UCRL-18225

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TWO-NUCLEON TRANSFER REACTIONS

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April 1968

Berkeley, California

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UCRL-18225
Preprint

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

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Text of an invited paper presented at the Washington Meeting of the American Physical Society.

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ABSTRACT

Qualitative features of two nucleon transfer reactions are discussed. The role of the residual interaction and the single-particle spectrum in distributing the transition strength is emphasized, showing how, among states of the same spin, it is sometimes concentrated into the lowest states while other times is spread over several. The validity and success under appropriate circumstances, of the simple direct mechanism is illustrated. In some situations higher order effects are expected to play a dominant role. A novel means of including the effects of core excitation on transfer reactions in a practical calculation is outlined.

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*This work performed under the auspices of the U. S. Atomic Energy Commission.

Although a two-nucleon transfer reaction would present a formidable calculational problem if one tried to solve it with any rigour, in fact, a very simple treatment usually works quite well.[1] We could think of the simple treatment, in which all but the transferred nucleons play a passive role, as the first term in an expansion of the T-matrix, with successive terms referring to ever more complicated processes. The expectation is that each additional complication, such as more profound rearrangements, leads to poorer overlaps and hence to weaker contributions.

For definiteness we consider the (t,p) reaction depicted in Fig. 1. The first order process (shown on the left) will contribute to the excitation of any level of A+2 which has a significant parentage based on the ground state of A. The wave function for such a state is indicated on the figure in idealized form as the product of the ground state of A with a wave function for the additional neutron pair.

On the other hand, if the state in question has as its parent an excited state of the target, as indicated on the right side of the figure, then it cannot be excited in lowest order, and we must calculate some higher order terms. For example the triton may inelastically excite the parent state, in this case Ψ_2, and then be stripped of its neutrons. The post excitation route is also available.

Even if the parentage is, in large part, based on the ground state, the usual treatment of the first order process may fail. If the inelastic transition is very strong, then the optical model wave function in the ground channel may be a poor representation of the true function within the nucleus, just where it is needed for the reaction calculation. Moreover, given a much enhanced

inelastic transition, the second order process may interfer significantly with the first, through smaller components in the wave functions.

So there are two distinct limiting circumstances when it will be necessary to calculate terms of second order, the first connected with questions of parentage, and the second with the degree of enhancement of inelastic transitions.

Fortunately it appears that for nuclei whose collectivity is not greater than that of the so-called vibrational nuclei, the higher order processes need not be called in on the second count. Therefore we consider first of all the lowest order process, the simple direct transfer.

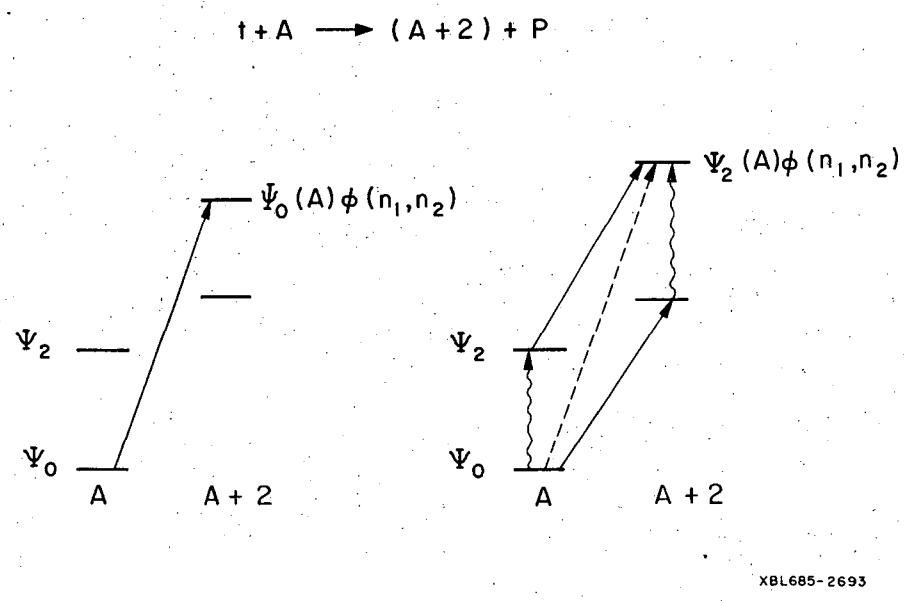

Fig. 1. First and second order processes are illustrated on left and right respectively. Wiggly arrows denote inelastic transitions and straight arrows denote two-nucleon transfer reactions.

