Title
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System Reliability of Flood Control Levees

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ABSTRACT
Systems of levees are present in many locations world-wide to provide flood protection for urban, industrial, and agricultural resources. In risk assessment of levee systems, the probability of demand (e.g., high water events, earthquakes, waves) exceeding capacity (e.g., freeboard, erodibility, liquefaction susceptibility) is evaluated across the system. We describe and compare two levee system reliability analysis frameworks for cases of seismic and high-water demand types. The first approach considers spatial correlations and distributions of demand and capacity between "segments" (i.e., elemental levee lengths, nominally 50 m in scale) through Monte-Carlo simulation. The capacity correlation model considered in this approach is empirically derived from seismic case histories in Japan. The seismic demand correlation model is also empirical and based on global ground motion data, whereas the high-water correlation is taken as unity. The second approach, which was developed and previously applied in the Netherlands, examines the distribution and correlation of capacities and demands between physics-based "reaches" (i.e., length of levee having uniform statistical distributions of capacity and demand, potentially hundreds of m in length). Statistics and spatial correlation of the limit state function, defined as capacity minus demand, are computed using a first-order reliability method (FORM) procedure based on the distribution functions and spatial correlation functions for capacity and demand. Having computed the distribution function and spatial correlation function for the limit state, the probability of failure of the reach is then computed using level-crossing statistics. We identify a hurdle in the implementation of the level-crossing statistics approach that is related to Markov-type correlation functions for levee capacity – this is overcome by developing a similar-performing Gaussian correlation function. We compute system failure probabilities from reach statistics by assuming statistical independence among reaches. We illustrate application of both methods for an example levee system subjected to realistic demand and capacity distributions. Our results show that characteristic lengths (defined as lengths of levee that can be considered as statistically independent) are comparable for high-water and seismic demands; our interpretation is that this result is driven by the use of similar capacity correlation models, whereas the differences in demand correlation models for the two hazards are not impactful.

1 INTRODUCTION

Levees are defined as man-made or natural embankments along rivers or water bodies. Their primary purpose is to provide protection against flood events. The performance of levees when subjected to natural and/or anthropogenic events (such as floods or earthquakes) is essential for the resilience of surrounding communities. Despite their critical function, many levees were not properly engineered at the time of their construction and are often founded on soft and weak soils. As a result, levees are frequently damaged during high-water events (e.g., Larson, 1996; Sills et al., 2008; Briaud et al., 2008) and following major earthquakes (Miller and Roycroft, 2004; Sasaki, 2009; Sasaki et al., 2012; Green et al., 2011; Kwak et al., 2016a).

For levees that continuously impound water, a single failure anywhere along their length will produce flooding, and hence comprises system failure. For levees that intermittently impound water, the seismic failure probability is related to the combination of seismic deformation risk and probability of high water during or shortly following the event, whereas the high-water failure probability is simply the single-segment failure probability during a high-water event. In either case (continuously or intermittently loaded), levees constitute a spatially distributed series system, which present particular challenges for risk assessment. This paper describes two conceptually similar approaches for analysis of levee risk, with an emphasis on the system probability of failure given knowledge of capacity and demand on a more local level. We defer to other documents for recommended analysis procedures for computing capacity at the segment, or cross-section, level (Zimmaro et al. 2017 for seismic, URS Corporation, Jack R. Benjamin & Associates Inc., 2008 for high-water).

Demands imposed on levee systems (e.g. high water related to flood events, earthquake shaking) are spatially correlated in a manner that reflects attributes of the event initiating the demand. Moreover, the available capacities of a portion of the levee to resist demands (e.g., erodibility, liquefaction susceptibility, etc.) are also spatially correlated due to the geologic depositional processes and the manner in which levee fills were constructed.

Several approaches can be used to consider spatial correlation of demand and capacity in levee systems. We take spatial demand correlation for high water events as unity (Vrouwenvelder, 2006). For seismic demands, models for spatial correlation of ground motions are applied (Jayaram and Baker, 2009). The correlation of capacity may be calculated based on spatial correlation of the soil properties that give rise to the levee capacity (e.g., Vrouwenvelder, 2006; Jongejan and Maaskant, 2015), or by back-calculation of the capacity distribution based on observed damage and demand distributions (Kwak et al., 2016b). We adopt the latter approach for the present work.

We present here a levee system reliability analysis framework applied at two levels of resolution. The first (Monte Carlo simulation) is computationally demanding, but flexible with respect to assumptions regarding the statistics of the limit state function. This approach generates random realizations of demand and capacity of levee segments compatible with spatial correlation models, and then numerically calculates the probability of failure. The second is less computationally demanding, but makes assumptions about the statistics of the limit state function. This approach is based on the First Order Reliability Method (FORM) and level-crossing statistics.

We present both approaches using consistent terminology, which is provided next. We describe the development of capacity distributions and correlation functions, which are required elements of both the Monte Carlo and FORM methods. For a hypothetical levee system subject to specified scenario demands, we then compare results of risk analysis for seismic and high water events using the two methods. This paper builds upon a previous paper (Kwak et al. 2017) that used a less developed version of the capacity correlation model, different levee configurations, and which considered only earthquake demands.

2 LEVEE SYSTEM RISK ASSESSMENT PROCEDURES

We apply the following terms for use in the engineering evaluation of levee risk (Kwak et al. 2017):

System: A length of levee that protects a particular region from flooding. A breach anywhere within the system constitutes system failure if the levee impounds water.

Reach (Physics-Based): A length of levee that exhibits uniformity in the statistical distributions of levee capacity (soil properties, geometry), and demand (flood level, earthquake shaking, etc.). Capacity and demand vary randomly within a reach, but their statistical distributions are uniform. A two-dimensional cross-section analysis must be interpreted in a manner that considers the out-of-plane variation in capacity and demand to draw meaningful conclusions about the probability of failure of a reach.

Reach (Legal/Jurisdictional): Levee systems are sometimes divided into "reaches" based on specific legal or jurisdictional boundaries, or other considerations that are unrelated to the physics that drive risk analysis. It is important to distinguish this definition from the physics-based definition, and to use the physics-based definition in risk analysis.

Characteristic length: A characteristic length is a specific length of levee for which the probability of system failure computed based on the assumption of statistical independence of each characteristic length is equal to the probability of system failure based on a more robust risk analysis that considers spatial correlation of capacity and demand within the system. The probability of system failure using the characteristic length method is computed based on a computationally simple product sum. However, the characteristic length can strictly only be defined by first computing the probability of system failure using a robust risk analysis framework, and subsequently calculating the characteristic length. The characteristic length depends on the spatial variations of capacity and demand within the system, and is different for different loading conditions as demonstrated subsequently in this paper. In practice, a specific characteristic length has been assumed from the outset to facilitate relatively simple analysis. Errors in the selection of characteristic length directly affect the computed probability of system failure.

