Title
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An Adaptive Learning System for Stepwise Automatisation of Multiplication Facts in Primary Education

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Abstract

We demonstrate an application for learning multiplication problems with an adaptive algorithm that is based on a computational cognitive model of the learner's memory. The application helps learners automatise and memorise multiplications through repeated practice over three levels of difficulty. In a naturalistic setting involving more than 500 primary school students (ages 6–10) who together recorded over 300,000 responses, we observed that performance improved as learners using the application progressed through the levels. A model-based analysis of performance revealed that learners' estimated *speed of forgetting* decreased from the second to the third level. This is consistent with a shift towards stronger declarative knowledge and/or more efficient computation procedures. The model also identified consistent differences in the difficulty of individual multiplication facts that persisted across levels. This study demonstrates the feasibility of using an adaptive fact learning application to help young learners master multiplication, an essential mathematical skill.

Keywords: Multiplication; Adaptive Learning system; Retrieval; Primary education

Introduction

Multiplication facts (such as $6 \times 7 = ?$) are a key component of the mathematics curriculum in primary education, as they form a basis for many more complex skills. In this paper, we describe an adaptive learning system for learning the tables of multiplication, using a cognitive model–based approach. The adaptive scheduling of multiplication problems is driven by a computational cognitive model of the learner's memory that is continually adjusted based on the responses made by the learner. We evaluate the usage, performance, and model-based assessments of this system in a pilot study in 11 primary schools.

Learning multiplication

The ability to fluently perform simple multiplications is an important goal in primary mathematics curricula. Like many cognitive skills, solving multiplication problems is complex and multifaceted. Typically, learners begin by using slower computational strategies, progressively transition to faster retrieval-based strategies, and develop the ability to selectively choose among strategies (Campbell & Graham, 1985; Siegler, 1988; Lemaire & Siegler, 1995; Van Der Ven, Boom, Kroesbergen, & Leseman, 2012; Zhang, Ding, Barrett, Xin, & Liu, 2014; Hofman, Visser, Jansen, Marsman, & Van Der Maas, 2018).

Some commonly used methods to teach multiplication have unclear efficacy, or are known not to be particularly effective. For example, many learners report using methods that involve passive study, such as listening to recordings or looking at the multiplication tables written down (e.g., Steel & Funnell, 2001). Such methods are most likely less efficient than active study methods, like reciting multiplication tables out loud or answering prompts (Ophuis-Cox, Catrysse, & Camp, 2023). Methods that impose a fixed structure on practice are also likely to be less effective than adaptive methods: when multiplication tables are studied in a fixed order, each individual problem is practised an equal number of times, despite some problems typically requiring more practice to master than others (e.g., Van Der Ven, Straatemeier, Jansen, Klinkenberg, & Van Der Maas, 2015).

Computer-based adaptive learning systems can address these shortcomings, for instance by promoting effective study methods like spaced practice with feedback, and by adapting to individual differences so that the learning experience is appropriately challenging (e.g., De Witte, Haelermans, & Rogge, 2015). Previous work in this domain centres on adaptive systems that use Item Response Theory (IRT; e.g., Klinkenberg, Straatemeier, & van der Maas, 2011; Van Der Ven et al., 2015; Faber, Luyten, & Visscher, 2017) to adaptively schedule exercises, by simultaneously estimating the learner's ability and the difficulty of individual problems.

Current study

Here, we present a different approach to adaptive learning of multiplications, which has several distinct advantages over IRT-based systems. The adaptive learning system in this study is based on a computational cognitive model of the learner's memory. The model is used to interpret learners' performance in terms of cognitive processes, capturing individual differences through a learner- and item-specific *speed of forgetting* (α) parameter, and adjusting problem scheduling accordingly. In contrast to the difficulty and ability estimates of IRT, the α parameter is cognitively meaningful and interpretable (Liefooghe & Van Maanen, 2023). This approach is also more flexible than IRT in capturing individual differences, by allowing relative rankings in problem difficulty to differ across learners. Furthermore, the cognitive model enables a model-based assessment of learners' mastery of each multiplication problem.