Two features of nuclear structure effect the intensity of these reactions, and they both really involve the parentage question. How much does the state of A+2 in question look like the ground state of A plus two neutrons, and how well are those two neutrons correlated in the way they are in a triton? Remember in the triton the relative angular momentum is dominantly S and the neutron spins are coupled to zero. For brevity I shall refer to this as the ^1S correlation by which will be implied also a spatial correlation dictated by the triton size.

The answer to the question is best provided, in this context by the object

$$\tilde{\phi}_J(\underset{\sim}{R}) = \langle \phi^{S=0}_{\lambda=0}(\underset{\sim}{r},\underset{\sim}{\rho}_1,\underset{\sim}{\rho}_2)\ \Psi_0(A) | \Psi_J(A+2) \rangle$$

In the ket stands the wave function for the state of A+2 under consideration. From this is projected the part which contains A nucleons in the ground state of the target. This projection leaves a function of the remaining two neutrons. Finally from this is projected the relative motion characteristic of the pair as they exist in the triton. The function $\tilde{\phi}(R)$ describes how the center of mass of the neutron pair moves in the nucleus, when they are correlated in the same way as in the triton. I shall refer to it as the projected wave function.

This discussion should not be taken to imply that the neutron pair exists as a cluster very much of the time. The residual two-nucleon interaction is after all weak compared to the central field in the nucleus. From a shell model treatment of ^{206}Pb we find that the ground state has 16% overlap with the ^1S correlation of the triton. However a pair of like nucleons can interact only in the singlet-even and triplet-odd states. The force is attractive in

the former and in the latter it is weak or possibly repulsive. So the interaction energy of a group of neutrons does arise from the singlet- even correlations. For this reason these reactions provide a stringent test for nuclear models, for to reproduce the intensities of the reaction leading to various states in the nucleus, the structure theory must correctly predict the degree and radial distribution of the ^1S correlation in each of them. What is meant by radial distribution of this correlation is illustrated in Fig. 2.

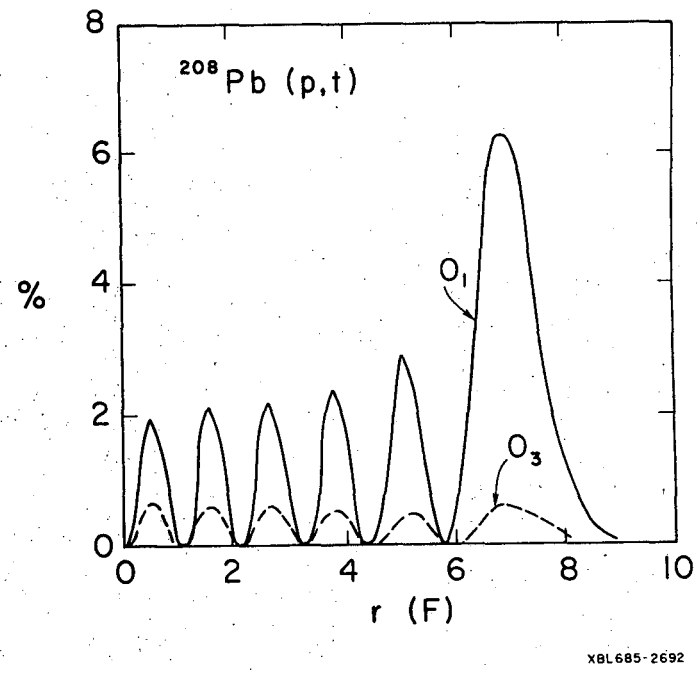

Fig. 2. The radial probability distribution $(r\,\tilde{\phi}(r))^2$ of the ^1S correlation in the ground and 0_3 state. Note the concentration in the surface for the enhanced ground state transition, beside the fact that the total probability for it is much higher.

Some qualitative features of transition intensities emerge very simply from a consideration of the correlations built up between nucleons by the residual shell model interaction. Take ^{206}Pb as an example, which has two holes in a doubly magic core. To avoid cluttered figures we consider only 0^+ states, though a similar discussion applies to any group of levels of the same spin and parity.

