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# NEUTRINO OSCILLATIONS AND THE LEPTONIC CHARGE OF THE UNIVERSE

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## ABSTRACT

The most stringent bounds on the individual lepton numbers of the universe arise from considerations of primordial nucleosynthesis. We consider the effect of resonant neutrino flavor oscillations on these limits. In particular, we include the contributions from neutrino-neutrino forward scattering to the effective mass of each neutrino flavor. We find that there can be significant modifications to the neutrino number densities and energy distributions for each flavor over a large range of neutrino vacuum masses and mixing angles. Modifications to the neutrino distributions occurring prior to the nucleosynthesis epoch can substantially alter the previously derived limits on the lepton number for each flavor and its associated degeneracy parameter.
*Subject headings:* cosmology — elementary particles — neutrinos — nucleosynthesis

## I. INTRODUCTION

It is normally assumed in studies of the early universe that the total lepton number of the universe is very small and effectively zero. In terms of the number density of charged leptons $n_i$, neutral leptons $n_{\nu_i}$, and photons $n_\gamma$, the lepton number for each generation is defined to be $L_i = (n_i - n_{\bar{i}} + n_{\nu_i} - n_{\bar{\nu}_i})/n_\gamma$. The total lepton number is the sum over the three generations of the individual lepton numbers, $L = \Sigma L_i$, $i = e, \mu, \tau$. Further, it is assumed in such studies that the individual lepton numbers $L_i$ are also very small. By very small we mean comparable to the baryon number $B$, where $B = (n_b - n_{\bar{b}})/n_\gamma$ ($n_b$ is the baryon number density). A number lepton number for the universe is somewhat "natural" in many grand unified models (Dimopoulos and Feinberg 1979; Schramm and Steigman 1979; Nanopoulos, Sutherland, and Yildiz 1980; Turner 1981). For instance, in an $SU(5)$ model (Georgi and Glashow 1974) where $B - L = 0$, the predicted value of $B$ is much smaller than the observed value of $B \sim 10^{-9}$ (for a discussion of universal baryon number see Trimble 1987). In other grand unified models where $B - L$ is not conserved, a nonzero value for $B - L$ can be generated prior to the weak phase transition (for a review see Kolb and Turner 1983). However, $B - L$ conserving, but $B + L$ violating, nonperturbative physics in the standard electroweak model $SU(2)_L \otimes U(1)_Y$ (Glashow 1961; Weinberg 1967; Salam 1968) may erase any initial net asymmetries between $|B|$ and $|L|$ (Kuzmin, Rubakov, and Shaposhnikov 1985). If $(B-L)_0$ is the initial value of $B - L$ then at scales much lower than the weak scale it is expected that $B \sim 28/79(B-L)_0$ and $L \sim -51/79(B-L)_0$ (Kuzmin, Rubakov, and Shaposhnikov 1987; Bochkarev, Khlebnikov, and Shaposknikov 1990). Observations would then indicate that the initially generated value of $L$ was $\sim 10^{-9}$.

These arguments apply only to the total lepton number of the universe and not to the individual lepton numbers. The difference between individual lepton numbers is conserved by the sphalerons of the electroweak model, and hence an initially large asymmetry between, say, the electron and tau number densities will remain even at temperatures far below that of the weak phase transition (Kolb and Turner 1987; Kuzmin, Rubakov and Shaposhnikov 1987). It is therefore possible that while the total lepton number of the universe is small ($|L| \sim 10^{-9}$), the individual lepton numbers are large, i.e., $|L_i| \gg 10^{-9}$ (we note that in some Majoron models with explicit lepton number violation the neutrino degeneracies can be reduced, eg., Langacker, Segre, and Soni 1982).

Due to the charge neutrality of the universe (Lyttleton and Bondi 1959) any net lepton number must reside entirely in the neutrino sector. It is important to emphasize that there is no firm experimental basis for postulating $L_i \sim 0$ since the present relic neutrino asymmetry is not directly observable. In fact, the best constraints on the individual lepton number of the universe arise from studies of primordial nucleosynthesis (for a review of primordial nucleosynthesis see Boesgaard and Steigman 1985).

Under the assumption that the distribution functions for the neutrinos are Fermi-Dirac in nature, then the lepton numbers can be characterized by a set of degeneracy parameters, $\zeta_i = \mu_i/kT$, where $\mu_i$ is the chemical potential of each neutrino species, $T$ is the temperature, and $k$ is Boltzmann's constant. Nonzero lepton numbers of the universe can effect nucleosynthesis primarily in two ways (Wagoner, Fowler, and Hoyle 1967; Yahil and Beaudet 1976; Beaudet and Yahil 1977; David and Reeves 1980; Fry and Hogan 1982; Steigman 1985; Terasawa and Sato 1985; Terasawa and Sato 1988). First, the excess energy density in a neutrino degenerate sea leads to an increased expansion rate for the universe which subsequently allows less time for the neutrons to decay into protons. This has the net result of increased $^4$He production relative to a standard big bang model which assumes $L_i = 0$. Second, the nonzero electron-neutrino degeneracy can directly effect the equilibrium $n/p$ ratio at weak freeze out. The altered neutrino chemical potential shifts the equilibrium of the

$$e^- + p \leftrightarrow \nu_e + n , \qquad \bar{\nu}_e + p \leftrightarrow e^+ + n \tag{1}$$



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1991ApJ...368....1S



reaction. The equilibrium $n/p$ ratio is related to the electron neutrino degeneracy by

$$\frac{n}{p} = \exp\left(\frac{-\Delta Mc^2}{kT_*} - \zeta_e\right), \tag{2}$$

where $T_*$ is the weak freeze out temperature for this reaction (which depends on the degeneracy parameters), and $\Delta M$ is the mass difference between the neutron and proton.

The most stringent constraint placed on any particular degeneracy parameter is that placed on the electron degeneracy, $\zeta_e$, from considerations of primordial nucleosynthesis. Although this constraint is dependent upon the accuracy assumed for the primordial abundance determinations for D, $^3He$, $^4He$, and $^7Li$, it is approximately given by (Beaudet and Yahil 1977; David and Reeves 1980)

$$-0.5 \leq \zeta_e \leq 1.5 . \tag{3a}$$

This constraint is more stringent than that derived from the age of the universe (the mass-energy limit), which is given by (Freese *et al.* 1983)

$$\left(\sum \zeta_i^2\right)^{1/4} \lesssim 140 . \tag{3b}$$

The role played by the neutrino mass on this constraint is discussed by Freese *et al.* (1983). for example, if $m_\nu \sim 15$ eV, the upper limit on the magnitude of the degeneracy is reduced to $\sim 5$. Freese *et al.* (1983) also consider the constraint

$$\left(\sum \zeta_i^4\right)^{1/4} \lesssim 50 , \tag{3c}$$

arising from considerations of the, still unresolved, formation of galactic structure (see also Steigman 1985).

The constraints placed by the nucleosynthesis studies on the other neutrino degeneracies, which do not directly influence the $n - p$ interactions, is less clear. However, the limit imposed by nucleosynthesis, which again depends on observational uncertainties, is generally somewhat better than that imposed by the mass-energy limit (see Fig. 3 of Beaudet and Yahil 1977). The nucleosynthesis constraint is found to be $|\zeta_{\mu,\tau}| \lesssim 20$.

