\circ}(r)$ is proportional to $[1 - g^{\circ}(r)]$, an increase in s causes $g(\underline{r})$ to tend toward unity. That is, as the disturbance becomes greater, the structure of the liquid becomes more random, an effect similar to that of increasing temperature.

Substitution of the assumed form of the perturbation into equation (6) yields an explicit expression for liquid viscosity which involves the pair potential function and the radial distribution function

$$\eta = A_1 \frac{\pi n^2 \delta}{u^*} \int_0^{\infty} r^3 \frac{d\phi(r)}{dr} [1 - g^{\circ}(r)] dr . \quad (15)$$

Thus these assumptions have reduced the problem of liquid viscosity to the same level of complexity as the problem of the thermodynamic properties. The direct calculation of the properties of a liquid now requires accurate representations of the function $\phi(r)$ and $g^{\circ}(r)$ which are not yet available. Nevertheless a useful conclusion can be drawn from equation (15).

It is assumed that the potential function $\phi(r)$ depends on two parameters, an energy parameter ϵ and a distance parameter δ . Then equation (15) can be made dimensionless to define a reduced viscosity

$$\eta^* = F(T^*, V^*) \quad (16)$$

where

$$\eta^* = \left[\frac{\eta \delta^2 N_0}{(MRT)^{1/2}} \right], \quad (17)$$

$$T^* = [kT/\epsilon], \quad (18)$$

and

$$V^* = [1/n\delta^3], \quad (19)$$

and where M is the molecular weight and N_0 is Avagadro's number. For a class of substances which obey the same potential energy function $g^0(r)$ should be a universal function of T^* , V^* , and $x=r/\delta$. Therefore, the reduced viscosity η^* should be a universal function of T^* and V^* . Furthermore, in the liquid region where the density is nearly constant, one would not expect $g^0(r)$ to depend very greatly on V^* . Thus according to equation (15), a good approximation ought to be that $\eta^*(V^*)^2$ is a universal function of T^* only,

$$\eta^* (V^*)^2 = G(T^*) \quad (20)$$

$$\text{where } G(T^*) = \frac{C_1}{T^*} \int_0^\infty \left[\frac{d(\phi(x)/\epsilon)}{dx} \right] [1 - g^0(x, T^*)] dx \quad (21)$$

and C_1 is a constant.