The unperturbed shell-model configurations are depicted in Fig. 3. Their positions are known from the single-hole states of ^{207}Pb. The percentage overlap of each pure configuration with the ^1S triton correlation is also shown. Note that the configurations of low angular momentum have a larger overlap than the high and they happen to lie lowest in this nucleus. Now when the residual interaction is turned on, and remember it is attractive in the singlet-even state, the configurations mix, that is to say correlations are set up, and the lowest resulting state will have more of the singlet-even correlation than its immediate neighbours. That is why it is lowest. Moreover the lowest state will have as its dominant components, the low-lying configurations which happen to be the ones most favoured by the transfer reaction. Thus not only will the ground state be more strongly populated than its near neighbours, but it will be the strongest 0^+ transition. The calculated spectrum of 0^+ states is shown at the right of the figure together with the dominant component of each level, and the percentage overlap with the ^1S correlation.[2]

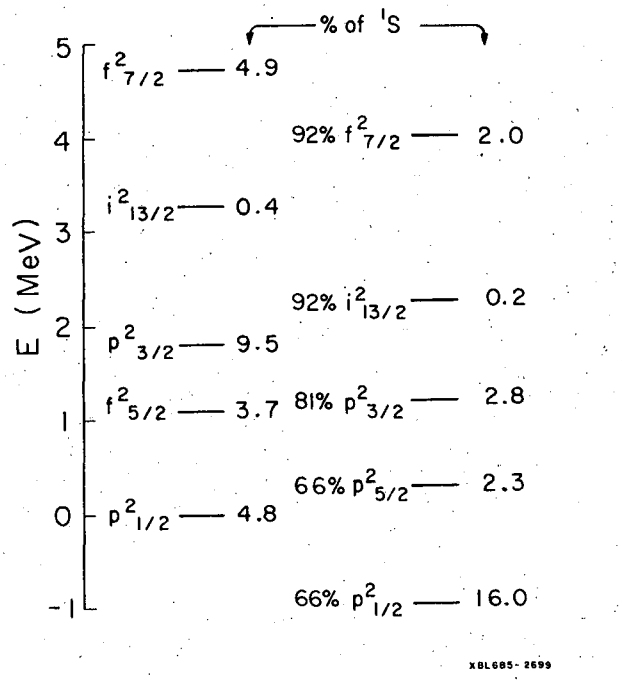

Fig. 3. For ^{206}Pb the pure and configuration mixed spectra of 0^+ states are shown on left and right respectively. The percentage of ^1S correlation is indicated and here is concentrated in the lowest state.

It is amusing that the opposite situation holds for such a close neighbour as ^{210}Pb. In this case the low-spin single-particle states lie higher in the spectrum as shown in Fig. 4. They are spread over an energy interval which is large compared to typical interaction energies, so that the ground state will have as its dominant components the high angular momentum configurations. The lowest 0^+, just because it is lowest, will possess more of the singlet-even correlation than any of the pure configurations which form its dominant components. And some of this singlet-even correlation will actually be the ^{1}S. However some higher level which is a coherent superposition of the other configurations will be comparable or more intense in the (t,p) reaction than the ground, because these configurations have more of the ^{1}S correlation.

A calculated spectrum of 0^+ states for ^{210}Pb is shown on the right of Fig. 4 with dominant configuration and percentage overlap with the ^{1}S correlation indicated for each level.[3] In this case the third level at about 3 MeV excitation has twice as much of the ^{1}S triton correlation as the ground state.

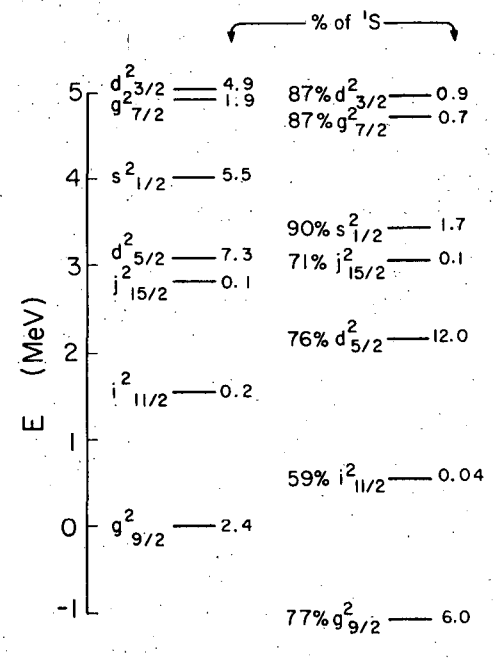

Fig. 4. For ^{210}Pb the pure and configuration mixed spectra of 0^+ states are shown on left and right respectively. The percentage of ^{1}S correlation is indicated and here is concentrated into two states, but most strongly in the 0_3.