It is the purpose of this paper to point out that if resonant neutrino oscillations (Mikheyev and Smirnov 1986; Wolfenstein 1978, 1979; for a review see Bahcall 1989) occur in the early universe, then the presently inferred limits on the $\zeta_i$ from nucleosynthesis studies may not represent the initial lepton asymmetries in the earliest epoch of the universe. Previous studies of resonant neutrino oscillations in the early universe have only considered the zero lepton number limit ($L_i \lesssim 10^{-9}$) (Langacker *et al.* 1987; see also Khlopov and Petcov 1981 where small lepton number generation via neutrino oscillations is discussed). In this work we extend these studies to nonzero lepton number universes. We show how the hitherto omitted neutrino-neutrino neutral current interactions make the dominant contribution to the difference between neutrino refractive indices.

The details of neutrino flavor transformation in resonant oscillations depend on the neutrino vacuum masses and mixing angles, neutrino interactions with the background plasma of baryons and leptons, and the expansion rate of the universe. These resonant transformations can result in significant alterations in the neutrino number densities and distribution functions over a broad range of vacuum mixing angles and mass differences.

In § II we discuss lepton number densities, neutrino distribution functions, neutrino interactions with the background plasma, the neutrino mass matrix, and resonant conditions for neutrino oscillations. In § III we describe the details of the neutrino transformation process including the adiabatic condition for resonant oscillations and the modification of the neutrino distribution function in four separate epochs prior to nucleosynthesis. The first epoch is when the characteristic time scale for resonant transformations is long compared with the weak interaction time scale during which resonant oscillations do not occur. The second and third epochs are when the time scale for resonant oscillations is short compared to the weak interaction time scale, and where oscillations do occur but the final state distribution functions thermalize (either rapidly or slowly) and hence are Fermi-Dirac when primordial nucleosynthesis commences. The fourth epoch is when the oscillations occur after weak decoupling and hence the distribution functions are not Fermi-Dirac upon entering into the nucleosynthesis epoch. In § IV the modification to the bounds on neutrino degeneracy parameters are discussed. Finally, in § V we present our conclusions.

## II. NEUTRINO OSCILLATIONS IN THE EARLY UNIVERSE

### *a) Energy Densities*

We begin by reviewing the definitions of number and energy distributions for the different particle species present in the early universe. Assuming a thermal distribution, the number density of particles of type $i$ with momentum between $p$ and $p + dp$ is given by

$$N_i(p)dp = \frac{1}{2\pi^2(\hbar c)^3}\, g_i p^2\left[\exp\left(\frac{E_i(p) - \mu_i}{kT}\right) \pm 1\right]^{-1} dp , \tag{4}$$

where $E_i(p) = \sqrt{p^2 + m_i^2}$ is the energy of the particle, the $\pm$ sign is negative for bosons and positive for fermions, and $g_i$ is the number of spin states of the particle ($g = 1$ for neutral leptons, $g = 2$ for charged leptons and photons).

Since photons do not carry any conserved quantum numbers they do not have a chemical potential, i.e., $\mu_\gamma = 0$. For particles that carry a conserved quantum number the chemical potential of a particle is equal and opposite to that of its antiparticle, i.e., $\mu_i = -\mu_{\bar{i}}$. Since observationally we know that $B \sim 10^{-9}$, the charge neutrality of the universe determines that presently the chemical

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1991ApJ...368....1S



potentials of baryons and charged leptons are $\sim 10^{-13}$ eV. However, as discussed in the introduction we have no direct information regarding $n_{\nu_i} - n_{\bar{\nu}_i}$ and consequently the chemical potentials of the neutrino distributions are undetermined. In the standard big bang scenario they are simply set identically to zero.

In the context of the standard big bang, with all the chemical potentials set to zero, the contribution to the energy density of the universe by the neutrinos is given by

$$\rho_{\nu_i+\bar{\nu}_i} = \int dq \sqrt{q^2 + m_i^2}[N_{\nu_i}(q) + N_{\bar{\nu}_i}(q)] , \tag{5a}$$

which can be shown to reduce to (assuming relativistic particles)

$$\rho_{\nu_i+\bar{\nu}_i} \approx \frac{7}{8} \frac{\pi^2}{15(\hbar c)^3} (kT)^4 , \tag{5b}$$

where $T$ is the temperature of the neutrino gas. For temperatures $T \gtrsim 1$ MeV, the neutrino gas and background plasma are in thermal equilibrium ($T_\nu = T_\gamma$). The contributions from photons $\rho_\gamma$ and from extremely relativistic charged leptons $\rho_{i+\bar{i}}$ are given by

$$\rho_\gamma = \frac{4}{7} \rho_{i+\bar{i}} = \frac{\pi^2}{15(\hbar c)^3} (kT)^4 . \tag{5c}$$

However, with nonzero neutrino chemical potentials the contribution to the energy density can be altered dramatically. In terms of the degeneracy parameters $\zeta_i$, the lepton number densities are given by

$$n_i - n_{\bar{i}} = \int dq[N_{\nu_i}(q) - N_{\bar{\nu}_i}(q)] = \frac{1}{2\pi^2} \left(\frac{kT}{\hbar c}\right)^3 [\mathscr{F}_2(\zeta_i) - \mathscr{F}_2(-\zeta_i)] , \tag{6a}$$

where the standard Fermi integrals are

$$\mathscr{F}_2(x) = \int_0^\infty dy\, y^2[\exp(y - x) + 1]^{-1} , \tag{6b}$$

and from which it can be shown that

$$L_i = \frac{n_i - n_{\bar{i}}}{n_\gamma} \approx 6.8 \times 10^{-2} \zeta_i(\pi^2 + \zeta_i^2) . \tag{7}$$

Also, it can be shown that the contribution to the energy density is

$$\rho_{\nu_i+\bar{\nu}_i} = \frac{\pi^2}{15(\hbar c)^3} (kT)^4 \left(\frac{7}{8} + \frac{15}{4\pi^2} \zeta_i^2 + \frac{15}{8\pi^4} \zeta_i^4\right) , \tag{8}$$

which in the limit of large degeneracy reduces to

$$\rho_{\nu_i+\bar{\nu}_i} \approx \frac{1}{8\pi^2(\hbar c)^3} (kT)^4 \zeta_i^4 . \tag{9}$$

It can be seen from equations (5) and (9) that in the limit of large degeneracies, the neutrinos dominate the energy density of the universe.

