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AIR INFILTRATION MEASUREMENT TECHNIQUES

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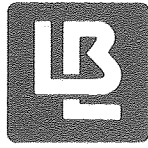
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## ENERGY & ENVIRONMENT DIVISION

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AIR INFILTRATION MEASUREMENT TECHNIQUES

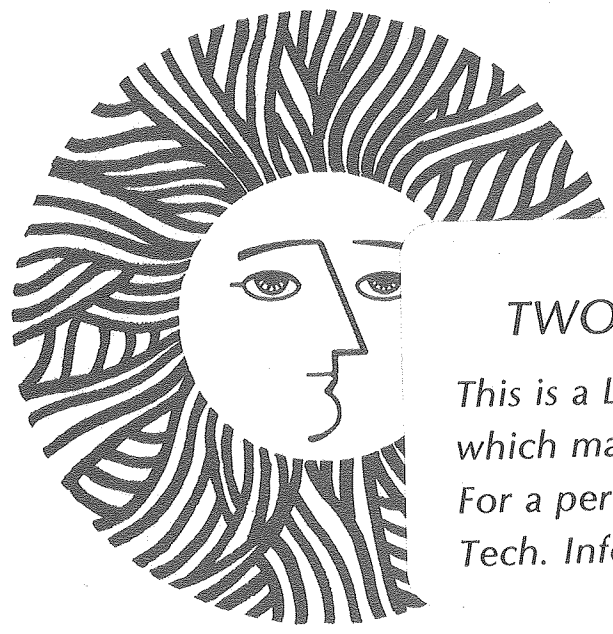
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Air Infiltration Measurement Techniques  
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This paper presents a survey of tracer-gas techniques for measuring air infiltration and includes a theoretical derivation of the equations, a description of each method, and a short description of the experimental procedure. A qualitative error analysis which concentrates on mixing problems is derived and used to compare the strengths and weaknesses of each method.

The theory of multi-chamber infiltration measurement is derived for use in situations involving many interconnected spaces (network type models). A set of measurement techniques analogous to the single chamber techniques is discussed along with qualitative error analysis.

The question of effective volume and mixing is addressed for both the single and multi-chamber cases. Also discussed is the general topic of non-tracer techniques for measuring infiltration.

Keywords: infiltration, instrumentation, measurement techniques, tracer gas, decay, continuous flow, long term average.

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This paper is to be presented at the first Symposium of the Air Infiltration Centre, entitled "Instrumentation and Measuring Techniques" Windsor, England, October 1980.



## ABSTRACT

This paper presents a survey of tracer-gas techniques for measuring air infiltration and includes a theoretical derivation of the equations, a description of each method, and a short description of the experimental procedure. A qualitative error analysis which concentrates on mixing problems is derived and used to compare the strengths and weaknesses of each method.

The theory of multi-chamber infiltration measurement is derived for use in situations involving many interconnected spaces (network type models). A set of measurement techniques analogous to the single chamber techniques is discussed along with qualitative error analysis.

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## INTRODUCTION

The measurement of air infiltration has become increasingly important in recent years. In typical United States houses air infiltration accounts for 1/3 of the total heating and cooling loads, As the average insulation value of houses is increased, this fraction will rise. Accordingly, efforts should be made to reduce the infiltration loads in line with the other load reductions. As houses are made tighter to reduce infiltration, air quality can deteriorate and minimum ventilation rates must be established. If we are to balance these competing demands of energy conservation and adequate air quality, it is essential that we be able to accurately measure infiltration.

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Current research in infiltration has its origin in the set of three papers published by Dick et al. in the late forties.<sup>1-3</sup> Many of the problems we struggle with today were identified in these papers, which remain a starting point for any serious study of research problems in infiltration.

## THEORY

One major change which has occurred since the Dick papers has been in instrumentation technology. Virtually all direct measurement schemes use the dilution of a tracer gas to estimate the infiltration; therefore, it is worthwhile to review the equations which govern tracer dilution techniques.

### Continuity Equation

If a tracer-gas is released into a space the rate of change of the amount of that gas will be governed by the amount of gas injected and the amount of gas lost.

$$\frac{dV_g}{dt} = F - \frac{dV_e}{dt} \quad (1)$$

### where

- t is the time [hr],
- $V_g$  is the volume of tracer in the space [ $m^3$ ],
- F is the injected flow of tracer from the source [ $m^3/hr$ ] and
- $V_e$  is the net volume of tracer lost to exfiltration [ $m^3$ ].

[Note that we have used the word "exfiltration" above. Since there is no net build-up of air within the space, the infiltration and exfiltration will must be equal. Therefore, in general, we will use the word infiltration to mean either air flowing out (exfiltration) or air flowing in (infiltration).]

We can define the average concentration of tracer-gas as being the ratio of the volume of tracer-gas in the space to the volume of the space itself.

$$C = \frac{V_g}{V} \quad (2)$$

where

C is the average concentration of tracer-gas and

V is the volume of the space [ $m^3$ ].

If the concentration of tracer-gas outside the test space is negligible, the rate of volume of tracer-gas lost will be the product of the infiltration and the concentration inside the test space.

$$\frac{dV_e}{dt} = Q C \quad (3)$$

where

Q is the infiltration [ $m^3/hr$ ].

Combining all of these equations together and rearranging terms gives the continuity equation in its standard form:

$$V \frac{dC}{dt} + Q C = F \quad (4)$$

A quantity often used to characterize infiltration is the ratio of the infiltration to the volume of the space, the infiltration rate.

$$A = \frac{Q}{V} \quad (5)$$

where

A is the infiltration rate [ $hr^{-1}$ ].



The infiltration rate is the number of volumes of room air displaced in a specified period of time (i.e. air changes per hour).

For a gas to be used as a tracer, it must fulfill certain requirements including ease of detectability, low ambient concentration, non-toxicity etc. A survey of commonly used tracer-gases and an experimental intercomparison, which includes a definition of a tracer gas, has been done by Grimsrud et al.<sup>4</sup>

### Mixing and Effective Volume

The continuity equation presented was derived using the average concentration and instantaneous flow as the measured quantities. While the injected flow of tracer-gas is usually quite easy to measure, the average concentration is not. Conventional practice has been to measure the concentration in a small number of locations and to assume that this concentration is representative of the average concentration in the test space; however, there are many instances when this assumption may not be adequate.