This reaction is being studied at Los Alamos, but the analysis of the data is incomplete. However one thing is very evident, and that is that the most intense transitions lie higher in the spectrum around 3 MeV.[4]

I stress again that this discussion applies to any group of levels of the same spin and parity.

Now I turn to the question of whether a calculation based only on the first order process discussed earlier, is justified. Clearly we have less reason to worry about it if we treat regions of the periodic table where very strong collective transitions are absent. The lead region satisfies this criterion.

It is difficult to separate a test of the reaction mechanism from questions of nuclear structure, but again the lead region is favourable. The angular distributions, not cross-sections, but angular distributions corresponding to the lowest 0^+, 2^+, 4^+ states in the $^{208}Pb(p,t)^{206}Pb$ reation are essentially independent of a detailed knowledge of the nuclear wave functions, and form a good test of the DWBA treatment of the first order process. Let me explain how this comes about. We know very well the positions of the unperturbed configurations in this region. An examination shows that the low-lying ones for ^{206}Pb all belong to the same oscillator shell. This means that the projected wave function for the C.M. motion in each of these configurations is of the same degree of complexity;-when expanded on an oscillator basis, the series termmates at the same place for each (see Table 1). The coherent lowest state will have such admixtures of these configurations as builds up the singlet-even correlation in the surface, since there is more volume there. That is, the last term in the expansion will dominate for the enhanced states. Thus the CM wave functions is known, aside from normalization, independent of

$^{208}Pb(p,t)^{206}Pb$

	G_4	G_5	G_6
$p^2_{1/2}$	.009	.006	.218
$f^2_{5/2}$	.003	-.094	.168
$p^2_{3/2}$	.013	.009	.308
$f^2_{7/2}$	.004	-.109	.194
0_1	.016	-.049	.397
0_2	.003	-.105	.111

Table 1. Expansion of projected wave function on oscillator bassis $\mathfrak{U}_L(R) = \sum_N G_{NL}\, u_{NL}(R)$.

a detailed knowledge of the configuration mixing in these coherent states, and so the angular distribution, though not the cross sections, can be calculated independent of nuclear wave functions. Such a calculation is shown in Fig. 5. The agreement with the experimental data obtained by the Minnesota group[5,6] is excellent. We interpret this as an indication that, at least under appropriate circumstances, it is sufficient to calculate the simple direct transfer process.

Although the Minnesota group was not able to resolve many of the higher levels, a group at MIT[7], using the Oak Ridge accelerator, obtained data on a number of levels, shown with angular distributions calculated by them using True and Ford's wave functions of ^{206}Pb. The remarkable thing here is that there are levels of spin ranging all the way from 0^+ to 9^-, and all are very well reproduced as seen in Fig. 6.

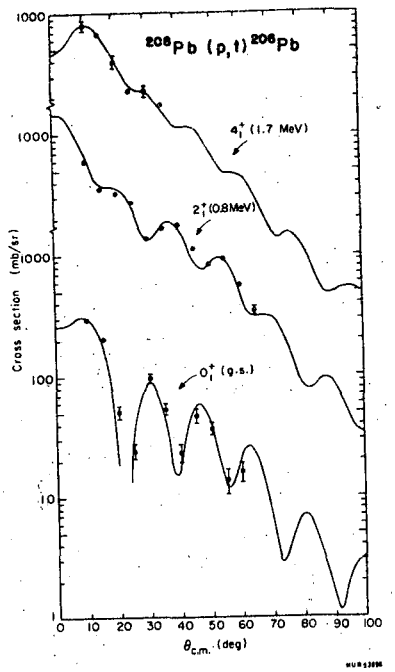

Fig. 5. Calculated angular distributions corresponding to 40 MeV protons are compared with the Minnesota data of Reynolds, Maxwell and Hintz.

