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CONTENTS

M.O. Rieger, Evolutionary stability of prospect theory preferences	1
I. Chakraborty and N.D. Sthiyamkumar, Revenue and efficiency ranking in large multi-unit and bundle auctions	12
I. Brocas, Countervailing incentives in allocation mechanisms with type-dependent externalities	22
A. Goenka, L. Liu and M.-H. Nguyen, Infectious diseases and economic growth	34
M.M.M. Buehimaier, Debt, equity, and information	54
Y. Hosoya, Identification and testable implications of the Ramsey–Cass–Koopmans model	63
J. Alcalde-Unzu and M. Vorsatz, Non-anonymous ballot aggregation: An axiomatic generalization of Approval Voting	69
V. Gretschko, A. Rasch and A. Wambach, On the strictly descending multi-unit auction	79
M. Magill and M. Quinzii, Anchoring expectations of inflation	86
D. Laussel and J. Resende, Dynamic price competition in aftermarkets with network effects	106
M. Hoelle, The relation between sunspot effects and multiplicity in incomplete markets models with numeraire assets	119
Q. Fu, J. Lu and Z. Wang, "Reverse" nested lottery contests	128

(Contents continued on Outside Back Cover)

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ABSTRACT

We propose an intermediation core for an economy that explicitly specifies how traders organize themselves into trade cooperatives (intermediaries) and how trade between them gets carried out. The intermediation core allocations are closely related to the equilibrium allocations of a non-cooperative intermediation game in Townsend (1983). We show that the intermediation core contains all subgame perfect equilibrium allocations of the intermediation game, similar to the inclusion of competitive equilibrium allocations in the core usually studied. We identify intermediation core allocations that are also subgame perfect equilibrium allocations of the intermediation game in terms of the supporting intermediary structures. These results help to characterize subgame perfect equilibrium allocations of the intermediation game and to analyze their welfare and stability properties.

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1. Introduction

The core of an exchange economy is based on coalitional rather than individualistic improvements that depend on what each coalition can achieve with its own members. The usual core is based on the assumption that any reallocation of coalition's total endowment among its members is feasible for the coalition. However, it is unclear how members organize themselves into the coalition and how they carry out the trade.

This paper has two purposes. First, we use the idea of intermediation to explicitly specify how economic traders organize themselves into trade cooperatives and how trade between them is carried out. This calls for the reformulation of what would be feasible for a coalition of traders to achieve. In this paper, an allocation is feasible for a coalition of traders if one of them acts as an *intermediary*, offering to buy and sell at a price vector, while the others act as *price-taking customers*. At each feasible allocation of a coalition, all members, possibly except for the intermediating trader, maximize their utility subject to budget constraints. For an allocation to be feasible for the economy,

however, we allow for the possibility that trade is carried out by multiple disjoint intermediaries. The core resulting from this formulation of coalitional feasible allocations will be referred to as the intermediation core.¹

Second, we relate intermediation core allocations with subgame-perfect equilibrium allocations under the approach taken in Townsend (1983). This approach provides an opportunity for each trader to play the role of an intermediary. One formalization of the approach works as follows. In the first stage, each trader individually and simultaneously offers to buy or sell commodities at a certain price vector and for a certain group of customers, subject to feasibility constraints. A trader may be offered a membership to multiple intermediaries. However, each trader must subsequently choose to trade with at most one intermediary in the second stage. Furthermore, a trader is obligated to intermediate under the announced terms should some of his potential customers choose to trade with him. Otherwise, he is free to act as the customer of an intermediary that includes him as a customer.² Because a trader's second-stage feasible choices

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¹ Feasible coalitional allocations in this paper are different from those in both Mas-Colell (1975) and Qin et al. (2006). In the former, the feasible allocations of a coalition are required to be in competitive equilibrium of the sub-economy composed of members of the coalition, whereas in the latter, no one is required to maximize utility subject to budget constraints.

² This is one of the several variants of the model in Townsend (1983). See Townsend (1978), Boyd and Prescott (1986) and Boyd et al. (1988) for applications of the intermediation games.

depend on the choices of other traders, the *social equilibrium* in Debreu (1952)³ is applied to the subgames in the second stage in the determination of a subgame perfect equilibrium (SPE).

An SPE of the intermediation game has the following properties. First, each non-intermediating trader maximizes utility by choosing an intermediary to trade with as well as the trade amount. Second, traders divide themselves into disjoint trading cooperatives, such that there is an active intermediating trader within each cooperative who specifies the terms of trade. Third, trade is stable in the sense that there is no entry of new intermediaries or exit of existing ones.⁴

We show that SPE allocations of the intermediation game are contained in the intermediation core under general conditions, similar to the inclusion of competitive equilibrium allocations in the usual core. We identify intermediation core allocations that are also SPE allocations in terms of the supporting intermediary structures. It is shown that an intermediation core allocation can be decentralized as an SPE allocation, whenever all intermediaries in the supporting intermediary structure have at least two customers. This stability of the intermediation core allocations resembles the *contestability* concept found in the industrial organization literature (Baumol et al., 1982). In particular, the two-customer requirement ensures that, for any active intermediary, there are always at least two contestable intermediaries that are ready to serve the other customers under the same terms.

Our paper also contributes to the literature on implementation of social choice correspondences using extensive form mechanisms and subgame-perfect equilibrium as a solution concept.⁵ A group of papers in this literature considers subgame-perfect implementation of cooperative game-theoretic solutions. For example, Serrano and Vohra (1997) and several others consider the core usually studied as the social correspondence and its subgame-perfect implementation. In a similar spirit to theirs, viewing the intermediation game (or the rules of the intermediation game) as an extensive form mechanism, our results establish a subgame-perfect implementation of intermediation core allocations that are supportable by intermediaries with two or more customers.

The remainder of the paper is organized as follows. Section 2 introduces the intermediation core, intermediation game, and subgame perfect equilibrium of the game. Section 3 establishes the main results and Section 4 concludes. Appendix A contains proofs of theorems, and Appendix B presents an example of an unequal treatment of the intermediation core.