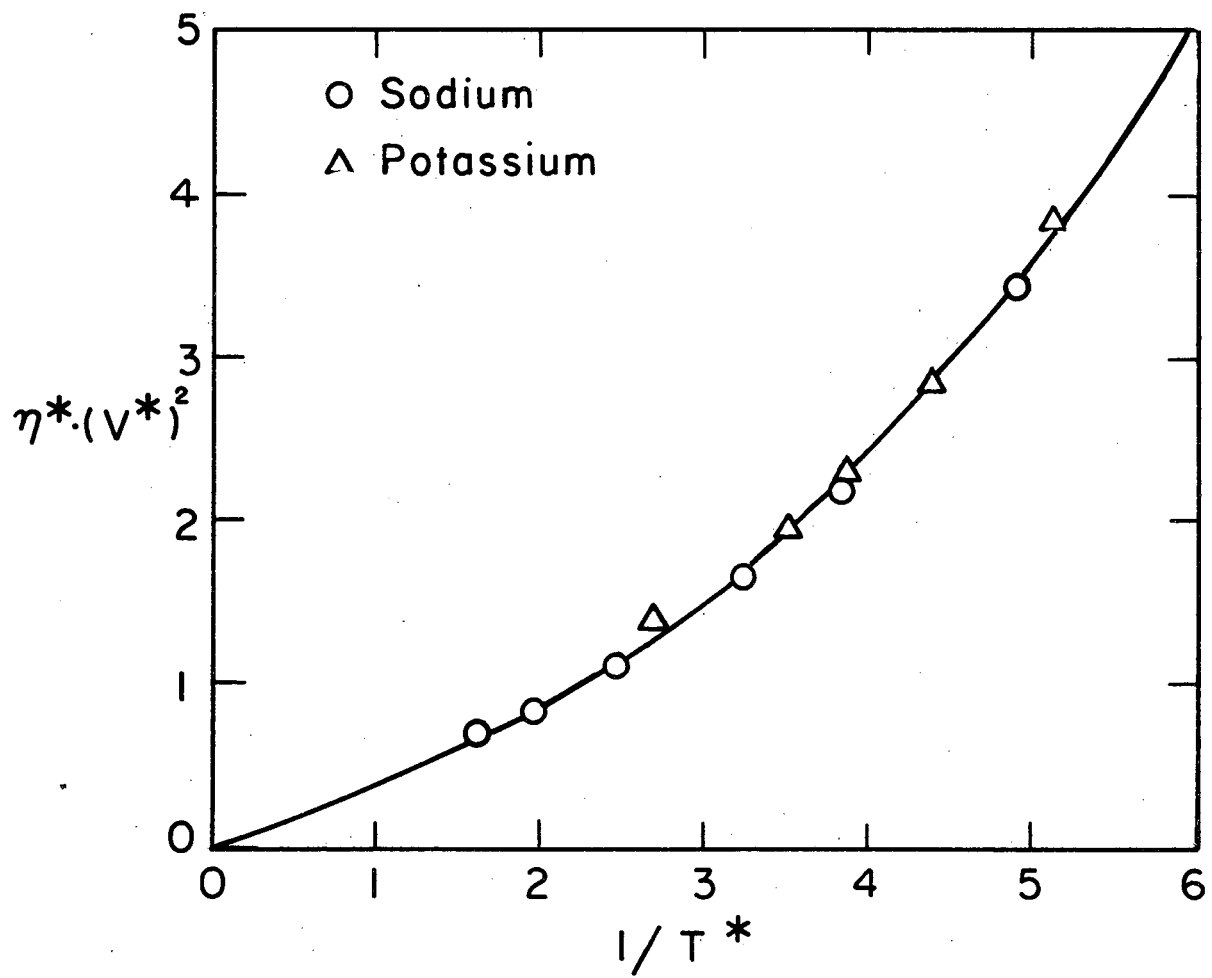
APPLICATION OF THE THEORY TO LIQUID METALS

In order to apply the results of the preceding analysis to develop a useful correlation of the viscosities

of liquid metals, it is necessary to have values for the atomic parameters ϵ and δ . Unfortunately there is little evidence to indicate the form of $\phi(r)$ for metals not to mention the actual values of the parameters. Determination from thermodynamic data such as heat of sublimation is complicated by the effects of the free-electron metallic binding in the condensed phase. Nevertheless, to establish a corresponding state plot of the function $\eta^*(V^*)^2$ versus T^* it is necessary only to assume that these substances all obey the same function $\phi(r)$ and then to choose some basis for relative values of ϵ and δ .

Ling [12] has determined effective Lennard-Jones parameters for liquid sodium and potassium from their experimental X-ray scattering curves. These parameters for the two metals have been used to plot their viscosity data in Figure 3 as $\eta^*(V^*)^2$ versus $1/T^*$. The ranges of temperature represented here are 350-650°K for potassium and 400-1000°K for sodium. The viscosity range is 0.605 to 0.165 cp for sodium, 0.514 to 0.010 cp for potassium, and the reduced volumes at a T^* of 0.25 are 0.80 for K and 0.75 for Na. It is seen that the data plotted in this way fall on a single line as predicted by the theory and that neglecting the implicit dependence on V^* is apparently a good approximation.

The viscosities of many other liquid metals have been measured over a wide range of temperatures. The available