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The application centres on learning multiplications through adaptive spaced retrieval practice. Retrieval practice is recognised for its efficacy within different domains, including in classroom environments (Schwartz, Son, Kornell, & Finn, 2011; Agarwal, Nunes, & Blunt, 2021), and there is some early evidence for its efficacy in learning multiplication facts, too (Ophuis-Cox et al., 2023). The adaptive core of the application has previously been demonstrated to be an efficient tool for learning various types of declarative material, such as vocabulary items and place names (e.g., van Rijn, van Maanen, & van Woudenberg, 2009; Wilschut et al., 2021). The current work involves a new type of material. While multiplication facts are also a form of declarative knowledge, they differ from simpler factual materials in that answers can also be derived through other methods.

The application is designed to reflect the multifaceted nature of multiplication skill development. Learning to perform multiplication typically involves starting with procedure-based computational strategies, and progressively perfecting these methods to enhance efficiency. Procedural strategies can include obtaining the answer from a previously established fact, or repeated addition (such as $3 + 3 + 3 = 9$). Over time, learners transition to faster strategies based on direct retrieval from memory, and develop the ability to selectively choose among strategies (Lemaire & Siegler, 1995). The design of the application should accomodate and encourage this shift from procedural to retrieval strategies (see also Chang, Sung, Chen, & Huang, 2008). Here, we aim to achieve this through a three-level structure (Table 1), in which learners progress from being able to solve each problem, through steadily improving their performance with repeated practice, up to being able to solve each multiplication via fast and accurate retrieval.

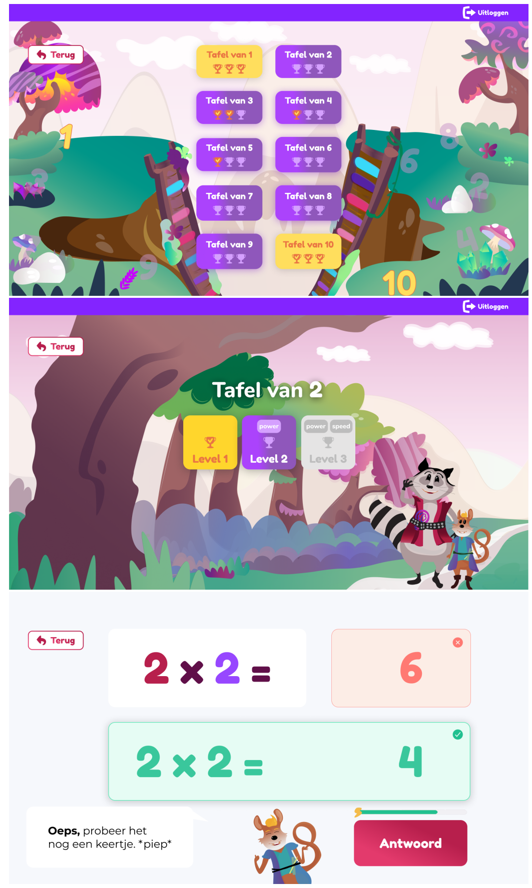

Figure 1: Screenshots of the application. **a**: Homepage. **b**: Level selection screen, indicating progress within the chosen times table. **c**: Practice screen showing error feedback.

Methods

Application design

The application enables learners to practice multiplication tables in three progressively more challenging levels (see Table 1). This design is intended to reflect a stepwise progression from the use of procedural knowledge to the recall of declarative knowledge. In all three levels, learners respond to a sequence of multiplication cues until they reach a mastery criterion. The levels differ in how items are scheduled and in how item mastery is assessed.

Adaptive scheduling Level 1 presents multiplications in a fixed order and is completed once the learner has correctly solved each item once. In Levels 2 and 3, the application uses an adaptive algorithm for spaced retrieval practice. This algorithm is based on an ACT-R model of declarative memory (Anderson & Schooler, 1991), and is described in detail in van Rijn et al. (2009) and Sense, Behrens, Meijer, and van Rijn (2016). It models the activation of each multiplication fact over time, and repeats items whenever their activation decays to a threshold value (see also Pimsleur, 1967; Pavlik & Anderson, 2008). The algorithm captures individual differences in difficulty and ability through an item- and learner-specific *speed of forgetting* parameter (α), which it estimates from the accuracy and speed of learner's responses (this can be seen as a kind of model tracing; e.g., Anderson, Corbett, Koedinger, & Pelletier, 1995). More difficult items will have a higher α value, which causes activation to decay faster, leading to more frequent repetition. In Level 2, α is adapted on the basis of response accuracy alone. There, α increases by 0.01 following an error, and decreases by 0.01 following a correct answer. In Level 3, which tests speed as well as accuracy, adaptation of α is based on both response accuracy and response time, with faster and more accurate responses leading to lower α (see Sense et al., 2016, for a detailed explanation of how α is adapted).