### *b) Neutrino Oscillations*

The neutrino mass eigenstates are related to the weak interaction flavor eigenstates by a unitary transformation. In the real world there are three neutrino flavors (Aarnio *et al.* 1989; Abrams *et al.* 1989; Adeva *et al.* 1989*a*, *b*; Akrawy *et al.* 1989; Decamp *et al.* 1989*a*, *b*; Dorfan *et al.* 1989) and hence three mass eigenstates. The unitary transformation $U$ contains four free parameters: three angles $\theta_k$, and one CP violating phase $\delta$ which is the analog of the Kobayashi-Maskawa matrix (Kobayashi and Maskawa 1973) that appears in the quark sector. For the purposes of this work, we treat the three neutrino system as an "effective" two neutrino system, $\nu_e$ and $\nu_x$ (i.e., $i = e, x$) with only one vacuum mixing angle $\theta_V$ (defined in the first quadrant), and hence ignore the possibility of CP violation in the neutrino sector. The vacuum mass eigenstates $|\nu_1\rangle, |\nu_2\rangle$ with masses $m_1$ and $m_2$, respectively, can be related to the flavor eigenstates $|\nu_e\rangle, |\nu_x\rangle$ by

$$|\nu_1\rangle = \cos\theta_V |\nu_e\rangle - \sin\theta_V |\nu_x\rangle , \tag{10a}$$

$$|\nu_2\rangle = \sin\theta_V |\nu_e\rangle + \cos\theta_V |\nu_x\rangle , \tag{10b}$$

and any arbitrary neutrino state $|\nu_a(p, t)\rangle$ can be written as

$$|\nu_a(p, t)\rangle = \nu_e(p, t)|\nu_e\rangle + \nu_x(p, t)|\nu_x\rangle , \tag{11a}$$

where

$$|\nu_e(p, t)|^2 + |\nu_x(p, t)|^2 = 1 \;\forall p, t . \tag{11b}$$

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1991ApJ...368....1S



The time evolution of the two neutrino system is given by

$$i \frac{\partial}{\partial t}\begin{bmatrix} \nu_e(p, t) \\ \nu_x(p, t) \end{bmatrix} = H(p, t)\begin{bmatrix} \nu_e(p, t) \\ \nu_x(p, t) \end{bmatrix}, \tag{12a}$$

where

$$H(p, t) = \left[p + \frac{1}{4p}(m_1^2 + m_2^2) + \frac{A}{4p}\right]\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{4p}\begin{pmatrix} \Delta_{\text{eff}} - \Delta\cos(2\theta_V) & \Delta\sin(2\theta_V) \\ \Delta\sin(2\theta_V) & \Delta\cos(2\theta_V) - \Delta_{\text{eff}} \end{pmatrix}, \tag{12b}$$

is the effective Hamiltonian of the neutrino system. The coefficient $A$ is proportional to the trace of the effective mass matrix that results from charged and neutral current interactions on the background plasma. The coefficients $\nu_i(p, t)$ have different values for different neutrino momenta and hence the extra argument $p$. We have also expanded the neutrino energy in the extreme relativistic limit as $E_i \approx p + m_i^2/2p$ for the eigenstates with mass $m_i$ and momentum $p$. The equation governing the time evolution of the two component antineutrino system can be obtained from equation (12) upon substitution of $\nu_i \rightarrow \bar{\nu}_i$ and $\Delta_{\text{eff}} \rightarrow -\Delta_{\text{eff}}$.

The difference between the square of the vacuum masses is defined as $\Delta \equiv m_2^2 - m_1^2$. The contribution to the difference between the effective masses for the two-component system due to the background plasma is $\Delta_{\text{eff}}$, which we now present. There are contributions from the weak charged currents $\Delta_{\text{eff}}^{W^\pm}$ mediated by $W^\pm$ bosons and also from weak neutral currents $\Delta_{\text{eff}}^{Z^0}$ mediated by $Z^0$ bosons. The contributions to $\Delta_{\text{eff}}^{W^\pm}$ arising from the forward scattering amplitudes of the Feynman diagrams shown in Figure 1 are found to be

$$\Delta_{\text{eff}}^{W^\pm} = 2\sqrt{2}G_F[n_{e^-}(t) - n_{e^+}(t) - n_{x^-}(t) + n_{x^+}(t)]p, \tag{13}$$

where $G_F$ is the Fermi coupling constant, and the number densities of charged leptons, $n_i(t)$, are independent of neutrino oscillations. These are the effective mass contributions considered in the *MSW* solution to the solar neutrino problem. Where there is an appreciable number density of neutrinos (e.g., in supernova cores) there is an additional contribution to the neutrino effective mass differences from neutral current processes (Fuller *et al.* 1987). The forward scattering amplitudes of the Feynman diagrams shown in Figure 2 compose $\Delta_{\text{eff}}^{Z^0}$, and we find that

$$\Delta_{\text{eff}}^{Z^0} = 2\sqrt{2}G_F[n_{\nu_e}(t) - n_{\bar{\nu}_e}(t) - n_{\nu_x}(t) + n_{\bar{\nu}_x}(t)]p, \tag{14}$$

where the $n_{\nu_i}(t)$, $n_{\bar{\nu}_i}(t)$ are the number densities of $\nu_i$ and $\bar{\nu}_i$ whose time dependence arises not only from the expansion of the universe but also from the oscillations themselves. The difference between the charged lepton densities is determined by the charge neutrality of the universe

$$n_{e^-} - n_{e^+} + n_{x^-} - n_{x^+} = n_p, \tag{15}$$

where $n_p$ is the proton number density. Since $n_p/n_\gamma \sim 10^{-9}$, we find that the neutral currents make the dominant contributions to the effective mass differences between the neutrino species, that is $\Delta_{\text{eff}}^{Z^0} > \Delta_{\text{eff}}^{W^\pm}$ for $L_i \gtrsim 10^{-9}$ (note that if $L_i = 0$, then, neglecting any higher order corrections, we have $\Delta_{\text{eff}}^{Z^0} + \Delta_{\text{eff}}^{W^\pm} = 0$). Our expressions for $\Delta_{\text{eff}}^{W^\pm}$ and $\Delta_{\text{eff}}^{Z^0}$ neglect finite temperature corrections of order $(T/M_W)^2$ where $M_W$ is the mass of the weak charged gauge boson, $\sim$80 GeV. At a temperature of 1 MeV such a correction becomes important only when $L_i \leq 10^{-9}$ for all species. Since we will be dealing with degeneracies many orders of magnitude greater than this, we neglect such terms.

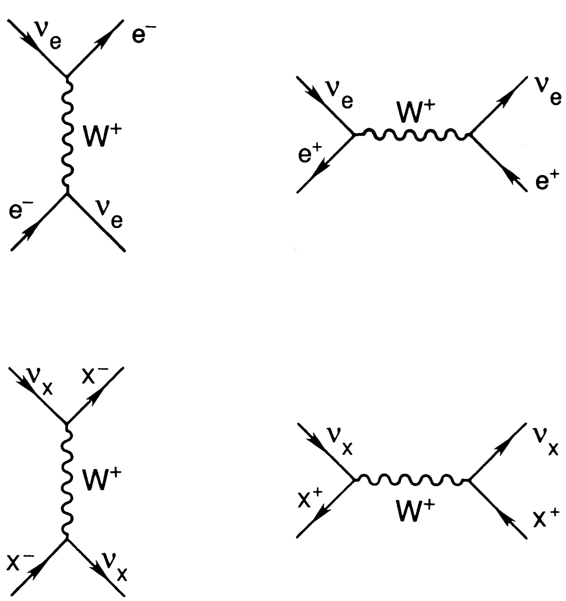


FIG. 1.—Feynman diagrams involving the weak charged current that contributes to the effective mass of neutrinos in a background plasma.

