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Congestion and tax competition in a parallel network[+]

By

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Abstract

This paper studies the effects of tolling road use on a parallel network when different governments have tolling authority on the different links of the network. The paper analyses the tax competition between countries that each maximise the surplus of local users plus tax revenues. Three types of tolling systems are considered: (i) toll discrimination between local and transit traffic, (ii) uniform tolls on local and transit traffic, (iii) only local tolls can be imposed. The paper characterises the optimal toll levels chosen in a Nash equilibrium for the three tolling systems. The numerical illustration shows that introducing transit taxes generates large welfare effects and that toll systems that only apply to local users only generate a low welfare gain. Nash equilibrium toll discrimination between local and transit traffic generates slightly higher welfare than the solution where both tolls have to be uniform.

Keywords: congestion pricing, transit traffic

JEL: H23, H71, R41, R48

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1. Introduction

Countries' road networks are usually publicly provided, they are congestible, and they are accessible to local and to transit users. Local users are those for which both costs and benefits of network usage matter for country welfare. Transit users typically contribute to local costs (congestion, pollution, etc.) but not to local benefits. Importantly, in a number of cases transit users have a choice between different jurisdictions' road networks. For example, there are two main routes from South-Central Europe (Switzerland, Austria, Italy) to the north (Belgium, Netherlands, Denmark, etc.), one through France, the other via Germany. Alternatively, consider the transalpine crossing between Germany and Italy, where Austria and Switzerland compete for transit traffic. In both cases transit has a choice of routes and it interacts with local traffic in each country.

In these circumstances, how would a local jurisdiction like to regulate access to its infrastructure?[1] If the jurisdiction has taxing power and when it cares more about local than about overall welfare, it has two incentives for introducing tolls, even if there is no revenue requirement as such. First, there is scope for efficiency-improving tolls, because individual users ignore their contribution to external costs such as congestion. Second, there is scope for tax exporting, by raising revenue from transit users and redistributing it to local voters. The purpose of this paper is to study the interaction between those incentives, under various assumptions on the type of allowable tolls, for given levels of infrastructure supply. More specifically, we look at a model with two parallel routes that are operated by two countries. Local traffic and transit traffic both contribute to congestion, and the two countries compete for revenue from transit. Assuming that countries maximise local welfare, consisting of local consumer surplus and tax revenues from local and transit traffic, we study strategic tolling by both countries under various conditions. First, we assume that local traffic and transit can be separately tolled and analyse the resulting Nash equilibrium tolling policies. Second, we look at the case where

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[1] Although the discussion is set in the context of congestible road infrastructure in two countries, similar issues arise in the public provision of e.g. health, educational and recreational services. In this sense, the ideas studied in this paper are not limited to the transport sector. The key feature of the analysis is that foreign (transit) users are not restricted to a particular jurisdiction but can choose between several, and that jurisdictions compete for revenue from transit.

only uniform tolls are possible or acceptable. Third, we consider the case where only local traffic can be tolled.

Despite the highly stylised setting, the examples refered to above show that the model does capture the main ingredients of a number of situations in Europe (North-South axe, transalpine crossing, etc.) and, potentially, the US where States may compete in the near future. The analysis of this paper then describes the potential tax competition between countries (France and Germany, Switzerland and Austria, etc.) or regions, in controlling local and transit transport through the use of taxes on both types of transport. All three types of tolling regimes considered may be relevant in this context. Toll differentiation is especially relevant if countries use different pricing instruments to control local and transit transport. The case of uniform tolls applies when the same instruments are used and toll differentiation is not allowed by, say, a federal government (as can be expected when both countries are members of the EU). The case of 'local tolls only' resembles the current situation, where fuel taxes and other taxes on car use are the main tolling instruments. High fuel taxes can easily be evaded by transit transport, especially in relatively small countries, so that the exclusive use of fuel taxes is similar to tolling local traffic only.

Given recent innovations in transport taxation within the EU, the analysis directly bears on current policy issues. New forms of transport taxes take the form of kilometre charges (implemented in Germany as of early 2003), tolls (already existing, amongst others, on French motorways), or of sophisticated time-of-day pricing. The idea of (no) tax differentiation and of tolling only part of the users then is highly relevant. On the one hand, the EU principle of fair and efficient transport pricing suggests that each user pays for the marginal costs imposed. On the other hand, the use of different instruments by Member States almost automatically implies that local and transit transport taxes will differ. At the same time, it is likely that some countries will start off by tolling local traffic only, if only because of the technical difficulties of tolling transit traffic. Against this background, the problem for each national government is to choose the best strategy. Should fuel prices be used (local tolls only), or are general tolls preferable; should tolls be uniform or differentiated? At the EU government level, there are parallel questions. Is toll harmonization required? How large are the welfare costs of tax competition and tax exporting behaviour? Our analysis sheds some light on these issues.

Although the specific topic of this paper has not been studied before, several strands of the economics literature are directly relevant. First, the tax competition literature studies the behaviour of individual jurisdictions in a multi-jurisdictional setting. The seminal paper by Mintz and Tulkens (1986) provides detailed descriptions of equilibrium concepts for tax competition between regions within a federal state, using a model that incorporates the provision and financing of public goods. The implications of asymmetries in country size for countries' strategic behaviour have been studied by Kanbur and Keen (1993) within the framework of a model with cross-border-shopping. Assuming that the objective of the country governments is to maximise revenues, they show that, under some plausible conditions, the optimal strategy for small countries is to undercut the price of the neighbouring country, and that a small country typically looses from tax harmonisation. More recently, Parry (2003) empirically illustrates the welfare effects of tax competition and finds them to be relatively small under some, but not all, scenarios. Finally, Sinn (2003) discusses various forms of 'systems competition', referring in general to competition between countries for mobile factors, e.g. within the EU or on a global scale. He finds the welfare effects to be detrimental in some, but not all, cases.

Second, the literature on the optimal pricing of road use in the presence of congestion has recently been extended to optimal tolling in simple parallel networks. For example, Verhoef et al. (1996), Braid (1996) and Liu and McDonald (1998) consider models with homogeneous users and a speed-flow representation of congestion. They study optimal second-best tolls on one link in the network, assuming that other links can not be optimally tolled for technical or political reasons. Both theoretical and empirical results suggest that the optimal second-best tolls on one link tend to be low, and could actually be negative. The welfare gains from this type of second-best tolls are low. However, more recent research shows that the results strongly depend on the assumption of homogenous road users. Small and Yan (2001) and Verhoef and Small (2003) allow for a heterogeneous population of road users, and find comparatively large benefits from second best tolls.

Third, a small but growing literature studies the role of different ownership regimes in models with parallel routes. For example, Verhoef et al. (1996) consider competition between a private road and a free-access road, and compare the second-best optimal tolls with those obtained when both roads are privately owned. De Palma and Lindsay (2000) use a bottleneck model of congestion and compare three types of ownership structure: a private road competing with a free access road, two competing private roads, and competition between a private and a public operator. Interestingly, they show that a private duopoly can be more efficient than a mixed private-public duopoly.

Finally, a few recent studies have looked specifically at tax exporting in the transport sector, within a serial network setting[2]. Levinson (2001) analyses US States' choice of instruments for financing transportation infrastructure. Theory predicts, and an econometric analysis confirms, that jurisdictions are more likely to opt for toll-financing instead of e.g. fuel taxes, when the share of non-residential users is large. Tolls become more attractive because they allow price discrimination and tax-exporting. De Borger et al (2003) apply a large-scale numerical optimisation model to study tax exporting behaviour by individual regions in a model with both domestic and international freight transport. Numerical illustrations of different Nash equilibria suggest that (of course) the Nash equilibrium produces lower welfare than the centralized optimum, but that the Nash-equilibrium performs much better than the reference equilibrium, in which externalities are not reflected in prices.

At the theoretical level, our analysis fills two gaps in the literature. First, although competition between operators has been considered before, a common feature of this work is the absence of transit users that can choose between routes. In contrast, our analysis incorporates route choice for transit, and it focuses on the interaction between local and transit traffic when governments compete for revenue from transit. The

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[2] For an early influential contribution on tax exporting by local governments, see Arnott and Grieson (1981).

distinction between local and transit traffic also allows us to explicitly consider a wider range of tolling instruments and to look at the implications of pricing only part of the users (for example, only local traffic). Second, our analysis focuses on competition in a parallel network between two local welfare-maximising governments. This type of competition seems highly relevant in the context of transport policy and has not been studied in detail in the literature, as governments have other preferences than private operators.

The theoretical analysis is complemented by a numerical illustration, based on a stylised dataset, allowing us to pin down orders of magnitude for the various effects. What happens to pricing of local trips, transport volumes and welfare when transit traffic can be charged differently from local traffic? What are the welfare effects of tax exporting and tax competition in this case? What are the implications for taxes and welfare of tax harmonisation and of taxing local transport only? How sensitive are the results to country size and transit shares? Among others, the numerical results suggest that despite a substantial amount of tax exporting, the efficiency costs of tax exporting are fairly small under most scenarios. Also, tax harmonization (uniform tolls) has a limited effect on overall welfare. To the contrary, allowing local tolls only (the current situation) is quite costly in welfare terms.

The structure of the paper is as follows. Section 2 presents the general theoretical model. We specify the characteristics of the network and derive optimal tax rules for a given country (implicitly defining the country's reaction functions) under three sets of assumptions on the policy instruments: differentiated tolls on local and transit transport, uniform tolls on local and transit traffic, and the case where only tolls on local transport are used. In Section 3 we simplify by assuming linear demand and cost functions; this allows us to explicitly analyse the properties of the reaction functions, as well as the resulting Nash equilibrium for each of the three cases. Mathematical details for sections 2 and 3 can be found in the appendices. Section 4 reports on a numerical illustration. Seven equilibria are numerically evaluated: the no-toll equilibrium, Nash with differentiated tolls, Nash with uniform tolls, Nash with local tolls only, a centralised solution with differentiated tolls, a centralised solution with local tolls only and, finally, a scenario with collusion between the countries. The role of the share of transit and of demand and cost asymmetries between countries is evaluated. Section 5 concludes.

2. The theoretical model

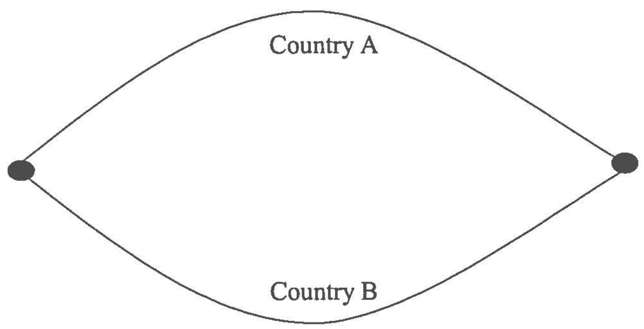
In this section we study optimal tolls on local and transit traffic in a simple parallel network. We first present the structure of the model and provide an overview of the tolling systems analysed. We then study the optimal behaviour of an individual country for each of the three tolling systems considered. Throughout this section we focus on the economically most relevant steps; the derivations are relegated to appendices.

2.1 Structure of the model and the pricing schemes considered

We consider the simplest possible setup for the analysis of tax competition between governments in a parallel network. The network analysed is depicted in Figure 1. It consists of two parallel links, and it is assumed that pricing of each link is the responsibility of a different government. Each link carries local traffic, which cannot change routes, and transit traffic, which can. Link capacities are given and both links are congestible.

Both governments are assumed to maximise a welfare function that reflects two concerns, viz. (i) the travel conditions of its local users and the associated welfare, and (ii) total tax revenues on the link it controls. We assume that all traffic flows are uniformly distributed over time and are equal in both directions, allowing us to focus on one representative unit period and one direction.

