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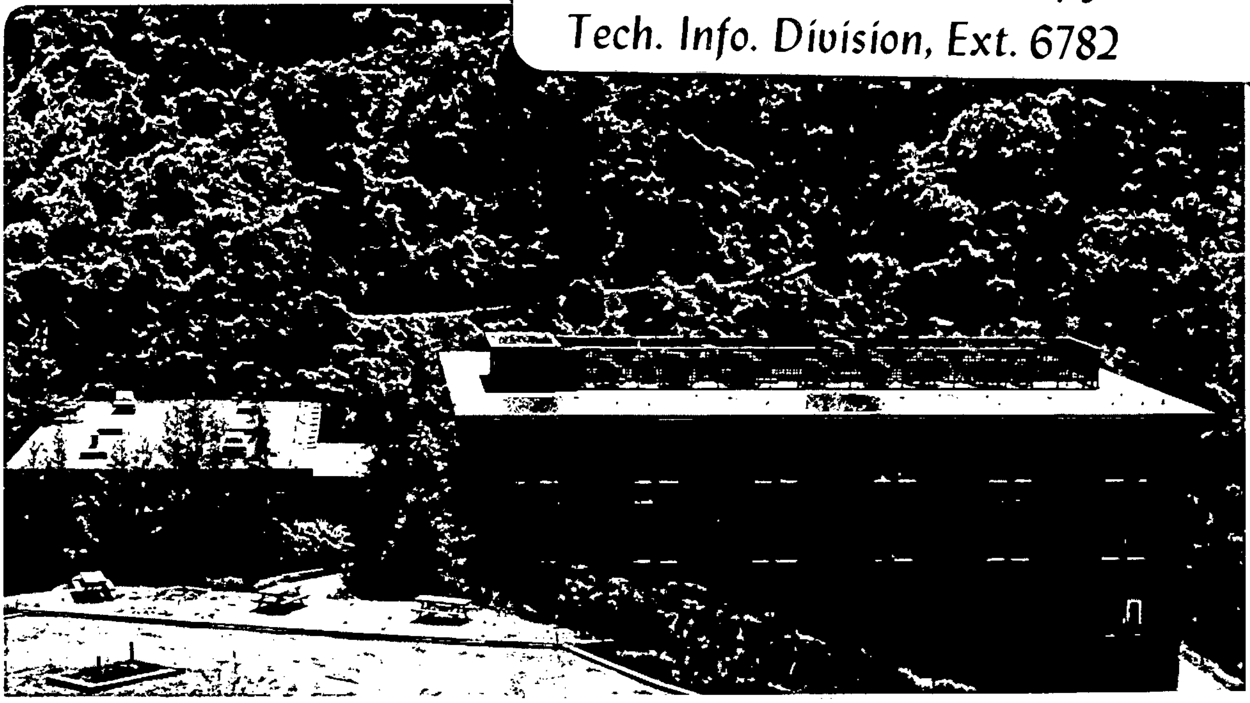

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APPROACH TO THE QUANTUM NOISE LIMIT IN THE DC SQUID

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ABSTRACT

Tunnel junction dc SQUIDs designed to approach the quantum noise limit in the temperature range 1 to 4K were fabricated with an inductance of about 2 pH and a capacitance per junction of about 0.5 pF. The lowest measured noise energy was $3.2\hbar$ at 1.4K at a frequency of 202 kHz. When the 1/f noise was subtracted, the white noise energy decreased from around $3\hbar$ at 4.2K to below $2\hbar$ at 1.4K.