Most of the inadequacies that arise can be broadly categorized as mixing problems. Mixing problems can be broken down into three types: mixing of fresh air into the space; mixing of tracer-gas into the space; and circulation of air within the space. Each of these mixing problems can have a different effect on the measured concentration.

When fresh air enters the space it may not be dispersed evenly, and this can cause the concentration of tracer-gas to vary from point to point. In a residence, this behavior can cause the average concentration to vary from room to room; the leakier room will have lower concentrations than the tighter rooms, and interior rooms will have higher concentrations than exterior rooms. This effect is not limited to rooms; any area may be affected (e.g. the concentration near walls may well be lower than the interior). If these variations are significant, it may be necessary to use a multi-chamber analysis to properly interpret the measurements.

The second effect that may occur is that air will infiltrate into the space and then exfiltrate out again without mixing. This air infiltration will not affect the concentration and hence will not be reflected in the calculated infiltration. However, in most cases, this type of air exchange is not of interest. If the air enters and leaves without any mixing then it can not affect the heat load nor the indoor air quality.

Another mixing problem arises because the injected tracer-gas does not instantaneously mix within the space. In general, injection and sampling are spatially separated, and there is a delay between the time a volume of tracer is injected into the space and the time the additional volume of tracer is reflected in the concentration. Because of this delay, the infiltration to appear to rise until the gas is well mixed; conversely, if the tracer-gas injection is decreased, incomplete mixing will show up as an apparent increase in the infiltration rate.

The two quantities, delay time and mixing time, can be used to define an effective flow. The effective flow is the flow rate that would account for the observed concentration if the flow were instantaneously mixed; it is related to the actual injected flow by the mixing function.

$$F_{\text{eff}}(t) = \int_0^{\infty} g(t'; t_m, t_d) F(t - t') dt' \quad (6)$$

where

$F_{\text{eff}}$  is the effective flow [ $\text{m}^3/\text{hr}$ ],  
 $g$  is the mixing function,  
 $t_m$  is the mixing time [hr] and  
 $t_d$  is the delay time [hr]

The mixing function is normalized to unity.

$$\int_0^{\infty} g(t'; t_m, t_d) dt' = 1 \quad (7)$$

Figure 1 shows a typical response of the concentration to a step function in flow. If there were perfect mixing, the increase in concentration would follow the flow (i.e. it would be a step function); however, when the mixing is not perfect the rise in concentration is slower and lags behind the flow. The rise-time of the concentration is another characteristic mixing time, and may be called the circulation time. Figure 1 also shows the approximate (normalized) mixing function for the measured site.

The third type of mixing difficulty arises because the physical volume of the space may not be the volume participating in the air exchange. In the derivation of the continuity equation we replaced the volume of tracer-gas by the concentration times the volume of the space. Since this is incorrect if the full volume of the space is not participating, the volume that appears in the continuity equation is not the physical volume of the space but, rather, the effective volume of the space.

The effective volume of the space may be smaller than the physical volume if there are parts of the space that don't communicate well with the rest — closets or cupboards, for example. It is relatively easy to recognize and compensate for these volumes; however, there can be other areas not participating in the air exchange that are quite difficult to identify: stratification may isolate large volumes near the ceiling; corners and alcoves may not communicate well with the interior etc.

Under some circumstances the effective volume may be larger than the physical volume. If there are attached spaces that can communicate with the rest of the living space, the attached volume may contribute to the effective volume. For example, if the air-distribution system goes through an unconditioned attic, basement, or crawlspace there may be significant air leakage, causing the attached space to participate in the air exchange and increasing the effective volume.

Furthermore, the effective volume may be a function of the type of experiment being done. If a particular sub-volume communicates with the rest of the interior, but does so very slowly, then it will be included as part of the effective volume only when the concentration is slowly

varying.

Consequently, we must consider the volume that appears in the continuity equation to be an unknown parameter, much as the infiltration is an unknown parameter. We can, however, begin with the plausible assumption that the effective volume should be roughly the same size as the physical volume.

### MEASUREMENT TECHNIQUES

When care is taken to include these consideration regarding mixing and effective volume problems, the continuity equation can be used to compute the infiltration from measured concentration data. There are, however, specific advantages and disadvantages to every technique: some techniques work better for short-term measurements while others work better for longer term measurements; some techniques are less sensitive to mixing problems than others and some techniques use simpler and are less expensive than others. Below is a short description of each of the most popular techniques including a discussion regarding relative advantages and disadvantages.

#### Decay Technique

The tracer decay technique is by far the most widely used method for measuring air infiltration. It makes use of the solution of the continuity equation for no injected flow assuming that the infiltration rate remains constant.

$$C(t) = C_0 e^{-At} \quad (8)$$

where

$C_0$  is the concentration at  $t=0$  and  
 $A$  is the infiltration rate [ $\text{hr}^{-1}$ ].

Procedure: The tracer detector is connected to a single channel chart recorder (or data may be taken by hand).

A volume of tracer sufficient to bring the concentration of tracer to near the full scale of the analyzer is released. Additional mixing may be used at this point to assure even distribution of the tracer.

The system is allowed to stabilize and data is recorded until the concentration drops well below its starting value ( 1/2 hr to 2 hrs).

The data is analyzed by fitting the concentration data to a simple exponential. The time constant (coefficient of the time variable) yields the infiltration rate.

If longer term measurements are required, the procedure is repeated.

This system is simple, uses the minimum amount of equipment and is well suited for making short term measurements or spot checks at many sites. The analysis calculates the infiltration rate - the ratio of the infiltration (in  $m^3/hr$  ) to the effective volume, which is used, in turn, to calculate the infiltration by multiplying it by the physical volume of the space. Therefore, the calculated infiltration will be in error by the ratio of the effective volume to the physical volume, a ratio which can be as large as 50%.