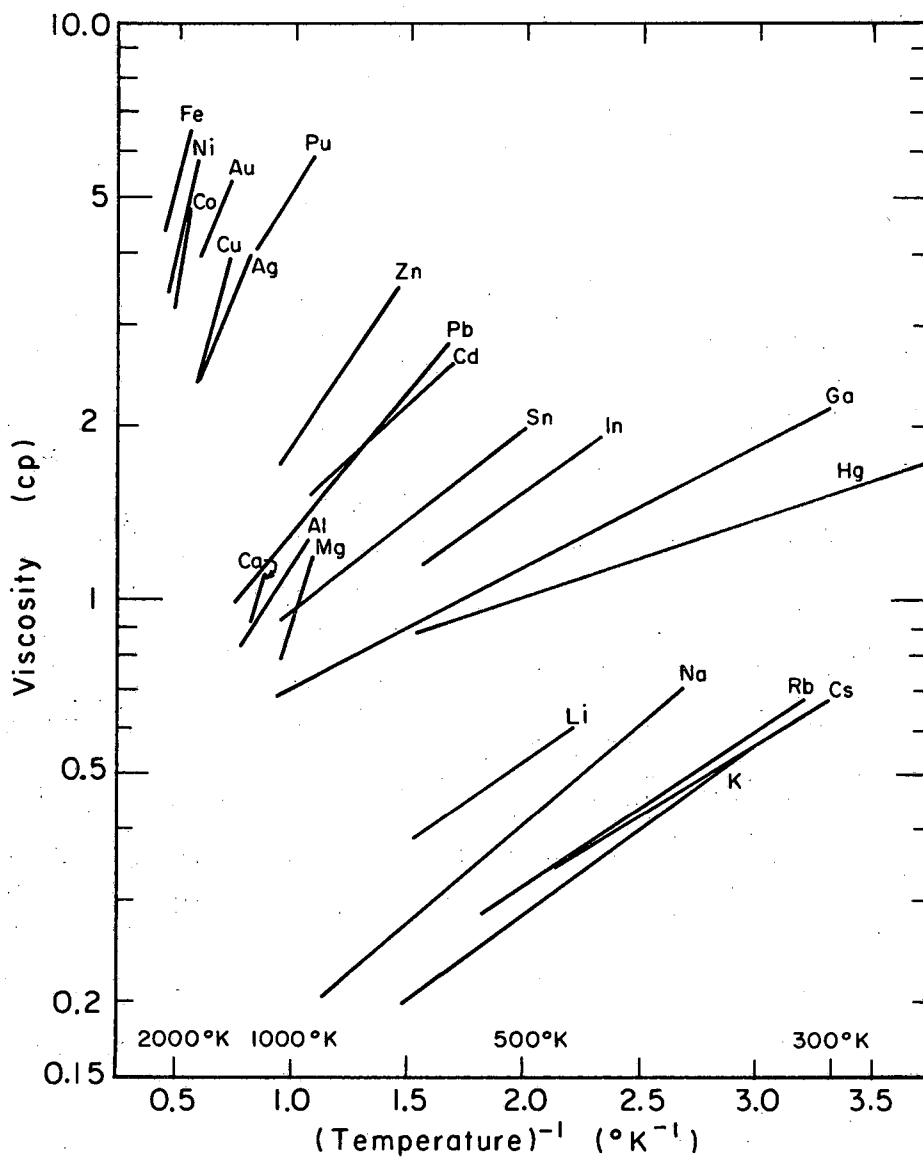
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Fig. 3

data are shown in Figure 4. It is seen that the temperature range is nearly 2000°K and that the viscosities range over nearly two orders of magnitude.