Figure 1 The network



The combinations of tolling instruments that we study are summarised in Table 1. The first case is that of full discrimination between local and transit traffic: each government (denoted by A and B) uses taxes on local users (t_A and t_B , respectively) and on transit traffic (τ_A and τ_B). In the second case discrimination between transit and local traffic is ruled out. In the third case, each government can tax local but not transit traffic. Note that the first case with discrimination may seem unrealistic because it runs against the non-discrimination rules in trade agreements. However, by choosing a particular toll structure, countries are able to price-discriminate against foreign users. Take as an example the yearly lump-sum fee for access to a country's network that is to be paid in Switzerland and in many other countries (the Eurovignette system): this in fact boils down to discrimination in favour of the local users as, almost by definition, they use the network more frequently. The third case can be seen as the situation where transit traffic can easily evade tolls or taxes; an example is a high fuel tax that can be evaded by transit traffic by fuelling abroad.

Table 1 only lists the three cases where both countries use the same type of tolling. In principle, we could also examine cases where the governments use different types of tolling systems. Indeed, these mixed cases exist in reality: France uses a uniform tolling system for motorways while Germany has no explicit toll, so uses a system similar to the case where only local traffic can be tolled. However, we focus on countries using the same instruments.

Table 1: The tolling systems studied

Description of tolling systems studied	**Tolling instruments**	**Example of practical relevance**
Differentiated tolls for local and transit transport	τ_i: transit toll region i (i=A,B) t_i: toll on local transport in region i (i=A,B)	Eurovignette (favors more intensive local users)
Uniform tolls for local and transit transport	θ_i: uniform toll in region i (i=A,B)	Current tolls on French highways
Tolls on local users only, no transit toll	t_i: toll on local transport in region i (i=A,B)	Fuel taxes, parking charges

Turning to the specification of the model, demand for local transport in A and B is represented by the strictly downward sloping inverse demand functions $P_A^Y(Y_A)$ and $P_B^Y(Y_B)$, respectively, where Y_A and Y_B are the local flows on both links. The generalised prices $P_i^j(.)$ include resource costs, time costs and tax payments or user charges. Similarly, overall demand for transit traffic is described by the strictly downward sloping inverse demand function $P^X(X)$, where X is the total transit traffic flow. By definition we must have

$$X_A + X_B = X\,, \tag{1}$$

where X_A and X_B are the transit flows via A and B, respectively. The two links are assumed to be perfect substitutes: transit users choose the route with the lowest generalised (money plus time) cost but have no specific preferences towards any of the routes.

Demand for the different types of transport on both links is specified as a function of the corresponding generalised costs. In what follows, we develop all specifications for the case of differentiated tolling; the cases of uniform tolls and local tolls only are easily derived by analogy. For example, the generalised user cost for transit via route A, denoted g_A^X, equals the sum of the time and resource costs of travel plus the transit toll on A :

$$g_A^X = C_A(X_A + Y_A) + \tau_A$$

In this expression, $C_A(.)$ is the time plus resource cost on route A; it obviously depends on both transit and local use of link A and we assume it is strictly increasing in the traffic volume. Similarly, the generalised user cost for local use of route A is given by

$$g_A^Y = C_A(X_A + Y_A) + t_A$$

User costs for route B are defined in an analogous way:

$$g_B^X = C_B(X_B + Y_B) + \tau_B$$

$$g_B^Y = C_B(X_B + Y_B) + t_B$$

Importantly, since we assume perfect substitutability between links for transit, in equilibrium the generalised cost for transit equals the generalised cost on the link with the lowest generalised cost. If both routes are used, transit traffic will be distributed across links so as to equalise generalised costs. Specifically, the Wardrop principle implies that

$$\begin{aligned} P^X(X) = g_A^X = C_A(X_A + Y_A) + \tau_A \text{ iff } X_A > 0 \\ P^X(X) = g_B^X = C_B(X_B + Y_B) + \tau_B \text{ iff } X_B > 0 \end{aligned} \tag{2}$$

Moreover, equilibrium for local traffic implies

$$P_A^Y(Y_A) = g_A^Y = C_A(X_A + Y_A) + t_A \tag{3}$$

$$P_B^Y(Y_B) = g_B^Y = C_B(X_B + Y_B) + t_B \tag{4}$$

In this paper we mainly focus on the case where all types of traffic exist in equilibrium, i.e., there is local and at least some transit in both countries. In theory, of course, this is just one of the many possibilities that exist in this model. When certain taxes are too high or there is too much other traffic using the same road, some types of transport demand may disappear, and this affects the structure of the remaining demand functions. This is a well-known problem in the tax competition literature (see Mintz and Tulkens, 1986). In fact, a complete analysis would have to distinguish 16 different regimes. Many of these, however, are not very interesting in practice (e.g., cases where there is no local traffic, cases where there is no transit in neither A or B). We therefore largely focus on the most relevant case where both types of transport exist in both countries, although we briefly touch upon the case with zero transit where appropriate.

2.2. Optimal tolls in a parallel network: the case of differentiated tolls

Let us consider the optimal behaviour of one of the countries, say A, assuming it can set different tolls on local transport and on transit. To do so, we first express local and transit demands in A as a function of all tax rates (the reduced-form demand functions), and then use this information in the first-order conditions describing optimal transport taxation in country A.

The reduced-form demand system is obtained by solving the equilibrium conditions (2), (3) and (4) for local and transit demands in the two countries as a function of all tax rates:

$$\begin{aligned} &X_A^r\left[\tau_A, t_A, \tau_B, t_B\right] \\ &X_B^r\left[\tau_A, t_A, \tau_B, t_B\right] \\ &Y_A^r\left[\tau_A, t_A, \tau_B, t_B\right] \\ &Y_B^r\left[\tau_A, t_A, \tau_B, t_B\right] \end{aligned} \tag{5}$$

The properties of these functions are analysed in detail in Appendix 1. For example, there it is shown that the demand functions for transit and local transport in countries A and B have the following properties:

$$\begin{aligned} &\frac{\partial X_A^r}{\partial \tau_A} < 0, \frac{\partial X_A^r}{\partial \tau_B} > 0 \frac{\partial X_A^r}{\partial t_A} > 0, \frac{\partial X_A^r}{\partial t_B} < 0 \\ &\frac{\partial Y_A^r}{\partial \tau_A} > 0, \frac{\partial Y_A^r}{\partial \tau_B} < 0 \frac{\partial Y_A^r}{\partial t_A} < 0, \frac{\partial Y_A^r}{\partial t_B} > 0 \end{aligned} \tag{6}$$

$$\begin{aligned} &\frac{\partial X_B^r}{\partial \tau_B} < 0, \frac{\partial X_B^r}{\partial \tau_A} > 0 \frac{\partial X_B^r}{\partial t_B} > 0, \frac{\partial X_B^r}{\partial t_A} < 0 \\ &\frac{\partial Y_B^r}{\partial \tau_B} > 0, \frac{\partial Y_B^r}{\partial \tau_A} < 0 \frac{\partial Y_B^r}{\partial t_B} < 0, \frac{\partial Y_B^r}{\partial t_A} > 0 \end{aligned} \tag{7}$$

To understand the intuition behind these effects, note that any tax change has two effects: first, it affects the distribution of transit over the two routes and, second, by affecting congestion levels in the two regions, it has an impact on the competition in each country between transit traffic and local traffic for the same road space. Consider two examples. First, take the effect of increasing the transit tax in *B* (τ_B). This tax increase will make route *B* less interesting for transit traffic so that X_B goes down, whereas demand for transit on route A rises. However, there are secondary effects. The direct positive effect on X_A raises congestion in *A* and hence the generalised user cost, whereas the lower volume of transit on route *B* decreases the generalised cost of using route *B*. The changes in congestion mitigate the initial transit effects described before. More importantly, the reduction of congestion in *B* raises the demand for local traffic in that country, whereas the increase of the generalised cost of route *A* implies a reduction in demand for local transport Y_A. Second, consider the effect of raising the charge t_A. The direct effect will be to decrease the demand for local traffic Y_A. However, this reduces the generalised cost of transit in *A* and hence increases transit traffic X_A, which mitigates the initial effects in *A*. Moreover, the local tax increase in *A* reduces transit demand in *B* and hence generalised user cost in that region. Consequently, demand for local transport in *B*, Y_B, increases.

Finally, note the following useful result, formally shown in Appendix 1, on the relative impact of a transit tax and a tax on local transport on the demand for transit:

$$\left|\frac{\partial X_A^r}{\partial \tau_A}\right| > \left|\frac{\partial X_A^r}{\partial t_A}\right|$$

Both taxes have opposite effects, but in absolute value the transit tax has a larger effect on transit demand than an increase in the tax on local traffic. This makes intuitive sense because a higher local tax only affects transit demand indirectly via the induced reduction in congestion. This finding will be useful for the interpretation later.

Using the reduced-form demand system we proceed to analyse the optimal behaviour of a given country, conditional on the tolls set abroad. We assume that the appropriate objective function used by each of the governments is a welfare function that consists of the sum of consumer surplus for the local users plus the total tax revenues earned on local and transit traffic on its territory. Consumer surplus for foreigners is assumed to be ignored. Consider, therefore, the problem of country A:

$$\underset{t_A,\tau_A}{Max} \quad W_A = \int_0^{Y_A} (P_A^Y(Y_A))dY_A - g_A^Y Y_A + t_A Y_A + \tau_A X_A, \tag{8}$$

where, see before, $g_A^Y = C_A(X_A + Y_A) + t_A$, and the reduced-form demands for X_A and Y_A depend on all four tax rates, see (5). Moreover, the country takes the tolls t_B, τ_B in country B as given.

The first-order conditions for an interior solution to (8) can be written, using the fact that in equilibrium generalised costs and generalised prices are equal, as:

$$\left(t_A - Y_A \frac{\partial C_A}{\partial V_A}\right)\frac{\partial Y_A^r}{\partial t_A} + \left(\tau_A - Y_A \frac{\partial C_A}{\partial V_A}\right)\frac{\partial X_A^r}{\partial t_A} = 0, \tag{9}$$

$$\left(t_A - Y_A \frac{\partial C_A}{\partial V_A}\right)\frac{\partial Y_A^r}{\partial \tau_A} + \left(\tau_A - Y_A \frac{\partial C_A}{\partial V_A}\right)\frac{\partial X_A^r}{\partial \tau_A} + X_A^r = 0, \tag{10}$$

where $V_A = X_A + Y_A$ is the total (local plus transit) traffic volume in country A. Solving (9) and (10) yields the optimal tax rules for country A. In Appendix 1 we show that the following results hold:

$$t_A = LMEC_A + X_A \frac{\partial C_A}{\partial V_A} \tag{11}$$

$$\tau_A = LMEC_A - X_A \left[\frac{\dfrac{\partial Y_A^r}{\partial t_A}}{\dfrac{\partial z_A}{\partial t_A}\dfrac{\partial X_A^r}{\partial \tau_A}}\right] \tag{12}$$

$$\tau_A > t_A. \tag{13}$$

Analogous results can be derived for country B. In these expressions, $LMEC_A$ is the *local* direct marginal external congestion cost, defined as:

$$LMEC_A = Y_A \frac{\partial C_A}{\partial V_A} = Y_A \frac{\partial C_A}{\partial X_A} = Y_A \frac{\partial C_A}{\partial Y_A}$$

It captures the effect of extra traffic on the generalised user cost in country A, multiplied by the number of local users of the link. It is a direct marginal external cost in that it does not take into account feedback effects on demand. Note that country A does not consider the time losses imposed on transit traffic through A as part of the relevant local marginal external cost.

We summarise our findings in Theorem 1.

THEOREM 1: In the case of a differentiated toll between local and transit traffic and assuming there is local and transit transport in both regions, (i) optimal local and transit tolls both exceed the local marginal external cost; (ii) the transit toll is strictly larger than the local toll.

The theorem immediately follows from the signs of the reduced-form demand price effects, see (6) and (7). Intuitively, the local toll exceeds LMEC because the true opportunity cost of an increase in local traffic not only covers the local direct marginal external cost but also the opportunity cost of the lost tax revenues on transit: more local traffic implies higher congestion and hence less transit demand[3]. The reason for the transit tax to exceed the tax on local transport is due to tax exporting behaviour: country A does not take into account the effect on congestion and tax revenues abroad.