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Recently, there have been substantial improvements in the sensitivity of dc Superconducting QUantum Interference Devices (SQUIDs).[1] Generally, the performances have been in reasonable agreement with the prediction for an optimized SQUID,[2] $\varepsilon/1\ \text{Hz} \equiv S_\Phi/2L \approx 10\ k_BT(LC)^{\frac{1}{2}}$, where $\varepsilon/1$ Hz is the flux noise energy. Here, S_Φ is the spectral density of the equivalent flux noise of a SQUID of inductance L, and C is the capacitance of each of the Josephson tunnel junctions. This result assumes that the flux noise arises from thermal noise in the resistors shunting the junctions. The lowest noise energy reported so far, approximately 6 $\hbar$, has been achieved by Voss _et al._[3] and by Cromar and Carelli.[4] Koch _et al._[5] have computed the noise energy when quantum corrections to the noise generated in the shunts near the Josephson frequency, ν_J, become important. This noise is mixed down to the measurement frequency ($\ll \nu_J$) by the non-linearity of the junctions. In the limit $T = 0$, the noise in the shunts reduces to zero point fluctuations, and the limiting noise energy is $\varepsilon/1\ \text{Hz} \approx \hbar$. In this Letter, we report measurements on dc SQUIDs designed to approach the quantum limit in the 1 to 4.2K temperature range. We have observed extrapolated white noise energies that, after subtraction of a 1/f component, are within a factor of two of the theoretical limit for $T = 0$.

The SQUIDs were designed to have very low values of L and C and to have the optimized values[5] $\beta \equiv 2LI_o/\Phi_o \approx 1$ and $\beta_c \equiv 2\pi I_oR^2C/\Phi_o \approx 1$, where I_o and R are the critical current and shunt resistance of each junction, and Φ_o is the flux quantum. The configuration is shown inset in Fig. 2, and is similar to that used by Cromar and Carelli.[4] Two Pb(20 wt. % In) - In_2O_3 - Pb tunnel junctions of nominal diameter 2 μm and separation 30 μm are defined by lifting off windows in an SiO insulating layer. Additional windows set the length of a 10 μm-wide CuAℓ resistance shunt for each junction. The capacitance of each junction is estimated to be 0.5 pF, while the SQUID loop, consisting of the insulating layer separating the upper and lower electrodes, has a self-inductance of about[6] 2 pH. For $\beta = \beta_c = 1$, we require $R \approx 1.1\ \Omega$ and $I_o = 0.5$ mA ($j_1 \approx 10^4$ A/cm^2). The SQUID may be flux-modulated by passing a current along the counter electrode.

Figure 1(a) shows the periodicity of the voltage, V, in the applied flux, Φ, for a typical SQUID at several values of bias current, I. The structure on the oscillations arises from resonances excited when the Josephson frequency is a subharmonic of the LC-resonance frequency. The maximum transfer function, $\partial V/\partial \Phi$, is obtained when the bias current is below the maximum critical current of the SQUID, so that the SQUID may be flux-modulated into the zero-voltage state. Figure 1(b) shows $\partial V/\partial \Phi$ and the dynamic resistance $R_D = \partial V/\partial I$ as a function of Φ. Both quantities show sharp peaks when the bias current is just above the critical current. This sharp response and the large values of $\partial V/\partial \Phi$ and R_D are obtained because the noise-rounding in the I-V and V-Φ characteristics is rather small for the high critical current junctions used. It is interesting to note that $|\partial V/\partial \Phi|$ scales almost exactly with R_D when $|\partial V/\partial \Phi|$ exceeds about 1.5 mV/Φ_o.

To measure the spectral density of the voltage noise, S_V, we coupled the SQUID to a low-noise preamplifier via one of two cooled LC-tank circuits. By connecting the two tank circuits together at the top of the cryostat we could obtain an intermediate resonant frequency. The noise at each measurement frequency was mixed down to frequencies below 1kHz, and the low-frequency spectral density was measured with a computer. The gain of the preamplifier-mixer-computer chain was calibrated against the thermal noise of a resistor at the preamplifier input. The Q of the tank circuit was determined by measuring the bandwidth at each bias point of the SQUID. The preamplifier noise temperature was about 2.8K. Since the effective output temperature of the SQUID, $S_V/4k_BR_D$, was typically 70K, the preamplifier noise was negligible, and we did not correct for it. The noise contributed by the 112 kHz and 202 kHz tank circuits was negligible, but the noise of the 149 kHz tank circuit, which contained leads at room temperature, was typically 10% of the SQUID noise, and was subtracted from the total measured noise. We measured $\partial V/\partial \Phi$ by applying a 1 kHz flux with an amplitude of 10^{-4} Φ_o or less, and measuring the voltage across the SQUID with a lock-in detector. We then computed $S_\Phi = S_V/(\partial V/\partial \Phi)^2$.