Because a given decay rarely lasts longer than a few hours, to get long-term information it is necessary to repeat the entire procedure frequently; a significant time period must be spent waiting for the mixing to be complete after each injection, making the system inappropriate to use for long term measurements.

Constant Concentration / Constant Flow

The best way to eliminate the problem of finding the effective volume is by finding a way to eliminate it from the continuity equation used to calculate infiltration. Since the coefficient of the effective volume is the time rate of change of the concentration it would be possible to remove any uncertainties due to the effective volume by maintaining a constant concentration. Thus, if there is automated injection with a feedback system to keep the concentration at a desired level the infiltration is simply,

$$Q = \frac{F}{C} \quad (9)$$

where  $F$  and  $Q$  may be time varying but  $C$  is a constant.

In practice, however, this technique is impossible because of the mixing function. The mixing function causes a delay in the response of the concentration to a change in the flow. When sensing no appreciable concentration change, the system continues to change the flow even further, causing an overshoot in the "constant" concentration level. Thus, if the automated system attempts to keep the concentration constant the delay will cause an (unstable) oscillation in the concentration and a breakdown in the experimental design.

The only way to prevent the unstable feedback system is to make the update time of the system long compared to any mixing time. If the update time is long enough (i.e. the loop gain of the system is small enough) the instabilities can be avoided. If this is done, the concentration can only be considered constant on time scales that are long compared to the update time which in turn must be long compared to the mixing times, and therefore, the assumption of constant concentration will be violated for all but the most highly mixed conditions. A modification to the constant concentration technique that might avoid this instability is to make the flow constant.

The constant flow technique minimizes mixing problems because after the initial warm-up period (a time long compared to any mixing time) the mixing function has no effect and the effective flow is equal to the

actual flow (See the mixing function in Fig. 1). However, since the concentration is not constant the volume term will be present; specifically,

$$Q = \frac{F}{C} - \frac{V}{C} \frac{dC}{dt} \quad (10)$$

where

V is the effective volume [ $m^3$ ].

Care must be taken in interpreting the volume used in any of these infiltration equations. In most cases, the volume referred to by the symbol, "V" is the effective volume and not the physical one.

If the system is near equilibrium, the concentration will be slowly varying and the time rate of change of concentration will be quite small. In this case the last term in the above equation is a small correction term and the error introduced by replacing the effective volume with the physical volume should be negligible.

Procedure: The tracer detector and a mass flow controller (or other flow metering device) are connected to a two pen chart recorder and started.

Enough tracer-gas is released to bring the concentration up to mid-scale on the analyzer.

An estimate of the infiltration is made and used to set the flow so that at equilibrium the concentration should be mid-range on the gas analyzer.

The system is allowed to run continuously and data is considered to be valid once the time rate of change of the concentration is small.

The data is analyzed by using the equation above.

This system has the advantage of being relatively insensitive to the mixing function and minimizes the effect of the (unknown) effective volume. It will run for several days and give continuous infiltration measurements. It does have some disadvantages, however; if the infiltration deviates significantly from the estimate, the gas analyzer will go off scale and data will be lost. If the infiltration changes rapidly, there will be a rapid change in the concentration that will emphasize any effects due to effective volume. The system requires some warm-up time (approximately 1 hr) before the data can be considered valid.

#### Long Term Average Technique

One of the tasks that needs to be done in order to characterize any nation's housing stock is to make a survey of infiltration rates averaged over long periods of time (i.e. a month). In principle we could use one of the systems above for a month and average the results; however, it would be much more desirable to have a simpler, low-cost system available for such a purpose.

To measure long term average infiltration rates, we average the continuity equation divided by the infiltration.

$$\left[ \frac{V}{Q} \frac{dC}{dt} \right] + [C] = \left[ \frac{F}{Q} \right] \quad (11)$$

where

$[ \dots ]$  indicates a time average

If the flow is kept constant,

$$\left[ \frac{F}{Q} \right] \rightarrow F \left[ \frac{1}{Q} \right] \quad (12)$$

If the averaging interval is long enough (i.e. long compared to the inverse infiltration rate),



$$\left[ \frac{V}{Q} \frac{dC}{dt} \right] \rightarrow 0 \quad (13)$$

Combining terms gives an expression for the average of the reciprocal infiltration.

$$\left[ \frac{1}{Q} \right] = \frac{\left[ C \right]}{F} \quad (14)$$

If the infiltration were constant, then the inverse of the average infiltration would be equal to the average of the inverse infiltration; however, over the long term the infiltration will not be constant. We must, then, define a correction factor,  $k$ , that allows us to convert the average of the inverse infiltration to the average infiltration:

$$\left[ \frac{1}{Q} \right] = \frac{k}{\left[ Q \right]} \quad (15)$$

where

$$k = \sqrt{1 + 2\sigma^2} \quad (16)$$

and  $\sigma$  is the fractional standard deviation of either the infiltration or the inverse infiltration.

Since the inverse average weights the small infiltration values more than the large ones (and more than they would be weighted if a simple average were taken), the correction factor will always be greater than unity.

Procedure: A two channel sampling pump and accessories are brought to the site of interest.

The pump is set so that one channel is used to inflate an initially empty bag with room air; and the other channel is used to evacuate a bag of tracer-gas.

The pump speed and tracer-gas concentration in the

initially full bag are set so that the concentration of gas in the initially empty bag will be within the range of the analyzer.

A volume of tracer-gas sufficient to bring the analyzer up to half scale is released and the pump started.

The equipment is left unattended on site for the duration of the experiment and then picked up and brought back to the laboratory for analysis.

The total amount of gas dispensed is measured by comparing the volume of the initially full bag to its volume after the experiment; the concentration in the initially empty bag is measured and the results used to calculate the average inverse infiltration.

We are currently using this system in the field. Because its concentration can be measured in the range of 1 part in  $10^{12}$  to 1 part in  $10^9$ , the tracer we have chosen for this use is  $SF_6$ , giving us both high sensitivity and large dynamic range, which are useful for long term average measurements. The analyzer costs about \$7000 but the total equipment cost for each site in the field is about \$500, which makes it inexpensive to do several long term average infiltration measurements simultaneously.

