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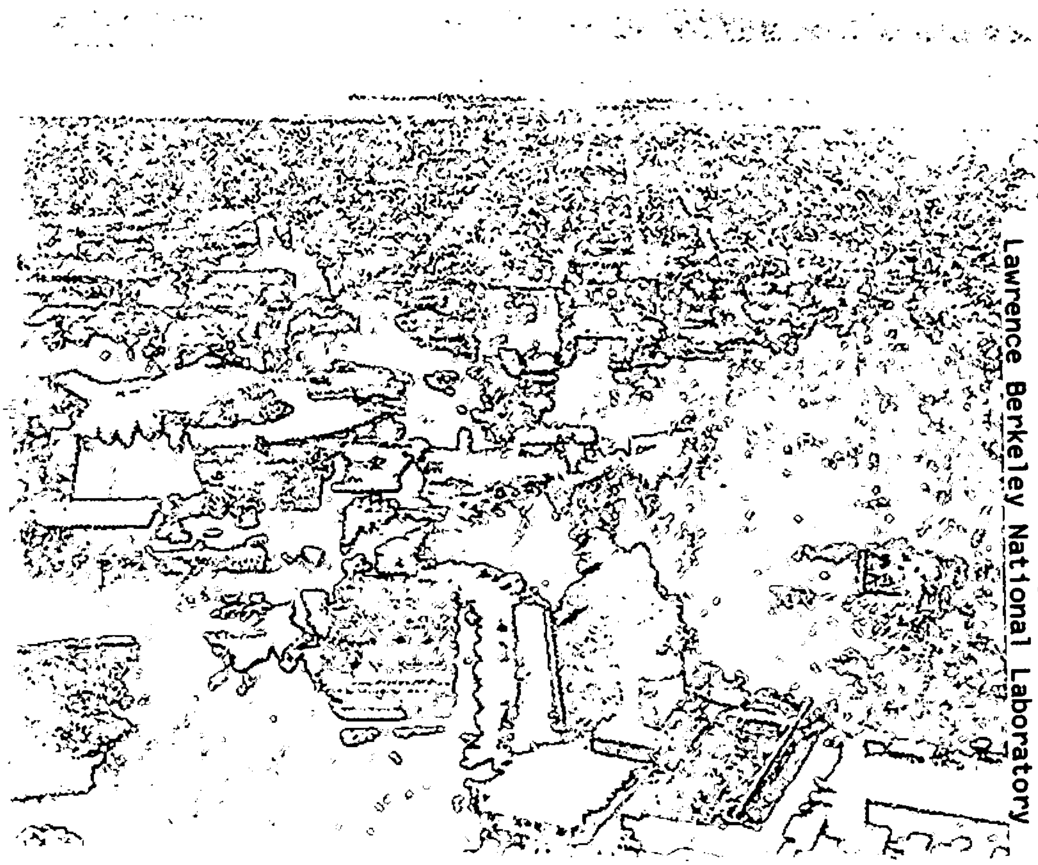

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Bell's Theorem Without Hidden Variables *

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Abstract

Experiments motivated by Bell's theorem have led some physicists to conclude that quantum theory is nonlocal. However, the theoretical basis for such claims is usually taken to be Bell's Theorem, which shows only that if certain predictions of quantum theory are correct, and a strong hidden-variable assumption is valid, then a certain locality condition must fail. This locality condition expresses the idea that what an experimenter freely chooses to measure in one spacetime region can have no effect of any kind in a second region situated spacelike relative to the first. The experimental results conform closely to the predictions of quantum theory in such cases, but the most reasonable conclusion to draw is not that locality fails, but rather that the hidden-variable assumption is false. For this assumption conflicts with the quantum precept that unperformed experiments have no outcomes. The present paper deduces the failure of this locality condition directly from the precepts of quantum theory themselves, in a way that generates no inconsistency or any conflict with the predictions of relativistic quantum field theory.

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1. Introduction.

A recent issue of Physics Today[1] has a bulletin entitled "Nonlocality Get More Real". It reports experiments at three laboratories (Geneva [2], Innsbruck[3], and Los Alamos[4]) directed at closing loopholes in proofs that quantum phenomena cannot be reconciled with classical ideas about the local character of the physical world.