Model-Based Mastery Levels 2 and 3 use a model-based assessment of longer-term retention somewhat analogous to Knowledge Tracing (Corbett & Anderson, 1995) that we call *Model-Based Mastery*. To determine mastery, the application predicts longer-term activation of each multiplication fact during practice. A level is completed once the predicted fu-

Table 1: Levels in the application.

Level	Goal	Scheduling algorithm	Mastery assessment	Time limit
1	Solve each multiplication once	Fixed order	1× correct	–
2	Repeated spaced practice	Adaptive (based on accuracy)	Model-based mastery	–
3	Fast and accurate retrieval	Adaptive (based on accuracy and RT)	Model-based mastery	8 s

ture activation of all facts exceeds a threshold value. Specifically, the model predicts activation of a fact f at time $t+24$ hours in the future, and evaluates whether that activation exceeds an activation threshold τ_M[1]:

$$M(f,t) = A(f, t+24h) \geq \tau_M \tag{1}$$

$$= \ln \sum_j ((t+24h) - t_j)^{-d} \geq \tau_M \tag{2}$$

Gamification Since the application was intended for young children, we developed an age-appropriate design. The interface (see Figure 1) incorporates gamification elements that are intended to motivate learners without impairing their learning process or mental well-being. The application uses a reward system that promotes intrinsic, rather than extrinsic, motivation (Deci, Koestner, & Ryan, 1999), by linking rewards with accomplishments that help learners approach their final goal (Reiners & Wood, 2015). Completing a level unlocks the next level (Figure 1b), and completing all three levels for a multiplication table leads to the corresponding number being coloured in on the learner's homepage (Figure 1a).

Learners receive immediate feedback on their performance. When combined with a goal (e.g., completing a level), such feedback can positively influence performance (Strang, Lawrence, & Fowler, 1978). The feedback is designed to encourage learners when they make mistakes, by providing positive reinforcement (e.g., *Oops, try again*; Figure 1c), thus embracing the concept of graceful failure (Kapur, 2008).

Procedure

We conducted an open-ended study in 11 primary schools in the Netherlands with students aged 6–10 years old. Teachers were encouraged to include the application in their lesson plan during school hours, but determined themselves when and how much they used it. At several points, teachers and students also participated in an online survey. Additionally, we conducted in-person moderated usability testing with 27 students in two schools, focusing on the design, usability, and users' subjective experience of the application.

[1] Since ACT-R's activation is based on the timing of encounters, and not the accuracy of the learner's responses at those times, we only include successful responses here, in order to prevent a sequence of errors leading to mastery. Ideally, the decay d would change depending on the item-specific α. In this initial version, however, the calculation uses a fixed decay of $d = 0.45$. The threshold τ_M is estimated from learning data using the method described in Van Der Velde, Sense, Borst, and Van Rijn (2022).

Results

Usage

Figure 2 summarises learners' usage of the application. Students and/or their teachers decided themselves when and what to study; we had no control over when students used the application and which multiplication tables and levels they selected. Over a 197-day period, there were 540 active learners who together completed 315,690 trials across 17,575 sessions (Figure 2a). On average, each learner spent a total of 66 minutes practising multiplications over 26 sessions (Figure 2b).

The duration of a session depended on the learner's performance, although a learner could also quit the session at any point before level completion. Level 1 sessions were typically shorter (a median session consisting of 10 trials) than sessions in Levels 2 and 3 (each with a median of 16 trials), as shown in Figure 2c. This is consistent with having to achieve a higher standard of mastery in the latter two levels (see Table 1).

Level completion rate Level completion was based on achieving a level-specific performance criterion (see Table 1). Figure 3 shows the level completion rate: of the learners who logged at least 10 trials in a level, what proportion went on to complete the level? Completion rates were very high in Level 1 (averaging 87%) and remained quite high in Levels 2 (83%) and 3 (73%).