2. Intermediation in an exchange economy

Let $N = \{1, 2, \dots, n\}$ be the set of traders and $l (< \infty)$ be the number of commodities. Trader $i \in N$ has consumption set $X^i \subset \mathfrak{R}_+^l$ and initial endowment $\omega^i \in X^i \cap \mathfrak{R}_{++}^l$. His preferences can be represented by an increasing utility function $U^i : X^i \rightarrow \mathfrak{R}$. An exchange economy is described by the list $\mathcal{E} = (X^i, U^i, \omega^i)_{i \in N}$.

2.1. Intermediation core

The core concept is based on what players can achieve by organizing themselves into coalitions. For the usual core of an

exchange economy, any allocation satisfying

$$\sum_{i \in C} x^i = \sum_{i \in C} \omega^i, \quad x^i \in X^i, \quad i \in C \quad (1)$$

is regarded as feasible for traders in coalition C . This feasibility condition does not explicitly specify how traders organize themselves into coalition C and how trade between them is carried out. In this paper, we make the organization of traders into a coalition and trade between them explicit by requiring one of these traders to intermediate for the rest of them.

Definition 1. Given $C \subseteq N$, a C -allocation $(x^i)_{i \in C}$ is *feasible* for coalition C if it satisfies (1) and there exists a price vector p such that x^i solves

$$\max_x U^i(y^i) \text{ subject to } p \cdot x = p \cdot \omega^i, \quad x \in X^i \quad (2)$$

for all $i \in C$ except for at most one member j in C , in which case j receives bundle $x^j = \sum_{i \in C} \omega^i - \sum_{i \in C-j} x^i$ with $C-j = C \setminus \{j\}$.

The set of all C -feasible allocations is denoted by $F(C)$. The trader whose bundle does not maximize utility subject to budget constraints at an allocation in $F(C)$ is the intermediating trader. The remaining members are the customers of the intermediary. If each member's bundle maximizes utility subject to budget constraints, any one of them can be the intermediating trader.

Definition 2. An allocation $x^* = (x^{*i})_{i \in N}$ is in the intermediation core if there is a partition $\{C^{*k}\}_{k=1}^m$ of N such that $(x^{*i})_{i \in C^{*k}} \in F(C^{*k})$, for $k = 1, 2, \dots, m$, and there is no coalition $C \subseteq N$ and $(x^i)_{i \in C} \in F(C)$ such that $U^i(x^i) > U^i(x^{*i})$ for all $i \in C$.

Given an intermediation core allocation x^* , we call the collection $(p^{*k}, C^{*k})_{k=1}^m$ a supporting intermediary structure for the allocation x^* if, for each coalition k , price vector p^{*k} supports allocation $(x^{*i})_{i \in C^{*k}} \in F(C^{*k})$. Note that to be in the intermediation core, we allow for an allocation of the economy to be achievable through multiple disjoint intermediaries in stead of just one grand intermediary. The intermediation core remains the same if for any coalition C , we modify $F(C)$ by allowing trade between members in coalition C to be achievable through multiple disjoint intermediaries. The reason for this is that if a partition of C can improve upon a given allocation, then any sub-coalition in the partition can also improve upon the allocation.

The following example illustrates that the intermediation core of an economy is not included in its usual core.

Example 1. Consider an exchange economy with two commodities and three traders. The traders' endowments are $\omega^1 = \omega^2 = (6, 0)$ and $\omega^3 = (0, 12)$. Their utility functions are $u^i(x^i) = x_1^i x_2^i$ for $x^i \in X^i = \mathfrak{R}_+^2$, $i = 1, 2, 3$. The allocation $\bar{x} = (\bar{x}^1, \bar{x}^2, \bar{x}^3)$ with $\bar{x}^1 = (3, 1) = \bar{x}^2$, and $\bar{x}^3 = (6, 10)$ is not in the usual core of this economy because it is not Pareto optimal.

However, \bar{x} is in the intermediation core. Consider the following supporting intermediary structure. Let trader 3 be an intermediary with a price ratio $\rho = p_1/p_2$ and with traders 1 and 2 as customers. The demand of each customer is $(3, 3\rho)$. As a result, the allocation of the intermediary is $(12, 12) - 2 \times (3, 3\rho) = (6, 12 - 6\rho)$. It is clear that allocation \bar{x} can be supported by price ratio $\rho = 1/3$. We will show that this allocation cannot be improved upon. It is clear that no individual trader alone can improve allocation \bar{x} .

We now consider grand coalitions. If trader 3 is an intermediary, then it is impossible to make traders 1 and 2 better off without making trader 3 worse off. If trader 1 is an intermediary with a price

³ Yannelis (2009) generalizes this social equilibrium concept by allowing for asymmetric information and a continuum of agents.

⁴ An existing intermediary with revised terms is regarded as a new intermediary.

⁵ The interested reader is referred to Moore and Repullo (1988) and Abreu and Sen (1990), which were among the earlier papers in the literature.

ratio ρ , traders 2 and 3 demand $(3, 3\rho)$ and $(6/\rho, 6)$, respectively. This leaves trader 1 with bundle $(9 - 6/\rho, 6 - 3\rho)$. A simple calculation shows that to make traders 2 and 3 better off, ρ must satisfy $1/3 < \rho < 3/5$. However, as the intermediating agent, trader 1 is worse off over this range of the price ratio. Due to symmetry, a grand coalition with trader 2 as the intermediary cannot improve upon allocation \bar{x} .

Now consider a two-member coalition consisting of traders 1 and 3. If trader 3 intermediates, then trader 1 demands $(3, 3\rho)$ at price ratio ρ leaving trader 3 with bundle $(3, 12 - 3\rho)$. It follows that trader 3 is necessarily worse off should trader 1 become better off. If trader 1 intermediates, however, trader 3 demands $(6/\rho, 6)$ at price ratio ρ , leaving trader 1 with bundle $(6 - (6\rho), 6)$. For trader 3 to be better off, the price ratio must be $\rho < 3/5$, which would result in a negative quantity of good 1 for trader 1. This is clearly not feasible. Due to symmetry, the same conclusion can be drawn from a coalition including traders 2 and 3. A coalition including traders 1 and 2 cannot improve upon \bar{x} because they are endowed with good 1 only.

In conclusion, allocation \bar{x} cannot be improved upon by any coalition. \square

Coalition improvements for the intermediation core are more restrictive than those for the usual core. As a result, because a competitive equilibrium allocation is in the core, it must be in the intermediation core as well. We summarize this result in the following proposition.