The experiment reported in the first of these papers [2] confirms the existence of a classically unexplainable connection between phenomena appearing at essentially the same time in two villages separated by a distance of more than 10km, and the paper begins with the provocative statement "Quantum theory is nonlocal." The longer version [5] says "Today, most physicists are convinced that a future loophole-free test will definitely demonstrate that nature is indeed nonlocal."

The theoretical basis for such claims is usually taken to be Bell's Theorem, which, however, shows only that if certain predictions of quantum theory are correct, *and if a certain hidden-variable assumption is valid,* then a locality condition must fail. This locality condition expresses the physical idea, suggested by the theory of relativity, that what an experimenter freely chooses to measure in one spacetime region can have no effect of any kind in a second region situated spacelike relative to the first.

The experimental results conform closely to the predictions of quantum theory in such cases, but the most natural conclusion to draw is not that locality fails, but rather that the hidden-variable assumption is false. For the "hidden-variable assumption" of Bell's original theorem[6] is basically the assumption that a set of possible outcomes can be simultaneously defined for each of the alternative mutually incompatible experiments that the experimenters might choose to perform. This assumption violates the precepts of quantum philosophy by assigning definite, albeit unknown, values to the outcomes of mutually incompatible measurements.

Bell[7] later introduced a seemingly weaker local hidden-variable assumption, but it can be shown[8,9] that this later form entails the original one, apart from errors that tend to zero as the number of experiments tends to infinity. Thus both forms of the hidden-variable assumption contradict the basic quantum precept that one cannot, in general, consistently assign possible outcomes to unperformed measurements. Hence, from the viewpoint of orthodox quantum theory, Bell's hidden-variable assumption, in either form, is more likely to fail than the locality condition. Consequently, these hidden-variable theorems provides no adequate basis for the claim that "Quantum theory is nonlocal", or that "nature is indeed nonlocal."

The present paper describes a fundamentally different kind of proof. It shows that a violation of the locality condition described above follows logically from assumptions that formalize the precepts of orthodox of quantum theory itself. This proof is logically rigorous. But like all proofs and theorems of this general kind it is based essentially on the use of statements about contrary-to-fact situations.

I have just mentioned that the basic principles of quantum philosophy forbid the unrestricted use of contrary-to-fact, or counterfactual notions. So it might be thought that the new proof is basically no better than the ones employing hidden-variables. However, there is an essential logical difference between the two proofs: Bell's hidden-variable assumption was an ad hoc assumption that had no foundation in the quantum precepts. Indeed, it *directly contradicted the quantum precepts.* Moreover, it led, when combined with the predictions of quantum theory, to *logical inconsistencies.* This latter fact vindicates the quantum precept that bans the hidden-variable assumption, and thus undermines the significance of any conclusions derived from that assumption.

The proof to be presented here differs from the hidden-variable theorems in two crucial ways. First, as just mentioned, the assumptions are *direct expressions* of the precepts of orthodox quantum precepts themselves, rather than being, as regards the hidden-variable aspect, a direct violation of those precepts. Second, the assumptions used in the present proof, lead only to a very restricted set of true counterfactual statements, and these lead neither to any logical contradiction, nor to any conflict with the predictions of relativistic quantum field theory. Thus the proof to be presented here lies on a logical level different from that of the proofs that follows Bell's hidden-variable approach.

What assumptions are used in the present approach?

The first assumption is that, for the purposes of understanding and applying quantum theory, the choice of which experiment is to be performed can be treated as a free variable. Bohr repeatedly stressed this point, which is closely connected to his "complementarity" idea that the quantum state contains complementary kinds of information pertaining to the various alternative mutually exclusive experiments that might be chosen. Of course, only one *or* the other of two mutually incompatible measurements can be actually performed, not both, and only a measurement that is actually performed can be assumed to have a definite outcome.

