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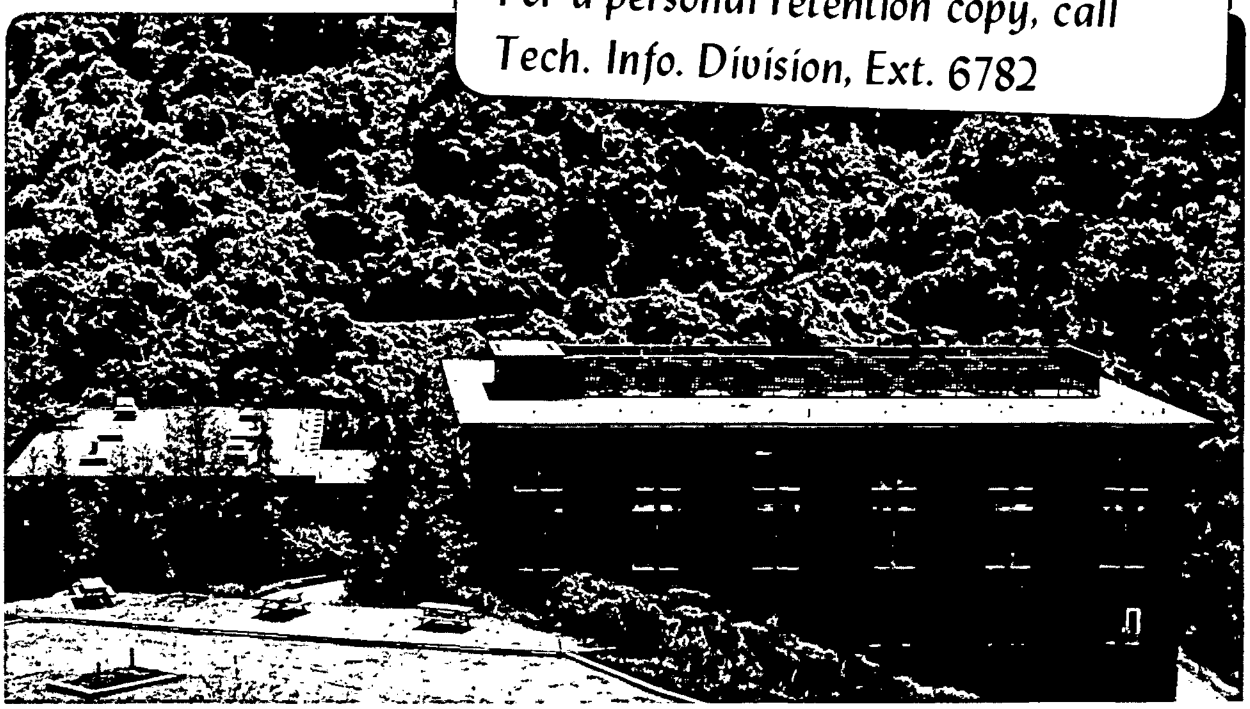

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QUANTUM NOISE THEORY FOR THE RESISTIVELY SHUNTED JOSEPHSON JUNCTION

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Abstract

We show that the low frequency spectral density of the voltage noise in a current-biased Josephson junction with critical current I_0, shunt resistance R, and small capacitance is $eI_0^2R^3/\pi V$ in the limit $eV >> k_BT(I/I_0)^2$, where V is the voltage and I is the current. The noise arises from zero-point current fluctuations in the shunt resistor. The rounding of the current-voltage characteristic caused by the quantum fluctuations and the effects of non-zero junction capacitance are calculated.

In this Letter we report calculations of the voltage noise in a current-biased resistively shunted[1] Josephson[2] junction (RSJ) when quantum corrections to the noise are taken into account. We show that the limiting noise in a specified region of the current-voltage (I-V) characteristic is set by zero-point fluctuations in the shunt resistor[3]. We predict that measurements of the noise in a junction with appropriate parameters should allow a direct observation of zero-point fluctuations. Furthermore, the calculated noise should enable one to estimate the limiting sensitivity of SQUIDs and Josephson video detectors and mixers of high frequency electromagnetic radiation.