Proposition 1. *Competitive equilibrium allocations are intermediation core allocations.*

An immediate implication of this proposition is that the intermediation core of an economy is non-empty under general conditions. For example, if the consumption set is non-empty, closed, convex, bounded below, and unbounded above, and the utility function is increasing and concave, then a competitive equilibrium exists. Because the proof of the existence of a competitive equilibrium is standard in the literature, it is omitted here (see, for example, Starr, 2011).

Proposition 2. *The intermediation core is non-empty.*

2.2. Intermediation game

Following Townsend (1983), we consider a non-cooperative intermediation game with endogenous intermediaries. A trader can try to gain market power by offering to intermediate for a group of traders. However, the degree of market power is weakened by competition between intermediaries. Specifically, the game has the two following stages.

Stage 1

Each trader i announces a subset $C^i \subseteq N$ with $i \in C^i$ and a price vector $p^i \in \mathfrak{N}_+^\ell$. The pair $s^i = (p^i, C^i)$ represents trader i 's offer to buy or sell at price vector p^i for customers in $C_{-i}^i = C^i \setminus \{i\}$. We use $s^i = \emptyset$ to denote the announcement such that $C_{-i}^i = \emptyset$ in which case, trader i forgoes the opportunity to intermediate.

Stage 2

Given Stage-1 announcements $s = (s^1, \dots, s^n)$, trader i 's feasible choices are as follows.

(i) $s^i = \emptyset$

In this case, trader i can either choose to trade with an intermediary offered by a trader in $\{j \in N : i \in C^j\}$ or stay

autarkic. This choice is denoted by $d^i(s)$. Here, $d^i(s) = j$ means that i chooses to trade with j , whereas $d^i(s) = 0$ means that he chooses to stay autarkic. When $d^i(s) = j$, trader i also chooses a net-trade vector $z^i(s)$ subject to

$$p^j \cdot z^i(s) \leq 0 \quad \text{and} \quad z^i(s) + \omega^i \in X^i. \quad (3)$$

(ii) $s^i \neq \emptyset$

In this case, if $d^l(s) \neq i$ for all $l \in C_{-i}^i$, trader i can act as if $s^i = \emptyset$ for the reason that none of his customers trades with him. If $d^l(s) = i$ for some $l \in C_{-i}^i$, however,

$$d^i(s) = i \quad \text{and} \quad z^i(s) = - \sum_{k \in C_{-i}^i; d^k(s)=i} z^k(s). \quad (4)$$

Given $(s, d_{-i}(s), z_{-i}(s))$, we say that it is feasible for i to intermediate if $s^i \neq \emptyset$, $d^i(s) = i$ for some $j \in C_{-i}^i$, and $\omega^i + z^i(s) \in X^i$ where $z^i(s) = - \sum_{k \in C_{-i}^i; d^k(s)=i} z^k(s)$. When it is not feasible for i to intermediate, both he and his customers stay autarkic. Thus, trader i 's consumption bundle is given by

$$x^i(s, d(s), z(s)) = \begin{cases} \omega^i + z^i(s) & \text{if } d^i(s) \neq 0, \\ \omega^{d^i(s)} + z^{d^i(s)}(s) & \text{if } d^i(s) = 0, \\ \omega^i & \text{otherwise.} \end{cases} \quad (5)$$

The last part in the first line of the right-hand-side of (5) ensures that it is feasible for the chosen intermediary $d^i(s)$ to intermediate.

As described above, the feasible choices of the traders in the second stage depend on the choices of the other traders. Specifically, given s and $(d_{-i}(s), z_{-i}(s))$, trader i 's Stage-2 choice $(d^i(s), z^i(s))$ is feasible if (i) $z^i(s)$ satisfies (3) when $d^i(s) \neq 0$, i and it is feasible for $d^i(s)$ to intermediate; (ii) $z^i(s)$ satisfies (4) when $d^i(s) = i$ and it is feasible for i to intermediate; and (iii) $z^i(s) = 0$ otherwise. We say $(d^i(s), z^i(s))$ is maximal if there is no other feasible Stage-2 choice for trader i that results in a more favorable consumption bundle, given s and $(d_{-i}(s), z_{-i}(s))$. We say that trader i 's Stage-1 announcement s^i is maximal if there is no other Stage-1 announcement that yields greater utility to trader i , given s_{-i} and $(d(\cdot), z(\cdot))$.

Because the Stage-2 feasible choices of the traders are mutually dependent, we apply Debreu's (1952) social equilibrium concept to this stage in our determination of an SPE of the two-stage game.

Definition 3. An SPE is a strategy profile $(s^*, d^*, z^*) = (s^{*i}, d^{*i}, z^{*i})_{i \in N}$ such that for each trader i

- (i) $(d^{*i}(s), z^{*i}(s))$ is feasible and maximal given $(d_{-i}^*(s), z_{-i}^*(s)) = (d^{*j}(s), z^{*j}(s))_{j \neq i}$, for all Stage-1 choices s ,
- (ii) s^{*i} is maximal, given $s_{-i}^* = (s^{*j})_{j \neq i}$ and (d^*, z^*) .

An allocation $x^* = (x^{*1}, \dots, x^{*n})$ is an SPE allocation if there is an SPE (s^*, d^*, z^*) such that $x^{*i} = \omega^i + z^{*i}(s^*)$ for all $i \in N$.

Property (i) means that given Stage-1 choice profile s , the profile $(d^*(s), z^*(s))$ is a social equilibrium for the Stage-2 subgame. Trade in an SPE is stable in the sense that there is no entry of new intermediaries or exit of existing ones, nor is there any customer who wants to switch between the existing intermediaries or any existing intermediary who wants to change its terms. Thus, an SPE induces a natural partition of the traders into stable trading cooperatives.

3. Main results

We begin with a result that competition between endogenous intermediaries is sufficiently strong that all SPE allocations are contained in the intermediation core.

Theorem 1. *The subgame perfect equilibrium allocations of the intermediation game are contained in the intermediation core.*

Proof. See Appendix A. \square

The converse of Theorem 1 is not true. The following example provides an illustration.