Segment: A segment is a length of levee with uniform capacity, and can be considered as an elemental length. A segment may be represented as a two-dimensional cross-section in engineering analysis. Soil properties may vary within a segment due to stratigraphy and depositional variability, but the capacity of the segment is constant because the size of the failure mass is large enough to average out the spatial variations in soil properties. Segments are shorter than reaches, and reaches may be analyzed as a collection of segments. The capacity among various segments is spatially correlated due to similarities in the depositional environment of the foundation soils and levee construction practices.

Note that different definitions may be found in literature for similar concepts (e.g., ‘reach’ as ‘section’, Jongejan and Maaskant, 2015).

Figure 1 shows a schematic of a levee system that is divided into multiple reaches. Each reach can be subdivided into segments, and a characteristic length may be computed from a risk analysis. In this case, we assume that a reach > characteristic length > segment, though reaches are not necessarily longer than characteristic lengths by definition.

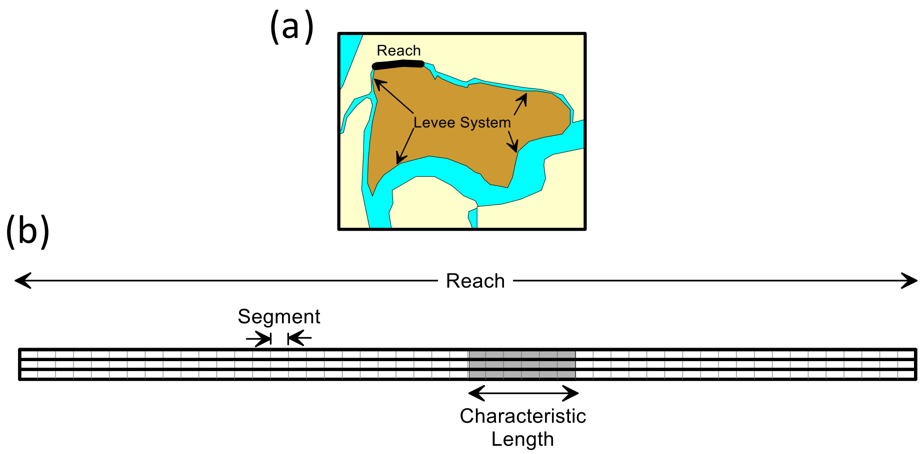

Figure 1. Definition of (a) levee system and reach; (b) levee segment and characteristic length within a reach. Adapted from Kwak et al. (2017).

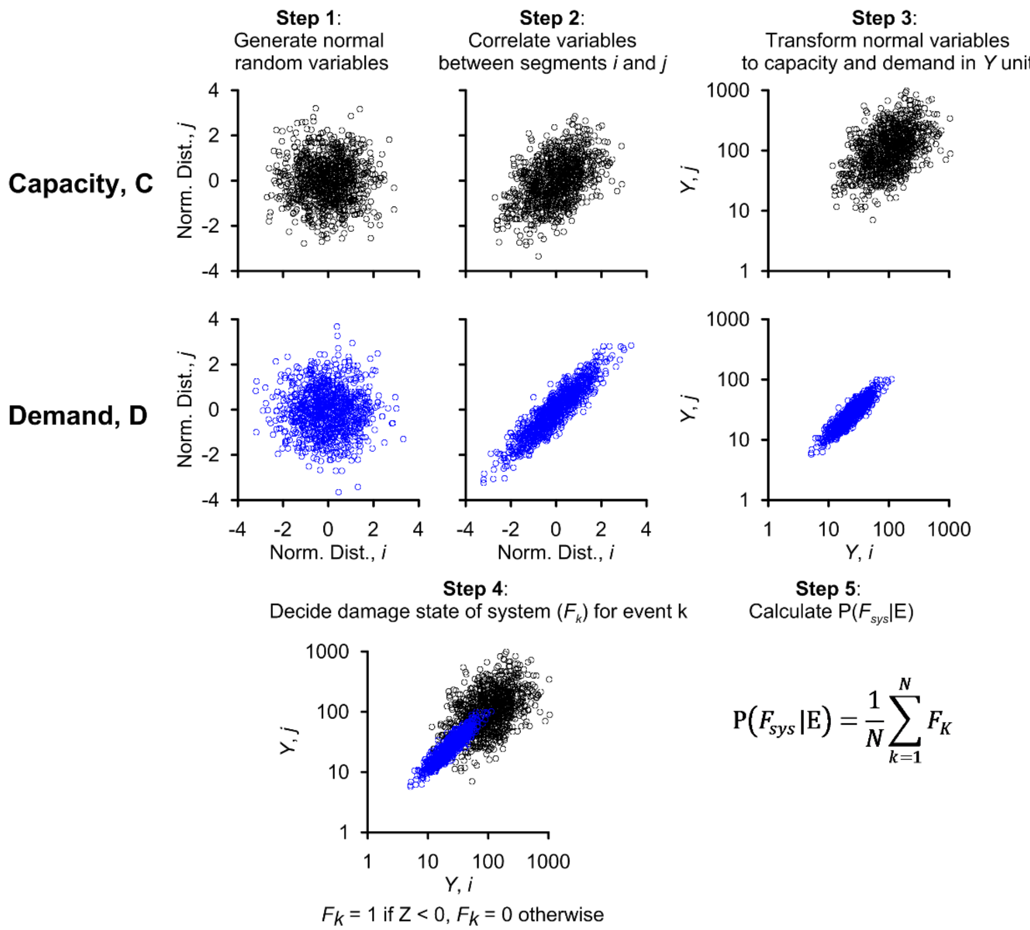

Figure 2. Illustration of procedure for system risk analysis using Monte-Carlo simulation (adapted from Kwak et al., 2017).