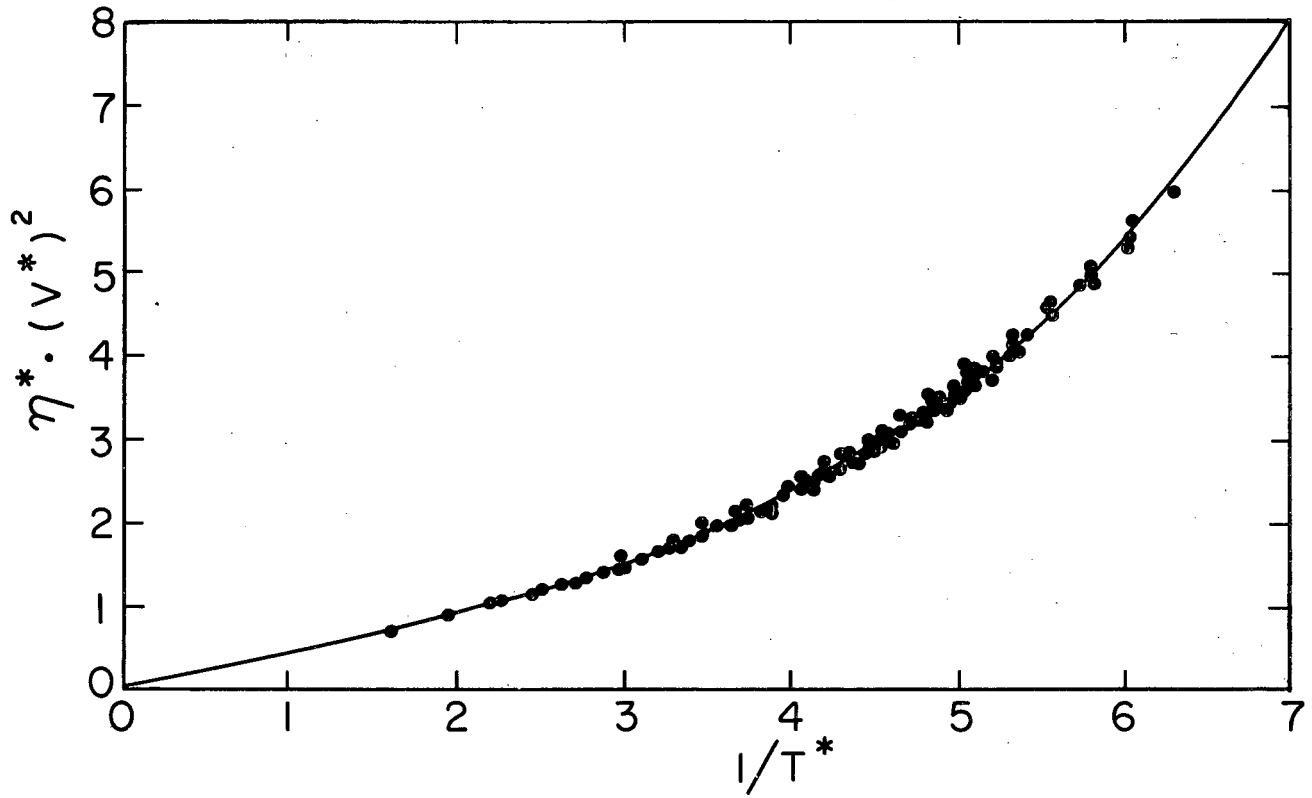
Since the parameter δ is supposed to be the distance to the minimum of the pair potential energy function, it should be possible to estimate its value from crystallographic data. Therefore we have chosen the value of δ to be the interatomic distance in the close-packed crystal at 0°K. These values are known for nearly all metals [13]. With this estimate for δ , $\eta^*(V^*)^2$ for all the data in Figure 4 can be calculated. Then from the general $\eta^*(V^*)^2$ versus $1/T^*$ curve which is established by the data for Na and K it is possible to evaluate ϵ for the rest of the metals. That is, we adjust ϵ to fit the data to the empirical curve in Figure 3. This has been done, and the resulting correlation is shown in Figure 5. The values of the parameters are given in Table 1.

In Figure 5 are plotted the data for the twenty-one different metals, from lithium and mercury to iron and plutonium, with a range of densities from 1.8 to 18 gm/cm³ and atomic weights from 6.9 to 242. It is seen that this method of plotting the data accounts very well for the dependence of liquid viscosity on the physical variables. The viscosities of liquid metals do indeed obey a corresponding states law.