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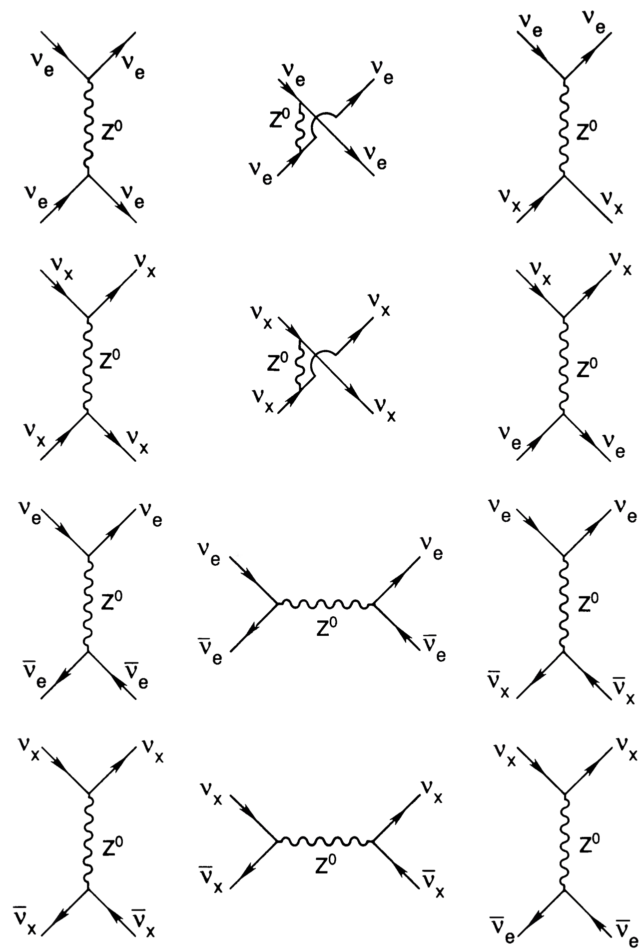


FIG. 2.—Feynman diagrams involving the weak neutral current that contributes to the effective mass of neutrinos in a background plasma. Contributions from neutral current interactions on the charged leptons are neglected since they are identical for each neutrino flavor.

The initial conditions determine whether the neutrinos or the antineutrinos resonantly transform. If $\Delta > 0$ and $\zeta_x > 0$, or $\Delta < 0$ and $\zeta_e - \zeta_x < 0$, then the neutrinos will resonantly transform, $\nu_e \leftrightarrow \nu_x$. The formalism that follows can be used to analyze all these scenarios in a similar fashion. We consider the scenario where $\Delta > 0$ and $\zeta_e > \zeta_x \gtrsim 10^{-9}$ initially for the purposes of our discussion. The condition for resonant oscillations to occur is that the diagonal elements of the mass matrix in equation (12) are equal; that is,

$$\Delta_{\rm eff}^{W^\pm} + \Delta_{\rm eff}^{Z^0} = \Delta \cos(2\theta_V) , \tag{16}$$

from which we see that the neutrino energy (to leading order in the masses) at which resonance occurs is given by

$$E^{\rm res} = \frac{\Delta \cos(2\theta_V)}{2\sqrt{2}G_{\rm F}\, Y_\nu(t)} , \tag{17}$$

where

$$Y_\nu(t) = n_{\nu_e}(t) - n_{\nu_x}(t) - n_{\bar{\nu}_e}(t) + n_{\bar{\nu}_x}(t) . \tag{18}$$

The width, $\Gamma$ (in units of mass$^2$), of the resonance is determined by the magnitude of the off-diagonal elements and is given by

$$\Gamma = \Delta \sin(2\theta_V) . \tag{19}$$

We note that resonant neutrino transformations may change $Y_\nu(t)$, and this in turn changes the resonance energy in equation (17). To see how the neutrino number densities and distributions change we need to examine the probability of transformation at the resonance. This depends on the degree to which the transformations are adiabatic.

## III. ADIABATIC CONDITIONS

So far we have only discussed the kinematic conditions required for resonant oscillation to occur. For a significant fraction of one type of neutrino to transform into another type, the system must change adiabatically when the resonant conditions are satisfied. There are two intrinsic time scales which determine how the neutrinos transform at resonance. Let $t_r$ be the time of one oscillation in

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the neutrino system when the resonance conditions are satisfied. Also, let the energy time scale, $t_E$, be the time for the kinematic conditions to change such that $\Delta_{\rm eff}$ is removed from its resonance value by the width of the resonance, $\Gamma$, that is,

$$|\Delta_{\rm eff} - \Delta_{\rm eff}^{\rm res}| = \Gamma \, . \tag{20}$$

For complete transformation of one type of neutrino into the other type it is required that

$$t_r \ll t_E \, . \tag{21}$$

We can find $t_r$ from the mass matrix of equation (12); viz.,

$$t_r = 8.3 \times 10^{-9} \frac{E}{\Delta \sin(2\theta_V)} \, {\rm s} \, , \tag{22}$$

where the neutrino energy $E$ is in MeV and $\Delta$ is in eV$^2$. Finding the time scale for variation in the resonant condition is somewhat more complicated. We can reexpress equation (18) as

$$Y_\nu(t) = \left[\frac{R(t_0)}{R(t)}\right]^3 Y_\nu^0(t) \, , \tag{23}$$

where $Y_\nu^0(t)$ represents the change in the number densities due to oscillations alone, $R(t)$ is the scale factor of the universe, and $t_0$ is some time prior to the onset of resonant transformations.

We can now identify four regions in the subsequent evolution of the neutrino distribution functions, as shown in Figure 3. At very high temperatures the weak equilibrium time scale, $t_w$, is is much shorter than $t_r$ and no oscillations take place (region I of Fig. 3). The second region of Figure 3 corresponds to $t_r < t_w < t_t$, where $t_t$ is the neutrino transformation time scale defined as the time required for the resonance condition to sweep through a significant fraction of the neutrino distribution. A third region occurs when $t_w > t_t > t_r$. Although resonant transformations occur in both the second and third regions, the total number of neutrinos with momentum between $q + dq$ is conserved during the transformation process in the latter region, whereas it is not in the former. Finally, if the neutrino transformations occur after decoupling (region IV) but prior to nucleosynthesis, the neutrino distribution functions will not be Fermi-Dirac upon entering the nucleosynthesis era.