Example 2. Consider an exchange economy with 2 commodities and 3 traders. Trader preferences can be represented by utility function $U^i(x^i) = \min\{x_1^i, x_2^i\}$ for $x^i \in X^i = \mathfrak{R}_+^2$ and $i = 1, 2, 3$. Their endowments are given by $\omega^1 = (10, 0)$, $\omega^2 = (0, 9)$, $\omega^3 = (1, 0)$. We show that the allocation $\bar{x} = (\bar{x}_1, \bar{x}_2, \bar{x}_3)$ with $\bar{x}^1 = (5.5, 4.5)$, $\bar{x}^2 = (4.5, 4.5)$, and $\bar{x}^3 = (1, 0)$ is in the intermediation core with supporting intermediary structure $(\bar{p}^k, \bar{C}^k)_{k=1}^2$, where $\bar{p}^1 = \bar{p}^2 = (1, 1)$, $\bar{C}^1 = \{1, 2\}$, and $\bar{C}^2 = \{3\}$. To this end, notice that $(\bar{x}^1, \bar{x}^2) \in F(\{1, 2\})$ and $\bar{x}^3 \in F(\{3\})$. In addition, no single trader alone can improve upon the allocation. Because neither trader 1 nor trader 3 is endowed with good 2, they cannot jointly improve upon the allocation. Traders 2 and 3 cannot jointly improve upon the allocation because the maximum amount of good 1 that they are endowed with is 1 unit. Traders 1 and 2 cannot jointly improve upon the allocation because their bundles already consist of a Pareto optimal allocation relative to their endowments. Next we show that a grand coalition cannot improve upon \bar{x} . To illustrate this, consider an arbitrary allocation x . For $U^1(x^1) > U^1(\bar{x}^1)$ and $U^2(x^2) > U^2(\bar{x}^2)$, it must be that $x_2^1 > 4.5$ and $x_2^2 > 4.5$ which implies $\sum_{i \in N} x_2^i > \sum_{i \in N} \omega_2^i$. Hence, $x \notin F(N)$. This shows that the grand coalition cannot improve upon allocation \bar{x} . In summary, we have shown that \bar{x} is an intermediation core allocation.

We now show that \bar{x} cannot be an SPE allocation. To this end, consider any strategy profile (s, d, z) that results in allocation \bar{x} .⁶ As previously noted, trader 3 cannot intermediate. Furthermore, trader 3 cannot be a customer of either trader 1 or trader 2, because otherwise he would have demanded a bundle different from \bar{x}^3 . Consider a deviating strategy $\tilde{s}^1 = (\tilde{p}, \tilde{C})$ with $\tilde{C} = \{1, 2, 3\}$ and $0 < \tilde{\rho} = \tilde{p}_2/\tilde{p}_1 < 7/9$. Because $d^3(s) = 0$, we have $d^3(\tilde{s}^1, s_{-1}) = 1$, and trader 1 is bound to intermediate. By (5), trader 1 receives

$$\begin{aligned} x^1 &= (11, 9) - \left(\frac{9\tilde{\rho}}{1+\tilde{\rho}}, \frac{9\tilde{\rho}}{1+\tilde{\rho}} \right) - \left(\frac{1}{1+\tilde{\rho}}, \frac{1}{1+\tilde{\rho}} \right) \\ &= \left(\frac{10+2\tilde{\rho}}{1+\tilde{\rho}}, \frac{8}{1+\tilde{\rho}} \right). \end{aligned}$$

As a result, trader 1's utility level is

$$U^1(x^1) = \min \left\{ \frac{10+2\tilde{\rho}}{1+\tilde{\rho}}, \frac{8}{1+\tilde{\rho}} \right\} = \frac{8}{1+\tilde{\rho}}.$$

Because $U^1(\bar{x}^1) = 4.5$ and $\tilde{\rho} < 7/9$, the preceding equation implies that $U^1(x^1) > U^1(\bar{x}^1)$. This shows that trader 1 has an

⁶ Note that the prices of the intermediating trader must be strictly positive. This can be illustrated as follows. Suppose that trader 1 is a customer. Then, the price of good 1 must be strictly positive for him to be able to afford (5.5, 4.5). If the price of good 2 is zero, then trader 1 would demand bundle (10, 10) which is not feasible. If trader 2 is the customer, however, then the price of good 2 must be positive for him to be able to afford bundle (4.5, 4.5). Hence, if the price of good 1 is zero, then trader 2 would demand bundle (9, 9), which is not compatible with trader 1 receiving (5.5, 4.5).

incentive to deviate. Because (s, d, z) is arbitrary, the allocation \bar{x} cannot be an SPE allocation. \square

A partial converse of Theorem 1 is given in the following theorem, which shows that an intermediation core allocation is also an SPE allocation of the intermediation game, provided that all intermediaries in the supporting intermediary structure have at least two customers.

Theorem 2. *Let $x^* = (x^{*i})_{i \in N}$ be in the intermediation core with supporting intermediary structure $(p^{*k}, C^{*k})_{k=1}^m$. If $|C^{*k}| \geq 3$ for all $k = 1, 2, \dots, m$, then x^* is a subgame perfect equilibrium allocation.*

Proof. See Appendix A. \square

Recall that C^{*k} contains the intermediating trader. Thus, $|C^{*k}| \geq 3$ means that there are two or more customers. This condition ensures that, for any active intermediary, there are always two contestable intermediaries who are ready to serve all customers at the same price vector. As illustrated in Example 2, this condition is indispensable.

4. Conclusion

In this paper we applied the approach proposed by Townsend (1983) to consider trading in an exchange economy through endogenous intermediaries. Under this approach, each trader has the opportunity to form an intermediary by offering to buy and sell commodities at a certain price vector for a certain group of customers. We introduced an intermediation core by reformulating coalitional feasible allocations. Like the inclusion of the competitive equilibrium allocations in the usually studied core, we showed that the subgame perfect equilibrium allocations of an intermediation game of Townsend (1983) are contained in the intermediation core. Furthermore, an intermediation core allocation is a subgame perfect equilibrium allocation if each supporting intermediary has two or more customers. This paper contributes to the literature on intermediation by providing tools for the characterization of the subgame perfect equilibrium allocations of intermediation games and for analyzing their welfare and stability properties.