This technique for measuring S_Φ can lead to substantial errors if the I-V characteristic is slightly hysteretic, because one can obtain an artificially high value of $\partial V/\partial \Phi$. To guard against this problem, we applied a very small flux noise (comparable to the intrinsic flux noise) to the SQUID at the measurement frequency, and observed the increase in the voltage noise. In this way, the SQUID was operated as a small-signal amplifier so that the response to very small signals could be determined. For SQUIDs with large values of $\partial V/\partial \Phi$, we were able to measure S_Φ directly: The values so obtained were within ± 10% of the values obtained by the alternative method. However, for small values of $\partial V/\partial \Phi$, where it was necessary to apply a larger amplitude of flux noise to produce an observable effect, there was significant inductive coupling of the noise to the leads attached to the SQUID. In this case, the method could not be used to measure S_Φ, but could still be used to check for the absence of hysteresis.

We report here on only two of several SQUIDs studied, all of which exhibited the same qualitative features. The parameters at 4.2K are given in Table I. For both SQUIDs, the critical current increased by about 20% when the temperature was lowered to 1.4K. In addition, both the maximum value of $\partial V/\partial \Phi$ and R_D increased as the temperature was lowered, largely as a result of the reduction in the noise rounding of the I-V characteristic. The characteristic of SQUID B became hysteretic at about 3K, so that data could not be obtained at temperatures much below 3.4K.

The measured values of $\varepsilon/1$ Hz vs. T for SQUID A are shown in Fig. 2. The noise at each frequency decreases roughly linearly as T is lowered. We estimate the accuracy of each point to be ± 5%. The lowest measured value of $\varepsilon/1$ Hz obtained at 1.4K and 202 kHz was $3.2 \pm 0.2\ \hbar$, corresponding to an equivalent flux noise of $S_\Phi^{\frac{1}{2}} = (1.7 \pm 0.1) \times 10^{-8}\ \Phi_o \mathrm{Hz}^{-\frac{1}{2}}$. It is evident from Fig. 2 that there exists a substantial frequency-dependent contribution to the SQUID noise. From noise measurements at three frequencies, listed in Table I, we determined that the spectral density of the excess noise exhibited a power law variation close to 1/f. This conclusion is supported by low frequency measurements which show $\varepsilon/1$ Hz scaling accurately as 1/f over four decades of frequency between 0.1 and 1000 Hz. The measured magnitude of the 1/f contribution below 1 kHz agrees with the value extrapolated from the high frequency data to within 20%. If we assume that the measured noise contains white noise together with a contribution varying as 1/f, we can separate the two components. These values are listed in Table I, where the 1/f noise energy is given for 100 kHz. The large errors in the white noise energy are the result of the large magnitude of the 1/f noise and the need to extrapolate to frequencies much higher than the measurement frequencies.

In Fig. 3, we plot ε/1 Hz (including 1/f noise) vs. Φ for SQUID A. Although $\partial V/\partial\Phi$ is sharply peaked in Φ, with a peak width less than 10^{-3} Φ_o, we see that ε/1 Hz is roughly constant for a range of order 0.1 Φ_o. This result is expected since S_Φ varies as $R_D^2/(\partial V/\partial\Phi)^2$, and R_D scales closely with $\partial V/\partial\Phi$[Fig. 1(b)]. Thus, although we have not attempted to do so, it should be possible to operate these SQUIDs in a flux-locked loop without significant loss of sensitivity.