User experience ratings We evaluated students' and teachers' subjective experiences through questionnaires. After approximately four weeks of using the application, 79 students from four different schools completed a version of the User eXperience Kids Questionnaire (UXKQ; Wöbbekind, Mandl, & Womser-Hacker, 2021). Through eight paired differentials set on a five-point scale, we measured hedonistic aspects (e.g., *boring — fun*), as well as perceived learning quality (e.g., *bad for learning — good for learning*). Across all pairs, students gave an average rating of 4.35, indicating a positive evaluation of their experience. Eight teachers were also asked to assess the usability of the application while observing its use in their classroom. Their percentile scores on the System Usability Scale (SUS; Brooke, 1996), a widely used measure in the field of human-computer interaction, averaged 80.9 (range: 57.5–97.5), reflecting a positive rating of the application's user-friendliness for young children.

Performance

Figure 4 shows the accuracy and speed of learners' responses in each of the levels. To be included in the analysis, learners

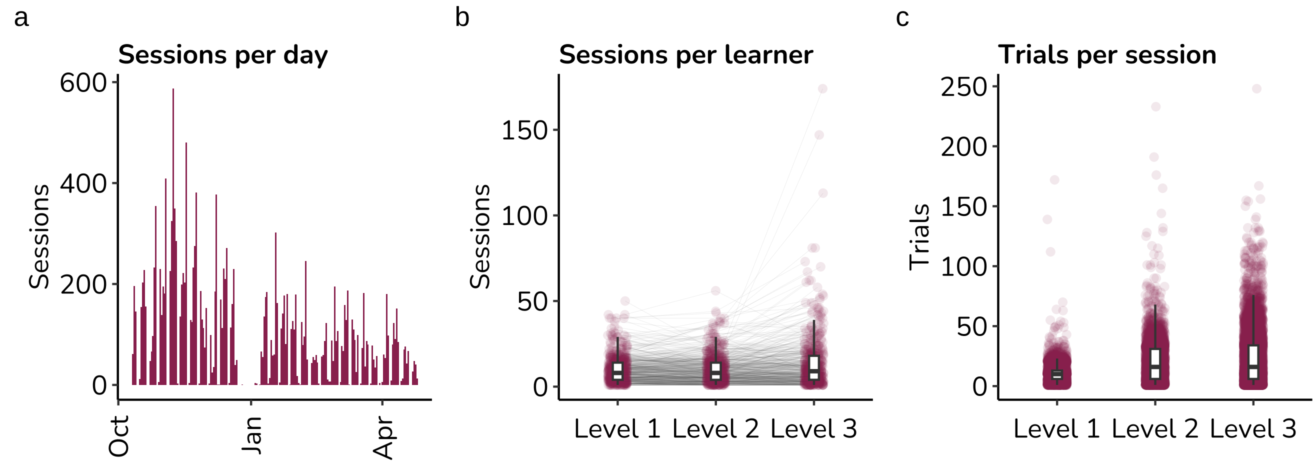

Figure 2: Usage of the application. **a**: The total number of sessions initiated each day. **b**: The number of sessions initiated by each learner, per level. **c**: The number of trials in each session, per level.

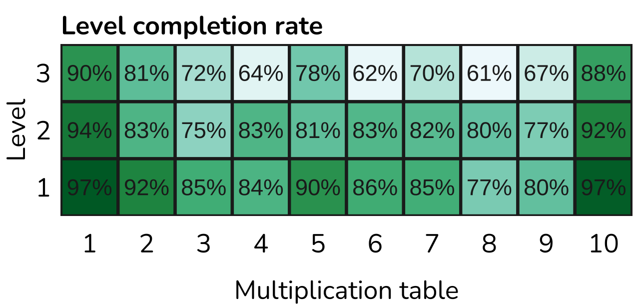

Figure 3: Level completion rate for each of the three levels, per table of multiplication.

needed to have recorded at least ten responses in a level.

As Figure 4a shows, learners' mean accuracy was generally high in all three levels, averaging 85.9% (SD = 12.6) in Level 1, 90.1% (SD = 10.9) in Level 2, and 86.1% (SD = 10.7) in Level 3. A generalised mixed-effects model fitted to the data indicated that there were small, but significant, changes in accuracy from Level 1 to Level 2 (estimated difference: 2.6pp, $p < .05$) and from Level 2 to Level 3 (estimated difference: -4.6pp, $p < .001$).