In the final two time epochs (regions III and IV of Fig. 3) $Y_\nu^0(t)$ can be written as

$$Y_\nu^0(t) = n_{\bar{\nu}_x}(t_0) - n_{\bar{\nu}_e}(t_0) + \int dq[|v_e(q, t)|^2 - |v_x(q, t)|^2][N_{\nu e}(t_0, q) + N_{\nu x}(t_0, q)] \, , \tag{24}$$

where $v_i(q, t)$ are the coefficients from equation (12). We cannot write a simple analytic expression for $Y_\nu^0(t)$ in regime II because during this period explicit solution of the relevant Boltzmann equation is required. Since $\Delta_{\rm eff}$ is proportional to $Y_\nu(t)$, we find that, for small variations in $\Delta_{\rm eff}$,

$$\frac{\delta\Delta_{\rm eff}}{\Delta_{\rm eff}} = \frac{1}{Y_\nu(t)} \frac{dY_\nu(t)}{dt} \delta t \, . \tag{25}$$

Evaluating at the resonance conditions, letting $|\delta\Delta_{\rm eff}| = \Gamma$, and identifying $\delta t$ with $t_E$, we find that

$$t_E = \left| \left[\frac{1}{Y_\nu(t)} \frac{dY_\nu(t)}{dt}\right]^{-1} \right| \tan(2\theta_V) \, . \tag{26}$$

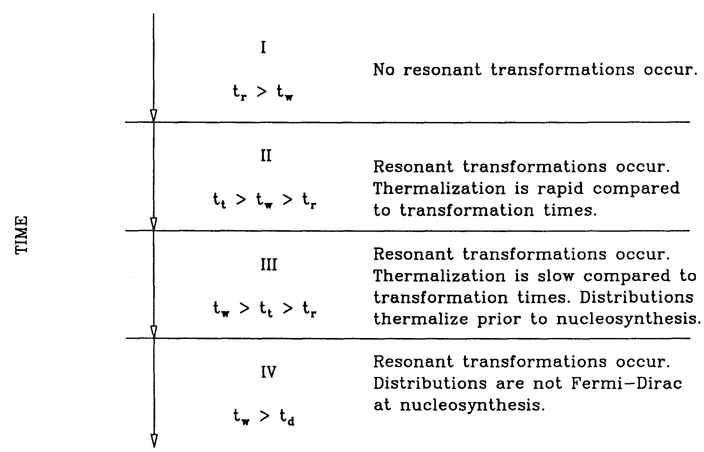


FIG. 3.—Modifications of the neutrino distribution functions in four different time epochs. Here $t_w$, $t_r$, $t_t$, and $t_d$ are weak interaction, resonant, transformation, and universal expansion time scales, respectively.

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1991ApJ...368....1S



Therefore the adiabatic condition we require for complete transformation between neutrino species with energy $E$ is

$$\frac{4\pi E}{c\Delta \sin(2\theta_V)} \ll \left| \left[ \frac{1}{Y_\nu(t)} \frac{dY_\nu(t)}{dt} \right]^{-1} \right| \tan(2\theta_V) . \tag{27}$$

If equation (27) is satisfied, neutrinos with energy between $E[1 - \tan(2\theta_V)]$ and $E[1 + \tan(2\theta_V)]$ are transformed with unit probability.

It follows from equation (23) that

$$\frac{1}{Y_\nu(t)} \frac{dY_\nu(t)}{dt} = -3 \frac{1}{R(t)} \frac{dR(t)}{dt} + \frac{1}{Y_\nu^0(t)} \frac{dY_\nu^0(t)}{dt} . \tag{28}$$

The first term on the right-hand side of equation (28) is the contribution from the expansion rate to the variation in number densities. This term is completely determined from the Einstein field equations, which are not modified by the transformation of one neutrino flavor into another because the energy density is conserved. In a radiation-dominated epoch it is easy to show from the Einstein field equations that

$$\frac{1}{R(t)} \frac{dR(t)}{dt} = \sqrt{\frac{8\pi G\rho}{3}} , \tag{29}$$

where $G$ is the gravitational constant, and $\rho$ is the energy density of the universe (although at this point we are discussing an "effective" *two* neutrino oscillating system, we will assume three neutrino species contribute to $\rho$). The second term on the right-hand side of equation (28) is due to the modification in number densities arising from neutrino transformations.

The adiabaticity requirement of equation (27) is only applicable at the onset of neutrino transformations and necessarily breaks down as the transformations proceed. To show this we consider the case of large chemical potentials (i.e., neglecting antineutrinos). Further, assume that initially $n_{\nu_e} - n_{\nu_x}$ is large enough so that $E^{\rm res}$ is small compared with the chemical potential,

$$E^{\rm res} = \frac{\Delta \cos(2\theta_V)}{2\sqrt{2}G_{\rm F}[n_{\nu e(t_0)} - n_{\nu x(t_0)}]} \ll \mu . \tag{30}$$

The flavor transformations decrease $n_{\nu_e} - n_{\nu_x}$ and hence increase $E^{\rm res}$. Therefore, the energy of resonance sweeps through the neutrino energy distributions, transforming one type of neutrino into the other if the adiabatic condition holds. However, when $(n_{\nu_e} - n_{\nu_x}) \to 0$ the energy process ceases. This means that for a distribution with an initially large chemical potential for which the transformations started out adiabatically, somewhere in the distribution this condition failed and $f(n_{\nu_e} - n_{\nu_x})$ of the $\nu_e$ transformed $(0 < f < \frac{1}{2})$. It follows that the maximal transformation scenario results in equal number densities of both neutrino species (i.e., $f = \frac{1}{2}$).[1] This is to be contrasted with the scenario where the adiabatic condition had been satisfied throughout all the transformations, in which case the energy distributions would have simply swapped ($f = 1$).

Let us now estimate the temperature above which the weak interactions occur sufficiently often to scramble the phase information in the neutrino system and prevent resonant oscillations from occurring. This is the first regime depicted in Figure 3. The rate of weak interactions in the plasma is $\Gamma_w \sim \Sigma \sigma_w^i n_i c$, where $\sigma_w^i$ is the cross section for the weak interaction of a neutrino with particles of type $i$ in the plasma. In the case where $\Sigma \zeta_i^3 \lesssim 30$ we find that for temperatures much lower than the mass of the muon

$$\Gamma \approx \frac{G_F^2}{\hbar^7 c^6} (kT)^4 E . \tag{31}$$

The time scale for weak interactions to occur in this regime is then

$$t_w \approx \frac{2.5}{ET^4}\ {\rm s} , \tag{32}$$

where $T$ and $E$ are subsequently in MeV. For the resonance not to be "washed out" by weak interactions we require that

$$t_w \gg t_r , \tag{33}$$

where $t_r$ is defined in equation (22). We introduce the parameter $y$ such that $E = yT$ is the energy of the neutrino and find that

$$T \ll 30 \left[ \frac{\Delta \sin(2\theta_V)}{y^2} \right]^{1/6} . \tag{34}$$

Therefore, we see that a broad range of vacuum mixing angles and vacuum masses allows resonant neutrino oscillations to occur prior to nucleosynthesis.[2] If they occur before the neutrinos thermally decoupled, which takes place at temperature $T_d$, then the distributions will thermalize to Fermi-Dirac distributions (regions II and III of Fig. 3). In a universe with zero lepton numbers the decoupling temperature for the $\nu_e$ is $\sim$2 MeV, and for the $\nu_{\mu,\tau}$ is $\sim$3.5 MeV (Freese, Kolb, and Turner 1983). However, these

[1] Physically this represents the fact that when the number densities are equal, the two neutrino flavors have identical interactions with the plasma, and therefore there is no effective mass difference between the flavors.

[2] The onset of nucleosynthesis is a function of the initial degeneracies. However, nucleosynthesis generally takes place between $0.1 \lesssim T \lesssim 1$ MeV, and for the point of our discussion we will assume that it all occurs instantaneously at $T = 0.5$ MeV.