Data for SQUID B were obtained only at two frequencies, so that we had to assume that the spectral density of the excess noise was proportional to 1/f. For this SQUID, the magnitudes of the white noise and 1/f noise at 100 kHz were comparable. The lowest observed noise energy was $(3.7 \pm 0.2)\hbar$ at 118 kHz and 3.4K, a value that yields a white noise energy of $(1.7 \pm 0.4)\hbar$.

Given the substantial errors, the measured values of the white noise energy are in reasonable agreement with predictions.[5] For a device with $L = 1$ pH, $C = 0.5$ pF, $\beta_c = 1$, and $\beta = 1$ we expect ε/1 Hz to be about $3\hbar$ at 4.2K, decreasing to a little more than $\hbar$ at 1K. We believe that the best measured white noise energies are less than $2\hbar$, but, although zero point fluctuations may have contributed significantly, we cannot claim that they clearly dominated the measured noise. Because the peak in $\partial V/\partial\Phi$, which determines the operating voltage, occurs at voltages typically a factor of 10 times lower than the characteristic value I_oR (see Table I of Ref. 5), one must achieve values of $\kappa \equiv eI_oR/k_BT$ roughly 10 times higher than the values (on the order of unity) needed to observe zero point fluctuation effects in single junctions. Thus, if SQUID A, for which $\kappa \approx 5.5$ at 1.4K, could have been cooled to (say) 0.3K (and had not become hysteretic), we expect that it would have attained a quantum-limited noise energy. To achieve the quantum limit convincingly in the He^4 temperature range one will need even smaller values of LC.

The present series of SQUIDs is not appropriate for quasistatic or low frequency measurements in view of the high level of 1/f noise and very small input inductance. However, they may be useful as high frequency amplifiers, in a stripline configuration. Since the Josephson frequency at the operating voltage is typically 25 GHz, one might expect the high sensitivity to extend to frequencies of at least several GHz.

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4. M. W. Cromar and P. Carelli, Appl. Phys. Lett. 38, 723 (1981).
5. R. H. Koch, D. J. Van Harlingen, and J. Clarke, Appl. Phys. Lett. 38, 380 (1981).
6. The SQUID inductance, L, was estimated in two ways. In the first, we measured the mutual inductance with respect to a current along the counter electrode, and assumed this value to be L/2. In the second, we measured the maximum and minimum critical currents, and estimated β from Fig. 4 of Ref. 2. The two methods agreed to within 4% and 7% for SQUIDs A and B. We have used the first method to obtain the values quoted in Table I.

Table I. Parameters and Performances of Two SQUIDs.

SQUID A: $L = 1.89$ pH, $C = 0.5$ pF, $I_o = 0.55$ mA, $R = 1.00\Omega$, $\beta = 1.00$, $\beta_c = 0.83$

T (K)	$\partial V/\partial \Phi$ (mV/Φ_o)	R_D (Ω)	ε/1Hz ($\hbar$) 202kHz	149kHz	112kHz	white	1/f (100kHz)
4.2	6.8	4.7	14.4	18.2	23.2	3.6 ± 3.0	21.6 ± 4.7
2.8	9.4	6.4	8.6	10.6	13.2	2.8 ± 1.8	11.6 ± 28
1.8	12.5	8.9	4.5	--	6.9	1.5 ± 1.0[a]	6.0 ± 1.5
1.4	13.9	11.1	3.2	4.0	5.1	0.8 ± 0.7	4.8 ± 1.0

SQUID B: $L = 2.50$ pH, $C = 0.5$ pF, $I_o = 0.38$ mA, $R = 1.25\Omega$, $\beta = 0.92$, $\beta_c = 0.90$

T (K)	$\partial V/\partial \Phi$ (mV/Φ_o)	R_D (Ω)	ε/1Hz ($\hbar$) 118kHz	34kHz	white[a]	1/f[a] (100kHz)
4.2	6.5	3.2	5.2	11.5	2.7 ± 0.6	3.0 ± 0.4
3.8	11.3	4.5	4.5	10.6	2.1 ± 0.5	2.7 ± 0.4
3.4	22.0	7.7	3.7	8.7	1.7 ± 0.4	2.3 ± 0.3

[a] Measurements made at two frequencies only: Excess noise is assume to scale as 1/f.