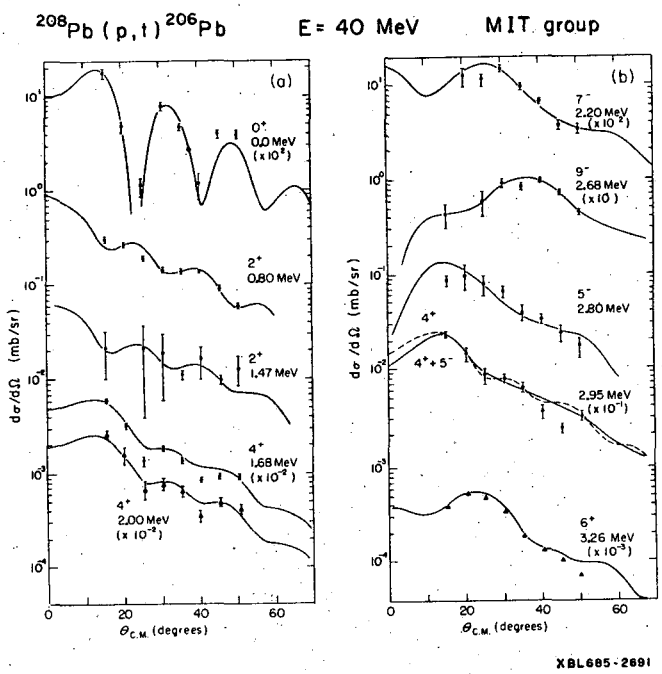

Fig. 6. Calculated angular distributions corresponding to 40 MeV protons to states ranging from spin 0 to 9 are compared with the MIT data of Smith, Moazed and Bernstein, and Roos.

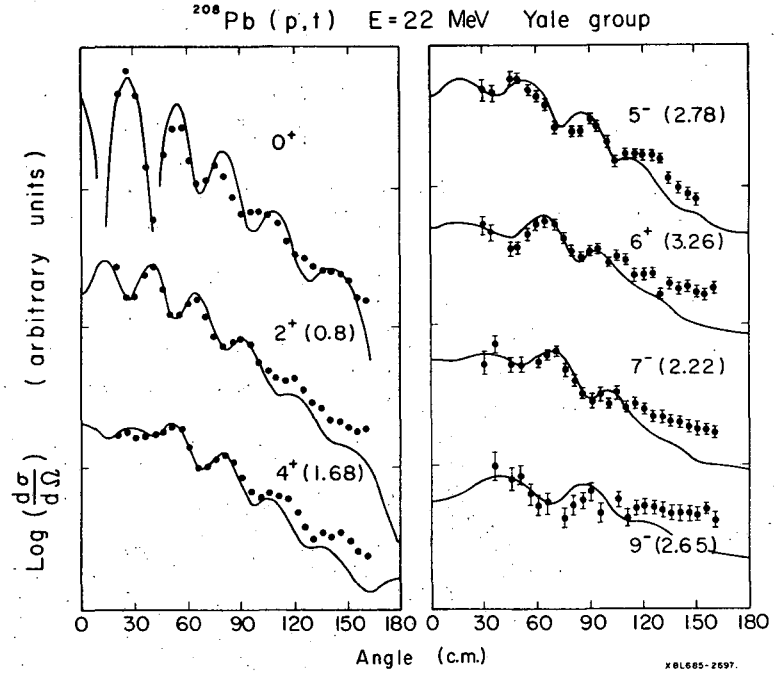

Fig. 7. Calculated angular distributions corresponding to 22 MeV protons are compared with the Yale data of Bromley, Holland and Stein.

More recently the same reaction was studied at 20 and 22 MeV by a Yale group,[8] and their data together with calculated angular distributions is shown in Fig. 7. Again excellent agreement is obtained for this very wide range of spins.

However angular distributions of reactions which are localized, to a high degree, in the surface, are never a very good test of nuclear structure, and sometimes as we saw, are independent of the details.

Relative cross sections constitute a more rigorous test of nuclear models. A comparison with the 22 MeV data of the Yale group is shown in Fig. 8 where the calculated cross sections again are based on the True-Ford shell model calculation.