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Fig. 4



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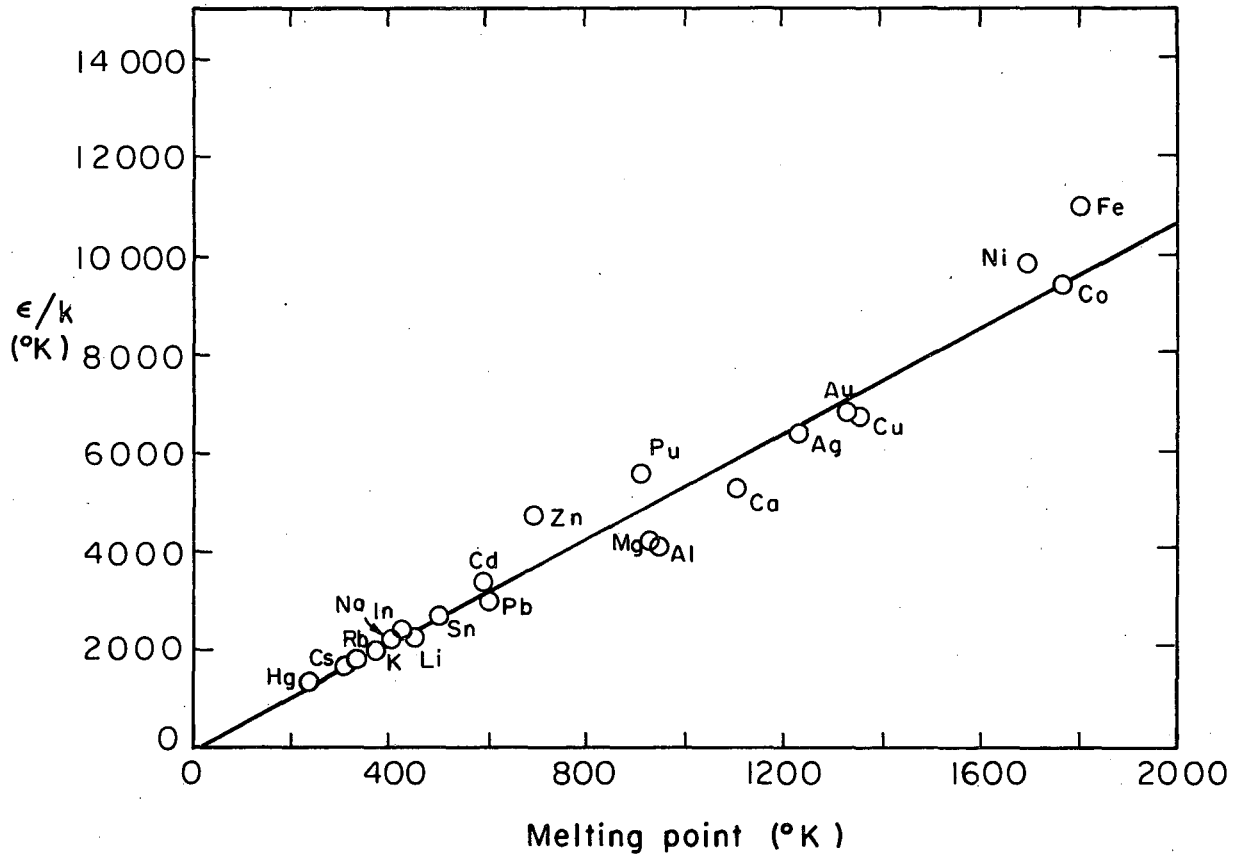
Fig. 5

However, in spite of the fact that the data can be reduced to a single curve, one might wonder whether this plot has fundamental significance because the parameter ϵ has been determined empirically. If the theory presented here is valid, then the values of ϵ obtained should in fact be the energy characteristic of the atomic interaction and should be related to other physical properties. Normally one finds that ϵ for a class of substances correlates with the critical temperatures, but unfortunately these are not known for most of the metals. However, various theories of melting such as that of Lennard-Jones and Devonshire [14] indicate that the melting point should be proportional to ϵ . Such a relationship has been observed for other classes of substances [15].

In Figure 6 the values of ϵ determined here are plotted versus the melting point, and it is seen that a very satisfactory correlation exists. On this basis, it is concluded that ϵ does in fact have the proper physical significance. Thus it should be possible to estimate ϵ for other metals from their melting points in order to predict viscosity from Figure 5. An illustration of the use of these correlations is given in the Appendix.

DISCUSSION

It has been seen that some rather crude assumptions about the behavior of the microscopic liquid structure under nonuniform conditions have made it possible to



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Fig. 6

develop a formal and rigorous, though incomplete, theory of liquids into a corresponding states correlation of viscosity for engineering use. Since it may not be clear at this point why much simpler dimensional analysis was not used to accomplish the same purpose, some of the advantages of this approach will be noted.

First, besides defining a reduced viscosity, equation (15) makes it possible to account for most of the density dependence of viscosity explicitly. This makes the correlation simpler.