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[3] Note that, for the specific model structure considered here, it turns out that the local tax equals the *global* direct marginal external cost of a traffic increase in country A, defined as

$$GMEC_A = (Y_A + X_A)\frac{\partial C_A}{\partial V_A} = V_A \frac{\partial C_A}{\partial V_A}.$$

The global marginal external cost is the increase in generalised cost from an extra unit of traffic, multiplied by the total number of road users in A. That the local tax exceeds the local marginal external cost is a general result, that it precisely equals the global marginal external cost is an artefact of the model structure. The intuition can be understood by the definition of the generalised cost in combination with the structure of the objective function. Transit traffic is indifferent between paying one Euro more in time costs and one Euro more in transit tolls. The government that hosts the transit traffic obviously prefers the transit toll. Therefore, the opportunity cost of allowing one more unit of local traffic equals the local marginal external cost plus the total transit revenue foregone through the increase in average costs for transit traffic. The definition of generalised costs implies that the increase in average costs (the marginal external cost of the transit traffic) equals the total transit revenue foregone.

2.3. Optimal tolls in a parallel network: uniform tolls

Suppose countries are limited to uniform tolls, i.e., the toll is restricted to be the same for local and transit trips. Denote the uniform tolls by θ_A and θ_B in regions A and B, respectively, where $\theta_i = \tau_i = t_i \ (i = A, B)$.

Solving the equilibrium conditions (2), (3) and (4) for the case of uniform tolls now yields the system:

$$\begin{array}{l} X_A^r[\theta_A, \theta_B] \\ X_B^r[\theta_A, \theta_B] \\ Y_A^r[\theta_A, \theta_B] \\ Y_B^r[\theta_A, \theta_B] \end{array} \qquad \text{(5bis)}$$

In Appendix 2 we show that the reduced-form demand functions for A (analogous results hold for B) have the following properties:

$$\frac{\partial X_A^r}{\partial \theta_A} < 0, \frac{\partial X_A^r}{\partial \theta_B} > 0, \frac{\partial Y_A^r}{\partial \theta_A} < 0, \frac{\partial Y_A^r}{\partial \theta_B} < 0$$

To understand these effects note that, in principle, an increase in the uniform tax in a region is expected to have a double effect on transit (local) demand in that region: a direct negative effect, and an indirect positive effect due to the lower volume of local (transit) traffic. The above results show that the former effect dominates the indirect feedback effect[4]. Moreover, we also find that an increase in the uniform tax abroad (e.g. in B) raises transit demand but reduces local demand (e.g., in A). The reason is simply that overall transit demand is shifted from B to A, which in turn raises congestion in A and hence lowers local demand in A.

Using these reduced-form effects, the optimal uniform toll for country A is easily derived. The first-order condition to the problem

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[4] For transit, this is in line with our earlier finding that, in the case of differentiated taxes, in absolute value the effect of the transit tax exceeded that of the tax on local transport.

$$\underset{\theta_A}{Max} \quad W_A = \int_0^{Y_A} (P_A^Y(Y_A))dY_A - g_A^Y * Y_A + \theta_A(Y_A + X_A)$$

can be written, after simple manipulation (see Appendix 2):

$$\theta_A = Y_A \frac{\partial C_A}{\partial V_A} - \frac{X_A^r}{\frac{\partial Y_A^r}{\partial \theta_A} + \frac{\partial X_A^r}{\partial \theta_A}}$$

This immediately tells us that:

$$\theta_A > LMEC_A$$

unless transit in A is zero. The optimal uniform toll exceeds the local direct marginal external cost, and it rises with transit[5]. In some sense, this finding can be considered a generalisation of a result by Arnott and Grieson (1981). Intuitively, the toll balances the distortion on the local transport market and the revenue opportunities on transit. We summarise this result in Theorem 2.

THEOREM 2: If countries are restricted to the use of uniform tolls on local and transit transport, the optimal uniform toll exceeds the local marginal external cost. Moreover, it will be higher the more important is transit traffic through the country.

2.4. Optimal tolls in a parallel network: the case of local tolls only

Suppose the government cannot tax transit ($\tau_i = 0\ (i = A, B)$). For example, this may be the case when it is limited to the use of fuel taxes. The equilibrium conditions (2), (3) and

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[5] Note that the above result implies that the toll exceeds LMEC even at very small shares of transit transport. This might seem counterintuitive. Indeed, intuitively one could argue that when the transit share is very small compared to local traffic, the decrease in average time costs due to a higher tax on local transport might in turn more than compensate the toll increase for transit, so that the full effect of a toll increase on transit may be positive. Interestingly, this intuitive argument is wrong. Wardrop equilibrium implies equality between the generalised prices of transit via A and B. An increase in the uniform toll in A raises the generalised price for transit in A. Hence, overall transit declines, and a proportionally larger share uses the route via B. The implication is that transit through A must necessarily decline.

(4) can then be solved for the system of reduced form demand functions that depend on the local tolls in both countries:

$$\begin{array}{l} X_A^r[t_A, t_B] \\ X_B^r[t_A, t_B] \\ Y_A^r[t_A, t_B] \\ Y_B^r[t_A, t_B] \end{array} \qquad (5ter)$$

The signs of these demand equations are identical to the reduced demand functions of the differentiated toll case. Own price effects are negative, cross price effects positive.

The first-order condition to the problem for country A:

$$\underset{t_A}{Max} \quad W_A = \int_0^{Y_A} (P_A^Y(Y_A))dY_A - g_A^Y * Y_A + t_A Y_A$$

implies, see Appendix 3:

$$t_A = Y_A \frac{\partial C_A}{\partial V_A}\left[1 + \frac{\dfrac{\partial X_A^r}{\partial t_A}}{\dfrac{\partial Y_A^r}{\partial t_A}}\right]$$

where the term between square brackets is shown to be positive. Using the signs of the demand functions this implies:

$$0 < t_A < LMEC_A$$

We summarize this finding in Theorem 3.

THEOREM 3: If only local traffic can be tolled, the optimal toll is positive but smaller than the local marginal external cost.

To understand the intuition, note that an increase of the toll on local traffic has two effects. On the one hand, the toll reduces local transport demand, a welfare-raising correction for the externality this traffic imposes. If there were no transit, a toll equal to LMEC would be optimal. However, on the other hand, the reduction in local traffic reduces the average time cost for transit and attracts more transit; this decreases local welfare and induces a tax below LMEC. If transit traffic reacts very strongly to an average travel time cost decrease, it may be optimal to set the tax very low so as to avoid attracting too much transit.

2.5. Summary

It is instructive to summarise the findings of this section in the following Table 2. The results show that depending on the policy instruments available, a wide range of optimal tolling schemes is possible. Some of these may well be consistent with observed practice. The use of vignettes in some countries comes close to the idea of tax differentiation, and it indeed implies the potential for tax exporting to foreigners. It is also often observed that countries that have difficulties in taxing transit either are very slow in imposing local congestion taxes (consider the discussion on road pricing in the Netherlands), or they are even explicitly opposed to specific congestion taxes unless transit can also be taxed (Belgium, Luxemburg, etc.). The results presented here for the case 'local tolls only' are not inconsistent with this type of behaviour, especially if one takes account of implementation costs.

Table 2: Summary of optimal tolling rules

Tolling regime	**Results on optimal tolls**	**Interpretation**
Differentiated tolls	$\tau_i > LMEC_i$ $t_i > LMEC_i$ $\tau_i > t_i$	- Local and transit toll exceed local marginal external congestion cost - Transit toll exceeds local toll
Uniform tolls	$\theta_i = \tau_i = t_i$ $\theta_i > LMEC_i$	- Uniform toll exceeds local marginal external congestion cost
Local tolls only	$0 < t_i < LMEC_i$	- Tolls on local traffic are positive but below marginal external congestion cost

3. Nash equilibria for linear cost and demand functions

To formally study the properties of the reaction functions, implied by the optimal tax rules, and the resulting Nash equilibria, it is instructive to impose more structure on the problem. In this section we therefore focus on linear demand and cost functions in order to explicitly solve for reaction functions and to obtain more details on the different types of equilibria that may result. Moreover, these simplifications pave the way for the numerical analysis that follows in Section 4.

3.1 Model structure

Specifically, we use the following linear inverse demand functions:

$$\begin{aligned} &P^X(X) = a - bX \\ &P_A^Y(Y_A) = c_A - d_A Y_A \\ &P_B^Y(Y_B) = c_B - d_B Y_B \\ &\text{with } a, b, c_A, d_A, c_B, d_B > 0 \end{aligned} \tag{14}$$

Cost functions for transport time (and resources) are specified as:

$$\begin{aligned} &C_A(X_A + Y_A) = \alpha_A + \beta_A(X_A + Y_A) \\ &C_B(X_B + Y_B) = \alpha_B + \beta_B(X_B + Y_B) \\ &\text{with } \alpha, \beta > 0 \end{aligned} \tag{15}$$

We only consider the general case where both regions have transit and domestic transport. We first discuss the case of differentiated tolls and continue with uniform tolls and the case of local tolls only.

3.2 Reaction functions and Nash equilibrium for the different tolling regimes

The algebraic derivations to arrive at the reaction functions and to show the existence of a Nash equilibrium for the various tolling regimes are conceptually simple, but somewhat tedious. We have therefore delegated the derivations to Appendix 4 and limited the discussion here to the economic implications of our findings.

Differentiated tolls

The reaction functions for country A are given by the following linear expressions:

$$\begin{aligned}
\tau_A &= c_A^\tau - \left(\frac{1}{2}\frac{\gamma_2^A}{\gamma_1^A}\right)\tau_B - \left(\frac{1}{2}\frac{\gamma_4^A}{\gamma_1^A}\right)t_B \\
t_A &= c_A^t + \left(\frac{1}{2}\frac{\gamma_2^A}{\gamma_1^A}K^A\right)\tau_B + \left(\frac{1}{2}\frac{\gamma_4^A}{\gamma_1^A}K^A\right)t_B
\end{aligned} \tag{16}$$

where the coefficients are explicitly defined in Appendix 4. Here it suffices to note:

$$\gamma_1^A < 0, \gamma_2^A > 0, \gamma_4^A < 0$$

$$\left|\gamma_2^A\right| > \left|\gamma_4^A\right|$$

$$-1 < K^A < 0$$

Interpretation of the signs of the foreign taxes on optimal local taxes in A is then clear. We find that an increase in the transit tax abroad induces country A to optimally adjust both its transit tax and the tax on local traffic upwards, but that the impact on the transit tax is larger than the effect on the local tax. Why is this the case? The higher tax on transit in B reduces transit there and raises transit demand in A. This increases local congestion in A. The optimal response in A is therefore to raise both taxes. Similarly, a higher local tax in B induces country A to optimally reduce transit as well as local taxes in A. The higher tax in B reduces congestion in B and makes B relatively more and A relatively less attractive to transit traffic. This also reduces both congestion and tax revenues in A. To compensate country A raises its tax rate on local traffic; this increases congestion but raises tax revenues.

Reaction functions for B have the same structure. In Appendix 4 we formally show, always assuming that all types of transport are positive at the equilibrium, existence of a Nash equilibrium and explicitly solve for the four equilibrium tax rates. However, not surprisingly, the solution for the Nash equilibrium itself is too complicated to yield extra economic insights. Therefore, to study the properties of the equilibrium in function of a number of crucial parameters describing the tax competition problem (e.g., the size of the country, the importance of transit etc.), we resort to numerical analysis in Section 4 below.

Uniform tolls

The reaction function for country A as a function of the uniform tax rate in B is given by the linear relation:

$$\theta_A = \frac{c_2^{tuA}}{c_1^{tuA}} + \frac{c_3^{tuA}}{c_1^{tuA}}\theta_B \tag{17}$$

where, see Appendix 4, $c_1^{tuA} > 0$, $c_2^{tuA} > 0, c_3^{tuA} > 0$. An analogous result holds for B. This shows that the reaction functions are upward sloping. Given our assumption that both types of traffic exist in the equilibrium in both countries, a Nash equilibrium can again be shown to exist.