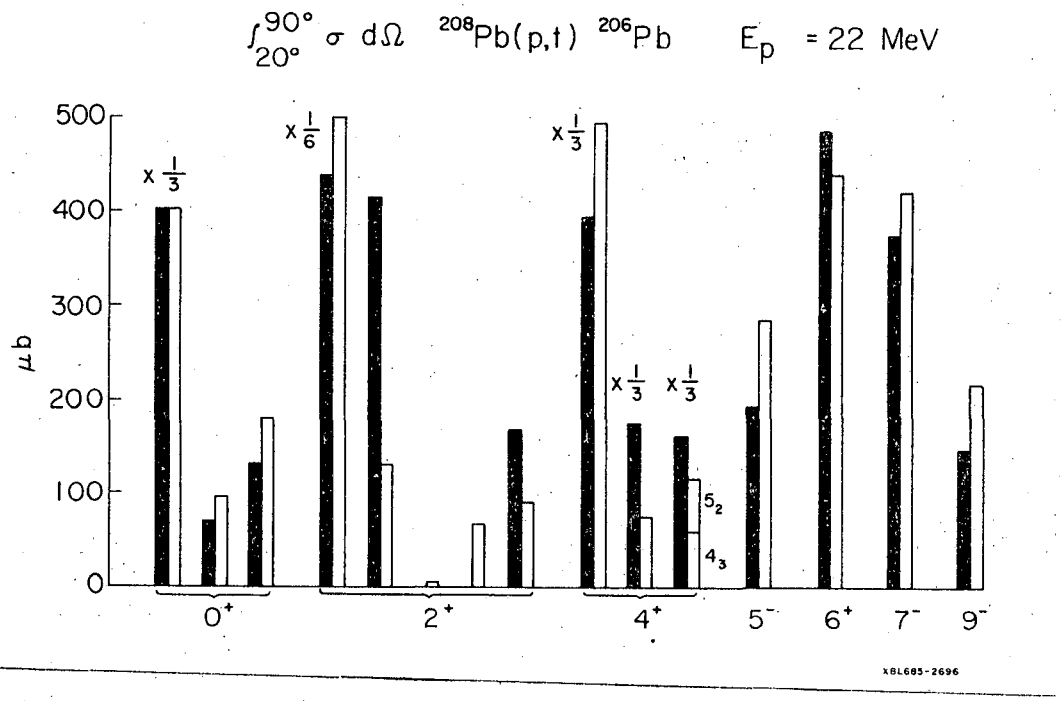

Fig. 8. Integrated cross sections for 22 MeV protons to a number of levels are shown. Black bars denote the experimental results of the Yale group and open bars denote calculations based on True and Ford wave functions for ^{206}Pb.

Overall the agreement is rather good, especially as concerns the strongest states in the spectrum. The worst results are obtained for the second 2^+ and 4^+ states.

Since there is data on this reaction at 20 and 40 MeV the question naturally arises whether the theory of the reaction, extrapolates correctly from one end to the other. That is to say, if the theory is normalized to the ground state reaction at 40 MeV does it correctly predict the 20 MeV cross section.

Unfortunately I cannot answer this question because of uncertainties in the optical model parameters. Of course little is known about the triton parameters, but even the proton parameters after a decade of work, are not well enough known. We need them here at 20 and 40 MeV but what is more, we must be

satisfied that they evolve one into the other as continious functions of energy. They exist for heavy nuclei at 40, 30 and 17 MeV, but at each energy different geometries were used, and in one case surface absorption, in the other volume.

The situation is depicted in Table 2 for optical potentials based on the work of several authors, and extrapolated to the energy needed here in several ways as regards the inaginary part. The 20 MeV cross section for the ground state is shown in the right column to be very sensitive to the imaginary part of the optical potential. From this we can only conclude that within uncertainties of the optical potential it is possible to obtain agreement.

	40 MeV		20 MeV		
	W	W_D	W	W_D	$\sigma_{p,t}$ in μb at 20 MeV
Satcher (30 MeV)	2.3	7.7	2.3	7.7	360
Frick-Satchler (40 MeV)	8		4		800
Perry (17 MeV Au)	4	0	4	0	360
	6		4		960
	10		4		1400
			Expt.	⟶	1500

Table 2. The real part of the optical potential is extrapolated to the energy of interest by 0.3 per MeV. The original geometry was retained. Imaginary parts were treated in the various ways shown in this table. Resulting integrated cross sections (0-90°) are compared with the experimental value.