Appendix A. Proofs

Proof of Theorem 1. Let (s^*, d^*, z^*) be an SPE of the intermediation game and $x^* = (x^{*i})_{i \in N}$ be the corresponding allocation. Suppose that the allocation is not in the intermediation core. Then, there exists a coalition C and a C -allocation $(x^i)_{i \in C} \in F(C)$ such that

$$U^j(x^j) > U^j(x^{*j}), \quad \forall j \in C. \tag{A.1}$$

Let p be the price vector that supports $(x^i)_{i \in C} \in F(C)$ and choose $i \in C$ whose bundle x^i does not maximize U^i subject to budget constraint at price vector p .⁷ Now, consider $s^i = (p, C)$. By (A.1), each trader $j \in C$ with $j \neq i$ chooses to trade with i due to the maximality of Stage-2 choices. That is, j chooses

$$d^{*j}(s^i, s_{-i}^*) = i, \quad \forall j \in C_{-i}. \tag{A.2}$$

Because $(x^j)_{j \in C} \in F(C)$ and p is the supporting price vector, (1) and (2) imply that $z^{*j}(s^i, s_{-i}^*) = x^j - \omega^j$ for all $j \in C$. Thus, by (3) and (A.2), trader $j \in C$ receives bundle x^j at

⁷ In case everyone's bundle maximizes utility subject to budget constraint, choose j arbitrarily.

(s^i, s_{-i}^*) , $d^*(s^i, s_{-i}^*)$, $z^*(s^i, s_{-i}^*)$). Because $U^i(x^i) > U^i(x^{*i})$, x^* cannot be an SPE allocation of the intermediation game. This is a contradiction. \square

Proof of Theorem 2. Given price vector p , we use $x^i(p)$ to denote the solution for utility maximization problem (2). For $k = 1, 2, \dots, m$, let $j_1^k \in C^{*k}$ be the intermediating trader and $j_2^k, j_3^k \in C^{*k}$ be two other members. Now, set $J^k = \{j_1^k, j_2^k, j_3^k\}$ and

$$s^{*j_1^k} = s^{*j_2^k} = s^{*j_3^k} = (p^{*k}, C^{*k}), \tag{A.3}$$

$$s^{*i} = \emptyset, \quad i \in C^{*k} \setminus J^k, \tag{A.4}$$

$$d^{*i}(s^*) = j_1^k, \quad i \in C^{*k}. \tag{A.5}$$

Let $z^{*i}(s^*) = x^{*i} - \omega^i$ for all i . Because $(x^{*i})_{i \in N}$ is an intermediation core allocation, it follows from the constructions of s^* in (A.3) and (A.4) that $d^*(s^*)$ in (A.5) together with $z^*(s^*)$ form a social equilibrium for Stage-2 following s^* . In addition, any strategy profile that has $(s^*, d^*(s^*), z^*(s^*))$ as the path of play can implement allocation x^* . Thus, it suffices to show that $(s^*, d^*(s^*), z^*(s^*))$ is an SPE path.

To this end, considering $(s^*, d^*(s^*), z^*(s^*))$ as a candidate SPE path, we only need to specify traders' choices at the off-equilibrium paths for cases, in which a single trader deviates. Let j be the trader who contemplates deviating from his Stage-1 choice s^{*j} to $s^j = (p, C) \neq s^{*j}$, while the other traders stick to theirs in s_{-j}^* . In what follows, we first construct maximal choices for all non-deviating traders. Recall that j will stay autarkic if it is not feasible for him to intermediate. In that case, he cannot be better off deviating. Thus, without loss of generality, we assume that it is feasible for j to intermediate.

There are six mutually exclusive and jointly exhaustive cases depending on the structure of the set of excluded members, $C^{*k} \setminus C$, and the identity of the deviating trader j .

- Case 1: $|C^{*k} \setminus C| > 1$ and $j \neq j_1^k$.

In this case, it is maximal for each trader $i \in C^{*k} \setminus C$ such that $i \neq j_1^k$ to trade with j_1^k . As a result, trader j_1^k is bound to intermediate. On the other hand, trader $i \in C^{*k} \cap C$ such that $i \neq j_1^k, j$ can trade either with j_1^k or j . Thus, maximal choices for non-deviating traders in C^{*k} can be constructed as follows:

$$d^{*i}(s^j, s_{-j}^*) = \begin{cases} j_1^k, & i \in (C^{*k} \setminus C) \cup \{j_1^k\}; \\ j, & U^i(x^i(p)) > U^i(x^i(p^{*k})) \\ & \text{for } i \in C^{*k} \cap C : i \neq j_1^k, j; \\ j_1^k, & U^i(x^i(p)) \leq U^i(x^i(p^{*k})) \\ & \text{for } i \in C^{*k} \cap C : i \neq j_1^k, j. \end{cases} \tag{A.6}$$

- Case 2: $|C^{*k} \setminus C| > 1$ and $j = j_1^k$.

In this case, choosing to trade with j_2^k is maximal for all traders in $C^{*k} \setminus C$ and hence, j_2^k is bound to intermediate. Since traders in $C^{*k} \cap C$ different from j_2^k, j do not intermediate, maximal choices for non-deviating traders in C^{*k} can be constructed as follows:

$$d^{*i}(s^j, s_{-j}^*) = \begin{cases} j_2^k, & i \in (C^{*k} \setminus C) \cup \{j_2^k\}; \\ j, & U^i(x^i(p)) > U^i(x^i(p^{*k})) \\ & \text{for } i \in C^{*k} \cap C : i \neq j_2^k, j; \\ j_2^k, & U^i(x^i(p)) \leq U^i(x^i(p^{*k})) \\ & \text{for } i \in C^{*k} \cap C : i \neq j_2^k, j. \end{cases} \tag{A.7}$$

- Case 3: $|C^{*k} \setminus C| = 1$ and $j \neq j_1^k$.