Furthermore the analysis has led to a tractable expression for liquid viscosity which takes into account the fundamental features of liquid structure and molecular behavior which appear to be responsible for the macroscopic phenomena. Although it remains to be seen how well this equation does represent the exact viscosity function, it should be noted that the substitution of approximate forms of $g^0(r)$ and $\phi(r)$ leads to a function which agrees qualitatively with the empirical curve shown in Figure 5. Thus there is evidence that equation (15) provides a good approximation to the correct viscosity function. On this basis it is suggested that similar consideration of the effect of a flow field on the structure of a liquid mixture could lead to an appropriately modified form of equation (15) which might indicate a method for interpolating or estimating viscosities of molten alloys.

In any case it is felt that the general theory based on the pair distribution function provides the most

meaningful approach to the problem of liquid properties now available and that consideration of its implications leads to a better understanding of the nature of irreversible processes in liquids.

Another important consequence of this development in addition to the correlation of viscosity, is that it has led to values of the effective atomic interaction energy parameters for a large number of metals. It is suggested that corresponding states correlations of other properties of the metals might be established on the basis of these parameters. For example, the parameters determined here have been used successfully by the author to correlate the configurational heat capacities of several liquid metals [16] and thus to explain the behavior of liquid heat capacity with temperature.

SUMMARY

In order to develop a useful correlation of the viscosities of liquid metals, the formal expression for liquid viscosity derived by Born and Green has been considered. Assumptions about the primary effects of a velocity gradient on the equilibrium liquid structure have led to an approximate form for the pair distribution function under nonuniform conditions. Substitution of this expression into the Born and Green equation yielded an equation for the viscosity in terms of the radial distribution function and the interatomic potential

energy function. Although these functions have not yet been determined accurately, it was possible to use this equation to establish the basis for a corresponding states correlation of the viscosities of liquid metals.

The viscosity data of twenty-one metals were shown to fall on a single curve when plotted in the manner indicated by the theory. Since the correlation involved two atomic interaction parameters, it was necessary to show that the values which fit the data were consistent with the physical significance ascribed to them. The distance parameter was estimated independently from crystallographic data, and the energy parameter was determined empirically from the generalized viscosity curve based on sodium and potassium. It was found that the empirical energy parameter correlated well with melting point as the true energy parameter would be expected to do.

Thus it was concluded that the parameters had the proper significance and that the theory offered a valid approximation to the true behavior of the liquid structure. Therefore the established correlation can be used with confidence for estimating the viscosities of other liquid metals. It was suggested that an extension of this theory might make it possible to predict the viscosities of molten alloys. Also the parameters determined here might be used to correlate other properties of the metals.

ACKNOWLEDGEMENT

The author is grateful to Professor J. M. Prausnitz for his support and encouragement in this work.

Computer facilities for processing data were provided by the University of California Computer Center and by the Lawrence Radiation Laboratory, Berkeley.

Table 1. Goldschmidt atomic diameters and energy parameters of the metals.

Metal	$\delta(\text{\AA})$ [13]	ϵ/k ($^{\circ}\text{K}$)
Na	3.84	1970 [12]
K	4.76	1760 [12]
Li	3.14	2350
Mg	3.20	4300
Al	2.86	4250
Ca	4.02	5250
Fe	2.52	10900
Co	2.32	9550
Ni	2.50	9750
Cu	2.56	6600
Zn	2.74	4700
Rb	5.04	1600
Ag	2.88	6400
Cd	3.04	3300
In	3.14	2500
Sn	3.16	2650
Cs	5.40	1550
Au	2.88	6750
Hg	3.10	1250
Pb	3.50	2800
Pu	3.1	5550

APPENDIX

At the time that the correlation presented in this paper was developed no data were available for the viscosity of liquid thallium. Very recently, however, Cahill and Grosse have reported values for the viscosity of thallium over a wide range of temperatures [17]. In order to demonstrate the procedure for using the correlation as well as to check the accuracy of its prediction in this case we shall compare the estimated viscosity with the experimental values.