Likharev and Semenov[4] (LS) and Vystavkin *et al.*[5] have calculated the voltage spectral density of a current-biased RSJ with zero capacitance when the current noise of the shunt is in the classical limit $\hbar\omega_J << k_B T$, where $\omega_J = 2eV/\hbar$ is the Josephson frequency at the average voltage V. We extend these calculations to the quantum limit $\hbar\omega_J >> k_B T$ where zero-point energy fluctuations in the shunt become significant. The calculation by Stephen[6] of fluctuations in the pair current of a junction biased near a self-resonant step can predict the linewidth of the Josephson radiation emitted by a voltage-biased tunnel junction in the quantum limit[6,7]. This calculation does not apply to the case of present interest because the apparent pair shot noise predicted arises from photon number fluctuations in a lossy resonant cavity coupled to the junction and is not intrinsic to the RSJ.

We consider a Josephson tunnel junction with critical current I_0 and capacitance C shunted with resistance R. We assume that V always lies below $2\Delta/e$, where Δ is the energy gap, so that the Riedel singularity[8] is unimportant. Furthermore, we take the temperature T to be well below the transition temperature, where the quasiparticle tunneling current is small compared with the current in the shunt resistance, so that we can neglect noise from the quasiparticle tunneling current[7]. The only significant noise source is the current noise, $I_N(t)$, in the resistor, which has a spectral density, including zero-point fluctuations,[9]

$$\mathcal{S}_I(\omega) = (\hbar\omega/\pi R) \coth (\hbar\omega/2k_B T) \qquad (1)$$

at angular frequency ω. We compute the spectral density of the voltage noise, $\mathcal{S}_V(\omega)$, for a current-biased RSJ at an angular frequency ω.

It is convenient to introduce the dimensionless units and parameters $i=I/I_0$, $v=V/I_0R=\omega_J/(2\pi I_0 R/\Phi_0)$, $\Gamma = 2\pi k_B T/I_0\Phi_0$, $\theta = \omega/(2\pi I_0 R/\Phi_0)$, $S_i(\theta) = \mathcal{S}_I(\omega)(2\pi R/I_0\Phi_0)$, $S_v(\theta) = \mathcal{S}_V(\omega)(2\pi/I_0\Phi_0 R)$, $\beta_c \equiv 2\pi I_0 R^2 C/\Phi_0$, and $\kappa \equiv eI_0R/k_BT$.

The phase difference $\delta(t)$ across the junction evolves with dimensionless time $t/(\Phi_0/2\pi I_0 R)$ according to the Langevin equation

$$\beta_c \ddot{\delta} + \dot{\delta} + \sin\delta = i + i_N \quad . \tag{2}$$

We first consider the limit[10] $\beta_c \ll 1$ in which the term $\beta_c \ddot{\delta}$ may be neglected in Eq.(2). In the limit in which noise-rounding effects are negligible, the I-V characteristic is[1] $v = (i^2-1)^{1/2}$ and Eq. (2) may be solved analytically using the LS method[4]. One calculates the Fourier components of the voltage fluctuations taking into account the mixing down of high frequency noise at harmonics of the Josephson frequency and finds the spectral density[4]

$$S_v(\theta) = \sum_{k=-\infty}^{\infty} |z_k|^2 \, S_i(\theta - kv). \tag{3}$$

Here, k is an integer, and [4]

$$|z_k| = \left| \delta_{k,o} + \frac{ki(i-v)^{|k|}}{\theta - kv} - \frac{1}{2}\left[\frac{(k-1)(i-v)^{|k-1|}}{\theta - (k-1)v} + \frac{(k+1)(i-v)^{|k+1|}}{\theta - (k+1)v}\right] \right| . \tag{4}$$

Evaluating the z_k in the limit $\theta/v \to 0$, that is, when the measurement frequency is much lower than the Josephson frequency, we find

$$S_v(0) = [\, i^2 S_i(0) + \tfrac{1}{2} S_i(v)]\, v^{-2} + O\left[\frac{\theta^2}{v^2} \sum_{k\geq 2} \frac{S_i(-kv)}{k^4}\right] . \qquad (\theta \ll v) \tag{5}$$

Even in the extreme quantum limit in which $S_i \sim kv$, the sum still converges, so that the last term is of order θ^2/v, which is negligible in the limit $\theta \ll v$.