In this case, if $j_1^k \notin C^{*k} \setminus C$, then it is optimal for the trader in $C^{*k} \setminus C$ to trade with j_1^k . This implies that j_1^k is bound to intermediate. The maximal choices for non-deviating traders in C^{*k} can be constructed as follows:

$$d^{*i}(s^j, s_{-j}^*) = \begin{cases} j_1^k, & i \in (C^{*k} \setminus C) \cup \{j_1^k\}; \\ j, & U^i(x^i(p)) > U^i(x^i(p^{*k})) \\ & \text{for } i \in C^{*k} \cap C : i \neq j_1^k, j; \\ j_1^k, & U^i(x^i(p)) \leq U^i(x^i(p^{*k})) \\ & \text{for } i \in C^{*k} \cap C : i \neq j_1^k, j. \end{cases} \tag{A.8}$$

If $j_1^k \in C^{*k} \setminus C$ and $j = j_2^k$, then j_1^k is the only trader in $C^{*k} \setminus C$ and it is maximal for j_1^k to trade with j_3^k when j_1^k is not bound to intermediate. Thus, we make j_3^k bound to intermediate by having j_1^k choose j_3^k and all other traders in $C^{*k} \cap C$ choose between j and j_3^k . Recall that j_1^k and j_3^k offer the same terms of trade. Maximal choices for non-deviating traders in C^{*k} can be constructed as follows:

$$d^{*i}(s^j, s_{-j}^*) = \begin{cases} j_3^k, & i = j_1^k, j_3^k; \\ j, & U^i(x^i(p)) > U^i(x^i(p^{*k})) \\ & \text{for } i \in C^{*k} \cap C : i \neq j_3^k, j; \\ j_3^k, & U^i(x^i(p)) \leq U^i(x^i(p^{*k})) \\ & \text{for } i \in C^{*k} \cap C : i \neq j_3^k, j. \end{cases} \tag{A.9}$$

If $j_1^k \in C^{*k} \setminus C$ and $j = j_3^k$, maximal choices for non-deviating traders in C^{*k} can be constructed as in the case with $j = j_2^k$ by replacing j_2^k with j_3^k . That is,

$$d^{*i}(s^j, s_{-j}^*) = \begin{cases} j_2^k, & i = j_1^k, j_2^k; \\ j, & U^i(x^i(p)) > U^i(x^i(p^{*k})) \\ & \text{for } i \in C^{*k} \cap C : i \neq j_2^k, j; \\ j_2^k, & U^i(x^i(p)) \leq U^i(x^i(p^{*k})) \\ & \text{for } i \in C^{*k} \cap C : i \neq j_2^k, j. \end{cases} \tag{A.10}$$

If $j_1^k \in C^{*k} \setminus C$ and $j \neq j_2^k, j_3^k$, optimal choices for non-deviating traders in C^{*k} can be constructed as in (A.10).

- Case 4: $|C^{*k} \setminus C| = 1$ and $j = j_1^k$.

In this case, either $j_2^k \notin C^{*k} \setminus C$ or $j_3^k \notin C^{*k} \setminus C$. Without loss of generality, assume $j_2^k \notin C^{*k} \setminus C$. Maximal choices for non-deviating traders in C^{*k} can be constructed by letting j_2^k be bound to intermediate for the trader in $C^{*k} \setminus C$:

$$d^{*i}(s^j, s_{-j}^*) = \begin{cases} j_2^k, & i \in (C^{*k} \setminus C) \cup \{j_2^k\}; \\ j, & U^i(x^i(p)) > U^i(x^i(p^{*k})) \\ & \text{for } i \in C^{*k} \cap C : i \neq j_2^k, j; \\ j_2^k, & U^i(x^i(p)) \leq U^i(x^i(p^{*k})) \\ & \text{for } i \in C^{*k} \cap C : i \neq j_2^k, j. \end{cases} \tag{A.11}$$

- Case 5: $|C^{*k} \setminus C| = 0$ and $j \notin J^k$.

In this case, $C^{*k} \subseteq C$. The construction of maximal choices for non-deviating traders in C^{*k} is easy when j_1^k is bound to intermediate. When j_1^k is not bound to intermediate, however, we need to guarantee that j_1^k 's choice is compatible with choices of the other traders.⁸ This can be done by letting trader j_2^k be bound to

⁸ For example, when j_1^k is not bound to intermediate and $U^{j_1^k}(x^i(p)) \leq U^{j_1^k}(x^i(p^{*k}))$, j_1^k can choose to trade with j_2^k or j_3^k . Whichever trader j_1^k chooses between j_2^k and j_3^k to trade with, the chosen trader cannot trade with j even if it is better for him to do so.

intermediate:

$$d^{*i}(s^j, s_{-j}^*) = \begin{cases} j, & U^i(x^i(p)) > U^i(x^i(p^{*k})) \\ & \text{for } i \in C^{*k} : i \neq j_1^k, j_2^k, j; \\ j_1^k, & U^i(x^i(p)) \leq U^i(x^i(p^{*k})) \\ & \text{for } i \in C^{*k} : i \neq j_1^k, j \text{ or} \\ & i = j_1^k \text{ and } \exists l \in C^{*k} \setminus \{j_1^k\} : \\ & U^l(x^l(p)) \leq U^l(x^l(p^{*k})); \\ j_2^k, & i = j_1^k, j_2^k, U^l(x^l(p)) > U^l(x^l(p^{*k})) \\ & \text{for } l \in C^{*k} \setminus \{j_1^k\}, \\ & \text{and } U_{j_1^k}^i(x_{j_1^k}^i(p)) \leq U_{j_1^k}^i(x_{j_1^k}^i(p^{*k})); \\ j, & i = j_1^k, j_2^k, U^l(x^l(p)) \\ & > U^l(x^l(p^{*k})) \text{ for all } l \in C^{*k}; \\ j, & i = j_2^k, U^{j_2^k}(x^{j_2^k}(p)) \\ & > U^{j_2^k}(x^{j_2^k}(p^{*k})) \text{ and} \\ & \exists l \in C^{*k} \setminus \{j_1^k, j_2^k\} : U^l(x^l(p)) \\ & \leq U^l(x^l(p^{*k})). \end{cases} \quad (\text{A.12})$$

- Case 6: $|C^{*k} \setminus C| = 0$ and $j \in J^k$.

Let $j = j_h^k \in J^k$ and let $j_{h'}^k$ and $j_{h''}^k$ denote the other two trades in J^k . In this case, maximal choices for non-deviating traders in C^{*k} can be constructed as in (A.12) by replacing j_1^k with $j_{h'}^k$ and j_2^k with $j_{h''}^k$.