We turn finally to the higher order processes. There exist no calculations but they will come soon. They will open the way to studying nuclei in which collective inelastic transitions are very strong. But also in vibrational like nuclei there are interesting possibilities because the higher order process is important for any state which possess anything like a two-phonon character as indicated in Fig. 9. Such states are not easily excited by the simple direct transfer, but transfer from the one-phonon excited state is strong, and of course the inelastic transition to the one-phonon level is enhanced.

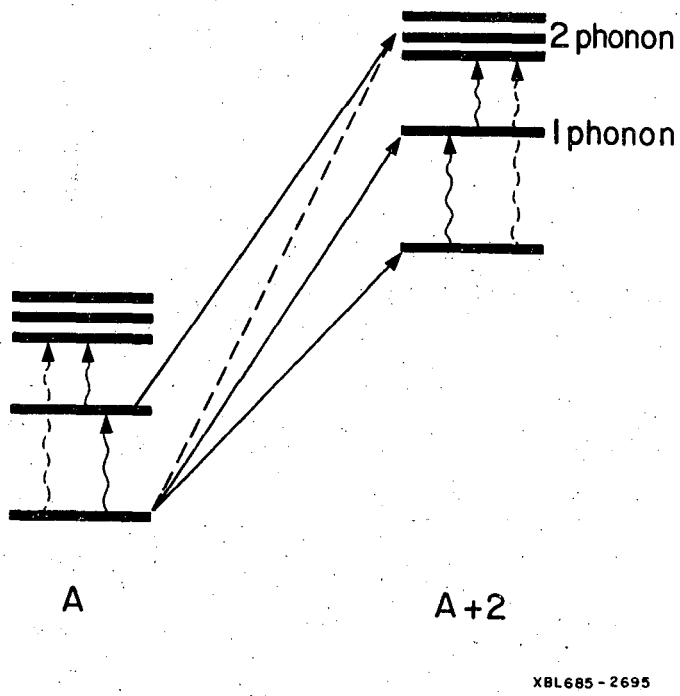

Fig. 9. Strong inelastic transitions in a vibrational like nucleus are illustrated by solid wiggly arrows, and weak by dashed. Strong two-nucleon transfer reactions are illustrated by solid straight arrows and weak by dashed. Second order processes obviously can play an important role in exciting two-phonon states.

We are working in Berkeley on a novel method which will handle higher order processes.[9] Again we take as an example the (t,p) reaction. On the left of Fig. 10 the triton channels are depicted, and t_1, for example denotes the whole collection of channel quantum numbers including the internal nuclear quantum numbers as well as those describing the relative motion in the channel. The arrows denote the feeding of this channel by inelastic scattering from the ground, and other excited states. The equation describing the triton motion in this channel is shown. If the right side of this equation were zero, it would describe the steady-state elastic scattering from the excited state. The source terms represent the inelastic processes. They couple the various triton channels. These are the usual coupled channel equations. They are to be solved subject to the physical conditions that there is a beam from the accelerator in the ground channel, and at the most only outgoing waves in other channels.

On the right we depict the channels in the residual proton system. Looking at a particular one of them, say p_1, it is fed by inelastic processes from other proton channels, but as well as that, it may be fed by the transfer reaction taking place in the various triton channels. Which of these contributes and with what intensity, is a question of parentage and correlations such as earlier discussed.

The form of the source term for a given triton channel leading to the proton channel in question is shown. The equation governing the motion of the proton in this channel is shown in the figure. It contains source terms corresponding to all the arrows, and it is coupled to all the other proton channels. It must be solved subject to the boundary condition that there are only outgoing protons. The amplitude of the outgoing waves yields directly the S-matrix elements from which the (t,p) cross section can be calculated in the usual way.