It is worth recalling in this connection that Bohr did not reject the argument of Einstein, Podolsky, and Rosen out of hand, simply because it involves considering two mutually incompatible experiments. Bohr accepted that feature of the argument, and was therefore forced to find another, much more subtle, ground for rejecting the argument of those authors, which, like the one to be presented here, but unlike those of Bell, scrupulously avoids any direct assumption that an unperformed experiment has a definite outcome.

The second assumption of the present work is that an outcome that has appeared to observers in one spacetime region, and has been recorded there, can be considered to be fixed and settled by the time the observations and recordings are completed: this outcome is treated as being independent of which experiment will be freely chosen and performed by another experimenter acting in a spacetime region *lying later than* the first region. This no-backward-in-time-influence property is assumed to hold in *at least* one Lorentz frame, which I shall call LF.

These two assumptions immediately entail the truth of a very limited class of counterfactual statements. These are statements of the form: If experiments E and F are freely chosen and performed in earlier and later regions, respectively, and the outcome in the earlier region is $E+$, then the outcome in the earlier region would (still) be $E+$ if everything in nature were the same except for (1), *a different free choice* made in the later region, and (2), *the possible consequences* of making that alternate later free choice *instead of* the actual free choice. The no-backward-in-time-influence condition asserts that the possible consequences of the experimenter's making the alternate (i.e., counterfactual) later free choice do not include any change of an outcome that has already appeared to observers in the earlier spacetime region, and has been recorded there.

This assumption of no backward-in-time action is the expression of a theoretical idea: it can never be empirically confirmed. On the other hand, it is completely compatible with all the predictions of quantum theory, and with all the properties of relativistic quantum field theory. This will be shown later. Thus it provides for an enlargement of the quantum-theoretical framework that allows one to consistently consider alternative possible "free choices."

Any physicist is certainly free to deny this assumption that a later free choice cannot affect an already-observed-and-recorded outcome. Hence my proof must be regarded as an exploration of the logical consequences of making this foray into the realm of counterfactuals. This excursion is needed in order to formulate the concept of "no influence" that is under consideration, and to give some effective meaning to the notion of a "free choice."

Notice that there is no direct assumption that some unperformed experiment has an outcome. In the first place, the effective replacement assumption is that there is no influence backward in time. This assertion is part of the very assumption that is under examination, rather than a direct negation of a basic quantum precept. In the second place, the measurement whose outcome is counterfactually specified to be $E+$ is exactly the locally defined experiment E that is assumed to be the experiment that is actually performed at the earlier time. This earlier experiment and its outcome is "unperformed" or "counterfactual" only in a theoretical nonlocal and atemporal sense, when it is considered in combination with a never-to-be-performed later experiment. The no-backward-in-time-influence assumption is essentially the assumption that this earlier locally characterized measurement E is one and the same measurement no matter which free choice is made later on. In this local sense there is no assumption about the outcome of any unperformed measurement: the single locally defined earlier measurement has one single outcome no matter which free choice is made later on.

Logicians have developed rigorous logical frameworks for consistent and unambiguous reasoning with statements involving counterfactual conditions. Those frameworks incorporate certain ideas about the world that are concordant with a deterministic classical-physics conception of nature. The present proof can be carried out within such a classical framework. However, the indeterminism of quantum physics leads naturally to a quantum logic for counterfactual reasoning. I shall, in Appendix A, present a rigorous formal proof within this quantum counterfactual logic, which is described in Appendix B.

This proof shows that in a certain (Hardy) experimental setup a statement can be constructed whose truth value (true or false) is *defined* in terms of the truth values of some statements pertaining to possible outcomes of possible experiments confined to a certain spacetime region R, but that this statement is, by virtue of certain predictions of quantum theory, *true* if one experiment is chosen and performed by an experimenter in an earlier (in LF) region L that is spacelike separated from region R, but is *false* if a different experiment is chosen and performed in that earlier region L. This non-trivial dependence upon a free choice made in one region of the truth of a statement specified by the truth or falsity of statements pertaining to possible events in a spacelike separated region, constitutes, within this logical framework, some kind kind of faster-than-light influence, as will be discussed.