Response times were not recorded in Level 1, but learners' median response times on correct trials in Levels 2 and 3 (Figure 4b) were reasonably fast, averaging 3.079 s (SD = 1.205) in Level 2 and 2.238 s (SD = 0.555) in Level 3. A linear mixed-effects model fitted to log-RT confirmed that successful responses were significantly faster in Level 3 than in Level 2 (estimated difference: -1.389 s; $p < .001$). As such, while participants became substantially faster at giving the correct answers to multiplication problems, their responses became only became marginally less accurate overall.

Model-based performance assessments

The memory model used in the application enables a model-based assessment of learners' performance.

Speed of forgetting

During practice, the memory model estimates a separate *speed of forgetting* (α) value for each fact that a learner encounters, based on the accuracy and speed of the learner's responses. Every time a new response is made, this new observation is incorporated in the estimate, leading to an increasingly data-informed estimate with each additional practice attempt. Crucially, the model interprets each response in context, taking into account the timing and performance of previous practice attempts. We use the final α estimate, which incorporates the entire learning history of a fact, to quantify a fact's difficulty for a learner. As a model-based measure, α enables comparison of fact difficulty, as well as learner ability, on a single scale[2].

Fact difficulty By aggregating final α estimates across learners, we get a general measure of the difficulty of each multiplication fact: the higher the average speed of forgetting, the more difficult the fact is. Figure 5 shows mean final α estimates for each of the one hundred multiplication facts, derived from responses made in Level 3. Each difficulty estimate is based on learning data from between 88 and 425 different individuals. As the figure illustrates, the model finds substantial variability in estimated difficulty between facts.

By comparing difficulty estimates across levels, we can

[2]This capacity to express difficulty and ability on a single scale is shared with Item Response Theory (IRT; e.g., Klinkenberg et al., 2011), although unlike IRT's estimates, α has a concrete interpretation in the memory model.

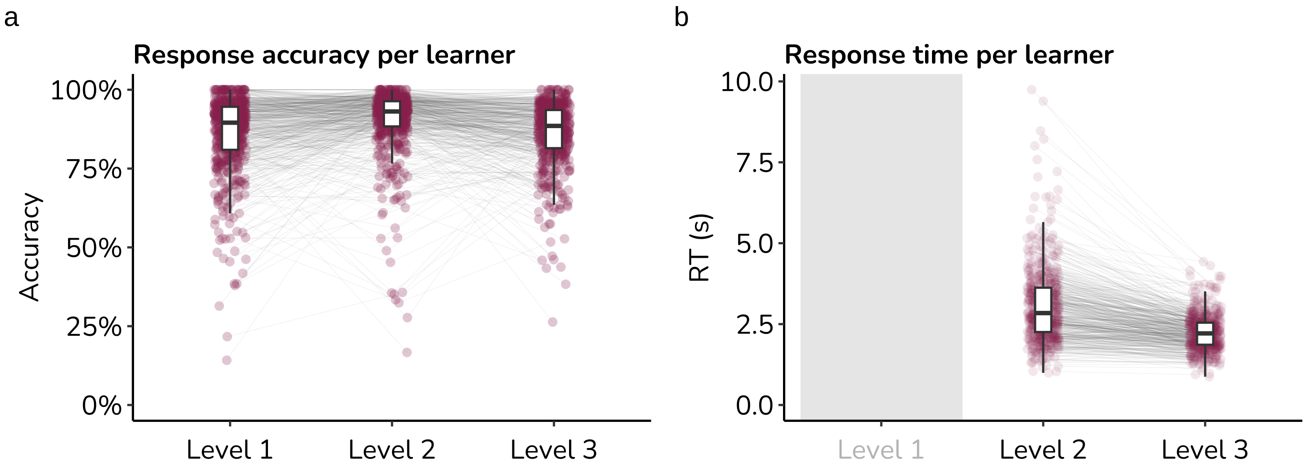

Figure 4: Performance on the individual study trials, per level. **a**: Mean response accuracy per learner. **b**: Median response time on correct answers per learner. Response times were only recorded in Levels 2 and 3.

also quantify changes in fact difficulty as learners progress through the levels. Figure 6a shows difficulty estimates by fact, based on mean α values from Levels 2 and 3. The α parameter was estimated independently in each of the levels (i.e., the Level 3 estimate did not take the learning history in Level 2 into account). Nevertheless, there was a very strong correlation in the estimated difficulty of individual facts ($r = 0.89$, $p < 0.001$), showing that relative differences in difficulty among facts persisted from Level 2 to Level 3. We also found that facts generally had a lower speed of forgetting in Level 3 compared to Level 2: all points lie below the diagonal. A linear mixed-effects model confirmed that this effect was significant ($\beta = -0.039$, $p < .001$). This finding is consistent with multiplication facts becoming easier as learners gained more experience.