3 LEVEE SYSTEM RISK ASSESSMENT PROCEDURES

System risk analysis consists of calculating the probability that one or more segments within the system experience failure due to a stressing event. In this paper, we focus on failure probabilities conditioned on the stressing event (denoted E), not the failure probability itself. Important aspects of this calculation are the distribution functions of capacity and demand for the segments, and spatial correlation of capacity and demand among segments. To illustrate the importance of spatial correlation on system risk analysis, consider two extreme cases: the capacity and demand distributions of two different segments are either perfectly correlated or statistically independent. For the case of perfect correlation, the capacity of each segment is a uniform number of standard deviations above or below the mean value, as is the demand. Hence, the conditional probability of system failure [i.e, $\mathrm{P}(F_{sys}|\mathrm{E})$] is equal to the maximum of the conditional probabilities of failure of the individual segments in the system. In the case of statistical independence, $\mathrm{P}(F_{sys}|\mathrm{E})$ is equal to the complement of system survival, which in turn is the product of each individual reach surviving. The conditional probability of failure associated with these scenarios lies between the two extremes, which are known as uni-modal bounds for a series system (Ang and Tang, 2007):

$$\max\left[\mathrm{P}\left(F_{Seg,i}|\mathrm{E}\right)\right] \leq \mathrm{P}\left(F_{sys}|\mathrm{E}\right) \leq 1 - \prod_{i=1}^{n}\left(1 - \mathrm{P}\left(F_{Seg,i}|\mathrm{E}\right)\right) \qquad (1)$$

where arguments F_{sys} and $F_{Seg,i}$ indicate failure of the system and segment i,

The range of failure probabilities provided by Eq. (1) is often wide. For example, a system composed of 10 segments each with $\mathrm{P}(F_{Seg}|\mathrm{E}) = 0.05$ will have $\mathrm{P}(F_{sys}|\mathrm{E}) = 0.05$ for perfect correlation and $\mathrm{P}(F_{sys}|\mathrm{E}) = 0.40$ for statistical independence. In general, $\mathrm{P}(F_{Seg}|\mathrm{E})$ will vary within the system, but is selected to be constant for this simple illustration. Where the actual value of $\mathrm{P}(F_{sys}|\mathrm{E})$ falls between these uni-modal bounds depends strongly on capacity and demand correlations among segments. The following sections describe two approaches for analysis of this probability. Both approaches fundamentally consider segment fragility and correlations, but in different ways.

3.1 Monte-Carlo Simulation-Based Approach

The approach begins with the definition of limit state function, *Z*:

$$Z = C - D \qquad (2)$$

Where *C* is capacity and *D* is demand. Note that *C* and *D* are spatially correlated random variables assumed to be log-normally distributed. Moments of the log normal distributions are constant within a reach, but there are between-segment variations in capacity and demand, which are driven by the respective correlation functions. The system failure probability is evaluated as follows:

1. Populate two sets of uncorrelated normal random variables, which will be used later for capacities and demands, with a sufficient number of realizations (here: 50,000) for each segment.
2. Construct symmetric matrices of correlation coefficient for demand and capacity, as given in Eq. 16 of Kwak et al. (2016b).
3. Use Cholesky decomposition (e.g., Baecher and Christian, 2003) to modify the realizations generated in (1) to exhibit the desired spatial correlation structure.
4. Transform the random variables from (3) to demands and capacities with appropriate units. This can be expressed in terms of a generic variable (*Y*) that both represents demands (e.g., ground shaking level for earthquakes, water elevation for high-water events) and the output of capacity functions.
5. Compute the limit state *Z* (Eq. 2) for each segment for each realization.
6. Compute the damage state of the system for each realization, which is 1 if any segment has capacity lower than demand ($Z < 0$) within the system.
7. Calculate the fraction of realizations for which $Z < 0$, which is an estimate of the system probability of failure.

Figure 2 illustrates the procedure for evaluating the system failure probability using the Monte-Carlo simulation-based approach.

3.2 Level-Crossing Statistics Method

The Monte Carlo simulations presented in the previous section are computationally demanding for large systems. A conceptually similar alternative that is less computationally demanding is described here. It involves computing the statistics of the limit state function (i.e., the distribution function and spatial correlation function) for segments within a reach using the first-order reliability method (FORM; Rackwitz, 2001) and then computing the reach failure probability using level-crossing statistics. Reach failure probabilities can then be extended to uni-modal bounds on system failure probabilities. The steps involved in this method are outlined below (see Vrouwenvelder, 2006 or Jongejan and Maaskant, 2016 for further details).

1. For each reach, define a representative segment having defined probability density functions (PDFs) for capacity and demand. Limit state function, *Z*, is the difference between capacity and demand (Eq. 2), and is assumed to follow a normal distribution (and is therefore formulated for *C* and *D* in natural log space).
2. Given the demand and capacity PDFs from (1), calculate the conditional failure probability [P(F_{seg}|E)], reliability index (β_{Seg}), and influence coefficients of the segment using FORM. Reliability index and failure probability are related as:

$$\mathrm{P}\left(F_{Seg}|E\right) = \mathrm{P}(Z < 0) = \Phi\left(-\beta_{Seg}\right) \quad (3)$$

where Φ is the standard normal cumulative distribution function. The influence coefficients (Hasofer and Lind, 1974) describe the relative weight of the demand (α_D) and capacity (α_C) distributions on the limit state function. This can be expressed by a linearized version of the limit state function at the design point[1] as follows:

$$Z = \beta_{Seg} + \alpha_D \varepsilon_D + \alpha_C \varepsilon_C \quad (4)$$

where ε_C and ε_D are independent, standard normal variables. The squared sum of α_C and α_D is unity (i.e. $\alpha_D^2 + \alpha_C^2 = 1$).
3. Calculate the failure probability of the reach on the basis of level-crossing statistics. In this step an approximate version of the *correlation function*[2] of the limit state function is taken as the weighted sum of the correlation functions for capacity and demand, as follows:

$$\rho_Z(x) = \alpha_C^2 \rho_C(x) + \alpha_D^2 \rho_D(x) \quad (5)$$

where *x* is distance between two points. The failure probability of a reach can now be approximated by:

$$\mathrm{P}(F_R|E) = 1 - \left(1 - \mathrm{P}\left(F_{Seg}|E\right)\right) \times \exp\left(-\frac{L}{2\pi}\sqrt{-\frac{d^2\rho_Z(0)}{dx^2}} \times exp\left(-\frac{\beta_{Seg}^2}{2}\right)\right) \quad (6)$$

where *L* is the reach length. Eq. (6) indicates a reach may be thought of, approximately, as a series system of independent, characteristics lengths, with a length (L_{Char}) given by:

$$L_{Char} = \mathrm{P}\left(F_{Seg}\right) \times \frac{2\pi}{\sqrt{-\frac{d^2\rho_Z(0)}{dx^2}}} \times \exp\left(\frac{\beta_{Seg}^2}{2}\right) \quad (7)$$

4. Calculate the conditional failure probability of the system combining reach conditional failure probabilities. Uni-modal bounds of system failure probability can be computed from reach failure probabilities using Eq. (1). When characteristic lengths are appreciably shorter than reach lengths, we consider it acceptable to assume zero correlation between reaches, as discussed further in Section 5.4.