Local tolls only

The reaction function for country A is shown to be:

$$t_A = \frac{c_2^{tlA}}{c_1^{tlA}} + \frac{c_3^{tlA}}{c_1^{tlA}} t_B \tag{18}$$

where $c_1^{tlA} > 0$, $c_2^{tlA} > 0$, $c_3^{tlA} > 0$.

Again, the slope of the reaction functions is positive, and (assuming both types of traffic exist at the equilibrium) existence of a Nash equilibrium can be shown, see Appendix 4.

4. Numerical illustration

4.1 Central scenario

In order to illustrate the theoretical analysis, a numerical model is used that fully corresponds to the linear model developed in the previous section. The data represent realistic orders of magnitude for the situations modelled above, but do not correspond to a particular real-world example. The central scenario uses a fully symmetric version of the model, with identical congestion functions and local demand functions for both countries. The congestion function is a linear approximation to the French functional form for highways (Quinet, 1998: 139), at a reasonably congested traffic volume. This is combined with linear demand functions for both local and transit travel. The precise parameterization of all functions is chosen so as to yield reasonable generalized price elasticities and congestion levels (including marginal external congestion cost); cf. more detail below.

In addition to symmetry, the central case also assumes a 50/50 distribution of transit and local traffic in each country, in the zero-toll situation (the parameterization is constructed for that zero-toll case). Table 3 shows some basic properties. Transit demand is twice local demand in A or B, and it is equally distributed over both countries (endogenously). The time cost is taken to be 50% of the generalized price. The non-time component is fixed across simulations.

Table 3 Zero-toll symmetric equilibrium (central case parameterization)

	Intercept	Slope	Level	Unit
Local demand, A=B	1690	-5.96	1300	Trips
Transit demand	3380	-11.92	2600	Trips
Time cost function, A=B	1.617	0.012	32.7	Euro/trip
Generalized price, A=B			65.4	Euro/trip
Local MEC, A=B			15.5	Euro/trip
Global MEC, A=B			31.1	Euro/trip

Note: all trips are taken to be 100km long; the trip levels are hourly levels

In Tables 4 and 5 some relevant findings for the above parameterization are summarized; the following cases are distinguished:

- S1: No toll equilibrium, to which the model is calibrated;
- S2: Nash equilibrium with differentiated tolls;
- S3: Nash-equilibrium with uniform tolls on local and transit traffic;
- S4: Nash equilibrium with local tolls only;

- S5: Centralized solution with differentiated tolls (aggregate welfare maximization of transit and non transit countries);
- S6: Centralized solution with local tolls only.

In each scenario, the toll revenue is allocated to the two tolling countries. Note that, by construction, we obtain interior solutions for the counterfactual scenarios. Corner solutions are not analysed here.

The results lead to the following observations. First, note that the numerical illustration broadly confirms the results of the theoretical analysis. For example, in the Nash equilibrium with differentiated tolls (S2), the local toll is equal to the global marginal external congestion cost, and the transit toll exceeds the local toll. Imposing uniformity (S3) leads to a toll that is between the differentiated tolls of S2. Second, we find that the maximal attainable welfare gain from congestion tolls, i.e., the welfare gain for the centralised solution with differentiation (S5), compared to the no-toll equilibrium (S1) to be 1.58%.[6] Note that, while toll differentiation is allowed in scenario S5, the resulting tolls are equal because marginal external costs are equal for both trip types. Third, the Nash-equilibrium with differentiated tolls improves overall welfare by 1.47%, or 93% of the maximal attainable gain.[7] Not only is the relative gain from the differentiated Nash equilibrium high, also the shares of both countries and of transit in total welfare are fairly close to that of the centralised solution. In both cases the shares of local traffic in welfare increase substantially compared to the no-toll situation, while that of transit traffic diminishes.[8] Since the Nash equilibrium with differentiated tolls improves welfare substantially and brings us close to the social welfare optimum, one could argue that the welfare costs of the lack of cooperation between countries seem to be relatively modest.

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[6] This order of magnitude is in line with that of earlier studies (see the results for the Trenen-model in De Borger and Proost (2001)) .

[7] These results can be compared to the case where the roads are owned by two profit-maximizing firms. The resulting Nash-Bertrand equilibrium would produce 90.4% of the welfare of the centralised social welfare optimum. The welfare costs of private ownership in this model are high because each firm has a monopoly over local traffic, and competition only concerns transit. If both roads are owned by one monopolist, the resulting welfare level is 80.9% of the centralised social welfare optimum.

[8] The resulting welfare loss for transit could be expected, as transit trips are priced below marginal social costs in the reference equilibrium. A toll is needed for reasons of efficiency, but transit does not share in the toll revenues.

At any rate, tolling with no coordination is better than no tolling (for overall welfare, not for transit).

Fourth, comparison of the Nash equilibrium with and without toll differentiation (S2 and S3) suggests that the uniformity constraint actually implies an overall welfare loss, be it small (0.06%-point). The effect on the local toll is large, however; it increases from 27.1 Euro/trip to 36.8 Euro/trip. That the impact on local welfare is small is due to the fact that tax revenues have the same welfare weight as consumer surplus. In that respect, note that the uniform toll in S3 is close to the transit toll in S2. Transit experiences a welfare loss that is very close to that of the differentiated toll case, meaning that the uniformity restriction does not protect them from welfare losses. Overall, the example indicates that the welfare effects of the Nash outcome with uniform tolls are quite similar to the outcome with differentiation.

Fifth, things are very different with the assumption that transit trips cannot be tolled: the performance of both the Nash and the centralised outcome (S4 and S6) is substantially worse than in the cases where transit is tolled. The Nash equilibrium without transit tolls (S4) generates 23% of the welfare gain in the Nash equilibrium with differentiated tolls (S2). The centralised solution with zero transit tolls produces 49% of the gain with optimal differentiation (compare S5 and S6). Note also that the centralised solution with zero transit tolls performs worse that the Nash equilibrium with or without toll differentiation.

Table 4 Results from symmetric model – Levels, index (S5=100)

Note: these scenarios produce symmetric outcomes, so the distinction between countries A and B is suppressed; variables not defined on the country level are shown in italics

			S1		S2		S3		S4		S5		S6	
			No tolls		NE - differentiation		NE - uniform		NE - local toll only		Centralised - diff.		Centralised - loc. only	
	Variable	Unit	Level	Index	Level	Index	Level	Index	Level	Index	Level	Index	Level	Index
1	Local demand	trips	1,300	112.5	1,163	100.6	1,108	95.9	1,262	109.2	1,156	100	1,146	99.2
2	Transit demand	trips	2,600	112.5	2,197	95.0	2,216	95.9	2,605	112.7	2,311	100	2,620	113.4
3	Trip volume, country level	trips	2,600	112.5	2,261	97.8	2,216	95.9	2,564	110.9	2,311	100	2,456	106.3
4	Transit volume, country level	trips	1,300	112.5	1,098	95.0	1,108	95.9	1,302	112.7	1,156	100	1,310	113.4
5	Generalised price, local	Euro/trip	65.4	73.0	88.5	98.7	97.7	109.0	71.8	80.1	89.7	100.0	91.2	101.8
6	Generalised price, transit	Euro/trip	65.4	73.0	99.3	110.7	97.7	109.0	65.0	72.5	89.7	100.0	63.7	71.1
7	Time cost	Euro/trip	32.7	111.8	28.7	97.9	28.1	96.1	32.3	110.3	29.3	100.0	31.0	105.9
8	**Local toll**	**Euro/trip**	**0.0**	**0.0**	**27.1**	**97.8**	**36.8**	**133.2**	**6.8**	**24.7**	**27.7**	**100.0**	**27.5**	**99.4**
9	**Transit toll**	**Euro/trip**	**0.0**	**0.0**	**37.9**	**137.0**	**36.8**	**133.2**	**0.0**	**0.0**	**27.7**	**100.0**	**0.0**	**0.0**
10	Local MEC	Euro/trip	15.6	112.5	13.9	100.6	13.3	95.9	15.1	109.2	13.8	100.0	13.7	99.2
11	Global MEC	Euro/trip	31.1	112.5	27.1	97.8	26.5	95.9	30.7	110.9	27.7	100.0	29.4	106.3
12	Local CS	Euro	141,748	126.5	113,422	101.2	102,948	91.9	133,558	119.2	112,029	100	110,214	98.4
13	Tax revenue, country level	Euro	0	0.0	73,069	114.3	81,596	127.6	8,606	13.5	63,926	100	31,521	49.3
14=12+13	Welfare, country level	Euro	141,748	80.6	186,492	106.0	184,544	104.9	142,164	80.8	175,955	100	141,735	80.6
15	Transit welfare (CS)	Euro	283,495	126.5	202,364	90.3	205,894	91.9	284,603	127.0	224,057	100	287,975	128.5
16=2*14+15	Overall welfare	Euro	566,991	98.4	575,348	99.9	574,982	99.8	568,931	98.8	575,968	100	571,445	99.2

Table 5 Distribution of welfare and total welfare gain (%)

	S1	S2	S3	S4	S5	S6
Share country A	25.00	32.41	32.10	24.99	30.55	24.80
Share country B	25.00	32.41	32.10	24.99	30.55	24.80
Share transit	50.00	35.17	35.80	50.02	38.90	50.39
Total (% change compared to S1)	100 (0.00)	100 (1.47)	100 (1.41)	100 (0.34)	100 (1.58)	100 (0.78)

It seems therefore, that welfare losses are much more substantial when transit remains un-tolled than when differentiated or uniform tolls are used. Moreover, from the welfare maximization scenarios (S5 and S6), it can be inferred that the large loss of performance due to zero transit tolls simply follows from the fact that a large portion of trips is un-tolled. Local traffic is taxed at less than the global marginal external cost, but the difference is small. Moving from a centralised (S6) to an uncoordinated solution (S4), under the zero transit toll constraint, introduces an additional source of efficiency loss: countries find it in their best interest to tax local traffic at far less than the global marginal external congestion cost. As countries care about local welfare only, they set local tolls at a low level, so encouraging local trip demand and indirectly discouraging transit trips. Transit is obviously the main beneficiate of these local tolls. They actually increase their welfare level.

The different scenarios with zero transit tolls are of interest because zero tolls on transit traffic mimics current conditions in Europe, at least for transit countries that are small enough to allow transit to pass without taking fuel. The impact of zero transit tolls is nicely summarised in Table 6. Comparing S2 and S4 shows that zero transit tolls induce countries to set very low local transport taxes; however, it implies very low welfare gains compared to the benchmark of zero tolls. Finally, note that Table 6 also reports on the case of collusion between countries A and B for the case of zero transit tolls, scenario S7. It does not fundamentally change the outcome, see the last column of Table 6.

Table 6 The impact of zero transit tolls

	S1	S2	S4	S7
	No tolls (benchmark)	NE tolls (both instruments)	NE for zero transit toll	Collusion for zero transit toll
Local toll	0	27.1	6.82	13.7
Transit toll	0	37.9	0	0
Local mecc	15.5	13.9	15.1	14.6
Global mecc	31.1	27.1	30.7	30.2
Local welfare	141,748	186,492	142,164	142,303
Transit CS	283,495	202,364	284,603	285,731
Total welfare	566,991	575,348	568,931	570,337
Welfare change vs. S1	0	1.47%	0.34%	0.59%

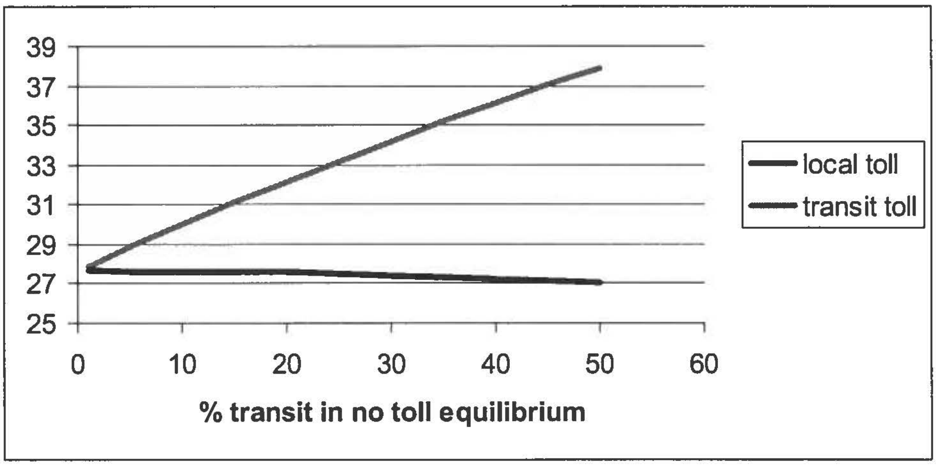
The numerical illustrations presented so far capture a situation where transit traffic forms a large share of total traffic, and where transit chooses between similar (here: identical) options. To summarise, the results indicate that:

- It is important to introduce some form of transit tolling; the welfare effects of tolling transit are large.
- The precise type of transit tolling (uniform local and transit tolls versus differentiated transit tolls) has relatively small effects on efficiency improvements compared to the no tolling situation.
- A toll on transit decreases the welfare level of transit because they do not share in the toll revenues. A uniformity restriction for local and transit tolls does not protect the transit from welfare losses. Transit prefers that only local traffic is tolled.