The quantum counterfactual logic described in appendix B is, I believe, interesting in itself, as is the formal proof given in Appendix A. They give precision and rigor to the argument. However, the argument is so intuitively obvious that I believe it is sufficient to state it in plain words. This is done in the next section. Then in section 3 it is shown, by referring to the Tomonaga-Schwinger formulation of quantum field theory, that the assumptions and conclusions of the proof are logically compatible with the predictions of relativistic quantum field theory. In section 4 I discuss the fact that the proved result is incompatible with the notion that the free choice made in one region can have *no influence of any kind* in a second region that is spacelike separated from the first.

2. The Informal Proof.

The argument is based on a Hardy-type [10] experimental set-up.

There are two experimental spacetime regions R and L, which are spacelike separated. In region R there are two alternative possible measurements, R1 and R2. In region L there are two alternative possible measurements, L1 and L2. Each local experiment has two alternative possible outcomes, labelled by + and − . The symbol $R1$ appearing in a logical statement stands for the statement "Experiment R1 is chosen and performed in region R." The symbol $R1+$ stands for the statement that "The outcome '+' of experiment R1 appears in region R." Analogous statements with other variables have the analogous meanings.

The detectors are assumed to be 100% efficient, so that for each possible world some outcome, either + or −, will, according to quantum mechanics, appear in R, and some outcome, either + or −, will appear in L.

Suppose Robert acts in region R, and Lois acts in region L. Then the first two predictions of QT for this Hardy setup are these:

(2.1): If Robert perform $R2$ and gets outcome $R2+$ and Lois performs $L2$, then Lois gets outcome $L2+$.

(2.2): If Lois performs $L2$ and gets outcome $L2+$, and Robert performs $R1$, then Robert gets outcome $R1-$.

Combining these two conditions with the no-backward-in-time-influence condition, which says that what Robert freely chooses to do in the later region R cannot disturb Lois's earlier outcome, one immediately obtains the conclusion that:

If Lois performs $L2$ then

"If Robert performs $R2$ and gets $R2+$ then if his choice had gone the other way he would have gotten outcome $R1-$."

This conclusion is expressed by Line 5 of the formal proof.

The second two predictions of QT for this Hardy setup are:

(2.3): If Lois performs $L1$ and get outcome $L1-$, and Robert performs $R2$, then Robert gets outcome $R2+$.

(2.4): *It is not true that* If Lois perform $L1$ and gets outcome $L1-$, and Robert performs $R1$ then Robert gets outcome $R1-$.

To deduce the desired conclusion one uses the fact that if Lois performs $L1$ then quantum theory predicts that she gets $L1-$ roughly half the time. Thus there are physically possible worlds in which Lois performs $L1$ and gets outcome $L1-$. In any such world if Robert chooses $R2$, then, according to (2.3), he will obtain outcome $R2+$. According to our no-backward-in-time-influence condition, the outcome $L1-$ observed earlier by Lois would be left unchanged if Robert had, later in R, made the other chioce, and performed $R1$. But then prediction (2.4) of quantum theory ensures that there are possible worlds in which Lois performs $L1$ and Robert performs $R2$ and gets outcome $R2+$, but in which Robert would not have obtained outcome $R1-$ of he had freely chosen to perform $R1$ instead of $R2$. This means that

If Lois performs $L1$ then *It is not true that*
"If Robert performs $R2$ and gets $R2+$ then if his choice had gone the other way he would have gotten outcome $R1-$."

This is exactly what the rigorous formal proof shows.

Notice that this second conclusion, like the earlier one, contains the statement SR:

"If Robert performs $R2$ and gets $R2+$ then if his choice had gone the other way he would have gotten outcome R1-."

This statement SR, whose truth or falsity is defined in terms of the truth or falsity of statements pertaining to possible events in region R, is true if Lois's free choice in region L is to perform $L2$, but is not true if Lois's free choice in region L is to perform $L1$. Thus the truth of this statement SR pertaining to region R depends upon what Lois freely chooses to do in a region L that is situated spacelike relative to region R. This dependence constitutes some kind of effect in region R of Lois's free choice made in region L. This is discussed in Section 4.