We now turn to the optimal choice for trader j , the deviating trader. He is bound to intermediate if some one in C chooses to trade with him. Otherwise, he can choose to trade with an intermediary that includes him as a member. Thus, his maximal choice can be constructed as follows:

$$d^{*j}(s^j, s_{-j}^*) = \begin{cases} j, & \exists i \in C_{-j} : d^{*i}(s^j, s_{-j}^*) = j, \\ j^{k'}, & d^{*i}(s^j, s_{-j}^*) \neq j \text{ for all } i \in C_{-j}, \end{cases} \quad (\text{A.13})$$

where $1 \leq k' \leq m$ such that $j \in C^{*k'}$ and $j^{k'} \in J^{k'}$ such that $d^{*i}(s^j, s_{-j}^*) = j^{k'}$ for all $i \in C^{*k'}.$

For each non-deviating trader i , let $z^{*i}(s^j, s_{-j}^*)$ be i 's excess demand at the price vector of the intermediary that includes i as a customer. For each trader i who is bound to intermediate, let $z^{*i}(s^j, s_{-j}^*)$ be determined as in Definition 1. By construction, stage 2 choices $(d^*(s^j, s_{-j}^*), z^*(s^j, s_{-j}^*))$ specified in (A.6)–(A.13) form a social equilibrium in the second stage following announcement profile (s^j, s_{-j}^*) . Thus, it only remains to show that trader j , the deviator, cannot be better off. The proof is completed if he does not have any customer. Suppose that he does have some customers. By construction, all of his customers are better off. Thus, because allocation x^* is in the intermediation core, trader j cannot be better off. \square

Appendix B. Unequal treatment of the intermediation core

Consider an economy in which there are three types of traders, a, b and c , set of which contain two identical traders. We name them as $a_1, a_2, b_1, b_2, c_1, c_2$. Preferences and endowments of type- b and type- c traders are given by $U^b(x_1, x_2) = \min(\frac{1}{2}x_1, x_2)$, $U^c(x_1, x_2) = \min(x_1, \frac{1}{2}x_2)$, $\omega^b = (1, 0)$, $\omega^c = (0, 1)$. Type a 's endowment is $\omega^a = (10, 10)$ and its preferences will be specified later.

We consider an allocation that is achievable by having two coalitions $\{a_1, b_1, c_1\}$ and $\{a_2, b_2, c_2\}$, each of which has trader a_i as an intermediary. For notational purposes, we call a coalition

$\{a_i, b_i, c_i\}$ an intermediary i . Let $\frac{1}{2} \leq \rho_i \leq \frac{2}{3}$ be the price ratios offered by an intermediary $i = 1, 2$ such that $\rho_1 \neq \rho_2$.

The dash-curve in Fig. B.1(a) is the utility frontier of b_i and c_i as customers, which is obtained by varying the price ratio offered by the intermediary. The bold-curve is the utility frontier of b_i and c_i if they jointly form a two-trader intermediary.¹⁰

Notice that each trader a_i receives the following bundle from intermediating with price ratio $\frac{1}{2} \leq \rho_i \leq \frac{2}{3}$ and customer set $\{b_i, c_i\}$:

$$\left(10 + \frac{\rho_i(\rho_i - 1)}{(1 + 2\rho_i)(2 + \rho_i)}, 10 + \frac{1 - \rho_i}{(1 + 2\rho_i)(2 + \rho_i)} \right). \quad (\text{B.1})$$

The locus of these bundles is represented by the G – H curve in Fig. B.1(b). We take the G – H curve and the dashed lines connected to it to be an indifference curve for type a .

We now show that an allocation achievable by the intermediary structure $(\rho_1, \{a_1, b_1, c_1\})$, $(\rho_2, \{a_2, b_2, c_2\})$ with $\frac{1}{2} \leq \rho_i \leq \frac{2}{3}$ is in the intermediation core. There are five cases with different conditional structures. Let $C \subset N$ be a candidate coalition.

- $C = \{b_i, c_j\}$: The utility level of b_i in this coalition is at most equal to $\frac{2}{5}$, which is lower than $\frac{3}{7}$, the minimum utility level of b_i as a customer in the proposed intermediaries (see point F).
- $C = \{a_i, b_1, b_2, c_j\}$: To make both type- b traders strictly better off, the price ratio must be smaller than $\min\{\rho_1, \rho_2\}$. This will make a type- c trader worse off. Hence, C cannot improve upon the candidate allocation. A similar argument applies to coalitions: $\{a, b, c_1, c_2\}$, $\{a, b_1, b_2, c_1, c_2\}$, $\{a_1, a_2, b_1, b_2, c_1, c_2\}$, $\{b_1, b_2, c\}$ or $\{b, c_1, c_2\}$.
- $C = \{a_i, b_j\}$: As shown in Fig. B.1(b), for a_i to be better off than serving both b_i and c_i , trader a_i must receive at least 10.054 units of good-2 (see point H on the indifference curve of an a_i in Fig. B.1(b)). This will leave b_j with at most $10 - 10.054 = -0.054$, which is not feasible.
- $C = \{a_1, a_2, b_i\}$: To make b_i better off relative to being a customer at a price ratio $\frac{1}{2} \leq \rho_i \leq \frac{2}{3}$, the resulting bundle left after satisfying b_i is at most $(20 + \frac{\rho_i}{2+\rho_i}, 20 - \frac{1}{2+\rho_i})$. Hence, with $\frac{1}{2} \leq \rho_i \leq \frac{2}{3}$, the maximum consumption level of good 2 for a type- a trader is strictly less than 10.054 (see point H in Fig. B.1(b)). This implies that no type- a traders can be better off.
- $C = \{a_1, a_2, b_i, c_j\}$: Suppose a type- a trader, say a_1 , intermediates. To attract one trader of type b and one trader of type c , the proposed price ratio must be $\frac{1}{2} \leq \rho \leq \frac{2}{3}$. See Fig. B.2(a)–(b). Let point \mathbf{K} in Fig. B.2(a) denote the resulting bundle of a_1 after b_i and c_j have completed their trade, and line \mathbf{K} – \mathbf{O} represents the budget line for trader a_2 , who acts as a customer. In addition to the G – H curve being an indifference curve, we also require that on the budget line \mathbf{K} – \mathbf{N} , bundle \mathbf{M} is optimal for each type a trader as a price-taking customer. To achieve such an allocation, her net-trade is given by segment \mathbf{O} – \mathbf{M} . Consequently, the opposite trade position relative to an allocation \mathbf{K} will be the net trade of a_1 as an intermediary (see Fig. B.2(b)). Using a similar-triangles argument, we can show that the segment \mathbf{O} – \mathbf{M} is always longer than segment \mathbf{K} – \mathbf{M} .¹¹ As a result, the consumption bundle of a_1 (allocation \mathbf{N}) is always below the G – H indifference curve of type a .