4 INPUT MODELS

4.1 Capacity Distributions

As illustrated in Figure 3, the capacity distribution for a segment with deterministic demand can be taken as the derivative of its fragility curve (Baker, 2008), which has demand parameter *Y* on its abscissa. We take the seismic levee fragility (and hence capacity distribution) from the empirical models of Kwak et al. (2016a), which use the demand parameter of peak ground velocity, PGV in units of cm/s. These empirical models inherently consider failure modes contributing to observed levee deformations. For the considered data set, these include liquefaction of foundation soils, seismic slope instability, and seismic compression. Figure 3 shows two example capacity distributions derived from these models. The distributions shown in Figure 3 are applicable for stiff soil (G_N=1 model) and for soft soil, high water, respectively.

For high-water conditions, possible failure mechanisms include underseepage (internal erosion), slope instability, and overtopping. In the application considered subsequently in this paper, we consider the internal erosion mechanism. We use the fragility relation shown in Figure 4 relating failure probability to vertical exit flow gradient, *i* (URS Corporation, Jack R. Benjamin & Associates Inc., 2008). Seepage analyses are used to relate water level (which comprises demand parameter *Y* in this case) to *i*, for the geometry and soil condition present in a particular levee reach. Analyses of this sort are illustrated for an example problem in Section 5.

4.2 Correlation Models

Demand correlation for ground motion is taken from an empirical model by Jayaram and Baker (2009):

$$\rho_D(x) = \exp\left(\frac{-3x}{\alpha_{DD}}\right) \quad (8)$$

where α_{DD} is a range parameter taken as 17.1 km for widely varying geologic conditions and 33 km for similar geologic conditions, and *x* is separation distance, as before. This form is referred to as a Markov correlation

[1] the point having the shortest distance from the limit state function to the origin in the standard normal space (Rackwitz, 2001)

[2] As used here, correlation functions describe correlation of a variable as a function of separation distance.

function. Figure 5 shows these two demand correlation models.

Demand correlation for high-water hazard is taken as unity, $\rho_D(x) = 1$ (Vrouwenvelder, 2006). This is used because flood events are considered to raise the water level in a rather uniform manner in the water bodies bounded by levees.

Capacity correlation, ρ_C, was estimated by Kwak et al. (2016b) using observations of the spatial correlations of damage states combined with correlations of seismic demand. This analysis resulted in Markov-type correlation model:

$$\rho_C(x) = \exp\left(\frac{-3x}{\alpha_{CC}}\right) \tag{9}$$

where α_{CC} is the range parameter, which is 8.1 km for level ≥ 1 damage (effectively any perceptible damage level) and 3.2 km for level > 2 damage (severe damage).

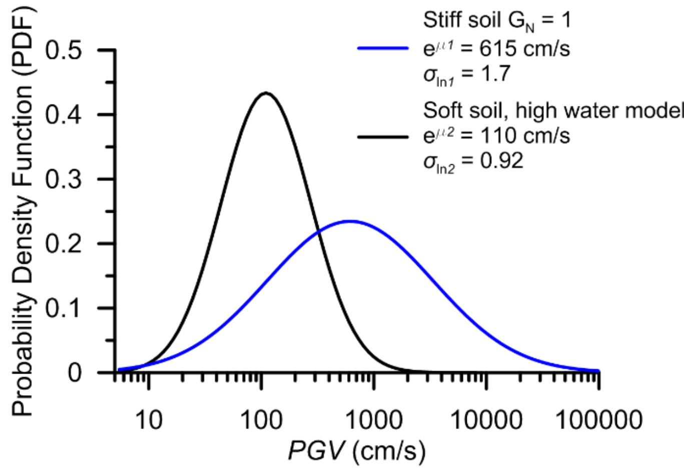

Figure 3. Example capacity distributions derived from empirical model of Kwak et al. (2016a). μ_1 and μ_2 represent natural log mean capacities (the exponent is taken to convert to arithmetic units) and σ_{ln1} and σ_{ln2} represent standard deviations of capacity distributions.

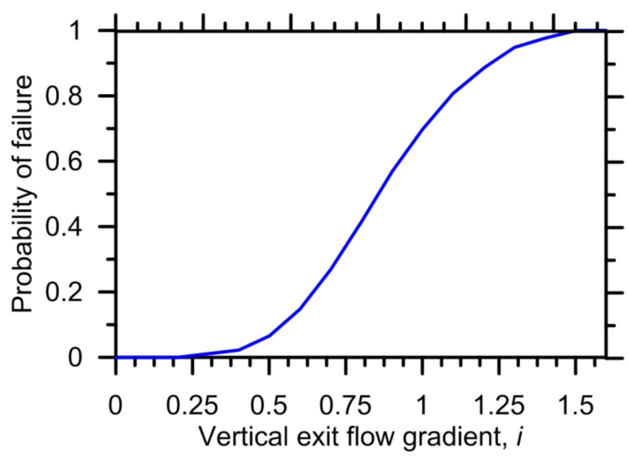

Figure 4. Fragility curve relating failure probability to vertical exit flow gradient, *i* (adapted from URS Corporation, Jack R. Benjamin & Associates Inc., 2008).

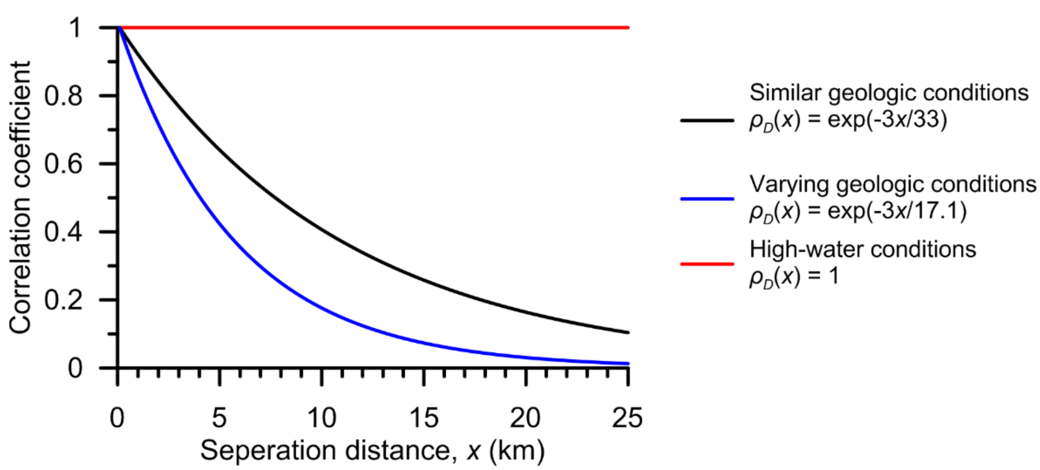

Figure 5. Demand correlation functions for both, seismic and high-water conditions.