3. Logical Consistency and Compatibility with Relativity.

The assumptions in this argument, unlike those of approaches based on hidden variables, are in line with the precepts of orthodox quantum theory, and when combined with the predictions of relativistic quantum field theory they lead to no logical inconsistencies. One can confirm this by simply noting that the no-backward-in-time-influence condition is satisfied in the formulation of relativistic quantum field theory given by Tomonaga[11] and by Schwinger[12], with their spacelike surfaces σ taken to be the constant-time surfaces in the special frame LF. This frame then defines the meaning of the evolving state $\Psi(t)$, which can be assumed to collapse to a new state when a measurement is completed, and new information is thus considered to have become specified.

There is, of course, no suggestion in the works of Tomonaga and Schwinger that some particular set of surfaces should be singled out as the "true" or "real" surfaces that define the real evolving state of the universe $\Psi(t)$ that is suddenly reduced to a new form when new information becomes available. Quite the opposite: they show that it does not matter which of the infinite collection of advancing sets of spacelike surfaces σ one uses to define the forward-evolving state of the system. They effectively show that the predictions of the theory will be independent of which such set of advancing spacelike surfaces σ one uses. This feature of their theory is, of course, completely concordant with the ideas of the theory of relativity.

My use of Tomonaga-Schwinger theory is a logical, not ontological, one. I merely claim that my no-backward-in-time-influence assumption is *logically compatible* with the predictions of relativistic quantum field theory: I make no claim that this causality assumption has any ontological significance, or that the particular frame LF is unique. On the other hand, I could not demonstrate compatibility with the predictions of relativistic quantum field theory if I tried to assert that the no-backward-in-time-influence condition held simultaneously in several frames: Tomonaga-Schwinger theory does not ensure the compatibility of that stronger condition with the predictions of relativistic quantum field theory.

4. Conclusion.

The two conclusions proved from our premises both involve the same assertion SR:

"If Robert performs RA and gets $RA+$ then if his free choice in R had gone the other way he would have gotten outcome $RC-$."

Here A stands for actual, and takes the value 2, and C stands for counterfactual, and takes the value 1.

What was proved is that this statement SR about the connections in R of the consequences of making alternative possible free choices in R is, by virtue of our explicity stated assumptions, true or false according to whether $L2$ or $L1$ is freely chosen in L.

This conclusion entails that information about which choice is freely made in region L must get to region R. This is because the truth value (true or false) of SR is defined in terms of the truth values of the elements of the quadruple of statement $(RA, RA+; RC, RC-)$: SR, with $A = 2$ and $C = 1$, is false if and only the set of truth values of that quadruple is $(t, t; t, f)$. For every other quadruple of truth values the statement SR is true. Thus the nontrivial dependence of the truth of SR on the free choice between $L1$ and $L2$ made in region L means that the truth or falsity of some statements about possible events in region R must, as a consequence of our assumptions, depend upon whether the free choice made in region L is to perform $L1$ or $L2$.

This result places a strong condition on theoretical models that reproduce the predictions of quantum theory. This condition is similar to the failure of locality associated with Bell's theorem. But here it is derived from the premises of "free choice" and "no backward in time influence" that are in line with the precepts of quantum theory, and that lead to no logical contradictions, and to no conflicts with the predictions of relativistic quantum field theory.

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APPENDIX A: The Formal Proof.

Each line of the following proof is a strict consequence of the predictions of quantum mechanics, (2.1)-(2.4), the general property that there are possible worlds W in which $L1$ is performed the outcome is $L1-$, the assumed property LOC1, and the properties of the rudimentary logical symbols. Line 6 is the one exception: it is just the same as line 5, but with L2 replaced by L1. The part of the proof from line 7 to line 14 shows that the statement on line 6 is false. Thus the proof shows that mentioned premises lead to the conclusion that line 5 is true, but that line 6 is false.