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1991ApJ...368....1S



decoupling temperatures are altered in the presence of nonzero lepton numbers, for example if $\zeta \gg 1$ (for the remaining discussion of this section only one degeneracy parameter is assumed nonzero), then

$$T_d = \beta\zeta^{-2/3} \exp\left(\frac{\zeta}{3}\right) \text{ MeV} , \tag{35}$$

where $\beta = 6$ for $\nu_e$, and 10 for $\nu_{\mu,\tau}$.

To determine what range of values $\Delta$ and $\theta_V$ can take in order for resonance conditions to be satisfied we set the diagonal elements in the mass matrix to be equal:

$$2\sqrt{2}G_F\, Y_\nu(t)p = \Delta \cos(2\theta_V) . \tag{36}$$

If we assume that the initial difference in number densities is dominated by only one of the flavors, then for a neutrino of energy $E = yT$ we find from equations (7) and (36) that

$$\Delta \cos(2\theta_V) = 5.4yT^4\zeta\left(1 + \frac{\zeta^2}{\pi^2}\right) \text{eV}^2 . \tag{37}$$

Further, for the adiabatic condition to be satisfied in this limit we have from equations (5), (8), (27), and (29),

$$\Delta \sin(2\theta_V) \tan(2\theta_V) \gg 2 \times 10^{-8} yT^3 \sqrt{1 + \frac{30}{43\pi^2}\zeta^2 + \frac{15}{43\pi^4}\zeta^4} \text{ eV}^2 . \tag{38}$$

The expression only holds if the expansion rate dominates the change of number densities, that is, only when the first term on the right-hand side of equation (28) makes the dominant contribution, such as at the beginning of neutrino oscillations. This condition will be modified when the neutrino number densities have undergone significant change resulting from resonant transformations.

We now wish to consider the range of $\Delta$ for which neutrino oscillations can occur over a significant range of the neutrino energy spectrum (i.e., significant range of $y$) prior to nucleosynthesis. For example, assuming $\zeta \sim 1$ and $\theta_V \ll 1$ in equation (37), we have

$$\Delta \approx 5.4yT^4 \text{ eV}^2 . \tag{39}$$

Equation (40) shows that for a given value of $\Delta$ the resonance condition is satisfied for ever increasing values of $y$ as the universe expands and cools. However, at $T \sim 0.5$ MeV nucleosynthesis commences, and any change in the neutrino distributions at temperatures much below this will not effect the nucleosynthesis calculations. For example, if $\Delta = 0.1$ eV$^2$ we see from equation (39) that the value of $y$, at the time of nucleosynthesis, which we call $y_c$, is given by $y_c \approx 0.3$. Alternatively, if $\Delta = 10^{-4}$ eV$^2$, then $y_c \approx 3 \times 10^{-4}$. Clearly in the former case a significant portion of the neutrino spectrum is affected prior to nucleosynthesis, whereas in the latter case a vanishingly small portion of the spectrum is affected by the neutrino oscillations.

This effect can be seen more clearly from Figure 4 where we have utilized equation (37) in plotting the value of $\Delta$ against $F$, the number fraction of neutrinos which transform prior to nucleosynthesis, for different values of $\zeta$ (again we have adopted $\theta_V \ll 1$). The number $F$ is defined as $F = y_c^3$ for $|\zeta| \lesssim 1$, and $F = (y_c/|\zeta|)^3$ for $|\zeta| \gtrsim 1$. Figure 4 shows that for large values of $\Delta$ ($\gtrsim 0.01$ eV$^2$) significant alteration over a large range of the neutrino spectrum can take place for a wide range of $\zeta$. However, for small values of $\Delta$ ($\lesssim 0.01$ eV$^2$) alteration of only a small portion of the neutrino spectrum occurs except in the regime of small initial degeneracy, $\zeta \lesssim 0.01$. Such small degeneracies have little effect on the nucleosynthesis calculations (e.g., Terasawa and Sato 1988). Therefore, although the region of small $\Delta$ and small $\zeta$ can affect the neutrino distributions, the subsequent effect on the nucleosynthesis will be unimportant.

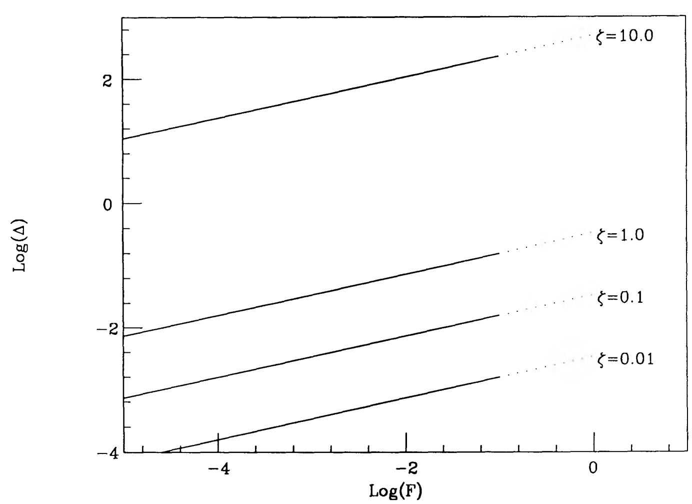


FIG. 4.—Value of the mass difference $\Delta$ (in eV$^2$) is shown plotted against the fractional quantity $F$ (a measure of the amount of neutrino transformation prior nucleosynthesis) for different values of the initial degeneracy parameter $\zeta$. Dashed lines indicate the approximate region where our analytical solution breaks down due to significant evolution of the neutrino system.

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1991ApJ...368....1S



We again stress the important point that due to the alteration of the neutrino spectrum by oscillations, the value of $\zeta$ used in equation (37) can only be representative of the initial spectrum. As the oscillations progress, the value of $\zeta$ will change. In the case of oscillations after weak decoupling, the distributions become non–Fermi-Dirac, and the meaning of $\zeta$ becomes ill defined. Such considerations will only be of importance, however, at large values of $F$ ($\gtrsim 0.1$), since for small $F$ the deviation from the initial neutrino spectrum will be small. That is, our analytical solution necessarily breaks down at large $F$ (indicated by the dashed lines of Fig. 4) due to significant evolution of the neutrino system.

In order for the oscillations to proceed we must also check that our temperature constraint of equation (34) and our adiabaticity condition of equation (38) must be satisfied. For the range of parameters shown in Figure 4 it can be seen from equation (34) that for $\theta_V \gg 10^{-6}$ the temperature constraint is satisfied for times before nucleosynthesis (Fig. 4 is valid for $\theta_V \lesssim 0.1$). In Figure 4 the adiabaticity condition is satisfied for $\theta_V \gtrsim 10^{-4}$, except in the lower right of the diagram ($F \gtrsim 10^{-2}$, $|\zeta| \lesssim 0.01$) where values of $\theta_V \gtrsim 10^{-2}$ are required.

The region $\Delta < 10^{-4}$ eV$^2$ (off scale in Fig. 4) represents the small values of $\Delta$ which would be consistent with the neutrino oscillation solution to the solar neutrino problem (Parke and Walker 1986). We can see that only in the somewhat less interesting region of low $\zeta$ would there be a regime which would allow for neutrino oscillations in both the solar interior and the early universe. Therefore, in this effective two-neutrino model significant alteration of both the neutrino spectrum and nucleosynthesis requires values of $\Delta$ much larger than those necessary for solar neutrino oscillations.