The trade off for the low cost comes is that a correction term is required to convert the average inverse infiltration into the average infiltration. We have used a large set of data<sup>5</sup> to find values for parameters from the previous equation:

Site	$\left[ \frac{1}{Q} \right]$	$[ Q ]$	$\sigma$	k
HTSG	1.22	.85	.21	1.05
HSLG	2.79	.38	.24	1.06

From this set of data we may conclude that the corrected factor is near unity for typical infiltration values, indicating that a independent measurement of the standard deviation of the infiltration will not be necessary.

### Continuous Flow

It has been our desire to have a highly accurate continuous infiltration monitoring system that is capable of running unattended for significant periods of time (e.g. a week). In order to accomplish this, we have developed a microprocessor-based continuous-flow infiltration system.

Continuous flow is quite similar to the constant flow technique save that the effective volume is treated as an unknown parameter rather than being approximated by the physical volume. Furthermore, non-linear search routines are used to find the best fit of the unknown parameter over a cycle period to help eliminate random error.

During a cycle the flow is held constant; if we treat the infiltration and effective volume as constants during one cycle period (typically half an hour), the continuity equation can be solved for the concentration as a function of time.

$$C(t) = \frac{F}{Q} + \left[ C_o - \frac{F}{Q} \right] e^{-\frac{Q}{V}t} \quad (17)$$

This expression contains three unknown parameters  $V$ ,  $Q$ ,  $C_o$ .  $C_o$ , the best fit to the initial concentration, is a parameter that is of no physical interest, but must be found simultaneously with the other two parameters.

Procedure: The flow rate for a cycle is set equal to a target concentration times the calculated infiltration from the previous cycle. This choice of flow assures that the concentration will always be near the target (usually chosen to be about 2/3 of full scale). If this is the

very first cycle the previous infiltration is assumed to be one physical volume per hour. This assumption is not critical since it is used only to decide what the flow rate for the first cycle should be.

For the first few minutes after the flow has been changed the analyzer is rezeroed using outside air. Aside from checking the zero drift of the analyzer, this time allows for the delay due to the mixing function. Since, in general, the flow will not change very much from cycle to cycle, the wait time necessary to overcome the mixing delays is quite small.

The flow rate is held constant by the use of a mass flow controller for the remainder of the cycle period and concentration data is collected at regular intervals from the tracer-gas analyzer.

At the end of the cycle period the concentration data is used in a simplex\* search routine to find the three parameters. The simplex algorithm contains constraints that do not allow the effective volume to vary much from cycle to cycle. The slow updating of the volume has the effect of "homing in" on the effective volume over a period of several cycles. If the effective volume changes slowly over time, this method can accurately follow these changes.

The calculated values are stored on a floppy disk for later use.

The computer monitors both the analyzer and mass flow controller, as well as, setting the mass flow controller and activating the solenoid

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\* The simplex algorithm is a standard type of nonlinear search routine that finds the set of parameters that have the maximum likelihood of fitting a set of data.

valves which are used to control the flow of sample or zeroing gas to the analyzer. While the infiltration system is in use, the computer can still be used to perform other functions, such as further data reduction or display. Figure 2 presents a block diagram of the continuous flow system.

The operating system for the microprocessor is a single-user time-sharing system called TORX (Task Oriented Real-time Executive), developed at the Lawrence Berkeley Laboratory for use on Z-80 based microprocessor systems.

This system solves the problems of effective volume, mixing and continuous operation; however, it costs about \$20,000 to build and therefore is unsuitable for widespread or short-term measurements.

#### Non-tracer Techniques

So far, the discussion of measurement techniques has been limited to those involving tracer-gas measurements. Recently much effort has been put into using indirect methods for estimating infiltration. The direct tracer-gas methods all require gas analyzers and on-site equipment for data acquisition. The indirect, non-tracer, methods attempt to correlate infiltration with the weather and structural parameters of the building.

Weather data can be obtained from nearby weather stations (e.g. airports) and requires no on-site equipment. The structural parameters (i.e. envelope leakage, geometry, shielding, and terrain factors) do not change significantly with time and, hence, need be measured only once.

Many researchers <sup>6-8</sup> including the authors <sup>9-11</sup> have worked on developing models to solve the problem of infiltration prediction. We will present one such model very briefly.

Weather driven infiltration is broken up into two parts: stack driven and wind driven. Equations calculating the stack effect ignore

the wind effect and vice versa.

$$Q_{\text{stack}} = f_s^* A_o \sqrt{\Delta T} \quad (18.1)$$

$$Q_{\text{wind}} = f_w^* A_o v \quad (18.2)$$

where

$Q_{\text{stack}}$  is the stack driven infiltration [ $\text{m}^3/\text{hr}$ ],

$Q_{\text{wind}}$  is the wind driven infiltration [ $\text{m}^3/\text{hr}$ ],

$f_s^*$  is the reduced stack parameter [ $\text{m}/\text{s}/\text{K}^{1/2}$ ],

$f_w^*$  is the reduced wind parameter,

$A_o$  is the total leakage area [ $\text{m}^2$ ],

$\Delta T$  is the inside-outside temperature difference [K] and

$v$  is the measured wind speed [m/s].

The model assumes that the leakage of the structure can be characterized by a leakage area (which can be measured using fan pressurization). The reduced stack parameter depends on the leakage distribution, the height of the structure and the absolute internal temperature. To combine the wind and stack effect into one equation, we use the assumption that the infiltration is dominated by turbulent flow.

$$Q = \sqrt{Q_{\text{stack}}^2 + Q_{\text{wind}}^2} \quad (19)$$

where

$Q$  is the total weather driven infiltration [ $\text{m}^3/\text{hr}$ ].

These models are very useful because of their experimental simplicity and their great potential for widespread medium accuracy predictions (approx. 20%). However, the models will never be able to substitute for the highly accurate direct measurement techniques. For a complete description and derivation see Ref. 11.