Proof:

1. $(L2 \wedge R2 \wedge L2+) \Rightarrow [R1\Box \rightarrow (L2 \wedge R1 \wedge L2+)]$ [(B.6)]
2. $(L2 \wedge R2 \wedge R2+) \Rightarrow (L2 \wedge R2 \wedge L2+)$ [(2.1)]
3. $(L2 \wedge R1 \wedge L2+) \Rightarrow (L2 \wedge R1 \wedge R1-)]$ [(2.2)]
4. $(L2 \wedge R2 \wedge R2+) \Rightarrow [R1\Box \rightarrow (L2 \wedge R1 \wedge R1-)]$ [1, 2, 3, (B.7)]
5. $L2 \Rightarrow [(R2 \wedge R2+) \rightarrow (R1\Box \rightarrow R1 \wedge R1-)]$ [4, LOC1, (A.5)]
6. $L1 \Rightarrow [(R2 \wedge R2+) \rightarrow (R1\Box \rightarrow R1 \wedge R1-)]$
7. $(L1 \wedge R2 \wedge R2+) \Rightarrow (R1\Box \rightarrow R1 \wedge R1-)]$ [6, (A.5)]
8. $(L1 \wedge R2 \wedge L1-) \Rightarrow (L1 \wedge R2 \wedge R2+)$ [(2.3)]
9. $(L1 \wedge R2 \wedge L1-) \Rightarrow (R1\Box \rightarrow R1 \wedge R1-)$ [7, 8, (B.5)]
10. $(L1 \wedge R2) \Rightarrow [L1- \rightarrow (R1\Box \rightarrow R1 \wedge R1-)]$ [9, (A.5)]
11. $(L1 \wedge R2) \Rightarrow [R1\Box \rightarrow (L1- \rightarrow R1 \wedge R1-)]$ [10, LOC1]
12. $(L1 \wedge R1) \Rightarrow \neg(L1- \rightarrow R1 \wedge R1-)$ [(3.4)]
13. $L1 \Rightarrow [R1 \rightarrow \neg(L1- \rightarrow R1 \wedge R1-)]$ [12, (A.5)]
14. $(L1 \wedge R2) \Rightarrow [R1\Box \rightarrow \neg(L1- \rightarrow R1 \wedge R1-)]$ [13, DEF.]

But the conjunction of 11 and 14 contradicts the assumption that the experimenters in regions R and L are free to choose which experiments they will perform, and that outcome $L1-$ sometimes occurs under the conditions that L1 and R1 are performed. Quantum theory predicts that if L1 and R1 are performed then outcome $L1-$ occurs half the time. Thus the falseness of the statement in line 6 is proved.

[Note that there is only one strict conditional [$\Rightarrow$] in each line. In an earlier brief description[13] of a theorem similar to the one proved above, but based on orthodox modal logic rather than the quantum logic developed above, some material conditionals standing to the right of this strict conditional were mistakenly represented by the double arrow $\Rightarrow$, rather than by $\rightarrow$. I thank Abner Shimomy and Howard Stein [14] for alerting me to this notational error.]

The logical structure can be expressed in terms of *sets*. This provides a compact method, accessible to interested physicists, for validating the various lines of the proof.

For any statement S expressed in terms of the rudimentary logical connections let $\{W : S\}$ be the set of all (physically possible) worlds W such that statement S is true at W (i.e., S is true in world W). Sometimes $\{W : S\}$ will be shortened to $\{S\}$.

A main set-theoretic definition is this: Suppose A and B are two statements expressed in terms of the rudimentary logical connections. Then $A \Rightarrow B$ is true if and only if the intersection of $\{A\}$ and $\{\neg B\}$ is void:

$$[A \Rightarrow B] \equiv [\{A\} \cap \{\neg B\} = \emptyset]. \qquad (A.1)$$

Equivalently, $\{A\}$ is a subset of $\{B\}$:

$$[A \Rightarrow B] \equiv [\{A\} \subset \{B\}]. \qquad (A.2)$$

Let $(S)_W$ mean that the statement S is true at W. Then

$$(A \rightarrow B)_W \equiv [(\neg A)_W \text{ or } (B)_W]. \qquad (A.3)$$

This entails that

$$\{A \rightarrow B\} \equiv (\{\neg A\} \cup \{B\}). \qquad (A.4)$$

Proof of (A.5)

Equation (A.5) reads:

$$[A \Rightarrow (B \rightarrow C)] \equiv [(A \cap B) \Rightarrow C]. \qquad (A.5)$$

This is equivalent to

$$[\{A\} \cap \{\neg(B \rightarrow C)\} = \emptyset] \equiv [\{A \cap B\} \cap \{\neg C\} = \emptyset]. \qquad (A.6)$$

But $\{\neg(B \rightarrow C)\}$ is the complement of $\{B \rightarrow C\}$. Using (A.4), and the fact that the complement of $\{\neg B\} \cup \{C\}$ is $\{B\} \cap \{\neg C\}$, one obtains the needed result.