Learner ability By aggregating final α estimates *within* learners, we can similarly calculate a measure of learner ability. Figure 6b shows learners' speeds of forgetting in Levels 2 and 3, once again based on independently estimated α values. Learners' average α was generally lower in Level 3 than in Level 2, as confirmed by a linear mixed-effects model ($\beta = -0.035$, $p < .001$). There was a weak correlation between levels ($r = 0.25$, $p < .001$), suggesting that while learners generally improved, the amount of improvement varied.

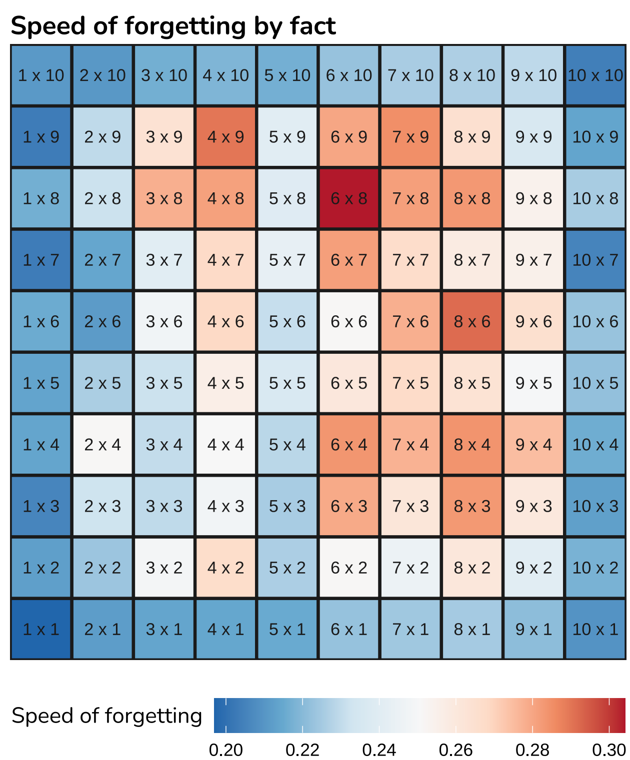

Figure 5: Estimated difficulty of each multiplication fact. Each estimate is the final *speed of forgetting* (α) of a fact in Level 3, averaged across learners. Higher values indicate more difficult facts.

Discussion

This paper describes an adaptive learning system for stepwise automatisation and memorisation of multiplication facts. We demonstrated in a naturalistic setting that this system can support young students in learning multiplications, that it can identify individual differences in learning ability and item difficulty, and that it encourages a shift from slower computational response strategies to faster retrieval methods.

It is well-established that learners use different strategies to solve multiplication problems, relying on procedural as well as declarative memory, and that the use of strategies changes with experience (e.g., Van Der Ven et al., 2012; Zhang et al.,