Markov correlation functions are not mean square differentiable (e.g., Fenton and Griffiths, 2008), which is undesirable because the second derivative of the function at zero separation distance is required in the level-crossing statistics method. The Gaussian correlation function provided in Eq. 10 is mean square differentiable, therefore finding a Gaussian correlation function that is "equivalent" to the Markov correlation function developed by Kwak et al. (2016b) is desirable. Ultimately, it would be desirable to re-derive demand and capacity correlation functions using the Gaussian function. In the meantime, an "equivalent" function is obtained by computing the probability of reach failure using the Markov function with the Monte Carlo method, then selecting a value of β_{CC} such that the same probability of reach failure is obtained using level-crossing statistics with the Gaussian function.

$$\rho_C(x) = \exp\left[-\left(\frac{3x}{\beta_{CC}}\right)^2\right] \tag{10}$$

As an example, consider Figure 6, which shows two limit state functions versus horizontal position along a 25 km linear levee system; one with a Markov correlation function and the other with a Gaussian function. The limit state function is selected to have a mean value of 0.5 and standard deviation of 0.2, and is assumed to be normally distributed. Failure is assumed to occur when the limit state function is lower than zero. The Markov type correlation function gives rise to high frequency variations in the limit state function, whereas the Gaussian limit state function is much smoother. The value of α_{CC} for the Markov function was set to 8.1 km following Kwak et al. (2016b), and the probability of failure was computed to be P_f = 0.23 using 1000 Monte Carlo simulations. The value of β_{CC} was then iteratively adjusted, and β_{CC} = 1.0 km was found to provide P_f = 0.23. These two correlation functions are therefore considered to be "equivalent". This approach is used to define appropriate equivalent Gaussian correlation structures for the capacity of each reach analyzed in Section 5 (Section 5.3).

5 EXAMPLE APPLICATION

5.1 Problem Description

We consider the levee system shown in Figure 7, which protects the town from flooding during high-water river flows. The river and levee are adjacent to the town through 'highland' (relatively firm soil conditions) and 'lowland' (soft soil) areas. The levee is 5 m in height and has a mean water level on the river side of 1 m above the levee base elevation – hence, the levee is assumed to be effectively continuously loaded. Due to the different foundation conditions, the highland and lowland levees have different side slopes of 1.5H:1V and 2H:1V, respectively, as shown in Figure 8. The time-averaged 30-m shear wave velocities in the two regions are 450 m/s and 200 m/s, respectively.

The study region is in an active seismic area, 45 km from a strike-slip fault having a scenario **M**6.5 earthquake. The area is also subject to water level rise in the river channel during storm events. Further details on the earthquake and high water demands are provided next.

5.2 Scenario Demands

In this paper, we consider log-normally distributed scenario-based high water level and seismic demands. Our failure probabilities are conditioned on those demand levels. We recognize that a more complete risk analysis would convolve uncertain demands with levee fragilities (described here) to evaluate return periods on levee failure, but our work has not evolved yet to that point.

The scenario high-water event is assumed to result from a severe storm in the river watershed. The median water level rise (ΔD_W) for both reaches in the river near the subject town from this event is assumed to be 1.2 m (Figure 8a), with a natural log standard deviation of 0.2.

We take the scenario ground motion as the within-event PGV distribution along the levee alignment. There is some change with coordinate *x* due to varying site-source distance (taken as distance to surface projection of fault, R_{JB}) and site condition. Figure 9 shows the variation of 16^{th}, 50^{th}, and 84^{th} percentile demands (using the Boore et al. 2014 ground motion model) with location along the levee. The origin of coordinate *x* is shown in Figure 7.

5.3 Monte Carlo Approach

We apply the Monte Carlo approach (Section 3.1) using the following inputs:

- Seismic and high water demand distributions are as described in Section 5.2.
- Seismic capacity distributions for highland and lowland areas are taken from the models of Kwak et al. (2016a) shown in Figure 3 as the G_N=1 model and the soft soil, high water model, respectively.
- High-water capacity distributions are described below.
- Spatial correlation models for demand and capacity are as described in Section 4.2.

We develop fragility for high water level by combining the hydraulic gradient-based fragility (Figure 4) with reach-specific seepage analyses performed for both reaches (in lowland and highland areas). The steady-state seepage analyses were performed using the computer program Slide 7.0 (Rocscience, 2015) using the section geometry and hydraulic conductivities shown in Figure 10. These seepage analysis results may be conservative for short-term flooding events, for which transient analyses would be more appropriate. Figure 10 also shows the resulting flow velocities for the mean high water level of 2.2 m above levee base. Figure 11 shows the resulting internal erosion simulation-based data points along with fragility curves as a function of high water elevation relative to levee base (D_W+ΔD_W). Both fragility curves are obtained fitting the data with a log-normal functional form, using the maximum likelihood estimation method (Baker, 2015).

Monte Carlo simulations (50,000 in total) applied to the seismic and high water scenario events produce system failure probabilities of $P(F_{sys})$ = 0.12 and 0.11, respectively.

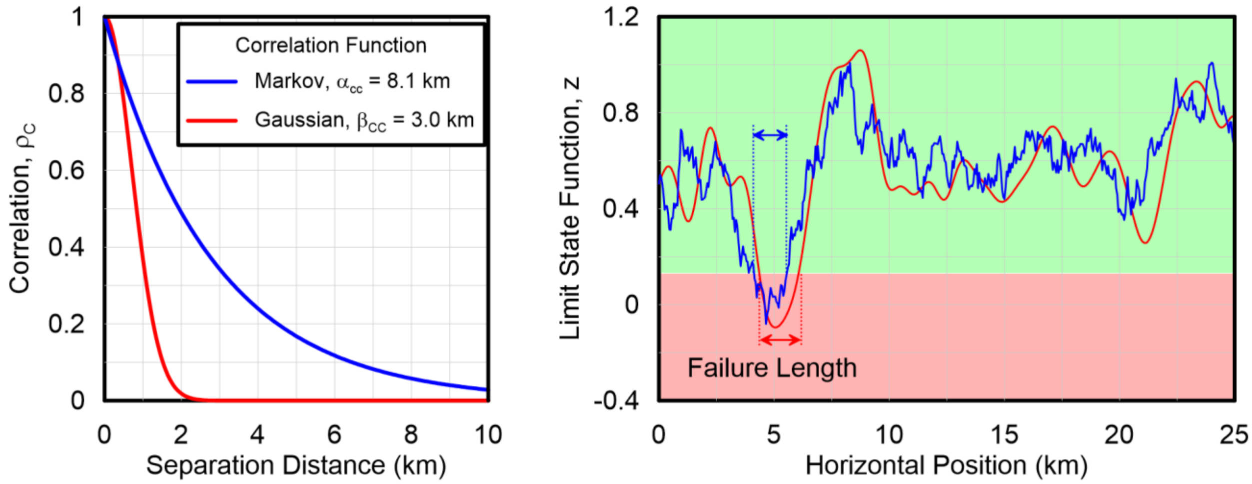

Figure 6. Capacity correlation functions (left) and their effect on the lateral distribution of limit state function *Z*.

