Title
On a Comment by J.P.B. Vreeburg

Journal
Microgravity - science and technology, 4(1)

Authors
Concus, P.
Finn, R.

Publication Date
1991

LBL-30101
Preprint

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Physics Division

Mathematics Department

To be submitted for publication.

On a Comment by J.P.B. Vreeburg

P. Concus and R. Finn

January 1991

Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098

DISCLAIMER

LBL-30101

ON A COMMENT BY J. P. B. VREEBURG *

Paul Concus
Lawrence Berkeley Laboratory
and
Department of Mathematics
University of California
Berkeley, California 94720

and

Robert Finn
Department of Mathematics
Stanford University
Stanford, California 94305

January 1991

* This work was supported in part by the Mathematical Sciences Subprogram of the Office of Energy Research, U. S. Department of Energy, under Contract Number DE-AC03-76SF00098, by the National Aeronautics and Space Administration under Grant NAG3-1143, and by the National Science Foundation under Grant DMS89-02831.

P. Concus and R. Finn

On a Comment by J. P. B. Vreeburg

This note addresses a comment on [5] by J. P. B. Vreeburg [6], which concerns our paper [2]. In [6] Vreeburg presents a discussion based on geometric considerations that gives conditions for a smooth surface to meet the walls of a wedge with prescribed angles γ_1 and γ_2 on the two faces. The discussion requires the assumption that the surface have a continuous normal up to the vertex from within the wedge domain and that the surface does not become vertical at the vertex. The necessary condition obtained in [6] for such a surface to be possible is

$$\pi \leq 2\alpha + \gamma_1 + \gamma_2 \leq 3\pi, \tag{1}$$

where 2α is the interior angle of the wedge. Although the discussion takes place in the context [5] of capillary surfaces (in zero gravity), the equation for a capillary surface is not invoked at all; thus (1) provides in fact a necessary condition that must be satisfied by a smooth surface in general, without regard to whether the surface also satisfies the capillary equation. Since a capillary surface need not be smooth at the vertex (even when (1) holds: the example of a spherical cap indicated in Fig. 1 becomes vertical), not all capillary surfaces are included in the discussion in [6], only *a fortiori* those that are also smooth.

In our initial study [1] of capillary surfaces in wedge domains the necessary condition

$$\alpha + \gamma \geq \pi/2 \tag{2}$$

is given for existence of a solution $u(x, y)$ of the capillary equation

$$\nabla \cdot \left(\frac{\nabla u}{\sqrt{1 + |\nabla u|^2}} \right) = \text{const.} \tag{3}$$

in the corner of a wedge domain with opening angle 2α, and meeting the bounding walls in the constant contact angle γ (measured within the fluid). Additional background information, other details on the discontinuous behavior of capillary surfaces in a wedge, and

Mail addresses: Prof. Paul Concus, Lawrence Berkeley Laboratory and Department of Mathematics, University of California, Berkeley, CA 94720, USA, Prof. Robert Finn, Department of Mathematics, Stanford University, Stanford, CA 94305, USA.

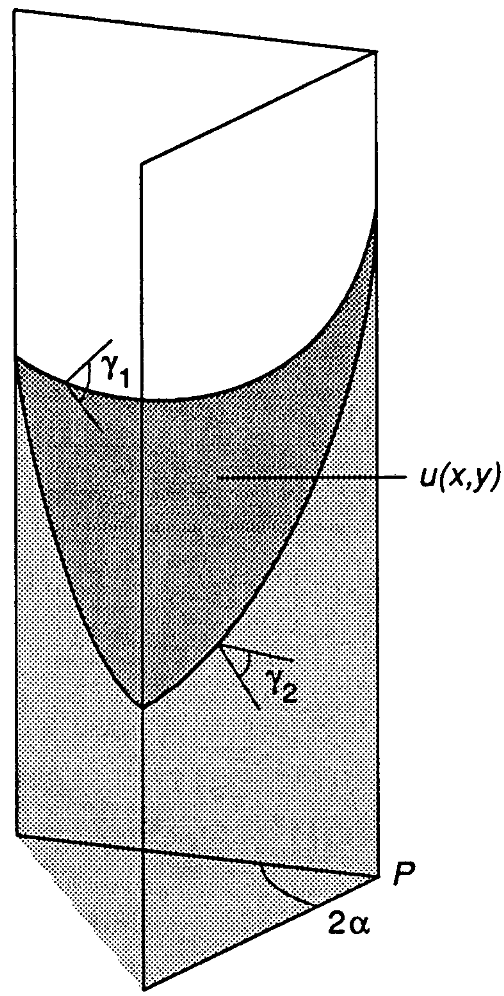

Figure 1. Capillary surface in a wedge with differing contact angles γ_1 and γ_2 on the two faces.

subsequent and more complete discussion can be found in [2] and in Chapters 1 and 5 of [4]. For (2) it is assumed that the liquid is wetting, i. e., $0 \le \gamma < \pi/2$; for non-wetting liquids ($\pi/2 < \gamma \le \pi$) the corresponding necessary condition is

$$\alpha \ge \gamma - \pi/2. \tag{2'}$$

Although (2), (2′) bear formal similarities to (1) when $\gamma_1 = \gamma_2 = \gamma$, an important feature of the result for capillary surfaces is that no assumptions are made on the behavior of a solution as the vertex P is approached; thus if (2), (2′) do not hold, there can be no solution of (3) near the vertex, not even one that oscillates or becomes infinite as P is approached from within the wedge domain.

The formal similarities between (2), (2′) and (1) can be attributed to the circumstance, that whenever a smooth surface with the prescribed data exists then it turns out that a smooth capillary surface also exists. It is of interest to have conditions that are both necessary and sufficient for the existence of a smooth capillary surface in a wedge domain with possibly differing contact angles γ_1 and γ_2 on the two faces. In a forthcoming paper [3] we give such conditions, along with modifications to include surfaces that become vertical at the vertex. In this connection it should be noted that (1) is necessary but not sufficient, e. g., it can be shown, for example, that if $\gamma_1 = \gamma_2 = \gamma = 150°$ and $\alpha = 30°$ then (1) is satisfied but there is no smooth surface (capillary or otherwise) meeting the walls with angle γ.

This work was supported in part by the Mathematical Sciences Subprogram of the Office of Energy Research, U. S. Department of Energy, under Contract Number DE-AC03-76SF00098, by the National Aeronautics and Space Administration under Grant NAG3-1143, and by the National Science Foundation under Grant DMS89-02831.

References

[1] Concus, P., and R. Finn, *On the behavior of a capillary free surface in a wedge*, Proc. Nat. Acad. Sci. U.S.A. **63** (1969), pp. 292–299.

[2] Concus, P., and R. Finn, *On capillary free surfaces in the absence of gravity*, Acta Math. **132** (1974), pp. 177–198.

[3] Concus, P., and R. Finn, *Capillary surfaces in wedge domains, II*, in preparation.

[4] Finn, R., *Equilibrium Capillary Surfaces*, Springer-Verlag, New York, 1986.

[5] Langbein, D., R. Grossbach, and W. Heide, *Parabolic flight experiments on fluid surfaces and wetting*, Appl. Microgravity Tech. **2** (1990), pp. 198–211.

[6] Vreeburg, J. P. B.,*Comment on the paper "Parabolic flight experiments on fluid surfaces and wetting"*, Microgravity Sci. Technol. **III** (1990), p. 125.