4.2 Varying the share of transit in the no-toll equilibrium

The transit share in the central scenario was 50% in both countries. In this sub-section we briefly consider the impact of changing the relative importance of transit; apart from that, the countries are still assumed to be symmetric. The following Figure shows the effects of varying the share of transit between 1% and 50%, while keeping the no-toll total traffic volumes at the levels of the central scenario (so this reflects 'constant congestion' compared to the central scenario).

Figure 2 Tolls as a function of no-toll transit share



The local toll decreases slightly as the share of transit increases, while the transit toll strongly increases. As the transit share goes to zero, the model converges to marginal social cost pricing; at the 1% transit share, the transit toll exceeds the local toll by just 0.87%. Higher transit shares lead to higher transit tolls; the local toll decreases somewhat because the higher transit toll leads to lower traffic levels, therefore lower (global) marginal external costs, therefore lower tolls.

It is interesting to find out if the qualitative results concerning the impacts of various pricing constraints change when transit shares are decreased. In order to do this, we reconsider scenarios S1 – S6 for a reference transit share of 10% (instead of 50%). The results are in Table 7 and Table 8. The maximal attainable welfare gain (S5) is the same as in the central scenario, because overall traffic and congestion conditions have not changed. The welfare loss from not coordinating between countries (compare S2 and S5) is even smaller than in the central scenario, as there is less transit and therefore less of a conflict between local and global welfare. The Nash equilibrium with toll differentiation and the overall welfare maximum are – for all practical purposes - identical in welfare terms. Furthermore, the welfare loss from uniformity is the same as in the central scenario (0.06%-point). As previously, the welfare level of transit decreases when it is tolled.

Not surprisingly, with low transit shares the inability to toll transit traffic is less detrimental than in the central scenario. In scenario S4, the Nash equilibrium with a local toll only, 62% of the gain from the gain in the Nash equilibrium with differentiation (S2) is obtained. In S6, overall welfare maximisation with a local toll, 90% of the maximal attainable gain (S5) results. It is still the case, however, that the Nash equilibrium with uniform tolls (S3) does better than welfare maximisation with local tolls only (S6).

Table 7 Results from symmetric model – Levels, index (S5=100); low transit share in S1 (10%)

Note: these scenarios produce symmetric outcomes, so the distinction between countries A and B is suppressed; variables not defined on the country level are shown in italics

			S1 No tolls		S2 NE - differentiation		S3 NE - uniform		S4 NE - local toll only		S5 Centralised - diff.		S6 Centralised - loc. only	
	Variable	Unit	Level	Index	Level	Index	Level	Index	Level	Index	Level	Index	Level	Index
1	Local demand	trips	2,340	112.5	2,081	100.0	2,062	99.1	2,222	106.8	2,080	100	2,077	99.8
2	Transit demand	trips	520	112.5	457	98.8	458	99.1	523	113.2	462	100	527	114.1
3	Trip volume, country level	trips	2,600	112.5	2,309	99.9	2,291	99.1	2,484	107.5	2,311	100	2,340	101.3
4	Transit volume, country level	trips	260	112.5	228	98.8	229	99.1	262	113.2	231	100	264	114.1
5	Generalised price, local	Euro/trip	65.4	73.0	89.6	99.9	91.3	101.8	76.4	85.2	89.7	100.0	90.0	100.3
6	Generalised price, transit	Euro/trip	65.4	73.0	92.0	102.6	91.3	101.8	64.1	71.5	89.7	100.0	62.3	69.5
7	Time cost	Euro/trip	32.7	111.8	29.2	99.9	29.0	99.2	31.3	107.1	29.3	100.0	29.6	101.2
8	**Local toll**	**Euro/trip**	**0.0**	**0.0**	**27.6**	**99.9**	**29.6**	**106.8**	**12.3**	**44.6**	**27.7**	**100.0**	**27.6**	**99.9**
9	**Transit toll**	**Euro/trip**	**0.0**	**0.0**	**30.0**	**108.4**	**29.6**	**106.8**	**0.0**	**0.0**	**27.7**	**100.0**	**0.0**	**0.0**
10	Local MEC	Euro/trip	28.0	112.5	24.9	100.0	24.7	99.1	26.6	106.8	24.9	100.0	24.9	99.8
11	Global MEC	Euro/trip	31.1	112.5	27.6	99.9	27.4	99.1	29.7	107.5	27.7	100.0	28.0	101.3
12	Local CS	Euro	255,146	126.5	201,761	100.1	198,221	98.3	230,180	114.1	201,652	100	201,006	99.7
13	Tax revenue, country level	Euro	0	0.0	64,345	100.7	67,718	105.9	27,403	42.9	63,927	100	57,361	89.7
14=12+13	Welfare, country level	Euro	255,146	96.1	266,106	100.2	265,939	100.1	257,582	97.0	265,579	100	258,367	97.3
15	Transit welfare (CS)	Euro	56,705	126.5	43,756	97.6	44,055	98.3	57,428	128.1	44,817	100	58,330	130.2
16=2*14+15	Overall welfare	Euro	566,997	98.4	575,968	100.0	575,933	100.0	572,592	99.4	575,975	100	575,064	99.8

Table 8 Distribution of welfare and total welfare gain (%); low transit share in S1 (10%)

	S1	S2	S3	S4	S5	S6
Share country A	45	46.20	46.18	44.98	46.11	44.93
Share country B	45	46.20	46.18	44.98	46.11	44.93
Share transit	10	7.60	7.65	10.03	7.78	10.14
Total (% change compared to S1)	100 (0.00)	100 (1.582)	100 (1.576)	100 (0.99)	100 (1.583)	100 (1.42)

4.3 The effects of asymmetry between countries

4.3.1 Asymmetrical local demand functions

In this sub-section, aggregate trip demand for the whole network in the no-toll equilibrium is held at the level of the central scenario, but the distribution of local traffic (and, consequently, of transit demand) between countries is changed. More precisely, (i) the transit demand function is assumed to be the same as before; (ii) the sum of local demand over both countries is the same as before; (iii) but, local demand in country A is decreased and that in country B is increased (in the reference case, the no-toll equilibrium). The local demand functions are adapted accordingly (implying both a shift and a change in slope, as the reference elasticity of demand is held constant). The scenario so obtained could be interpreted as the case of a densely populated vs. a sparsely populated country (countries B and A respectively).

The results are summarized in Table 9. The top part of the table describes the effects of the asymmetry on the reference equilibrium. From left to right we decrease the local demand in country A and correspondingly increase local demand in B. Since road capacity does not change, a larger share of (constant) transit demand is attracted to Country A. By construction, the local marginal congestion cost in Country A decreases, that in Country B increases, and the generalized cost and the global marginal congestion costs are the same as in the central scenario in both countries in the reference equilibrium.

The implications of the asymmetry for the Nash equilibrium with differentiated tolls are fairly straightforward. Implementing the differentiated tolls leads to smaller local demand reductions in Country A as the asymmetry increases, while local demand reductions in Country B become larger. The effect of the asymmetry on the total transit demand reduction is very small. Also, as Country A carries more transit flow (in relative and in absolute terms), the move to the Nash equilibrium implies a larger reduction in its share in total transit flow. Stronger asymmetry implies a lower local toll and a larger transit toll in Country A, with the opposite directions of change in Country B. The local marginal congestion costs in Country A decrease less as the asymmetry is larger, while the global marginal congestion cost decreases more strongly. In terms of welfare, the gain from the Nash differentiated tolls in Country A becomes larger as its local willingness to pay for trips is smaller, for constant road capacity. Correspondingly, the gains for Country B become smaller. Notably, the transit welfare reduction after introduction of the differentiated tolls hardly depends on the asymmetry.

This exercise suggests that a country which is in a position to attract a lot of transit traffic, because it has a lot of road capacity and/or little local demand, will benefit a lot from a differentiated toll on local and transit traffic. The competitive advantage that follows from having sufficient capacity that is not congested from local use, enables the country to raise substantial amounts of toll revenue from non-local users, so increasing local welfare. The welfare potential of the competing country decreases, but transit users do not suffer more or less. In short, this example suggests that countries have strategic incentives for provision of infrastructure. Endogenising the capacity provision decision seems to be a worthwhile extension of this paper.

Table 9 Asymmetrical local demand functions

	A: symmetry (central scenario)	B: asymmetric	C: asymmetric	D: asymmetric	E: asymmetric
Characteristics of reference situation					
Reference distribution of demand, % (aggregate demand and transit demand are constant across scenarios)					
Country A	25	21	15	8	ε
Country B	25	29	35	42	50-ε
Transit	50	50	50	50	50
Reference distribution of transit over countries					
Share Country A	50	57.7	69.2	84.6	100-ε
Share Country B	50	42.3	30.8	15.4	ε
Reference marginal congestion cost, Euro/trip, index					
Local MECC A	15.6 (100)	85	62	31	ε
Local MECC B	15.6 (100)	115	138	169	200-ε
Global MECC A	31.1 (100)	100	100	100	100
Global MECC B	31.1 (100)	100	100	100	100
Comparisons of reference situation and Nash equilibrium					
Percentage change of trip demand					
Local demand A	-10.5	-10.0	-9.2	-8.0	-6.8
Local demand B	-10.5	-11.1	-11.8	-12.7	-13.6
Transit demand	-15.5	-15.5	-15.4	-15.3	-15.0
Change in distribution of transit (%-point change in share of Country A = - value for Country B)					
Change share Country A	0	-2.0	-5.0	-9.1	-13.2
Optimal toll, Euro/trip, index					
Local toll A	27.1 (100)	97.9	94.5	89.9	84.9
Local toll B	27.1 (100)	102.1	105.0	108.8	112.2
Transit toll A	37.9 (100)	101.4	103.4	105.8	107.8
Transit toll B	37.9 (100)	98.5	96.0	92.3	88.2
Change in marginal congestion costs, %					
Local MECC A	-10.5	-10.0	-9.2	-8.0	-6.8
Local MECC B	-10.5	-11.1	-11.8	-12.7	-13.6
Global MECC A	-13.0	-14.9	-17.8	-21.8	-26.2
Global MECC B	-13.0	-11.2	-8.7	-5.4	-2.4
Welfare change, %					
Country A	31.6	42.0	67.2	157.4	To infinity
Country B	31.6	23.9	15.4	7.6	2.2
Transit	-28.6	-28.6	-28.5	-28.2	-27.7

4.3.2 Asymmetrical congestion functions

Here we test the sensitivity of results to differences in the congestion functions between countries. The congestion function for Country B is the same as in the central scenario. For Country A, the slope is decreased, simultaneously increasing the intercept in order to retain the volumes and travel times (in both countries) of the central scenario. In other words, the congestion function for Country A is tilted around the reference point for the central scenario. The congestibility of the road in Country A is decreased, but the fixed component of travel time is increased. This could be interpreted as making the road in Country A longer and larger. Note that this implies decreasing the congestibility of the parallel network, from the transit traffic point of view.