Proof of line 5

Line 4 has the condition L2 appearing to the right of the counterfactual condition R1. The counterfactual condition R1 changes R2 to R1, but leaves L2 unchanged. Hence the L2 appearing on the right can be omitted, since it appears already on the left. But then application of (2.1) gives the line 5.

Proof of line 12

Statement (3.4), expressed in the set-theoretic form, says there is some world in $\{L1 \wedge R1\} \cap \{L1-\}$ that is not in $\{R2-\}$. This entails that, in $\{L1 \wedge R1\}$, there some world in $\{L1-\}$ that is not in $\{R2-\}$. This is the form of (2.4) given in line 12.

Proof of line 14

By definition, the assertion that $[C\Box \rightarrow D]$ is true in world W is equivalent to the assertion that D is true in *every* possible world W' that differs from W only by possible effects of imposing condition C rather than whatever condition in world W is directly contradicted by condition C.

In line 14 the world W can be any world in which L1 and R2 hold. And W' can be any world that differs from W only by possible effects of changing R2 to R1. But no matter what these possible changes are, the world W' must be a world in which L1 and R1 hold, and in any such world the statement $(L1- \rightarrow R1 \wedge R1-)$ is false, by virtue of line 13. Thus line 14 is true.

Appendix B. Quantum Logic for Counterfactuals

Within orthodox quantum theory, with its notion of free choices on the part of the experimenters, and the notion of no-backward-in-time influence of these free choices, logical reasoning covers unambiguously some statements involving counterfactual conditions. The primary logical concept here is the notion of a "logically possible world." It will be enough to define it in the case under consideration.

This case involves two spacelike-separated spacetime regions R and L, and in each region two alternative mutually exclusive experiments, $R1$ and $R2$, and $L1$ and $L2$, respectively. Each possible experiment has two possible outcomes, $R1+$ and $R1-$, etc. Thus there are in this situation sixteen "logically possible worlds", each one labelled by one of the four globally defined experiments, $(R1, L1)$, $(R1, L2)$, $(R2, L1)$, and $(R2, L2)$, and by one of the four possible outcomes of that globally defined experiment, $(+, +)$, $(+, -)$,$(-, +)$, and $(-, -)$. This set of sixteen logically possible worlds is the logical universe under consideration here. Each such "world" might better be called a "world history".

A *physically possible world* is logically possible world that by virtue of the laws of nature (i.e., the predictions of quantum theory) has a non-null probability to occur. The physically possible worlds are called "possible worlds." Normally, I omit also the word "possible": unless otherwise stated a "world" will mean a "physically possible world".

The rudimentary logical relationships involve the terms "and", "or", "equal" and "negation". A rudimentary statement S involving these relations is said to be true in world W if and only if S is true by virtue of the set of conditions that define W and the laws of nature.

The concept of "implication" occurs, but it is important to distinguish between two different concepts.

The rudimentary relationship of implication is the so-called "material conditional". It is defined in terms of the rudimentary logical relationships defined above, and it will be represented here by the single arrow $\rightarrow$. By definition:

"$(A \rightarrow B)$ is true in world W" is equivalent to

" (A is false in W) or (B is true in W) " . (B.1)

This rudimentary relationship is different from the logical relationship called the "strict conditional", which is represented here by the word "implies", and a double arrow. The statement " 'A is true' implies 'B is true' " is sometimes shortened to "A implies B", and is represented symbolically here by

$$A \Rightarrow B. \tag{B.2}$$

By definition, $A \Rightarrow B$ is true if and only if for *every* (physically possible) world W either "A is false in W" or "B is true in W": i.e., for *every* (physically possible) world W, the rudimentary statement $(A \rightarrow B)$ is true in W.