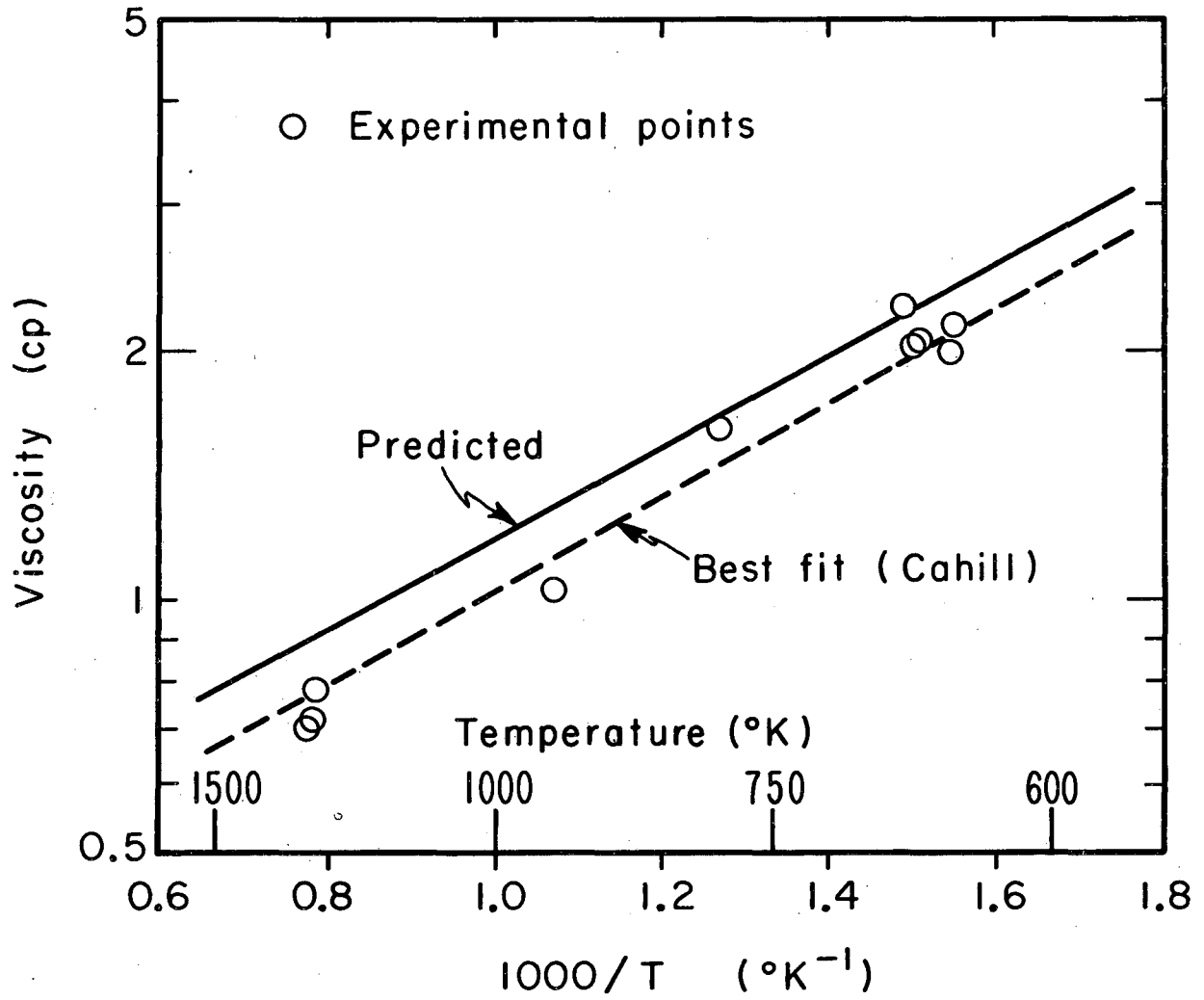
For thallium the atomic weight M is 204.4, the atomic diameter δ is 3.40 Å [13], and the melting point is 576 °K. The relationship between the parameter ϵ and melting point represented by the line in Figure 6 can be expressed as

$$\frac{\epsilon}{k} = 5.20 T_m \quad (22)$$

Therefore the energy parameter for thallium is estimated to be $(3000 k)^\circ\text{K}$. With this quantity known the reduced temperature corresponding to any absolute temperature can be calculated from equation (18). For example, at $T = 644^\circ\text{K}$, $T^* = 0.214$ and $1/T^* = 4.66$. From Figure 5 it can be determined that the reduced viscosity function $\eta^*(V^*)^2$ corresponding to this reduced temperature is 3.05. At $T = 644^\circ\text{K}$, the density of thallium is 11.18 gm/cm^3 . From equation (19) and the value of δ the reduced volume is calculated to be 0.78. Therefore $\eta^* = 5.00$.