The results are in Table 10. Column A is the central scenario. In columns B through D, the slope of the congestion function of Country A is reduced by 5 to 15% and the intercept is adapted to keep reference volumes and distributions constant (see top half of the table). In column E, the slope of the congestion function of Country A is reduced to epsilon, implying a virtual absence of congestion. The effects of the experiment on the impact of introducing Nash equilibrium tolls is limited, except in the extreme case of column E. Introducing the asymmetry reduces the optimal tolls, mainly in Country A. The transit toll in Country B is least sensitive to the asymmetry. Note that the local toll in Country A converges to zero as the road becomes congestion-free; this is entirely in line with theory, as the toll is still equal to the global marginal congestion cost. The local welfare gains from the tolls decrease, which could be expected as the initial inefficiency from congestion becomes smaller with the network capacity increase. More importantly, the country with the more congestible network (Country B) tends to gain relatively more from the introduction of the tolls than the one with the less congestible network (Country A). This effect is small, however.

Table 10 Asymmetrical congestion functions

	A: symmetry (central scenario)	B: asymmetric	C: asymmetric	D: asymmetric	E: asymmetric
Characteristics of reference situation					
Reference distribution of demand, % (aggregate demand and transit demand are constant across scenarios)					
Country A	25	25	25	25	25
Country B	25	25	25	25	25
Transit	50	50	50	50	50
Reference distribution of transit over countries					
Share Country A	50	50	50	50	50
Share Country B	50	50	50	50	50
Reference marginal congestion cost, Euro/trip, index					
Local MECC A	15.6 (100)	95	90	85	ε
Local MECC B	15.6 (100)	100	100	100	100
Global MECC A	31.1 (100)	95	90	85	ε
Global MECC B	31.1 (100)	100	100	100	100
Comparisons of reference situation and Nash equilibrium					
Percentage change of trip demand					
Local demand A	-10.5	-10.2	-9.8	-9.5	−ε
Local demand B	-10.5	-10.5	-10.4	-10.4	-7.4
Transit demand	-15.5	-15.2	-14.9	-14.6	-7.4
Change in distribution of transit (%-point change in share of Country A = - value for Country B)					
Change share Country A	0	0.42	0.87	1.34	18.0
Optimal toll, Euro/trip, index					
Local toll A	27.1 (100)	95.7	91.4	87.1	ε
Local toll B	27.1 (100)	99.8	99.5	99.3	87.3
Transit toll A	37.9 (100)	97.3	94.6	91.8	42.6
Transit toll B	37.9 (100)	98.5	97.6	95.4	62.4
Change in marginal congestion costs, %					
Local MECC A	-10.5	-10.2	-9.8	-9.5	0
Local MECC B	-10.5	-10.5	-10.4	-10.4	-7.4
Global MECC A	-13.0	-12.3	-11.6	-10.9	13.1
Global MECC B	-13.0	-13.2	-13.4	-13.6	-24.1
Welfare change, %					
Country A	31.6	30.1	30.2	29.5	18.6
Country B	31.6	31.0	30.5	30.0	18.6
Transit	-28.6	-28.1	-27.6	-27.1	-14.2

5. Summary, conclusions and directions for future research

In this paper we studied optimal and strategic pricing of local and transit traffic on a simple parallel network. The tolling authority on the individual links of the network was assumed to be assigned to different countries. We first theoretically analysed Nash equilibria in this setting for three types of pricing structures: differentiated tolls between local and transit traffic, uniform tolls, and local tolls only. Then a numerical model was used to illustrate the main results and to assess the welfare effects of various pricing constraints and of (the lack of) coordination between countries. Moreover, the relevance of the share of transit in total transport demand and of asymmetries between countries was numerically illustrated.

The conclusions are easily summarised. First, the welfare effects of tolling transit seem to be large, but there is almost no difference in global efficiency between uniform and differentiated tolling. Specifically, differentiation between local and transit tolls as compared to uniform tolls does not yield large welfare differences, although obviously tolls on transit may differ substantially. Allowing differentiated tolls in an uncoordinated setting tends to go at the expense of transit traffic. Second, the welfare effects of coordination between countries are relatively small in comparison with the welfare gains of tolling transit. The outcome when countries behave strategically but do tax transit (e.g., the Nash equilibrium with uniform tolls) yields higher welfare effects than the coordinated welfare optimum for the network as a whole when transit is not tolled. Third, the effect of higher transit shares on the Nash equilibrium with differentiated tolls is to strongly raise the transit toll and to slightly decrease the local toll. As the transit share goes to zero, the model converges to marginal social cost pricing for local traffic. Fourth, the impact of introducing asymmetries between countries is to raise welfare gains for the country with lower local demand (comparing the Nash-equilibrium to the no-toll equilibrium); welfare gains in the other country become less pronounced.

Finally, note that this paper could be extended along several lines. First, we have limited the analysis to cases where at all equilibria both local and transit transport occur in both regions. Although the case of zero local traffic is not very interesting, allowing corner solutions at zero transit does seem a relevant case to consider. Under specific conditions, countries could actually choose to eliminate all transit on their territory. Studying these conditions seems a relevant addition to the analysis of this paper. Second, different pricing instruments (road pricing, fuel taxes, vignettes, etc.) could be introduced explicitly. This would probably make the theoretical analysis intractable, but it would enrich the numerical results. Third, one could incorporate freight transport and analyse partial taxation of the network (e.g., toll trucks but not passengers). Fourth, the transition process of introducing tolling instruments sequentially could be explicitly studied. For example, given that one country moves from a system with local tolls only to a system with explicit transit tolling, how does this affect optimal responses by the other country? Alternatively, if a country moves from differentiated tolls towards uniform tolls, what is the optimal response for the other country? What do the resulting Nash equilibria look like?

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Appendix 1: Detailed analysis of the case of differentiated tolls

In this appendix we study in more detail the case of differentiated tolls on local and transit transport. We derive the reduced form demand system and discuss its properties, and we derive the optimal toll results presented in the main body of the paper.

The reduced-form demand system

Using (1) and focusing on the case where there is local and transit traffic in both regions, the system consisting of (2), (3) and (4) can be reformulated as

$$P^X(X_A + X_B) = C_A(X_A + Y_A) + \tau_A \tag{A.1}$$

$$P^X(X_A + X_B) = C_B(X_B + Y_B) + \tau_B \tag{A.2}$$

$$P_A^Y(Y_A) = C_A(X_A + Y_A) + t_A \tag{A.3}$$

$$P_B^Y(Y_B) = C_B(X_B + Y_B) + t_B \tag{A.4}$$

This system of four equations can easily be solved for the reduced form demand functions as functions of the four tax rates. A particularly instructive way to do this is to first solve (A.3) and (A.4) separately for the demands for local transport as a function of transit demands and local tax rates in a given region:

$$Y_A = z_A(X_A, t_A) \tag{A.5}$$

$$Y_B = z_B(X_B, t_B) \tag{A.6}$$

Note that application of the implicit function theorem to (A.3) implies:

$$\frac{\partial z_A}{\partial X_A} = \frac{\frac{\partial C_A}{\partial V_A}}{\frac{\partial P_A^Y}{\partial Y_A} - \frac{\partial C_A}{\partial V_A}} < 0 \tag{A.7}$$

$$\frac{\partial z_A}{\partial t_A} = \frac{1}{\frac{\partial P_A^Y}{\partial Y_A} - \frac{\partial C_A}{\partial V_A}} < 0 \tag{A.8}$$

where

$$V_A = X_A + Y_A$$

is the total transport volume in A. Using (A.4), an analogous result is derived for B. Interpretation is simple: an exogenous increase in transit in a given region reduces the demand for local transport, as it raises local congestion and hence generalised user cost. Raising the local tax, at a given transit level, reduces local demand for transport.

Substituting (A.5)-(A.6) into (A.1) and (A.2) yields two equations in X_A, X_B as a function of all four tax rates:

$$P^X(X_A + X_B) = C_A\left[X_A + z_A(X_A, t_A)\right] + \tau_A \tag{A.9}$$

$$P^X(X_A + X_B) = C_B\left[X_B + z_B(X_B, t_B)\right] + \tau_B \tag{A.10}$$

The solution of this system yields the reduced-form demand functions for transit, denoted in the main body of the paper as $X_A^r\left[\tau_A, t_A, \tau_B, t_B\right]$ and $X_B^r\left[\tau_A, t_A, \tau_B, t_B\right]$, respectively. To determine the signs of the various tax effects on transit demands, totally differentiate system (A.9)-(A.10) and write the result in matrix notation:

$$\begin{bmatrix} \frac{\partial P^X}{\partial X} - \frac{\partial C_A}{\partial V_A}(1+\frac{\partial z_A}{\partial X_A}) & \frac{\partial P^X}{\partial X} \\ \frac{\partial P^X}{\partial X} & \frac{\partial P^X}{\partial X} - \frac{\partial C_B}{\partial V_B}(1+\frac{\partial z_B}{\partial X_B}) \end{bmatrix} \begin{bmatrix} dX_A \\ dX_B \end{bmatrix} = \begin{bmatrix} \frac{\partial C_A}{\partial V_A}\frac{\partial z_A}{\partial t_A} dt_A + d\tau_A \\ \frac{\partial C_B}{\partial V_B}\frac{\partial z_B}{\partial t_B} dt_B + d\tau_B \end{bmatrix}$$

Applying Cramers' rule then yields, after simple algebra, the effects of tax changes on demand in A (analogous results hold for B):

$$\frac{dX_A}{dt_A} = \frac{1}{\Delta}\left\{\left(\frac{\partial C_A}{\partial V_A}\frac{\partial z_A}{\partial t_A}\right)\left[\frac{\partial P^X}{\partial X} - \frac{\partial C_B}{\partial V_B}(1+\frac{\partial z_B}{\partial X_B})\right]\right\} \quad \text{(A.11)}$$

$$\frac{dX_A}{d\tau_A} = \frac{1}{\Delta}\left[\frac{\partial P^X}{\partial X} - \frac{\partial C_B}{\partial V_B}(1+\frac{\partial z_B}{\partial X_B})\right] \quad \text{(A.12)}$$

$$\frac{dX_A}{dt_B} = -\frac{1}{\Delta}\left(\frac{\partial P^X}{\partial X}\frac{\partial C_B}{\partial V_B}\frac{\partial z_B}{\partial t_B}\right) \quad \text{(A.13)}$$

$$\frac{dX_A}{d\tau_B} = -\frac{1}{\Delta}\frac{\partial P^X}{\partial X} \quad \text{(A.14)}$$

where

$$\Delta = -\frac{\partial C_A}{\partial V_A}(1+\frac{\partial z_A}{\partial X_A})\left[\frac{\partial P^X}{\partial X} - \frac{\partial C_B}{\partial V_B}(1+\frac{\partial z_B}{\partial X_B})\right] - \frac{\partial P^X}{\partial X}\frac{\partial C_B}{\partial V_B}(1+\frac{\partial z_B}{\partial X_B})$$

Using (A.7) for k=A,B it follows:

$$(1+\frac{\partial z_k}{\partial X_k}) = \frac{\frac{\partial P_k^Y}{\partial Y_k}}{\frac{\partial P_k^Y}{\partial Y_k} - \frac{\partial C_k}{\partial V_k}} > 0$$

which immediately implies $\Delta > 0$. Note that (A.11)-(A.14) then imply:

$$\frac{dX_A}{d\tau_A} = \frac{\partial X_A^r}{\partial \tau_A} < 0,\ \frac{dX_A}{d\tau_B} = \frac{\partial X_A^r}{\partial \tau_B} > 0,\ \frac{dX_A}{dt_A} = \frac{\partial X_A^r}{\partial t_A} > 0,\ \frac{dX_A}{dt_B} = \frac{\partial X_A^r}{\partial t_B} < 0$$

Moreover, (A.8), (A.11) and (A.12) imply $\left|\frac{\partial X_A^r}{\partial \tau_A}\right| > \left|\frac{\partial X_A^r}{\partial t_A}\right|$.