MULTI-CHAMBER INFILTRATION

Whenever the communication between different parts of the interior space is poor, it may be necessary to treat the interior as a collection of separate well mixed spaces. Each space can communicate with every other space, as well as, the outside. To derive the multi-chamber continuity equation, we start with the same type of equation as for the single chamber case.

$$\left[ \frac{dV_g}{dt} \right]_k = F_k - \left[ \frac{dV_e}{dt} \right]_k \quad (20)$$

The subscript  $k$  denotes the  $k$ th chamber; hence, if there are  $N$  chambers there are  $N$  such equations.

The net amount of tracer that exfiltrates from each chamber is dependant not only on the total exfiltration but also on the infiltration from all the other chambers.

$$\left[ \frac{dV_e}{dt} \right]_k = \sum_{j \neq k} \left[ \frac{dV^e}{dt} \right]_{kj} - \left[ \frac{dV^i}{dt} \right]_{kj} \quad (21)$$

where

$$\left[ \frac{dV^e}{dt} \right]_{kj} \quad \text{is the exfiltration from room } k \text{ to room } j$$

$$\left[ \frac{dV^i}{dt} \right]_{kj} \quad \text{is the infiltration from room } j \text{ to room } k.$$

This definition of air flows includes the inherent redundancy that infiltration into room  $a$  from room  $b$  is also exfiltration out of room  $b$  to room  $a$ :

$$\left[ \frac{dV^i}{dt} \right]_{jk} = \left[ \frac{dV^e}{dt} \right]_{kj} \quad (22)$$

We now introduce the concentration, much as we did before, as the ratio

of the volume of tracer-gas to the volume of the test space:

$$C_k = \frac{\left[ \frac{V_g}{V_k} \right]_k}{V_k} \quad (23)$$

where

$V_k$  is the volume of the kth chamber.

If we define the air flow,  $Q_{kj}$ , as the negative of the air flow from room  $j$  to room  $k$ ,

$$Q_{kj} = - \left[ \frac{dV^i}{dt} \right]_{kj} = - \left[ \frac{dV^e}{dt} \right]_{jk} \quad \text{for } j \neq k \quad (24)$$

From these expressions we can rewrite the net exfiltration term,

$$\left[ \frac{dV^e}{dt} \right]_k = \sum_{j \neq k} \left[ Q_{jk} C_k - Q_{kj} C_j \right] \quad (25)$$

Since the total amount of air that flows into each room must be equal to the total amount of air that flows out, the total infiltration must equal the total exfiltration.

$$\sum_{j \neq k} Q_{kj} = \sum_{j \neq k} Q_{jk} \quad (26)$$

Up to now the diagonal elements of the infiltration matrix ( $Q_{kk}$ ) have been undefined. The above expressions suggest a convenient definition for them; the element  $Q_{kk}$  is the total air flow either in or out of the kth space:

$$Q_{kk} = \sum_{j \neq k} -Q_{ji} = \sum_{j \neq k} -Q_{kj} \quad (27)$$

where

$Q_{kk}$  is the total exfiltration out of room k.



This definition of the infiltration matrix has several interesting properties: all of the off-diagonal elements are negative and all of the diagonal elements are positive, making it a positive definite matrix; the sum of any row or any column is zero, which is an explicit statement of the fact that total amount of air flowing into each space is equal to the total amount flowing out.

The equation for the net exfiltration becomes,

$$\left[ \frac{dV_e}{dt} \right]_k = -V_k \sum_j Q_{kj} C_j \quad (28)$$

which leads to the expression governing multi-chamber infiltration.

$$V_k \frac{dC_k}{dt} + \sum_j Q_{kj} C_j = F_k \quad (29)$$

#### The "Zeroth" Chamber - Outside

Thus far, we have implicitly been treating the outside as one of the N chambers in our multi-chamber derivation; it is, however, a very special chamber and merits special consideration. Treating the outside as a chamber of infinite volume means that the concentration of tracer-gas will always be zero, trivializing its continuity equation:

$$V_0 = \infty \quad (30.1)$$

$$C_0 = 0 \quad (30.2)$$

$$V_0 \frac{dC_0}{dt} = F_0 \quad (30.3)$$

where

$C_0$  is the concentration of tracer-gas outside

$V_0$  is the volume of outside and

$F_0$  is the tracer flow injected outside.

These equations contain no useful information and therefore can be eliminated from the multi-chamber equations without loss if care is taken to insure that the definition of the diagonal elements of the infiltration matrix ( $Q_{kk}$ ) are properly defined. In the previous section we defined the diagonal elements of the infiltration matrix so that the sum of any row or column would be zero:

$$\sum_{k=0}^N Q_{kj} = 0 \quad \text{for all } j \quad (31.1)$$

$$\sum_{k=0}^N Q_{jk} = 0 \quad \text{for all } j \quad (31.2)$$

If we eliminate the zeroth chamber from the sums, the sums will no longer be zero but, rather, will yield the values of the infiltration and exfiltration from the outside to each chamber:

$$\sum_{k=1}^N Q_{kj} = -Q_{0j} \quad \text{for all } j \quad (32.1)$$

$$\sum_{k=1}^N Q_{jk} = -Q_{j0} \quad \text{for all } j \quad (32.2)$$

where

$-Q_{0j}$  is the infiltration from the outside to room  $j$  and

$-Q_{j0}$  is the exfiltration from room  $j$  to the outside.

We now stipulate the convention that all sums are to be taken from 1 to  $N$ , explicitly excluding the outside chamber. To recover the infiltration or exfiltration between a chamber and the outside, we can sum the appropriate row or column in the matrix.

Matrix Notation

The multi-chamber continuity equation can be put into matrix notation, but we must first define the volume matrix:

$$V_{kj} = V_k \delta_{kj} \quad (33)$$

where

$\delta_{kj}$  is the kronecker delta function.\*

Now the continuity equation becomes,

$$\bar{V} \frac{d\bar{C}}{dt} + \bar{Q} \bar{C} = \bar{F} \quad (34)$$

where

$\bar{V}$ ,  $\bar{Q}$  are two dimensional matrices and

$\bar{C}$ ,  $\bar{F}$  are vectors (one dimensional matrices).