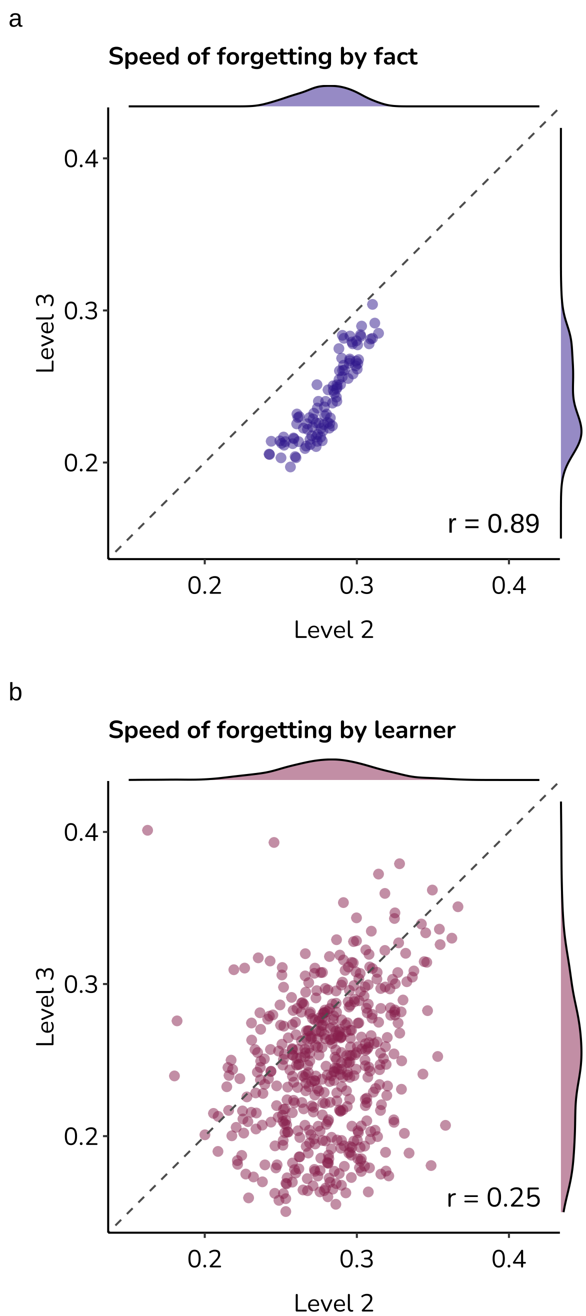

Figure 6: *Speed of forgetting* (α) in Levels 2 and 3. **a**: Mean α by fact. **b**: Mean α by learner.

2014). Knowing which strategy a learner uses to solve a multiplication problem is challenging. To a limited extent, we can tell retrieval and computation apart based on response time (Hofman et al., 2018), as well as by errors that are traceable to specific computational mistakes (Savi, Deonovic, Bolsinova, Van der Maas, & Maris, 2021). The application was designed to accommodate the use of a mix of strategies, and to encourage a shift towards retrieval. In the first two levels, response times do not factor in the adaptation of repetition scheduling, which means that learners get repeated exposure to problems without being penalised for the use of inefficient strategies. This repeated practice should lead to a stronger declarative representation (Anderson, Fincham, & Douglass, 1999), and/or to more efficient computational procedures (Taatgen, 2013), both of which would enable faster responses as learners gain more experience. The third level was intended to elicit predominantly direct retrieval, by capitalising both on a history of repeated exposure to the material in earlier levels, and by encouraging fast responses with a time limit. The finding that speed of forgetting estimates decreased in Level 3 is consistent with improvement along these lines, as is the observation of fairly stable differences in fact difficulty (Figure 6a). Although we cannot fully rule out the use of computational procedures, the Level 3 completion rate nonetheless suggests that learners were generally able to achieve the intended mastery of the material (i.e., the ability to consistently respond quickly and accurately).

The fact-level difficulty estimates obtained through the system (Figure 5) reveal individual differences between facts, and partially match previously described canonical effects. We see some evidence for a *problem size effect* (difficulty increases with larger operands; e.g., Campbell & Graham, 1985; Imbo, Duverne, & Lemaire, 2007), particularly when the first operand is large. Notably, no clear *tie effect* (problems with equal operands are easier; e.g., Van Der Ven et al., 2015) is visible in the α estimates. The *five effect* (problems with 5 as an operand are easier; e.g., Verguts & Fias, 2005) is also not consistently visible, seeming mainly to appear when 5 is the first operand. Within most of the multiplication tables, individual facts were seen to vary widely in difficulty. This highlights the potential for an adaptive system to identify and act on these differences. In the same vein, we saw large individual differences in learners' abilities (Figure 6b) that make a personalised approach fruitful.

Improving core mathematical skills like multiplication in young learners is particularly important, as PISA scores indicate a sustained decline in the mathematics performance of children in the Netherlands and other countries over recent years (PISA, 2023). An adaptive learning system can help in achieving this goal, by supporting effective study methods and by letting learners work at their own pace. In this light, it is encouraging that teachers and students rated their experiences with the application very favourably.

This study demonstrated an adaptive learning application for stepwise automatisation and memorisation of multiplication facts, built on a computational cognitive model of memory. The performance of learners using the system improved as they worked through the levels. Model-based analytics derived from the system offered actionable insights in the difficulty of individual problems and the ability of individual learners, supporting personalised learning of a crucial skill.

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