Figure Captions

Fig. 1(a) Voltage-flux characteristics for a typical SQUID at 4.2K for 8 values of bias current; (b) transfer function, $\partial V/\partial\Phi$ (——), and dynamic resistance, R_D(---), vs. Φ/Φ_o for the SQUID in (a) at 4.2K with I = 2.70 mA.

Fig. 2 Measured ε/1 Hz vs. T at three frequencies for SQUID A, with ± 5% error bars. Inset shows SQUID configuration.

Fig. 3 Measured ε/1 Hz vs. Φ/Φ_o for SQUID A at 202 kHz and 1.4K.

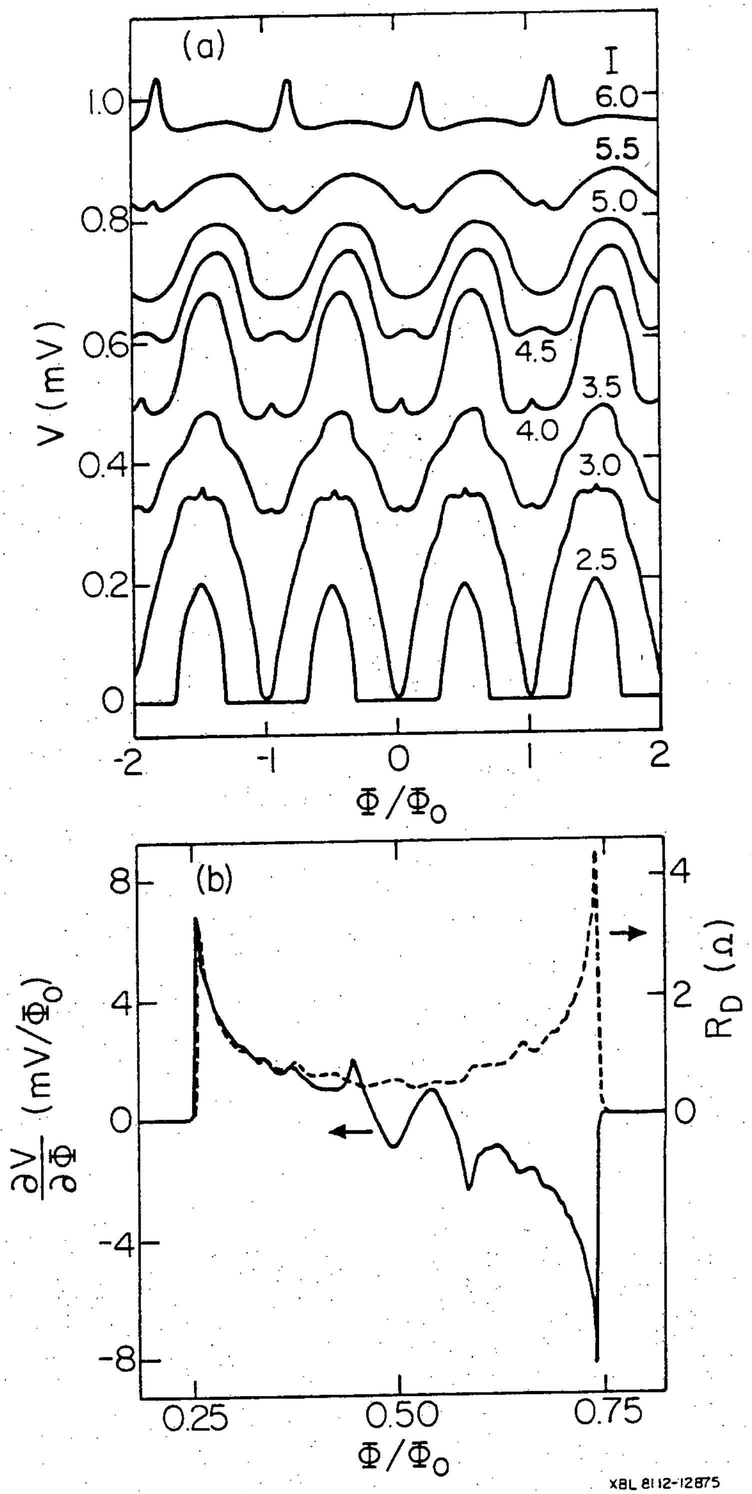

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Fig. 1