This notation offers great simplicity in expression, and we will use it whenever possible.

Mixing and the Volume Matrix.

In multi-chamber infiltration, all of the mixing problems (delay times, mixing functions, and effective volumes) exist for each chamber and can be handled in the same manner as in the single chamber case. However, there is one type of mixing that is peculiar to the multi-chamber problem: short-circuiting.

Short-circuiting occurs when flow that is injected to one chamber does not mix with the air in that chamber but, rather, flows directly into another chamber and adds to the concentration in that chamber. For example, this could happen if a small air current took part of the

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\*The kronecker delta is equal to 1 if  $k=j$  and is equal to zero if  $k \neq j$ .

injected tracer and blew it into an adjacent room. Another way this could happen is if there were a leak in the tracer distribution system that allowed the gas to appear in one chamber when it was attributed to another.

Since the effect of short-circuiting is to increase the concentration in one chamber caused by flow in another, it will appear as an off-diagonal element in both the infiltration matrix and the volume matrix. The appearance of additional off-diagonal elements in the infiltration matrix is difficult to detect; since there are no off-diagonal elements in the volume matrix without short-circuiting, the appearance of any off-diagonal elements in the volume matrix can be attributed to short-circuiting. This fact can be used to find the amount of short-circuiting and, if the volume matrix is measured, correct for it. Conceptually, short-circuiting of a measured flow rate is equivalent to injecting a different flow without short-circuiting. This relationship allows the definition of an effective flow rate.

$$\bar{V} \frac{d\bar{C}}{dt} + \bar{Q} \bar{C} = \bar{F}_{\text{eff}} \quad (35)$$

where

$\bar{F}_{\text{eff}}$  is the effective tracer flow.

We relate the effective flow to the actual flow by the short-circuiting matrix.

$$\bar{F}_{\text{eff}} = \bar{S} \bar{F} \quad (36)$$

where

$\bar{S}$  is the short-circuiting (S) matrix.

Substituting the measured flow and short circuiting matrix in and multiplying through by the inverse of the S matrix,

$$\bar{V}_m \frac{d\bar{C}}{dt} + \bar{Q}_m \bar{C} = \bar{F} \quad (37)$$

where

$\bar{V}_m$  is the measured volume matrix and

$\bar{Q}_m$  is the measured infiltration matrix.

The measured quantities are related to the actual quantities and the S matrix.

$$\bar{V}_m = \bar{S}^{-1} \bar{V} \quad (38.1)$$

or,

$$\bar{S} \bar{V}_m = \bar{V} \quad (38.2)$$

and

$$\bar{Q}_m = \bar{S}^{-1} \bar{Q} \quad (39.1)$$

or,

$$\bar{S} \bar{Q}_m = \bar{Q} \quad (39.2)$$

Since the real volume matrix is diagonal, the off-diagonal elements of the product of the short-circuiting and measured volume matrices must be zero.

$$\sum_k S_{ik} \left[ \bar{V}_m \right]_{kj} = 0 \quad i \neq j \quad (40)$$

This set of equations alone is not sufficient to determine the S matrix. We must make use of the fact that the total amount of tracer injected is the same regardless of any short-circuiting (i.e. the S matrix is normalized).

$$\sum_j S_{kj} = 1 \quad \text{for all } k \quad (41)$$

Once Eqs 41 and 42 have been used to find the elements of the short-circuiting matrix, Eq. 40 can be used to find the actual infiltration from the measured infiltration.

Measurement techniques

The continuity equation for multi-chamber infiltration has exactly the same form as that for single chamber infiltration. This suggests that the same type of measurement techniques could be used to find the infiltration.

Several authors<sup>12,13</sup> have made multi-chamber calculations but very little field data has been taken. While each of the single chamber analysis schemes may be used for multi-chamber analysis, much more data must be taken to validate the measurement schemes. Because the number of unknown parameters scales as the square of the number of chambers,  $N^2$  times as many independent data points must be taken to get the same results. In some cases, sufficient data may be acquired by increasing the length of time needed for the measurements but it is conceptually simpler to use  $N$  independent tracer-gases and measure the concentration of each gas in every chamber, thus giving the factor of  $N^2$  more data without increasing the length of the experiment. The infiltration is a function of time; therefore, the increased length of the single gas experiment is likely to be prohibitive given that the quantity measured is changing during the course of the experiment.

Decay: In a multi-chamber decay all of the flows are set to zero.

$$\bar{C}(t) = e^{-\bar{A}t} \bar{C}_0 \quad (42)$$

where

$\bar{A} = \bar{V}^{-1} \bar{Q}$  is the infiltration rate matrix and

$\bar{C}_0$  is the initial set of concentrations

Care must also be taken in defining of the infiltration rate matrix; while all elements are well defined, only the diagonal elements have easily interpretable meanings: they are the rate of total infiltration

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We have used a matrix as the exponent; this has meaning only through the context of an infinite series and cannot be expressed in simple terms otherwise.

from all spaces (including outside).

Constant Concentration/Constant Flow: In a constant concentration experiment, the concentration of gas in each space is held constant.

$$\bar{Q} \bar{C} = \bar{F} \quad (43)$$

This experiment cannot be done with only one gas, unless several completely independent runs are made. Using separate runs makes the rather poor assumption that the air flows will be the same for each run. In either case, we construct a concentration matrix and an injected flow matrix, whose columns in both cases are either different runs or different gases, and whose rows (as before) are different chambers and an injected tracer flow matrix having the same properties.

$$\bar{Q} \bar{C} = \bar{F} \quad (44)$$

Solving for the infiltration directly yields,

$$\bar{Q} = \bar{F} \bar{C}^{-1} \quad (45)$$

As can be seen from this expression, the requirement of independent runs or different gases are necessary to keep the concentration matrix from becoming singular.