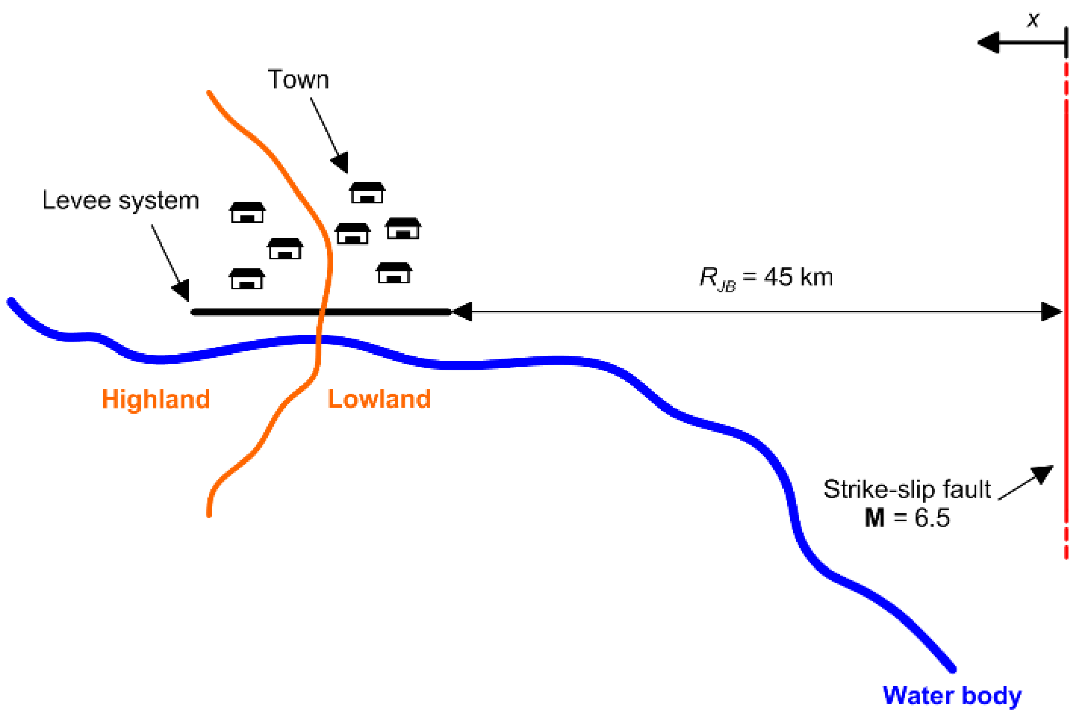

Figure 7. Schematic view of town protected by river-bounding levee passing over two geologic conditions and near an active fault.

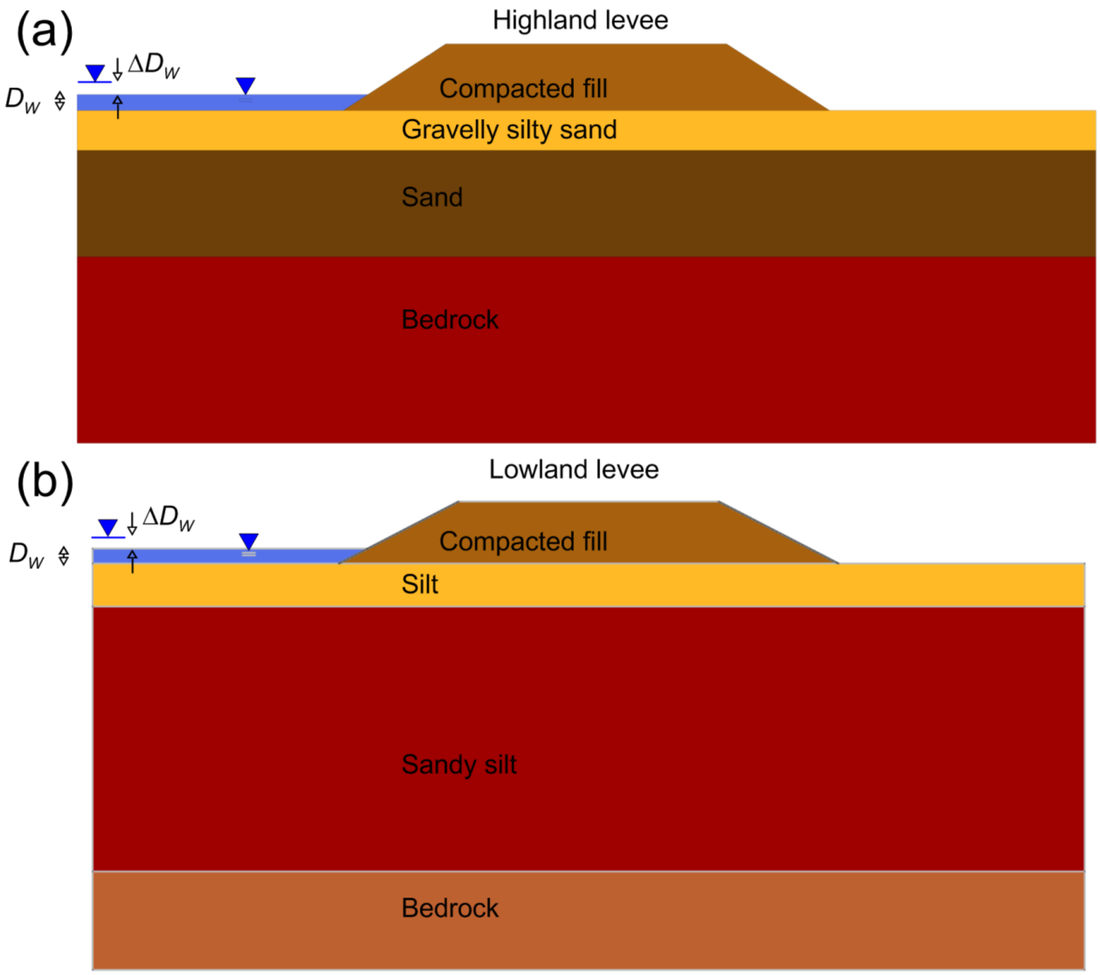

Figure 8. Cross sections of levees in: (a) highland area with mean water level plus mean water level rise, and (b) lowland area with mean water level.