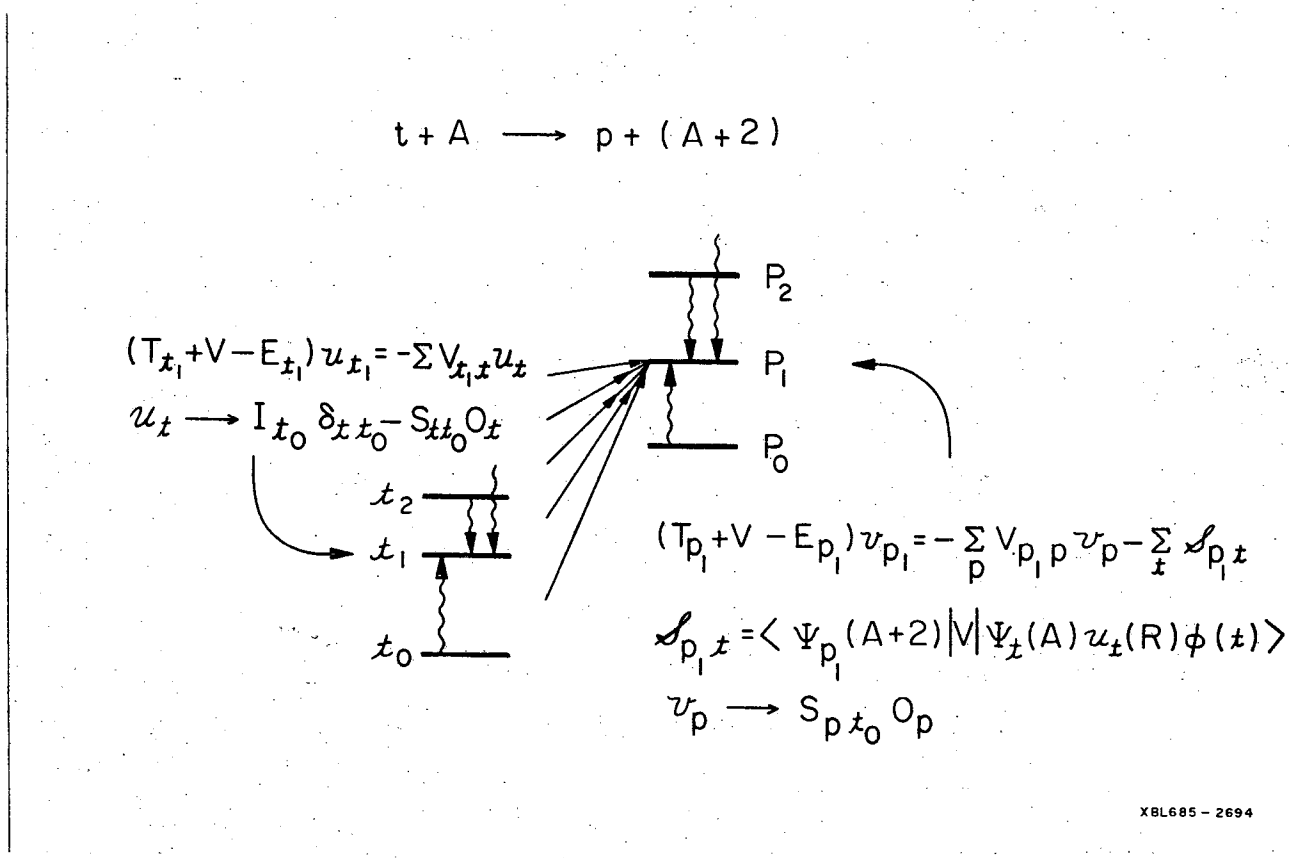

Fig. 10. On the left t+A channels are illustrated. Attention is concentrated on channel t_1 which is fed by inelastic transitions from other channels. The equation describing the triton motion is shown. On the right the proton channel p_1 is fed by transfer reactions as well as by inelastic transitions from other channels which themselves have already been fed by transfer reactions. The transfer source terms are denoted by $\mathcal{S}$ and their structure is shown.

To summarize, we must solve a set of coupled equations describing the inelastic scattering in the triton-target system. These solutions are used to construct the proton source terms arising from the transfer reactions. Finally the coupled equations describing the proton motion are solved.

This method computes the inelastic scattering effects to all orders among the retained channels, and treats the transfer process, as the weak processe it is, only in first order. It is easily verified that if we drop all the sources corresponding to inelastic processes, we retrieve the DWBA to the transfer reaction. This allows us to check all details of the construction of the transfer source terms, down to the last factor of π.

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8. I am indebted to D. A. Bromley, G. Holland and N. Stein for communication of their data before publication.
9. This work is in collaboration with R. Asquito.