Substituting Eq. (1) into Eq. (5) we find

$$S_V(0) = 4\Gamma r_D^2 \,[1 + (\kappa v/2i^2)\coth(\kappa v)]\,, \tag{6}$$

where $r_D = \partial v/\partial i = i/v$, or, in dimensioned units,

$$\mathcal{S}_V(0) = R_D^2\left[\frac{2k_BT}{\pi R} + \frac{eV}{\pi R}\left(\frac{I_o}{I}\right)^2 \coth\left(\frac{eV}{k_BT}\right)\right]\,, \tag{7}$$

where $R_D = \partial V/\partial I$ is the dynamic resistance. The inset of Fig. 1 shows the temperature dependence of $\mathcal{S}_V(0)$ for particular values of I_o (assumed to be independent of temperature) and R at fixed bias current. For comparison, the LS result in the classical limit is also shown.

It is instructive to consider several limits of Eq. (7): (i) $eV \ll k_BT$ ($\kappa v \ll 1$): We obtain the LS result[4] $\mathcal{S}_V(0) = (2k_BTR_D^2/\pi R)[1+\tfrac{1}{2}(I_o/I)^2]$. (ii) $eV \gg k_BT$ ($\kappa v \gg 1$): We obtain $\mathcal{S}_V(0) = R_D^2[2k_BT/\pi R + eVI_o^2/\pi RI^2]$. For $eV \ll k_BT(I/I_o)^2$ ($vr_D^2 \gg \kappa$) this yields the Nyquist result $\mathcal{S}_V(0) = 2k_BTR/\pi$, while for $eV \gg k_BT(I/I_o)^2$ ($vr_D^2 \ll \kappa$), we find the quantum limit

$$\mathcal{S}_V(0) = eV(I_o/I)^2R_D^2/\pi R = eI_o^2R^3/\pi V. \tag{8}$$

Thus, to observe quantum effects we require $\kappa \equiv eI_oR/k_BT \gg 1$. At the particular bias $V=I_oR$, Eq. (8) reduces to $\mathcal{S}_V(0) = eI_oR^2/\pi$, which is just the voltage spectral density of the shot noise due to a current I_o flowing through a resistance R. However, it should be clear from the derivation that Eq. (8) arises not from an intrinsic shot noise in the pairs tunneling through the barrier but rather from the zero-point fluctuations of the shunt resistance which have a current spectral density $\hbar\omega/\pi R$.

To compute the noise rounding of the I-V characteristics or to include the effects of a non-zero capacitance, we have used numerical techniques. Although we have computed the general case $T \neq 0$, we report here results only for the $T=0$ limit in which Eq. (1) becomes $\hbar\omega/\pi R$. We used an LSI-11 computer to integrate Eq. (2) using a noise driving term obtained by digitally filtering pseudo-random white noise. We obtained the mean voltage by averaging the instantaneous voltage over typically 10^4 Josephson cycles, and determined the low frequency spectral density of the voltage noise by averaging the fluctuations in the voltage after low-pass digital filtering. The accuracies of the average voltage and the spectral density are believed to be ±5% and ±10% for $i>i_0$, and ±10% and ±20% for $i \lesssim i_0$. Figure 2 illustrates the noise rounding of the I-V characteristics due to zero point fluctuations for[10] $\beta_c \approx 0.1$. For a given depression of the critical current below the noise-free value, the rounding extends to much larger values of voltage than in the equivalent thermal noise case[11] because the noise in the resistor at the Josephson frequency increases with voltage.

Figure 3 shows the effects of increasing β_c. The dynamic resistance increases markedly at low voltages [Fig. 3(a)] as β_c increases[12]; hysteresis occurs for $\beta_c \gtrsim 1$. Figure 3(b) shows the corresponding spectral densities of the voltage noise, with the dotted line taken from Eq. (6). For $v>0.5$ the noise rounding is small, and the computer and analytical results are indistinguishable. The increase in noise with increasing β_c for a given voltage at low voltages reflects the higher dynamic resistance, but for all β_c at very low voltages the noise decreases with decreasing voltage because of noise rounding. At high voltages, the noise decreases with increasing β_c at a given voltage because the noise currents are filtered out at frequencies above $\sim 1/RC$.