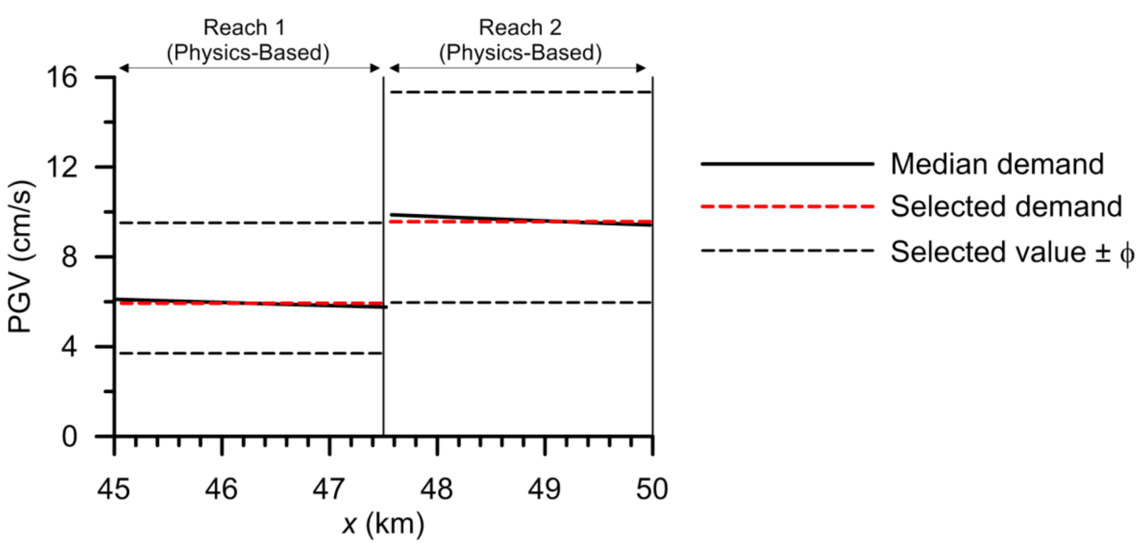

Figure 9. Variation of demand along the levee system. ϕ is within-event standard deviation (Boore et al. 2014).

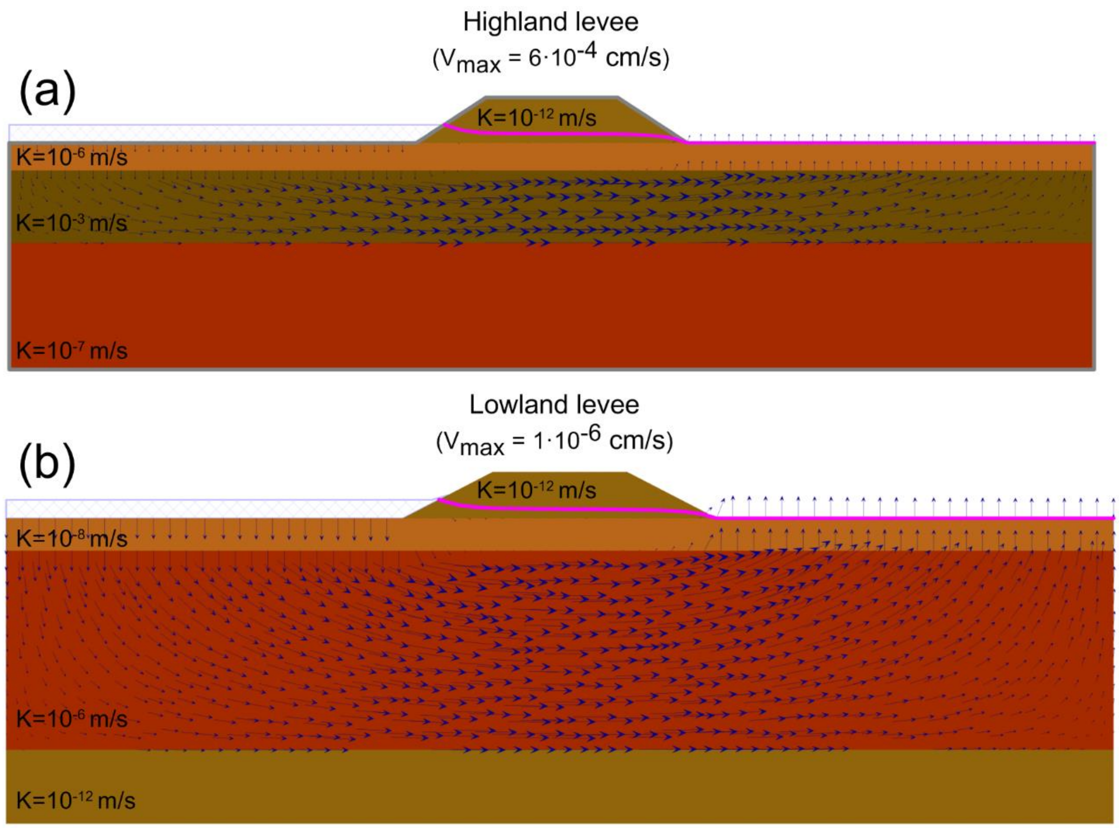

Figure 10. Computed fields of flow velocity beneath (a) highland and (b) lowland levees. Vector lengths are scaled relative to the maximum flow velocity (v_{max}).