In the constant flow case, the (diagonal) physical volume matrix is used as a correction term for the above expression.

$$\bar{Q} = \bar{F} \bar{C}^{-1} - \bar{V} \frac{d\bar{C}}{dt} \bar{C}^{-1} \quad (46)$$

where

$\bar{V}$  is the physical volume matrix

A version of this technique is being used by British Gas.<sup>14</sup>

Long Term Average: Although long term average multi-chamber infiltration can be considered, it must be done with separate gasses because it is impossible to do several independent runs over such a long period of time. It is the inverse of the infiltration that is calculated, as

in the single chamber case.

$$\left[ \bar{Q}^{-1} \right] = \bar{F} \left[ \bar{C}^{-1} \right] \quad (47)$$

There is as in the single chamber case, a correction factor,  $k$ ; however, in the multi-chamber case,  $k$  will be a matrix and, without actual measurements, it is difficult to interpret the importance of this correction factor.

Continuous Flow: The continuous flow method uses the full solution to the multi-chamber continuity equation.

$$\bar{C}(t) = \bar{Q}^{-1} \bar{F} + e^{-\bar{V}^{-1} \bar{Q} t} \left[ \bar{C}_0 - \bar{Q}^{-1} \bar{F} \right]$$

The single chamber algorithm could be adapted for use in the multi-chamber problem. In each chamber one and only one tracer-gas would be injected and it would be controlled using that diagonal element of the infiltration matrix as was done for the single chamber model. The simplex algorithm could be adapted to include the  $3N^2$  unknown parameters in the above equation.

## CONCLUSION

We have presented a spectrum of different types of infiltration measurement techniques. Each technique has certain advantages and disadvantages, and each technique is better suited for one type of task than another. Decay measurements are well suited for spot measurements; constant-flow measurements are well suited for medium-length, semi-automatic experiments; long term average measurements are good for low-cost low-to-medium accuracy data averaged over a period of time; and continuous flow is good for long-term, high-accuracy continuous infiltration measurements. Each type of measurement can, in principle be used in a multi-chamber mode; however, very little field data has been accumulated on multi-chamber infiltration.



Each technique handles the problems of mixing/effective volume in a slightly different way. The decay technique ignores the problem of effective volume and solves the mixing time problem by waiting; constant flow eliminates the mixing time and simultaneously minimizes the effective volume problem in steady-state as does the long-term average infiltration technique; and the continuous flow technique eliminates the effective volume problem by calculating it, and minimizes the mixing problem by having quasi-constant flow.

Infiltration-pressurization correlations are, potentially, the superior method for predicting infiltration loads and long term indoor air quality concerns. Such correlations involve a one-time set of measurements, for quantifying the structure leakage and terrain parameters. From these measurements the infiltration can be calculated at any time from readily accessible weather data.

In situations where the interior of a structure is divided into several communicating sub-sections it may be necessary to invoke a multi-chamber infiltration algorithm to analyze the air flows. We have developed the equations governing multi-chamber infiltration, and have indicated ways in which the single channel measurement techniques can be modified in order to incorporate multi-chamber features.

REFERENCES

1. J.B. Dick, "Experimental Studies in Natural Ventilation of Houses," J. Inst. Heat. Vent. Eng. 17,420 (1949)
2. J.B. Dick & D.A. Thomas, "Ventilation Research in Occupied Houses," J. Inst. Heat. Vent. Eng. 19,306 (1941)
3. J.B. Dick, "The Fundamental of Natural Ventilation of Houses," J. Inst. Heat. Vent. Eng., p. 123, (1950)
4. D.T. Grimsrud, M.H. Sherman, J.E. Janssen, A.N. Pearman, D.T. Harrje, "An intercomparison of Tracer Gases Used for Air Infiltration," ASHRAE Trans. 86, (1980), Lawrence Berkeley Laboratory Report, LBL-8394 (1979)
5. G. Reeves, M. McBride, C. Sepsy, "Air Infiltration Model for Residences," ASHRAE Trans. 85 Part 1, (1979)
6. A. Blomsterberg D. Harrje, "Approaches to Evaluation of Air Infiltration Energy Losses in Buildings," ASHRAE Trans 85, Part I:PH-79-10, (1979).
7. A.K. Blomsterberg, M.H. Sherman, D.T. Grimsrud, "A Model Correlating Air Tightness and Air Infiltration," Proc. Conf. Thermal Perf. Ext. Env. of Buildings, (1979)
8. D.K. Alexander D.W. Etheridge, "The British Gas Multi-cell Model For Calculating Ventilation," ASHRAE Trans. 86, Part I, (1979).
9. M.H. Sherman, D.T. Grimsrud, R.C. Diamond, "Infiltration-Pressurization Correlation: Surface Pressures and Terrain Effects," ASHRAE Trans. 85 Part II:DE-79-1, (1979)
10. M.H. Sherman, D.T. Grimsrud, "Infiltration-Pressurization Correlation: Simplified Physical Modeling," ASHRAE Trans 86 Part II,