¹⁰ This utility frontier when both of them are customers is represented by $V^b(\rho) = \frac{1-2V^c}{2-3V^c}$, whereas the frontier when both of them form an intermediary is represented by $V^b = \frac{1}{2} - \frac{V^c}{2}$ when $0 \leq \rho_i \leq 1$, and $V^b = 1 - 2V^c$ otherwise.

¹¹ We can create two similar triangles based on segments \mathbf{O} – \mathbf{M} and \mathbf{K} – \mathbf{M} . Because the height (perpendicular to the x -axis) of the \mathbf{O} – \mathbf{M} triangle is longer than the height of the \mathbf{K} – \mathbf{M} triangle, we can conclude that segment \mathbf{O} – \mathbf{M} is longer than segment \mathbf{K} – \mathbf{M} .

⁹ Notice that $j = j_1^k$ is possible, in which case, $j^{k'} \neq j_1^k$.

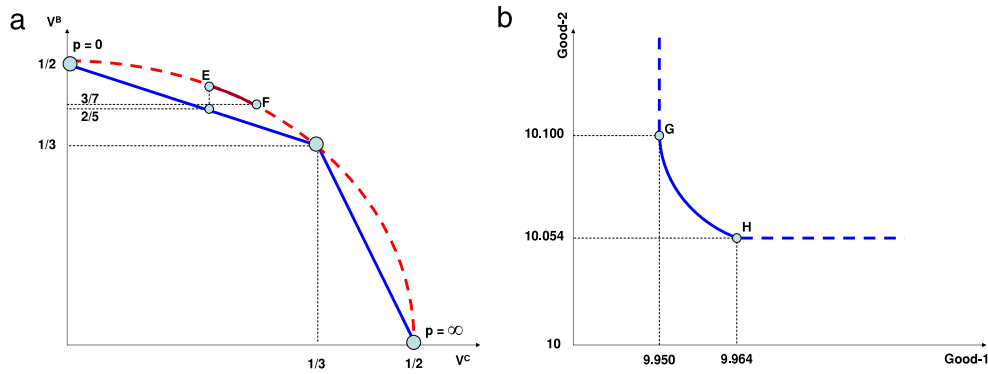


Fig. B.1. (a) A candidate allocation is on the E - F arc, with $\frac{1}{2} \leq \rho \leq \frac{2}{3}$. (b) An indifference curve of a type- a trader. The G - H portion corresponds to (B.1) given that $\frac{1}{2} \leq \rho \leq \frac{2}{3}$.

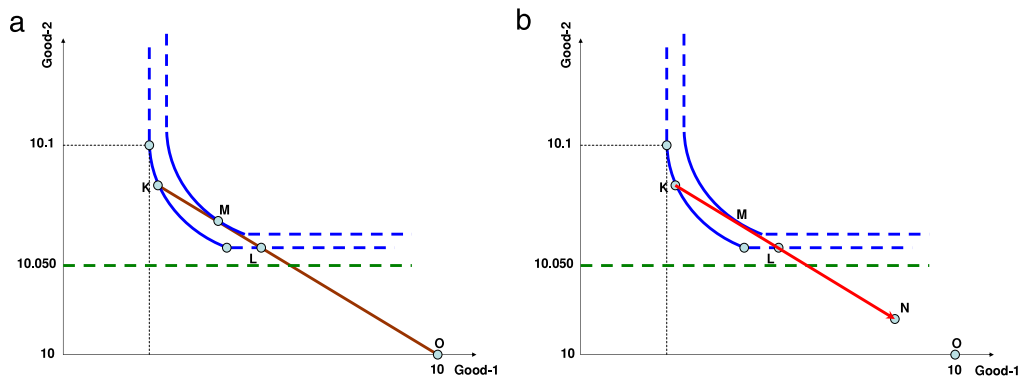


Fig. B.2. (a) The budget line and optimal consumption allocation for a customer type a . (b) The final allocation of the intermediary type a .

Again, we define type a 's preferences using indifference curves in Fig. B.2(b). This implies that the intermediating trader is worse off. The similar argument applies to cases in which b_i or c_j is the intermediating trader.

References

Abreu, D., Sen, A., 1990. Subgame perfect implementation: a necessary and almost sufficient condition. *J. Econom. Theory* 50 (2), 285–299.
 Baumol, W., Panzar, J., Willig, R., 1982. *Contestable Markets and the Theory of Industrial Structure*. Harcourt Brace Javanovich, New York.
 Boyd, J., Prescott, E., 1986. Financial intermediary-coalitions. *J. Econom. Theory* 38, 211–232.
 Boyd, J., Prescott, E., Smith, B., 1988. Organizations in economic analysis. *Canad. J. Econom.* 477–491.
 Debreu, G., 1952. A social equilibrium existence theorem. *Proc. Natl. Acad. Sci. USA* 38 (10), 886.
 Mas-Colell, A., 1975. A model of equilibrium with differentiated commodities. *J. Math. Econom.* 2 (2), 263–295.
 Moore, J., Repullo, R., 1988. Subgame perfect implementation. *Econometrica* 1191–1220.
 Qin, C., Shapley, L., Shimomura, K., 2006. The Walras core of an economy and its limit theorem. *J. Math. Econom.* 42 (2), 180–197.
 Serrano, R., Vohra, R., 1997. Non-cooperative implementation of the core. *Soc. Choice Welf.* 14 (4), 513–525.
 Starr, R., 2011. *General Equilibrium Theory: An Introduction*. Cambridge University Press.
 Townsend, R., 1978. Intermediation with costly bilateral exchange. *Rev. Econom. Stud.* 45 (3), 417–425.
 Townsend, R.M., 1983. Theories of intermediated structures. In: *Carnegie-Rochester Conference Series on Public Policy*, vol. 18. pp. 221–272.
 Yannelis, N., 2009. Debreu's social equilibrium theorem with asymmetric information and a continuum of agents. *Econom. Theory* 38 (2), 419–432.