The proof to be presented here is based on a causality condition called LOC1. It expresses the condition that there is at least one Lorentz frame, LF, such that if an experiment is performed and the outcome is recorded *prior* to some time T, as measured in LF, then this outcome can be regarded as fixed and settled, independently of which experiment will eventually be freely chosen and performed (faraway) at a time later than T.

It is assumed that the regions L and R lie earlier and later that this time T, respectively. Then LOC1 means that if Lois (acting in L) performs her experiment before Robert (acting in R), we can safely assume that her result does not depend on what Robert will do, but not vice-versa.

Logicians deal with statements of this kind by employing a third kind of implication. It uses the concept "instead of". This concept is of central importance in classical counterfactual reasoning, and it has an unambiguous meaning within our quantum context. That meaning is now explained.

Suppose that A represents some possible conditions that the experimenters could set up, and some conditions on the possible outcomes. [For example, A could be the condition that Lois and Robert perform $L2$ and $R2$, respectively, and that Lois gets outcome $L2+$]

Suppose condition C represents some free choice (by some experimenter) that could conflict with A. [In the example, C could be "Robert performs $R1$"].

Finally, suppose condition D represents some possible outcome that could occur if C were to hold "instead of", whatever condition C contradicts. [In the example, D could be "Lois gets outcome $L2+$," or perhaps "Robert gets outcome $R1-$."]

Then consider a statement of the form:

"A implies [If, instead, C then D]" (B.3)

The phrase "If, instead, C then D" is traditionally represented symbolically by $[C\Box \rightarrow D]$, and I shall use that symbolic form for the quantum version defined here. Like all rudimentary statements the assertion $[C\Box \rightarrow D]$ it is a statement that is made *in* one world, say W. But it is a statement *about* about an entire set of worlds W', namely the set of all (physically possible) worlds that differ from W only by possible consequences of choosing the experimental condition C instead of whatever condition in world W conflicts with C.

Given this definition of $[C\Box \rightarrow D]$ the statement

$$A \Rightarrow [C\Box \rightarrow D] \qquad (B.4)$$

expresses the condition that, for all (physically possible) worlds W, if A is true in W then then D is true in every (physically possible) world W' that differs from W only by possible effects of choosing condition C instead of whatever condition in W conflicts with C.

It is essential that this definition allows this statement to be combined with other logical statements in an unambiguous way. In particular, the usual laws of logic can be applied, *without any change*, to arguments involving statements of this kind. Suppose, or example, that one has, in addition to the truth of (B.4), also the truth of $(B \Rightarrow A)$, which asserts that, for all W, if B is true in W then A is true in W. Then one can immediately conclude from the meaning of $(B.4)$ that

$$B \Rightarrow [C\Box \rightarrow D]. \qquad (B.5)$$

The definition of [C $\Box \rightarrow$ D] is general. But in order to make use of it one must have some condition on the "possible effects of choosing condition C instead of whatever condition in world W conflicts with C."

This is where LOC1 comes in. Suppose the region L, where Lois acts, lies earlier than the region R, where Robert acts. Then LOC1 entails [with "and" represented by $\wedge$]

$$(L2 \wedge R2 \wedge L2+) \Rightarrow [R1 \Box\!\!\rightarrow (L2 \wedge R1 \wedge L2+)] \tag{B.6}$$

This statement is true by virtue of the LOC1 premise that the outcome that Lois gets, and also her free choice, do not depend on what Robert does later.

Another example of the logical rules is this. Suppose that (B.5) is true. And suppose that F is a condition on the outcome under the alternative condition C, and that $D \Rightarrow F$ is true. This is the condition that, for every W', if D is true in W' then F is true in W'. Then the meaning of (B.5), as described above (B.4), with B in place of A, ensures that the following statement is true:

$$B \Rightarrow [C \Box\!\!\rightarrow F]. \tag{B.7}$$

This result is used to get line 4 of the proof given in Appendix A.

All the other lines of the proof given in Appendix A can be strictly deduced, in a similar way, from just the meanings of the logical symbols, the predictions of quantum theory, and the property LOC1.

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