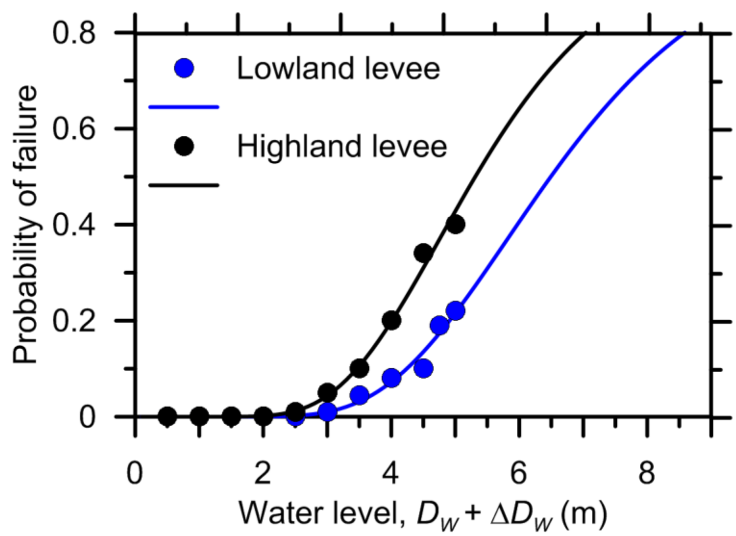

Figure 11. Internal erosion fragilities as a function of high water elevation relative to levee base. Dots represent data points obtained from seepage analyses, solid lines represent fitting curves obtained with a log-normal functional form, using the maximum likelihood estimation method.

5.4 Level-Crossing Statistics Approach and Characteristic Lengths

Recall that the level-crossing statistics approach is based on the assumption of constant limit state distributions within reaches. In the example problem, capacity distributions are constant within reaches, but demand distributions are variable with distance (x) as shown in Figure 9 (black solid lines). Accordingly, we assign constant (spatially invariant) distributions to the two reaches as shown in Figure 9 (red dashed lines).

Based on the demand and capacity distributions, results of the FORM analyses within the two reaches are given in Table 1. The higher failure probability for the highland, high water case results from the shorter flow path and faster flow velocities (Figure 10).

Table 1. FORM analysis results for segment-level performance in scenario events

Reach & demand	$P(F_{seg}\|E)$	β_{seg}	α_D	α_C
Highland, seismic	0.0044	2.63	0.26	-0.96
Highland, high water	0.011	2.28	0.52	-0.85
Lowland, seismic	0.009	2.36	0.45	-0.89
Lowland, high water	0.0025	2.80	0.52	-0.85

Application of level-crossing statistics to the segment-level FORM results produces the reach conditional failure probabilities and characteristic lengths in Table 2.

Table 2. Results of level-crossing statistics for reach-level performance in scenario events.

Reach & demand	$P(F_R\|E)$	L_{char} (km)
Highland, seismic	0.05	0.21
Highland, high water	0.09	0.33
Lowland, seismic	0.08	0.27
Lowland, high water	0.03	0.26

Having established the probability of failure for a single reach, we now turn our attention to computing the probability of failure of the multi-reach system. For simplicity, we assume that the limit state function for segments within one reach are uncorrelated with the limit state function for segments within an adjacent reach. Based on this assumption, the probability of system failure can be computed as:

$$P(F_{sys}|\mathrm{E}) = 1 - \prod_{i=1}^{N_R} \left[1 - P(F_{seg,i}|\mathrm{E})\right]^{L_i/L_{char}} \quad (11)$$

where N_R is the number of reaches. N_R is equal to two in the present application (Figure 9).

The assumption of statistical independence of the limit state function among reaches is justified when either of the following conditions is met:

1. The levee system exhibits an abrupt transition between reaches that occurs, for example, at the transition between two geologic units. This condition provides a physical justification for assuming that the limit state function is uncorrelated among reaches.
2. The characteristic lengths are significantly shorter than the reach length. In this case, any spatial correlation that exists at the contact between two reaches will not significantly influence the system failure probability.

For cases in which neither of these conditions are met, $P(F_{sys}|E)$ will be lower than computed using Eq. 11, which therefore provides a conservative estimate. Eq. 11 constitutes one of two unimodal bounds given in Eq. 1. The other unimodal bound assumes that the limit state function is perfectly correlated among reaches. We believe the solution will generally lie closer to that provided by Eq. 11 because the limit state function is likely closer to being statistically independent than perfectly correlated among reaches for typical levee systems.

Based on the use of Eq. 11, the conditional system failure probabilities for the two demand scenarios are:

- Seismic, $P(F_{sys}|E) = 0.13$
- High-water, $P(F_{sys}|E) = 0.10$

These failure probabilities compare favorably to results of Monte Carlo analysis (0.12 and 0.11 for seismic and high-water, respectively).

6 CONCLUSIONS

We describe two methods for risk analysis of spatially distributed systems subjected to spatially variable and uncertain demands. The intended application is levee systems used for flood protection, and the risk analyses are for high-water events (storm surge) and ground failure from earthquake shaking. The two methods are conceptually similar in that both utilize a limit state function defined as the difference between capacity and demand, which is described by its distribution and spatial correlation models.

One method randomly samples demands and capacities according to their respective distribution and correlation functions, computes limit states for levee segments, and computes failure probabilities on the basis of the number of realizations in which at least one segment fails divided by the total number of realizations (thousands). This method can consider spatially varying demands and capacities, but is computationally intensive.

The other method (referred to as the level crossing statistics method) discretizes a system into multiple reaches, calculates the probability of failure for each reach, and combines reach probabilities of failure to evaluate system probability of failure. The reach failure probability is calculated by estimating the limit state function for a representative segment (using the First Order Reliability Method), and then extending that result to the reach level using level-crossing statistics. We postulate that reach failure probabilities can be combined to evaluate system risk by assuming statistical independence between reaches, provided reach lengths exceed characteristic lengths. The FORM method is efficient and effective when the limit state function may reasonably be approximated as constant over fairly long reach lengths. However, the Monte Carlo method may be needed when demand and/or capacity varies significantly along the system length and the limit state function is non-stationary over short lengths.

Application of the two methods is illustrated using a two-reach levee system providing continuous flood protection to a town, and subject to high water and earthquake hazards. The high-water risk is assumed to result from internal erosion from underseepage. The seismic risk is driven by liquefaction and/or cyclic softening of levee and foundation soils.

Our example calculations show several attributes that reflect the characteristics of the input demand, capacity models, and correlation models:

1. The spatial correlation of the limit state function is much more strongly influenced by the capacity spatial distribution than the demand distribution. This reflects shorter correlation lengths for capacity.
2. Despite much stronger spatial demand correlations applied for the high-water scenario vs that for the seismic scenario, characteristic lengths for the two hazards are comparable.
3. For the example considered, results of the Monte Carlo and level-crossing statistics methods are generally comparable. It is unknown at this time how general this finding may be.

Two important aspects of the level crossing statistics method introduced in this paper are (1) the conversion of Markov-type spatial correlation models for demand to Gaussian functions, and (2) considerations in the analysis of system risk given failure probabilities for individual reaches.

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