From equation (17) the viscosity at this temperature is easily calculated to be 2.36 cp. This value is to be compared with the experimental value 2.11 cp; the error amounts to about 12%. Although this error is greater than the estimated experimental error of 0.05 cp. to 0.1 cp, it is seen that the correlation predicts a value which is quite acceptable for engineering purposes.

The viscosity of thallium has been estimated from the correlation over the experimental temperature range, and the predicted curve is compared with the experimental points in Figure 7. It is seen that although the predicted values are consistently on the order of 10% high, the predicted temperature dependence is quite satisfactory. These results are encouraging because most of the remaining metals whose viscosities have not been measured melt at temperatures well above 1000 or 1500°K. At these high temperatures experimental difficulties become considerable, and experimental errors of 10% and much more are to be expected. Thus it is felt that as experimental conditions become more severe, the predictive ability of this correlation becomes increasingly valuable.



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Fig. 7

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NOMENCLATURE

A_i	= Constant coefficient in function $F_i(r)$.
C_i	= Constant coefficient in function $G(T^*)$.
D	= Self-diffusion coefficient, cm^2/sec .
$F(T^*, V^*)$	= Universal function.
$F_i(r)$	= Coefficients in perturbation series.
$f(\underline{\xi})$	= Velocity distribution function.
$G(T^*)$	= Universal function.
$g(\underline{r})$	= Pair distribution function.
$g^\circ(r)$	= Equilibrium pair distribution function, radial distribution function.
$g^\circ(\underline{r})$	= Deviation of pair distribution function from equilibrium.
$\underline{\underline{I}}$	= Unit tensor.
k	= Boltzmann constant.
M	= Molecular weight.
m	= Mass of atom.
N_0	= Avagadro's number.
n	= Particle number density.
$\underline{\underline{P}}$	= Pressure tensor.
p	= Thermodynamic pressure.
R	= Gas constant.
\underline{r}	= Radial position vector.
r	= Radial component of position vector.
s	= Perturbation parameter.
T	= Temperature, $^\circ\text{K}$.
\underline{u}	= Macroscopic velocity vector.

- u_{δ} = Average macroscopic speed at $r=\delta$.
 u^* = Average thermal speed of atoms.
 V^* = Reduced volume.
 x = Reduced radial distance.

Greek letters

- δ = Distance parameter characteristic of pair potential energy function and radial distribution factor.
 ϵ = Energy parameter characteristic of pair potential energy function.
 η = Viscosity, centipoise.
 η^* = Reduced viscosity.
 θ, ϕ = Angles representing \underline{r} in spherical coordinates.
 κ = Subscript indicating kinetic term.
 $\underline{\xi}$ = Microscopic velocity vector.
 $\underline{\tau}$ = Stress tensor.
 $\phi(r)$ = Pair potential energy function.

FIGURE CAPTIONS

Figure 1. The experimental radial distribution function for liquid argon at various temperatures. [5]

Figure 2. Schematic diagram for a linear velocity gradient in a monatomic liquid shown on the macroscopic and microscopic scales. " u_0 " is the average velocity of an atom on the sphere $r=\delta$ relative to that of the central atom.

Figure 3. The reduced viscosities of sodium and potassium as functions of reduced temperature. The reduced quantities have been calculated on the basis of the atomic parameters determined by Ling. [12]

Figure 4. The viscosities of liquid metals and their dependence on temperature.

Figure 5. The reduced viscosities of liquid metals and their dependence on reduced temperature.

Figure 6. The correlation of the energy parameter ϵ with melting point for the liquid metals.

Figure 7. Predicted viscosity of liquid thallium compared with the experimental data of Cahill and Grosse. [17]

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