We have not included in our calculation the possibility of macroscopic quantum tunneling[13,14] (MQT), which would permit the junction to tunnel between states of metastable equilibrium and hence increase the noise rounding for for $I < I_0$. For an undamped junction in the limit $C \rightarrow 0$, the tunneling rate is expected to become infinite[14], and the classical model of the junction as a point particle moving in a potential is no longer valid. However, since MQT is predicted to decrease rapidly as the damping increases[13], we do not expect it to make a significant contribution in the highly damped limit considered here.

In conclusion, we note that the quantum effects calculated here should be observable provided one can obtain the limit $\kappa >> 1$. Writing $\kappa = (e/k_BT)(\beta_c\Phi_0 j_1/2\pi c)^{1/2}$, where j_1 is the critical current density and c is the capacitance per unit area of the tunnel junction, we see that the limit requires a high current density and/or a low temperature. At 1K, with $j_1 = 10^4 Acm^{-2}$, $\beta_c = 1$ and $c = 0.04\ pF\mu m^{-2}$, we find $\kappa \approx 10$, a value at which quantum corrections are considerable (see inset of Fig. 1). Our results for $\beta_c << 1$ should also be applicable to point contact junctions and micro-bridges to the extent that these devices can be represented by the RSJ model.

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References

1. W.C. Stewart, Appl. Phys. Lett. 12, 277 (1968); D.E. McCumber, J. Appl. Phys. 39, 3113 (1968).
2. B.D. Josephson, Phys. Lett. 1, 251 (1962).
3. J.C. Gallop and B.W. Petley [J. Phys. E. 9, 417 (1976)] have discussed this possibility.
4. K.K. Likharev and V.K. Semenov, J.E.T.P. Lett. 15, 442 (1972).
5. A.N. Vystavkin, V.N. Gubankov, L.S. Kuzmin, K.K. Likharev, V.V. Migulin, and V.K. Semenov, Phys. Rev. Appl. 9, 79 (1974).
6. M.J. Stephen, Phys. Rev. 182, 531 (1969).
7. A.J. Dahm, A. Denenstein, D.N. Langenberg, W.H. Parker, D. Rogovin, and D.J. Scalapino, Phys. Rev. Lett. 22, 1416 (1969).
8. E. Riedel, Z. Naturf. 19A, 1634 (1964).
9. H.B. Callen and T.A. Welton, Phys. Rev. 83, 34 (1951).
10. As $\beta_c \to 0$, the roll-off frequency of the noise, $\sim 1/RC$, increases, and the mean square current noise available to the junction, $\propto (1/RC)^2$, eventually becomes so large that the noise-rounded critical current is reduced to zero. In the analytical discussion we choose $0 < \beta_c \ll 1$, while for the computer results we choose $\beta_c \approx 0.1$.
11. V. Ambegaokar and B.I. Halperin, Phys. Rev. Lett. 22, 1364 (1969).
12. J. Kurkijärvi and V. Ambegaokar, Phys. Lett. 31A, 314 (1970); R.F. Voss (preprint).
13. A.O. Caldeira and A.J. Leggett (preprint).
14. J. Kurkijärvi (to be published in the Proceedings of the Second International Conference on SQUIDs, Berlin, May 6-9, 1980).

Figure Captions

Fig. 1 Low frequency spectral density $s_V(0)$ of the voltage noise vs. current for 5 values of $\kappa \equiv eI_0R/k_BT$ with $\beta_c \ll 1$. Inset shows $s_V(0)$ vs. T for I_0 = 1mA, I = 1.41mA, and R = 0.86Ω (chosen to give $V = I_0R$ and κ = 10 at 1K). Dashed line shows the LS classical result.

Fig. 2 I-V characteristics at T = 0 with $\beta_c \approx 0.1$ for 4 values of $\kappa\Gamma \equiv 2\pi eR/\Phi_0$, showing rounding due to zero-point fluctuations. Dashed line shows noise rounding in the thermal noise limit for a similar depression in critical current as for the case $\kappa\Gamma = 0.05$.