When we return to considering three flavors of neutrinos, however, a particularly interesting scenario arises. If $m_{\nu_\tau} \sim 10$ eV, $m_{\nu_\mu} \sim 10^{-2}$ eV, and $m_{\nu_e} \sim 10^{-4}$ eV, then this would give rise to $\Delta_{\tau,e} \sim \Delta_{\tau,\mu} \sim 100$ eV$^2$, whereas $\Delta_{e,\mu} \sim 10^{-4}$ eV$^2$. For small mixing angles, such $\Delta$'s lead to alterations in the nucleosynthesis limits ($\nu_{e,\mu} \leftrightarrow \nu_\tau$) on the $\zeta_i$ and would also be consistent with an MSW solution to the solar neutrino problem ($\nu_e \leftrightarrow \nu_\mu$).

## IV. CONTRAINTS ON THE DEGENERACY PARAMETERS

If resonant neutrino oscillations do occur prior to nucleosynthesis, then in general no constraints can be derived from the limits obtained from the degenerate universe nucleosynthesis studies, other than those imposed by neutrino number conservation and energy conservation. For Fermi-Dirac distributions these can be expressed as

$$T^3 \sum_i \zeta_i \left(1 + \frac{\zeta_i^2}{\pi^2}\right) = T'^3 \sum_i \zeta_i' \left(1 + \frac{\zeta_i'^2}{\pi^2}\right) \quad \text{(number conservation)}\,, \tag{40a}$$

$$T^4 \left(\frac{43}{8} + \frac{15}{4\pi^2} \sum_i \zeta_i^2 + \frac{15}{8\pi^4} \sum_i \zeta_i^4\right) = T'^4 \left(\frac{43}{8} + \frac{15}{4\pi^2} \sum_i \zeta_i'^2 + \frac{15}{8\pi^4} \sum_i \zeta_i'^4\right) \quad \text{(energy conservation)}\,, \tag{40b}$$

where the $\zeta_i'$ are the degeneracy parameters which exist after the resonance transformations have occurred. Equations (40a) and (40b) cannot be simultaneously satisfied for arbitrary $\zeta_i$ unless the temperature, $T'$, of the plasma after transformations is greater than that prior to transformations. That is, the neutrino oscillations give rise to a reheating of the universe as entropy is generated from the equilibrium of the chemical potentials. Note that such reheating does not occur when the transformations take place after decoupling, or in the case where initially $\zeta_e = \zeta_x$.

The extraction of limits on the initial degeneracy parameters from those derived from previous nucleosynthesis studies is complicated by the reheating mechanism. This is due to the perturbation in the temperature history of the universe, relative to that derived for nonevolving degenerate neutrino distributions. If resonance occurs after decoupling (and hence there is no reheating), then extraction of meaningful degeneracy limits remains complicated since the neutrino distributions present during nucleosynthesis are no longer Fermi-Dirac. In this case the concept of a chemical potential has no direct relation to the neutrino energy distributions. Since the limits provided by nucleosynthesis in previous studies have used Fermi-Dirac distributions throughout, their interpretation when oscillations are included becomes inapplicable.

In principle, one could derive new limits of the initial neutrino degeneracy from a much more detailed study than that reported here. For example, for given values of $\Delta$, $\theta_V$, and $\zeta_i$, one could numerically evolve the two-component neutrino system from the mass matrix in equation (12) and determine the final energy distributions and effective degeneracy parameters at the onset of nucleosynthesis. The abundances of the light elements could be found for each set of parameters and then compared with the observed abundances, thereby constraining the initial values for $\zeta_i$. However, it is clear from the previous discussions that the energy distributions of the neutrinos will be complicated, nonlinear functions of $\Delta$, $\theta_V$, and $\zeta_i$, all of which are unknown parameters. Such a procedure will clearly be cumbersome and computer intensive and at the present time is not warranted. It is the purpose of the present work simply to show that the previous constraints on $\zeta_i$ are not valid in the presence of neutrino oscillations.

Neutrino oscillations in the standard big bang have previously been analyzed by Langacker *et al.* (1987). These authors showed that the $\nu_e - \nu_\mu$ asymmetry generated by $e^+e^-$ annihilations prior to $\nu_e$ decoupling (but post $\nu_\mu$ decoupling) was altered by neutrino oscillations, thereby giving rise to small modifications in the $^4$He mass fraction ($\sim 0.001$). However, their analysis considered only the charged current contributions to the effective mass matrix. When the neutral current contributions are included we find from equations (13) and (14) that (to leading order) $\Delta_{\rm eff} = 0$ in a universe with $L_i = 0$ (see also Fukugita *et al.* 1988). In these circumstances no alteration in the $^4$He mass fraction would be expected. In order to obtain a change in the $^4$He abundance similar to that derived by Langacker *et al.* (1987), one must postulate an *initial* degeneracy $\zeta_i$ of order $10^{-3}$.

## V. CONCLUSIONS

We have investigated the effect of resonant neutrino oscillations on the previously derived limits for the leptonic charge of the universe. We have shown that for a large range of $\Delta$ and $\theta_V$ significant neutrino flavor oscillations can occur in the early universe

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1991ApJ...368....1S



which subsequently alter the degeneracy of individual neutrino species. We note that the hierarchy of masses in the three-neutrino model provide the possibility of neutrino oscillations in the parameter space which significantly effect nucleosynthesis, as well as simultaneously allowing for neutrino oscillations in the solar interior.

As noted in the introduction the effects of neutrino degeneracy can influence the yields of nuleosynthesis in two ways. The excess energy density associated with neutrino degeneracy and the consequences of electron neutrino degeneracy on the weak interaction rates both directly perturb the nucleosynthesis yields away from the standard big bang predictions. Consider the situation where $\Delta > 0$ and $\zeta_e > \zeta_x > 10^{-9}$, as discussed in the text. We have found from our analysis above that for $\zeta \gtrsim 0.1$ values of $\Delta \gtrsim 0.01$ eV$^2$ and $\theta_V \gtrsim 10^{-4}$ will allow both the temperature and adiabatic constraints to be satisfied. In such circumstances a significant fraction of the neutrinos will undergo oscillations into alternate flavors prior to the nucleosynthesis epoch. Our analysis further showed that in the limit of large degeneracy oscillations will tend to reduce any asymmetry between the degeneracies of each neutrino flavor, i.e., $\zeta_e \to \zeta_x$ and $\zeta_x \to \zeta_e$. It is known from previous studies (e.g., Terasawa and Sato 1988) that if the individual degeneracies are large and approximately equal, then no agreement between the calculated and observationally inferred primordial abundances can be obtained. Equal degeneracies at the onset of nucleosynthesis, however, was only obtained as an asymptotic limit in our analysis. We discussed how violation of the adiabatic conditions would most likely occur before the asymptotic solution of equal degeneracies was obtained. Prior to nucleosynthesis then, the neutrino oscillation will have reduced, but not completely eradicated, the asymmetry between the different neutrino degeneracies. As alluded to in the introduction, many studies of neutrino degeneracy, which include asymmetry between the degeneracies of each neutrino flavor, have previously been carried out. Such studies show that allowed regions of the degeneracy parameter space allow for compatibility with the observed primordial abundances. There is no point in repeating these calculations here. We simply wish to point out that if neutrino oscillations occurring in the early universe perturb the initial neutrino degeneracies, this in turn alters the final nucleosynthesis yields.