Finally, to determine the impact of taxes on local demands, note from (A.5)-(A.6) that

$$\frac{dY_A}{dt_A} = \frac{\partial z_A}{\partial X_A}\frac{dX_A}{dt_A} + \frac{\partial z_A}{\partial t_A}$$
$$\frac{dY_A}{d\tau_A} = \frac{\partial z_A}{\partial X_A}\frac{dX_A}{d\tau_A}$$
$$\frac{dY_A}{dt_B} = \frac{\partial z_A}{\partial X_A}\frac{dX_A}{dt_B}$$
$$\frac{dY_A}{d\tau_B} = \frac{\partial z_A}{\partial X_A}\frac{dX_A}{d\tau_B}$$

so that, using all previous results, it immediately follows:

$$\frac{dY_A}{d\tau_A} = \frac{\partial Y_A^r}{\partial \tau_A} > 0, \frac{dY_A}{d\tau_B} = \frac{\partial Y_A^r}{\partial \tau_B} < 0, \frac{dY_A}{dt_A} = \frac{\partial Y_A^r}{\partial t_A} < 0, \frac{dY_A}{dt_B} = \frac{\partial Y_A^r}{\partial t_B} > 0$$

For the reduced form demand functions for country B, the signs of the different tax effects are determined completely analogously.

Optimal tax rules

The first-order conditions to optimisation problem (8) are given by

$$P_A^Y \frac{\partial Y_A^r}{\partial t_A} - g_A^Y \frac{\partial Y_A^r}{\partial t_A} - Y_A\left[\frac{\partial C_A}{\partial V_A}\left(\frac{\partial Y_A^r}{\partial t_A} + \frac{\partial X_A^r}{\partial t_A}\right) + 1\right] + t_A \frac{\partial Y_A^r}{\partial t_A} + Y_A^r + \tau_A \frac{\partial Y_A^r}{\partial t_A} = 0$$

$$P_A^Y \frac{\partial Y_A^r}{\partial \tau_A} - g_A^Y \frac{\partial Y_A^r}{\partial \tau_A} - Y_A\left[\frac{\partial C_A}{\partial V_A}\left(\frac{\partial Y_A^r}{\partial \tau_A} + \frac{\partial X_A^r}{\partial \tau_A}\right)\right] + t_A \frac{\partial Y_A^r}{\partial \tau_A} + X_A^r + \tau_A \frac{\partial Y_A^r}{\partial \tau_A} = 0$$

Using the fact that in equilibrium generalised cost equals generalised price and rearranging, the system can be written as:

$$\left(t_A - Y_A \frac{\partial C_A}{\partial V_A}\right)\frac{\partial Y_A^r}{\partial t_A} + \left(\tau_A - Y_A \frac{\partial C_A}{\partial V_A}\right)\frac{\partial X_A^r}{\partial t_A} = 0 \qquad \text{(A.15)}$$

$$\left(t_A - Y_A \frac{\partial C_A}{\partial V_A}\right)\frac{\partial Y_A^r}{\partial \tau_A} + \left(\tau_A - Y_A \frac{\partial C_A}{\partial V_A}\right)\frac{\partial X_A^r}{\partial \tau_A} + X_A = 0 \qquad \text{(A.16)}$$

Writing the system in matrix notation and solving by Cramers' rule yields the tax rule for local traffic as follows:

$$t_A = \frac{1}{D}\left\{Y_A \frac{\partial C_A}{\partial V_A}[D] + X_A \frac{\partial X_A^r}{\partial t_A}\right\} \qquad \text{(A.17)}$$

where

$$D = \frac{\partial Y_A^r}{\partial t_A}\frac{\partial X_A^r}{\partial \tau_A} - \frac{\partial Y_A^r}{\partial \tau_A}\frac{\partial X_A^r}{\partial t_A} = \frac{\partial z_A}{\partial t_A}\frac{\partial X_A^r}{\partial \tau_A}$$

The last equality follows from the definition of the various demand effects derived before. Substituting D in (A.17) and slightly manipulating the result immediately leads to the result presented in the main body of the paper:

$$t_A = (Y_A + X_A)\frac{\partial C_A}{\partial V_A} = LMEC_A + X_A\frac{\partial C_A}{\partial V_A} \qquad \text{(A.18)}$$

Using similar procedures we find for the transit tax

$$\tau_A = Y_A\frac{\partial C_A}{\partial V_A} - X_A\left[\frac{\dfrac{\partial Y_A^r}{\partial t_A}}{\dfrac{\partial z_A}{\partial t_A}\dfrac{\partial X_A^r}{\partial \tau_A}}\right] \qquad \text{(A.19)}$$

Finally, comparison of (A.18) and (A.19) implies that the tax on transit exceeds the tax on local transport, implying tax exporting behaviour. To see this, note that our results imply

$$\tau_A - t_A = -X_A\left[\frac{\partial C_A}{\partial V_A} + \frac{\dfrac{\partial Y_A^r}{\partial t_A}}{\dfrac{\partial z_A}{\partial t_A}\dfrac{\partial X_A^r}{\partial \tau_A}}\right]$$

Substituting $\frac{\partial Y_A^r}{\partial t_A} = \frac{\partial z_A}{\partial X_A}\frac{\partial X_A^r}{\partial t_A} + \frac{\partial z_A}{\partial t_A}$, using $\frac{\partial z_A}{\partial X_A} = \frac{\partial C_A}{\partial V_A}\frac{\partial z_A}{\partial t_A}$ (see (A.7)-(A.8)) and rearranging yields

$$\tau_A - t_A = -X_A\left[\frac{1 + \dfrac{\partial C_A}{\partial V_A}\left[\dfrac{\partial X_A^r}{\partial \tau_A} + \dfrac{\partial X_A^r}{\partial t_A}\right]}{\dfrac{\partial X_A^r}{\partial \tau_A}}\right]$$

Using (A.11)-(A.12) and explicitly substituting Δ then yields, after some manipulation:

$$\tau_A - t_A = X_A \left[\frac{\frac{\partial P^X}{\partial X}\frac{\partial C_B}{\partial V_B}\left[1+\frac{\partial z_B}{\partial X_B}\right]}{\frac{\partial P^X}{\partial X}-\frac{\partial C_B}{\partial V_B}\left[1+\frac{\partial z_B}{\partial X_B}\right]} \right] > 0$$

Appendix 2: Detailed analysis of the case of uniform tolls

Reduced-form demand system

Using similar developments as in the differentiated tolling case we immediately obtain (the definition of $\Delta > 0$ is unchanged):

$$\frac{dX_A}{d\theta_A} = \frac{1}{\Delta}\left\{\left(1+\frac{\partial C_A}{\partial V_A}\frac{\partial z_A}{\partial \theta_A}\right)\left[\frac{\partial P^X}{\partial X}-\frac{\partial C_B}{\partial V_B}(1+\frac{\partial z_B}{\partial X_B})\right]\right\} < 0 \qquad \text{(A.20)}$$

$$\frac{dX_A}{d\theta_B} = \frac{-1}{\Delta}\left[\frac{\partial P^X}{\partial X}\left(1+\frac{\partial C_B}{\partial V_B}\frac{\partial z_B}{\partial X_B}\right)\right] > 0 \qquad \text{(A.21)}$$

Furthermore, analogous procedures as in the case of differentiated taxes immediately yield:

$$\frac{dY_A}{d\theta_A} < 0, \frac{dY_A}{d\theta_B} < 0$$

Optimal tax rules

The first-order condition to the problem

$$\underset{\theta_A}{Max} \quad W_A = \int_0^{Y_A} (P_A^Y(Y_A))dY_A - g_A^Y * Y_A + \theta_A(Y_A + X_A),$$

can be written as:

$$P_A^Y\frac{\partial Y_A^r}{\partial \theta_A} - g_A^Y\frac{\partial Y_A^r}{\partial \theta_A} - Y_A\left[\frac{\partial C_A}{\partial V_A}\left(\frac{\partial Y_A^r}{\partial \theta_A}+\frac{\partial X_A^r}{\partial \theta_A}\right)+1\right] + \theta_A(\frac{\partial Y_A^r}{\partial \theta_A}+\frac{\partial Y_A^r}{\partial \theta_A}) + (Y_A^r + X_A^r) = 0$$

Simplifying and solving for the tax yields:

$$\theta_A = Y_A\frac{\partial C_A}{\partial V_A} - \frac{X_A^r}{\frac{\partial Y_A^r}{\partial \theta_A}+\frac{\partial X_A^r}{\partial \theta_A}}$$

Appendix 3: Detailed analysis of the case 'local tolls only'

Reduced-form demand system

The derivatives of the reduced-form demand functions with respect to the local tolls are easily shown to be identical to those for the differentiated tolling case. Indeed, the only difference is that the transit toll is set to zero.

Optimal tax rules

The first-order condition to the problem

$$\underset{t_A}{Max} \quad W_A = \int_0^{Y_A} (P_A^Y(Y_A))dY_A - g_A^Y * Y_A + t_A Y_A$$

immediately yields, after simple manipulation:

$$t_A \frac{\partial Y_A^r}{\partial t_A} - \left(Y_A \frac{\partial C_A}{\partial V_A} \right) \left(\frac{\partial Y_A^r}{\partial t_A} + \frac{\partial X_A^r}{\partial t_A} \right) = 0$$

Solving for the optimal local toll leads to:

$$t_A = Y_A \frac{\partial C_A}{\partial V_A} \left[1 + \frac{\frac{\partial X_A^r}{\partial t_A}}{\frac{\partial Y_A^r}{\partial t_A}} \right]$$

Importantly, the term between square brackets can be shown to be positive (and smaller than one), implying the optimal tax is between zero and the local marginal external cost. To see this, remember that the derivatives of the reduced-form demand functions are given by the same expressions A.11 and A.13 as for the differentiated tolling case. Then substitute the definition of Δ and use A.7-A.8 to obtain, after straightforward manipulation:

$$1 + \frac{\frac{\partial X_A^r}{\partial t_A}}{\frac{\partial Y_A^r}{\partial t_A}} = \frac{M_1}{\left(\frac{\partial C_A}{\partial V_A} - \frac{\partial P_Y^A}{\partial Y_A}\right)\left\{\frac{\partial C_A}{\partial V_A}\frac{\partial z_A}{\partial t_A}[M_2]\right\} + M_1} \qquad \text{(A.22)}$$

where

$$M_1 = -\frac{\partial P^X}{\partial X}\frac{\partial C_B}{\partial V_B}\left[1 + \frac{\partial z_B}{\partial X_B}\right] > 0$$

$$M_2 = \frac{\partial P^X}{\partial X} - \frac{\partial C_B}{\partial V_B}\left[1 + \frac{\partial z_B}{\partial X_B}\right] < 0$$

It immediately follows that the both the numerator and the denominator of the right-hand side of A.22 are positive. .

Appendix 4: Details on the reaction functions and the Nash equilibria

1. The case of differentiated tolls

We consecutively derive the reduced-form demands, the reaction functions, and the Nash equilibrium.