Fig. 3 (a) I-V characteristics at T = 0 for $\kappa\Gamma = 2\pi eR/\Phi_0 = 0.0194$ (R=40Ω) with $\beta_c \approx 0.1$, = 0.5, 1. (b) Spectral density of the voltage noise for the curves in (a); dotted line is taken from Eq. (6). The dashed portions are of lower accuracy.

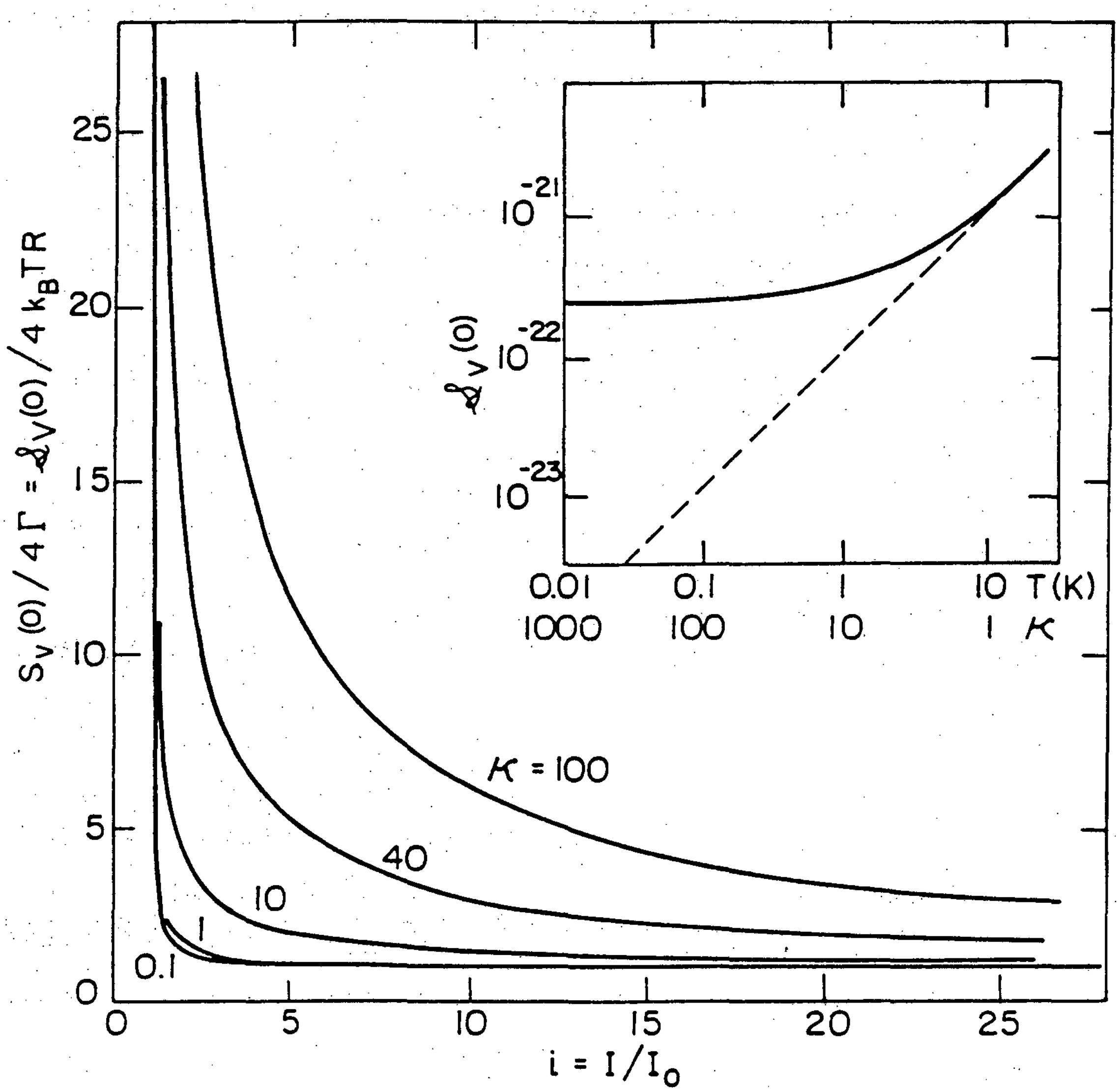

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Fig. 1

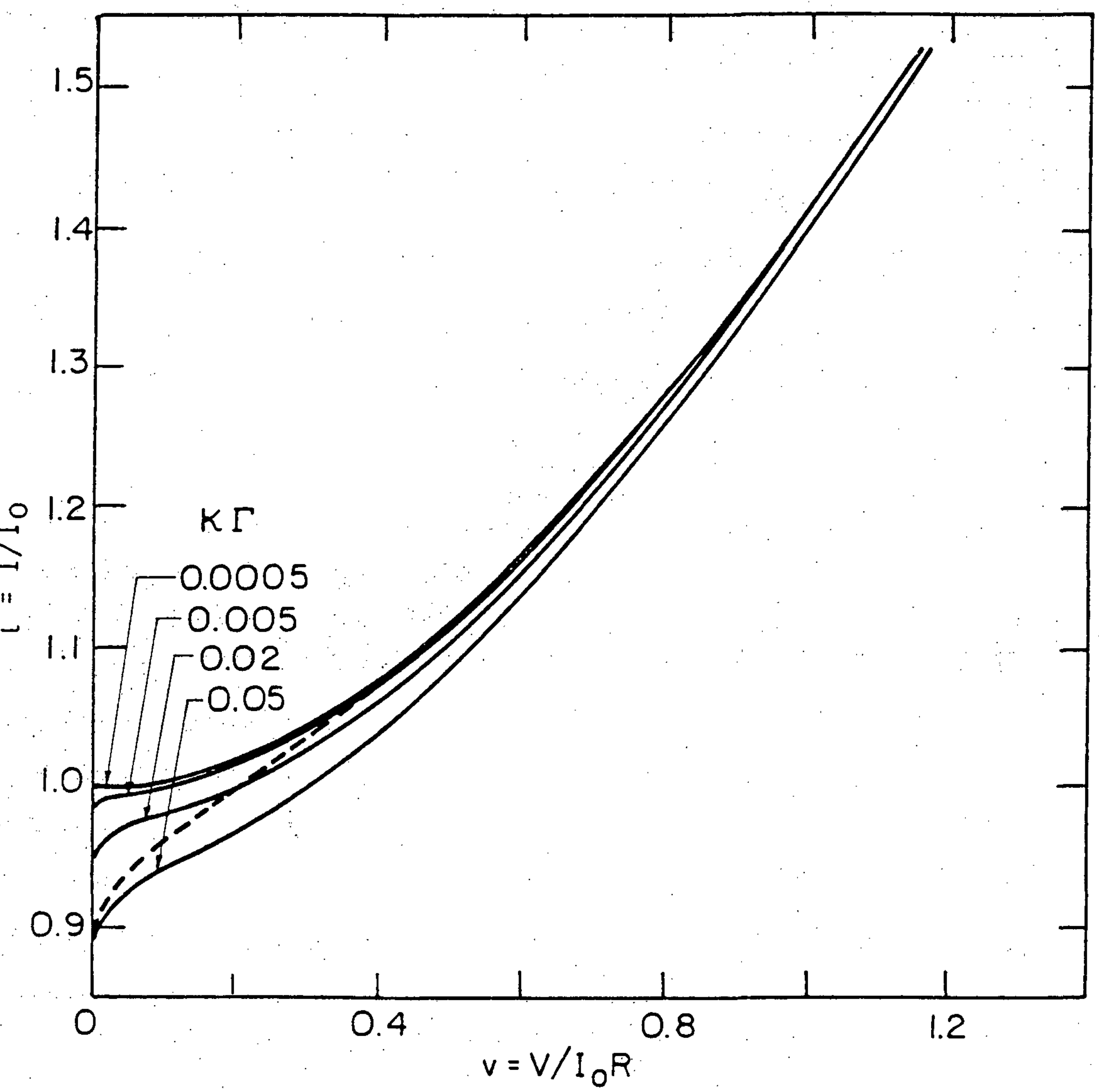

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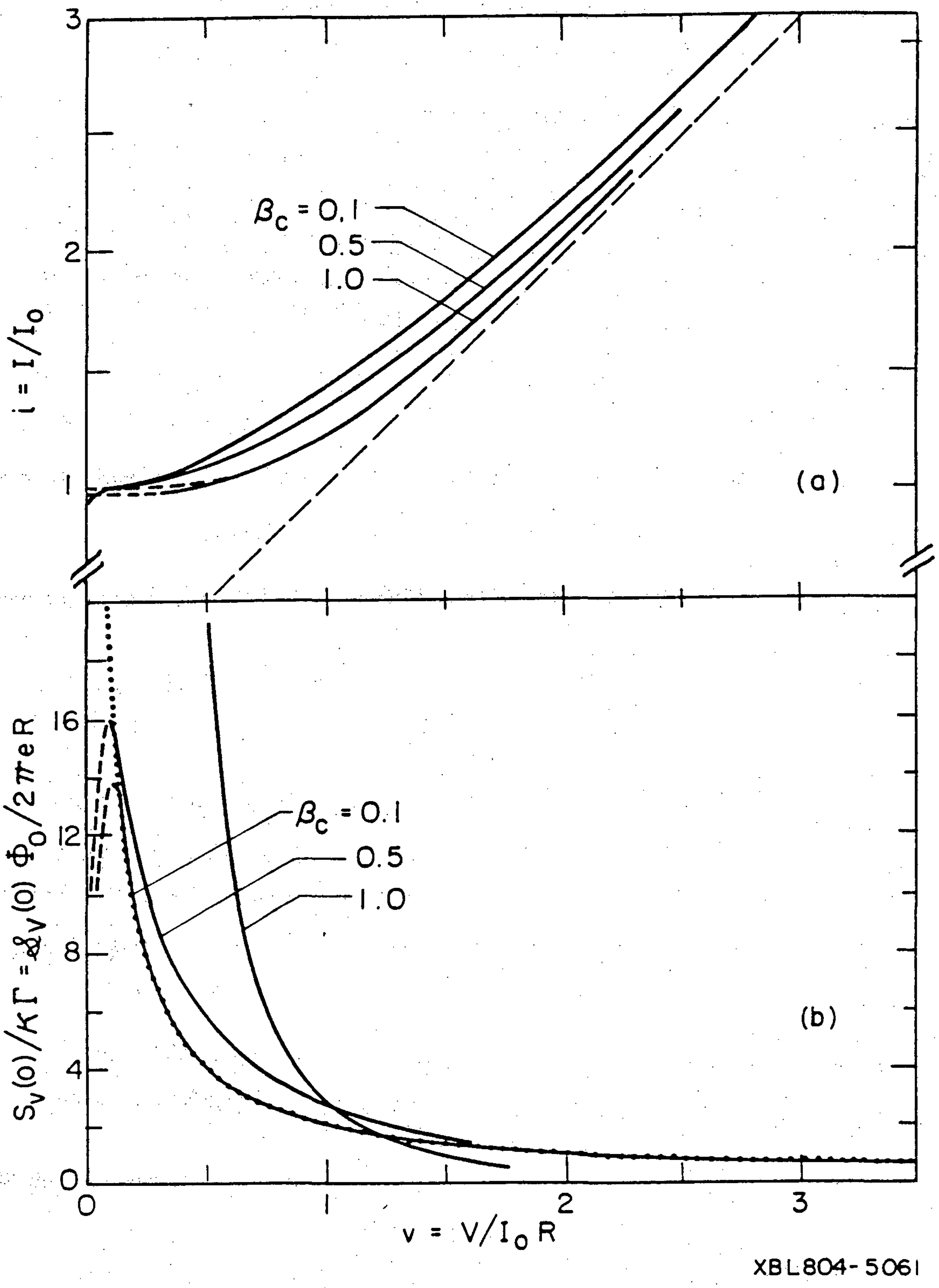
Fig. 2



3
2
1
$i = I/I_0$
$\beta_c = 0.1$
0.5
1.0
(a)
16
12
8
4
0
$S_V(0)/\kappa\Gamma = \mathcal{S}_V(0)\ \Phi_0/2\pi eR$
$\beta_c = 0.1$
0.5
1.0
(b)
0
1
2
3
$v = V/I_0R$
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Fig. 3

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