Since resonance conditions can occur prior to the nucleosynthesis epoch, it becomes difficult to determine limits on the original neutrino asymmetry of the universe from studies of primordial nucleosynthesis. This is compounded by two effects. The first of these is the reheating of the universe arising from the equilibration of the neutrino distribution functions. Second, the resonance may occur both before or after weak decoupling, and in the latter case the neutrino oscillations will give rise to non–Fermi-Dirac energy distributions. We conclude that in the presence of neutrino oscillations the best limits on the degeneracy of the universe are not in fact derived from nucleosynthesis studies but from the age of the universe, $|\zeta_i| \leq 140$. These latter constraints are significantly less stringent than those imposed by nucleosynthesis studies without neutrino oscillations.

Finally, we have found that in a degenerate universe the dominant contribution to the neutrino effective matrix arises not from the charged current interaction but from the neutral current interaction between neutrinos. It would be interesting to investigate the effect of these latter interactions in the study of neutrino interactions in other astrophysical contexts.

Following completion of this work we became aware of complimentary work by Enqvist, Kainulainen, and Maalampi (1990). They examine the effects of oscillations between sterile neutrinos (those that do not transform under the standard model gauge group) and electron-type neutrinos on nucleosynthesis. Although they follow a similar strategy in developing the analysis of the oscillations, their results cannot be compared with ours as we only deal with known neutrinos (a component of a weak isodoublet under the standard model gauge group).

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## REFERENCES


Aarnio, P., *et al.* 1989, *Phys. Letters*, B, **231**, 539.
Abrams, G. S., *et al.* 1989, *Phys. Rev. Letters*, **63**, 724.
Adeva, B., *et al.* 1989*a*, *Phys. Letters B*, **231**, 509.
———. 1989*b*, L3 preprint 004.
Akrawy, M. Z., *et al.* 1989, *Phys. Letters B*, **231**, 530.
Bahcall, J. N. 1989, *Neutrino Astrophysics* (Cambridge: Cambridge University Press).
Beaudet, G., and Yahil, A. 1977, *Ap. J.*, **218**, 253.
Bochkarev, A. I., Khlebnikov, S. Yu., and Shaposhnikov, M. E. 1990, *Nucl. Phys. B*, **329**, 493.
Boesgaard, A. M., and Steigman, G. 1985, *Ann. Rev. Astr. Ap.*, **23**, 319.
David, Y., and Reeves, H. 1980, in *Physical Cosmology*, ed. R. Balain, J. Audouze, and D. N. Schramm (Amsterdam: North Holland), p. 443.
Decamp, D., *et al.* 1989*a*, *Phys. Letters B*, **231**, 519.
———. 1989*b*, CERN-EP/89-169 preprint.
Dimopoulos, S., and Feinberg, G. 1979, *Phys. Rev. D*, **20**, 1283.
Dorfan, J. M., *et al.* 1989, *Phys. Rev. Letters*, **63**, 2173.
Enqvist, K., Kainulainen, K., and Maalampi, J. 1990, University of Helsinki preprint HU-TFT-90, No. 13, 22, and 29.
Faber, S., and Gallagher, J. 1979, *Ann. Rev. Astr. Ap.*, **17**, 135.
Freese, K., Kolb, E. W., and Turner, M. S. 1983, *Ap. J.*, **148**, 3.
Fry, J. N., and Hogan C. J. 1982, *Phys. Rev. Letters*, **49**, 1783.
Fukugita, M., Nötzold, D., Raffelt, G., and Silk, J. 1988, *Phys. Rev. Letters*, **60**, 879.
Fuller, G. M., Mayle, R. W., Wilson, J. R., and Schramm, D. N. 1987, *Ap. J.*, **322**, 795.
Georgi, H., and Glashow, S. L. 1974, *Phys. Rev. Letters*, **32**, 438.
Glashow, S. L. 1961, *Nucl. Phys. B*, **22**, 579.
Khlopov, M. Yu., and Petcov, S. 1981, *Phys. Letters, B*, **99**, 117.
Kobayashi, M., and Maskawa, T. 1973, *Progr. Theor. Phys.*, **49**, 652.
Kolb, E. W., and Turner, M. S. 1983, *Ann. Rev. Nucl. Part. Sci.*, **33**, 645.
———. 1987, *Mod. Phys. Letters, A2*, **5**, 285.
Kuzmin, V. A., Rubakov, V. A., and Shaposhnikov, M. E. 1985, *Phys. Letters, B*, **155**, 36.
———. 1987, *Phys. Letters, B*, **191**, 171.
Langacker, P., Petcov, S. T. Steigman, G., and Toshev, S. 1987, *Nucl. Phys. B*, **282**, 589.
Langacker, P., Segre, G., and Soni, S. 1982, *Phys. Rev. D*, **26**, 3425.
Lyttleton, R. A., and Bondi, H. 1959, *Proc. Roy. Soc. London A*, **252**, 313.
Mikheyev, S. P., and Smirnov, A. Yu. 1986, in *Proc. of the 12th Conference on Neutrino Physics and Astrophysics*, ed. T. Kitagaki and H. Yuta (Singapore: World Scientific), p. 177.
Nanopoulos, D., Sutherland, D., and Yildiz, A. 1980, *Lettere Nouvo Cimento*, **28**, 205.
Parke, S. J., and Walker, T. P. 1986, *Phys. Rev. Letters*, **57**, 2322.
Salam, A. 1968, in *Elementary Particle Physics: Relativistic Groups and Analyticity, Nobel Symposium 8*, ed. N. Svartholm (Stockholm: Almquist & Wiksell), p. 367.
Schramm, D., and Steigman, G. 1979, *Phys. Letters B*, **87**, 14.
Steigman, G. 1985, in *Nucleosynthesis and Its Implications on Nuclear and Particle Physics*, ed. J. Audouze and N. Mathieu (NATO ASI Ser. 163) p. 45.
Terasawa, N., and Sato, K. 1985, *Ap. J.*, **294**, 9.


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1991ApJ...368....1S



Terasawa, N., and Sato, K. 1988, *Progr. Theor. Phys.*, **80**, 3, 468.
Trimble, V. 1987, *Ann. Rev. Astr. Ap.*, **25**, 425.
Turner, M. 1981, *Phys. Letters B*, **98**, 145.
Wagoner, R. V., Fowler, W. A., and Hoyle, F. 1967, *Ap. J.*, **148**, 3.
Weinberg, S. 1967, *Phys. Rev. Letters*, **19**, 1264.
Wolfenstein, L. 1978, *Phys. Rev. Letters D*, **17**, 2369.
———. 1979, *Phys. Rev. Letters D*, **20**, 2634.
Yahil, A., and Beaudet, G. 1976, *Ap. J.*, **206**, 26.

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