To get the reduced-form demands, we follow the procedure outlined in Appendix 1 for the linear demand and cost functions given in the main body of the paper. The demands for local transport conditional on transit and the local tax are given by

$$\begin{aligned} Y_A &= z_0^A + z_1^A X_A + z_2^A t_A \\ Y_B &= z_0^B + z_1^B X_B + z_2^B t_B \end{aligned} \quad \text{(A.23)}$$

where

$$z_0^A = \frac{c_A - \alpha_A}{d_A + \beta_A}, z_1^A = -\frac{\beta_A}{d_A + \beta_A}, z_2^A = -\frac{1}{d_A + \beta_A} \quad \text{(A.24)}$$

$$z_0^B = \frac{c_B - \alpha_B}{d_B + \beta_B}, z_1^B = -\frac{\beta_B}{d_B + \beta_B}, z_2^B = -\frac{1}{d_B + \beta_B} \quad \text{(A.25)}$$

Substituting these functions in the Wardrop equilibrium conditions yields, after some manipulations, the reduced-form demands for transit transport. We find:

$$X_A^r = \gamma_0^A + \gamma_1^A \tau_A + \gamma_2^A \tau_B + \gamma_3^A t_A + \gamma_4^A t_B \quad \text{(A.26)}$$

$$X_B^r = \gamma_0^B + \gamma_1^B \tau_B + \gamma_2^B \tau_A + \gamma_3^B t_B + \gamma_4^B t_A \quad \text{(A.27)}$$

where the coefficients are given by

$$
\begin{aligned}
\gamma_0^A &= \frac{b^2(z_0^B - z_0^A) + (a - bz_0^A)T^B}{N} & \gamma_0^B &= \frac{b^2(z_0^A - z_0^B) + (a - bz_0^B)T^A}{N} \\
\gamma_1^A &= -\frac{\left[b + T^B\right]}{N} & \gamma_1^B &= -\frac{\left[b + T^A\right]}{N} \\
\gamma_2^A &= \frac{b}{N} & \gamma_2^B &= \frac{b}{N} \\
\gamma_3^A &= -\left\{\frac{z_1^A\left[b + T^B\right]}{N}\right\} & \gamma_3^B &= -\left\{\frac{z_1^B\left[b + T^A\right]}{N}\right\} \\
\gamma_4^A &= \frac{bz_1^B}{N} & \gamma_4^B &= \frac{bz_1^A}{N}
\end{aligned}
\tag{A.28}
$$

In these expressions $N = bT^A + bT^B + T^A T^B$, and $T^A = \beta_A(1 + z_1^A)$, $T^B = \beta_B(1 + z_1^B)$. Since, using (A.24)-(A.25), the T^i are easily shown to be positive, it immediately follows that N>0. Therefore, we have

$$\gamma_1^A < 0, \gamma_2^A > 0, \gamma_3^A > 0, \gamma_4^A < 0.$$

$$\gamma_1^B < 0, \gamma_2^B > 0, \gamma_3^B > 0, \gamma_4^B < 0.$$

Note that the reduced form demand functions have a straightforward structure. More precisely, observe that the coefficients of the local and the transit taxes are directly related in the following simple manner (i=A,B):

$$
\begin{aligned}
\gamma_3^i &= z_1^i \gamma_1^i \\
\gamma_4^i &= z_1^i \gamma_2^i
\end{aligned}
\tag{A.29}
$$

Moreover, using (A.24)-(A.25) it immediately follows that $-1 < z_1^i < 0$ so that:

$$
\begin{aligned}
|\gamma_3^i| &< |\gamma_1^i| \\
|\gamma_4^i| &< |\gamma_2^i|
\end{aligned}
\tag{A.30}
$$

Finally, note that reduced form demands for local traffic are obtained by inserting the demands for transit (equations (A.26)-(A.27)) into system (A.23).

The reaction functions are derived as follows. Using the linear demand and cost functions in the optimal tax rules for country A derived in Appendix 1, we find after some algebra:

$$t_A = \beta_A(Y_A + X_A) \tag{A.31}$$

$$\tau_A = \beta_A Y_A + \rho_A X_A \tag{A.32}$$

where

$$\rho_A = \beta_A + \frac{bT^B}{b+T^B}$$

Finally, substituting (A.23), (A.26) and (A.27) into (A.31)-(A.32) and solving for the tax rates in A as sole functions of the two tax rates in B yields, again after some algebra,

$$\begin{aligned} \tau_A &= c_A^\tau - (\frac{1}{2}\frac{\gamma_2^A}{\gamma_1^A})\tau_B - (\frac{1}{2}\frac{\gamma_4^A}{\gamma_1^A})t_B \\ t_A &= c_A^t + (\frac{1}{2}\frac{\gamma_2^A}{\gamma_1^A}K^A)\tau_B + (\frac{1}{2}\frac{\gamma_4^A}{\gamma_1^A}K^A)t_B \end{aligned} \tag{A.33}$$

where all coefficients have been defined before, except

$$\begin{aligned} c_A^\tau &= \frac{1}{2}\left(\beta_A z_0^A - \frac{\gamma_0^A}{\gamma_1^A}\right) \\ c_A^t &= \left[\beta_A z_0^A + \frac{1}{2}T^A\left(\beta_A z_0^A \gamma_1^A + \gamma_0^A\right)\right]\frac{K^A}{T^A\gamma_1^A} \\ K^A &= \frac{T^A\gamma_1^A}{1 - z_1^A(1+T^A\gamma_1^A)} \end{aligned}$$

Moreover, for purposes of the interpretation it is useful to note that $-1 < K^A < 0$. This is easily seen to be the case as follows. First,

$$T^A\gamma_1^A = -\frac{T^A(b+T^B)}{N} = -\frac{T^A(b+T^B)}{bT^A + bT^B + T^A T^B}$$

which implies $-1 < T^A\gamma_1^A < 0$. This in turn implies $-1 < K^A < 0$.

Importantly, since the tax competition problem considered in this section is a game with four tax rates, it is not obvious to prove the existence and uniqueness of Nash equilibrium in this general setting. Fortunately, the linear structure of the problem allows us to reduce the four-dimension game into a policy game in two dimensions; moreover, existence and uniqueness then immediately follow. To see this, consider the structure of the reaction functions (A.33) and note that the local and transit tax rates of each country can be written as a function of the same linear combination of the tax rates of the other country. Specifically, define:

$$\pi_A = \tau_A + z_B^1 t_A$$
$$\pi_B = \tau_B + z_A^1 t_B$$

Substituting this result in (A.33) we obtain:

$$\tau_A = c_A^\tau - r^A \pi_B$$
$$t_A = c_A^t - r^A K^A \pi_B$$

where $r^A = \frac{1}{2}\frac{\gamma_2^A}{\gamma_1^A}$. Similar expressions result for region B. Noting that only positive π_i make economic sense, we can then reformulate (A.33) and its equivalent for B as follows:

$$\pi_A = s^A + p^A \pi_B$$
$$\pi_B = s^B + p^B \pi_A$$

where

$$s^A = c_A^\tau + z_1^B c_A^t$$
$$s^B = c_B^\tau + z_1^A c_B^t$$
$$p^A = -r^A(1 + z_1^B K^A)$$
$$p^B = -r^B(1 + z_1^A K^B)$$

Simple algebra, using the definitions given before and realising that $-1 < K^i < 0$, then shows that the reaction functions have a positive intercept, are upward sloping, and have a slope less than one.

Finally, solving the reaction functions for the original four tax rates yields the Nash equilibrium in function of the various coefficients that describe cost and demand responses. The solution can be written as:

$$\tau_A = \frac{c_A^\tau\left[1-(1-K^B z_1^A)z_1^B r^A r^B K^A\right] + c_A^t\left[(1-K^B z_1^A)z_1^B r^A r^B\right] - c_B^\tau\left[r^A\right] - c_B^t\left[r^A z_1^A\right]}{1 - r^A r^B\left[(1-K^B z_1^A)(1-K^A z_1^B)\right]}$$

$$t_A = \frac{c_A^\tau\left[-(1-K^B z_1^A)r^A r^B K^A\right] + c_A^t\left[1-(1-K^B z_1^A)r^A r^B\right] + c_B^\tau\left[r^A r^B\right] + c_B^t\left[r^A z_1^A K^A\right]}{1 - r^A r^B\left[(1-K^B z_1^A)(1-K^A z_1^B)\right]}$$

where $r^i = \frac{1}{2}\frac{\gamma_2^i}{\gamma_1^i}$. Analogous expressions result for country B.

2. The case of uniform tolls

We follow the same steps and use the same definitions as in the previous case. The reduced-form demands for local transport conditional on transit and the local tax are given by

$$\begin{aligned} Y_A &= z_0^A + z_1^A X_A + z_2^A \theta_A \\ Y_B &= z_0^B + z_1^B X_B + z_2^B \theta_B \end{aligned} \tag{A.34}$$

The reduced-form demand functions for transit are now the following:

$$X_A^r = \gamma_0^A + (\gamma_1^A + \gamma_3^A)\theta_A + (\gamma_2^A + \gamma_4^A)\theta_B \tag{A.35}$$

$$X_B^r = \gamma_0^B + (\gamma_1^B + \gamma_3^B)\theta_B + (\gamma_2^B + \gamma_4^B)\theta_A \tag{A.36}$$

where the coefficients are defined as above.

To obtain the reaction function for region A, use

$$\begin{aligned} \frac{\partial Y_A^r}{\partial \theta_A} &= z_2^A + z_1^A(\gamma_1^A + \gamma_3^A) < 0 \\ \frac{\partial X_A^r}{\partial \theta_A} &= \gamma_1^A + \gamma_3^A < 0 \end{aligned}$$

in the optimal tax rule derived in Appendix 2:

$$\theta_A = Y_A \frac{\partial C_A}{\partial V_A} - \frac{X_A^r}{\dfrac{\partial Y_A^r}{\partial \theta_A} + \dfrac{\partial X_A^r}{\partial \theta_A}}$$

Solving explicitly for the optimal tax, we find the reaction function:

$$\theta_A = \frac{c_2^{tuA}}{c_1^{tuA}} + \frac{c_3^{tuA}}{c_1^{tuA}} \theta_B \tag{A.37}$$

where

$$c_1^{tuA} = 1 - z_1^A - (\beta_A z_1^A + \eta_A)(\gamma_1^A + \gamma_3^A)$$

$$c_2^{tuA} = (\beta_A z_1^A + \eta_A)\gamma_0^A + \beta_A z_0^A$$

$$c_3^{tuA} = (\beta_A z_1^A + \eta_A)(\gamma_2^A + \gamma_4^A)$$

and

$$\eta_A = -\frac{1}{(1+z_1^A)(\gamma_1^A+\gamma_3^A)+z_2^A} > 0$$

Tedious algebra shows that ($\beta_A z_1^A + \eta_A$)>0 so that $c_3^{tuA} > 0, c_1^{tuA} > 0$: the reaction functions are upward sloping. Moreover, a Nash equilibrium indeed exists. This requires the condition:

$$\frac{c_3^{tuA}}{c_1^{tuA}} \frac{c_3^{tuB}}{c_1^{tuB}} < 1$$

which, using straightforward algebra, can easily be shown to hold.

<u>3. Local tolls only</u>

Again we follow the same steps and use the same definitions as in the section for the differentiated tolls. The demands for local transport conditional on transit and the local tax are given by

$$\begin{aligned} Y_A &= z_0^A + z_1^A X_A + z_2^A t_A \\ Y_B &= z_0^B + z_1^B X_B + z_2^B t_B \end{aligned} \tag{A.38}$$

Reduced-form demands for transit are:

$$X_A^r = \gamma_0^A + \gamma_3^A t_A + \gamma_4^A t_B \tag{A.39}$$

$$X_B^r = \gamma_0^B + \gamma_3^B t_B + \gamma_4^B t_A \tag{A.40}$$

To get the reaction function for country A, use the above specifications in the optimal tax rule

$$t_A = Y_A \frac{\partial C_A}{\partial V_A}\left(1 + \frac{\frac{\partial Y_A^r}{\partial t_A}}{\frac{\partial X_A^r}{\partial t_A}}\right)$$

The result turns out to be:

$$t_A = \frac{c_2^{tlA}}{c_1^{tlA}} + \frac{c_3^{tlA}}{c_1^{tlA}} t_B \tag{A.41}$$

where $$c_1^{tlA} = 1 - \beta_A \delta_A (z_2^A + z_1^A \gamma_3^A)$$

$$c_2^{tlA} = \beta_A \delta_A (z_0^A + z_1^A \gamma_0^A)$$

$$c_3^{tIA} = \beta_A \delta_A z_1^A \gamma_4^A$$

Again, simple but long algebra shows that the slope of the reaction function is positive; moreover, assuming all types of transport exist in the equilibrium, the existence of a Nash equilibrium can be shown. This follows because one shows that $\frac{c_3^{tIA}}{c_1^{tIA}} \frac{c_3^{tIB}}{c_1^{tIB}} < 1$.