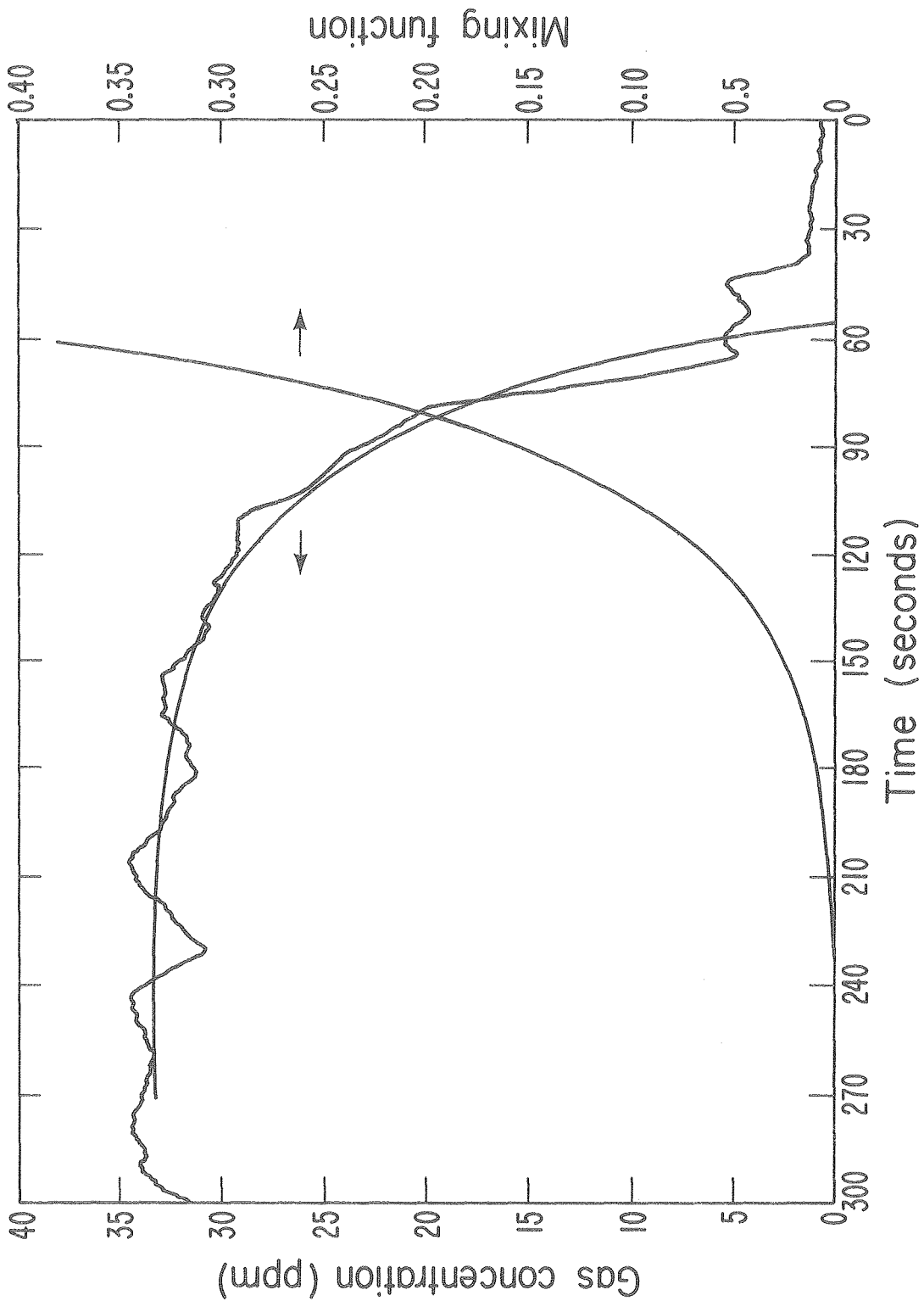
- (1980), Lawrence Berkeley Lab LBL-10163 (1980).
11. M.H. Sherman, D.T. Grimsrud, "Measurement of Infiltration Using Fan Pressurization and Weather Data," Proc. Air Infiltration Conference, Windsor, UK (1980), Lawrence Berkeley Lab LBL-10852 (1980)
  12. D.T. Grimsrud, M.H. Sherman, R.C. Diamond, P.E. Condon, A.H. Rosenfeld, "Infiltration-Pressurization Correlations: Detailed Measurement on a California House," ASHRAE Trans. 85 Part I (1979), Lawrence Berkeley Laboratory LBL-7824 (1978).
  13. F. Sinden, "Multi-chamber Theory of Air Infiltration," Build. Environ. 13, 21 (1978).
  14. D.K. Alexander, D.W. Etheridge R. Gale, "Theoretical and Experimental Studies of Heat Loss Due to Ventilation," Proc. XXI Int. Cong. Building Services Eng. (1980)

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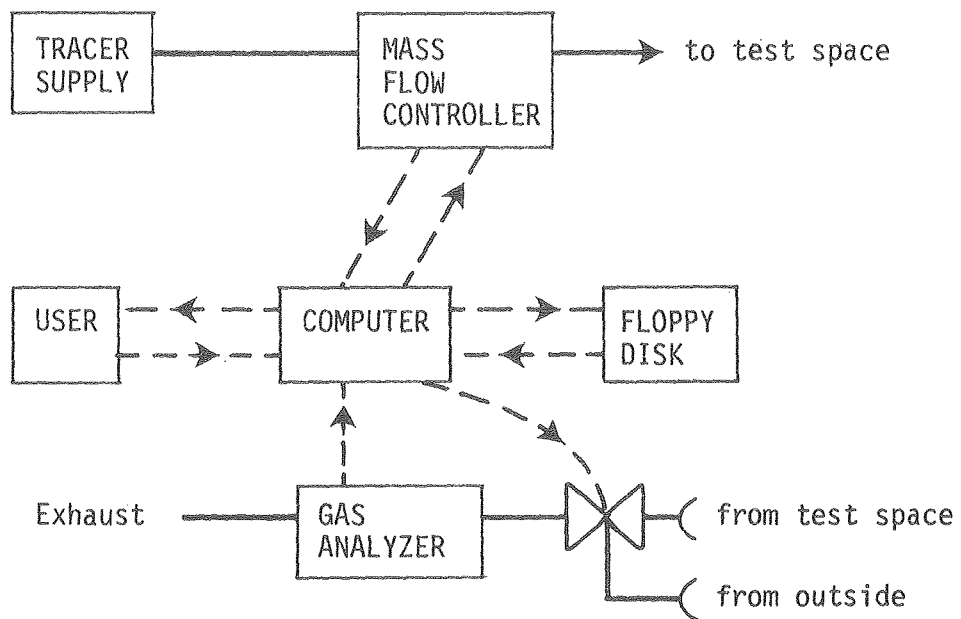
LIST OF FIGURES

- 1 The response of the concentration to a step function in flow is plotted vs. time. The flow was turned on at  $t=0$  and the concentration recorded as a function of time. Also shown is the approximate mixing function for this situation. Note there are two scales.
- 2 Block diagram of the continuous flow infiltration system using a Z80 microprocessor, floppy disk, mass flow controller and gas analyzer.



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