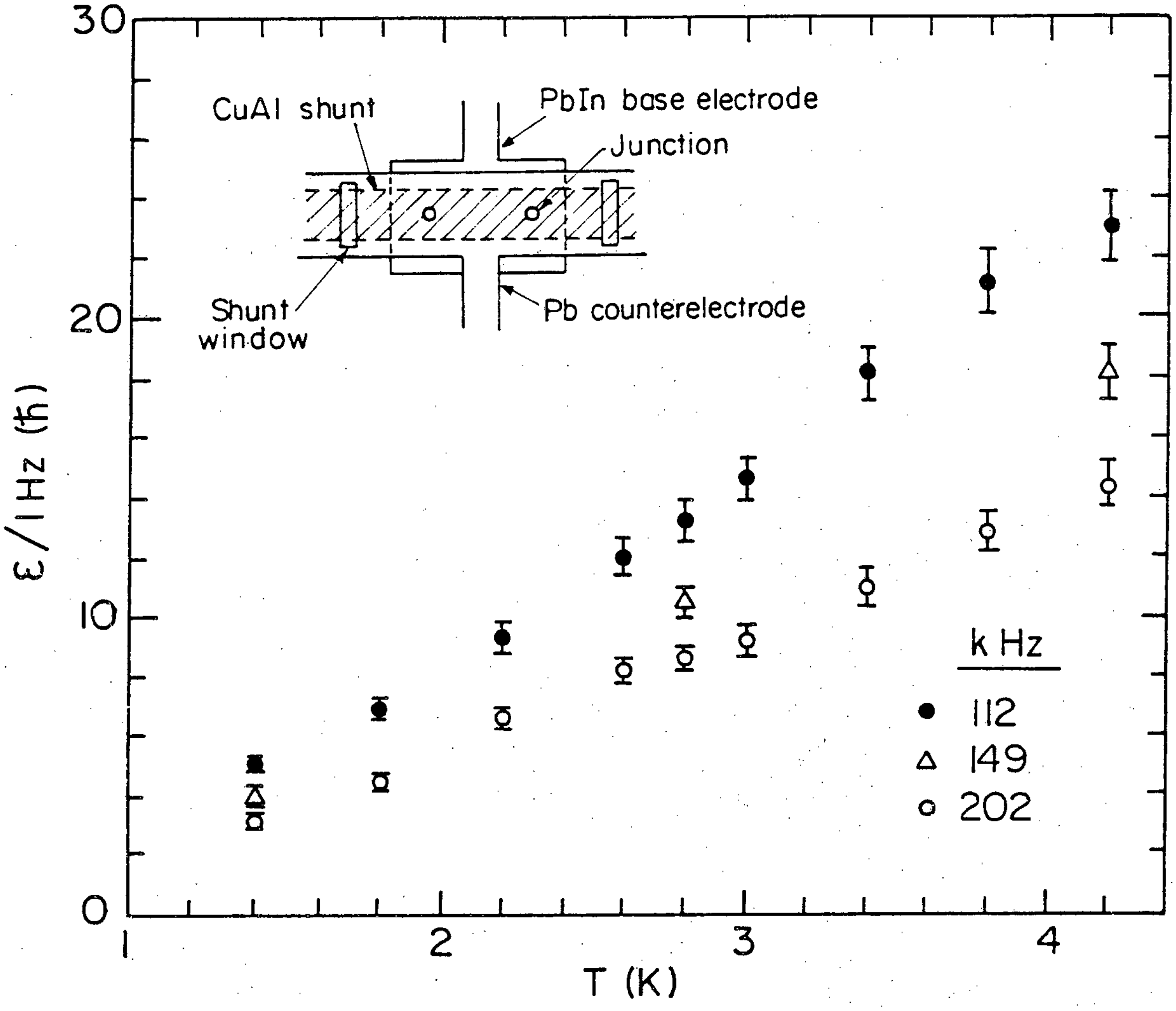

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Fig. 2

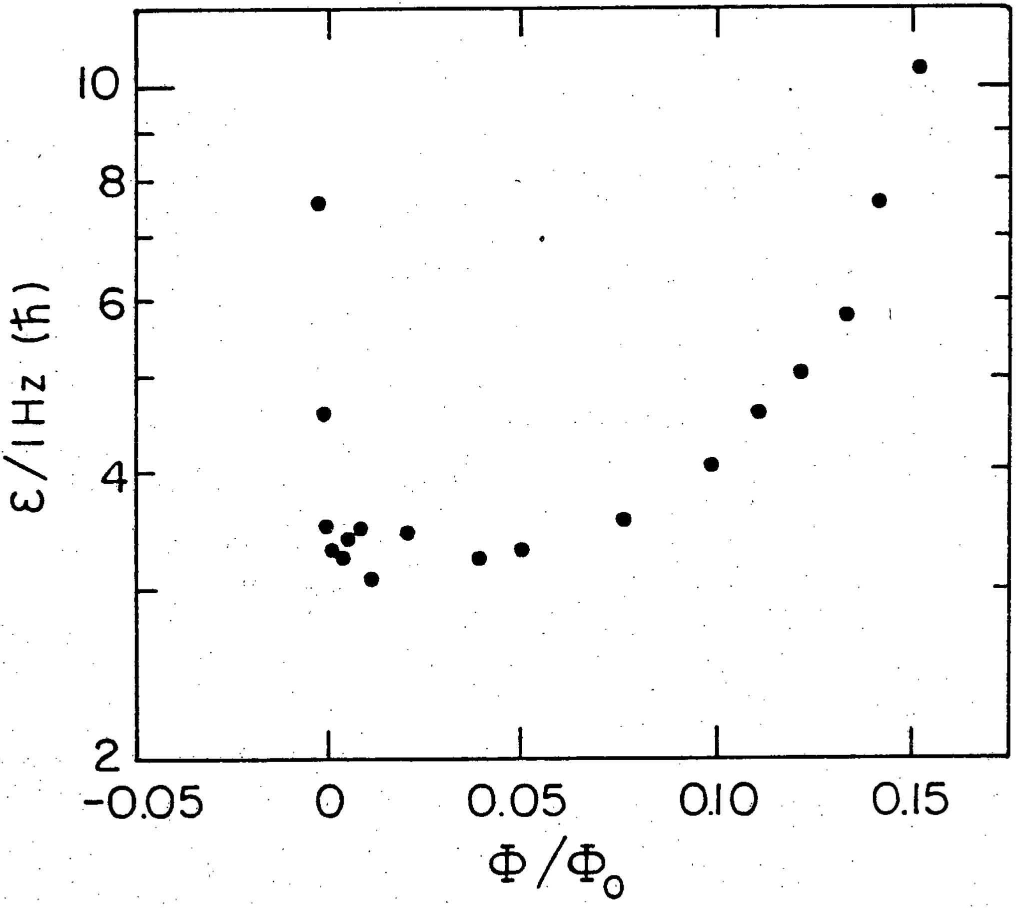

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Fig. 3

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