Journal
IEEE Transactions on Applied Superconductivity, 24(3)

ISSN
1051-8223

Authors
Rochepault, E
Arbelaez, D
Prestemon, SO
et al.

Publication Date
2014

DOI
10.1109/tasc.2013.2285095

Error Analysis and Field Correction Methods in Superconducting Undulators

E. Rochepault, D. Arbelaez, S. O. Prestemon, and R. D. Schlueter

Abstract—**In Free Electron Lasers (FEL), the electron trajectory through the undulator must meet stringent requirements in terms of trajectory wander and phase variation. This paper analyzes the feasibility of using line current pairs as correctors for superconducting undulators given a set of expected fabrication errors. A tolerance study has first been performed to investigate the impact of geometrical errors on the field quality. These errors are corrected with line currents that increase or decrease the magnetic field locally. Once the uncorrected trajectory is known, an algorithm finds the minimum number of correctors required to fulfill the trajectory specifications, and gives the corrector locations. All the correctors can be powered with the same current, greatly simplifying the implementation. The current then offers a degree of freedom to correct the trajectory and can be tuned dynamically as a function of the magnetic deflection.**

Index Terms—**Correction, free electron laser (FEL), magnetic design, superconducting, tuning, undulators.**

I. Introduction

FREE ELECTRON LASERS (FEL) produce high brightness X-ray beams greatly prized by synchrotron light sources. Undulators are used to generate the radiation wave and an operating field control allows tuning the wavelength. Over the last 25 years, a great interested has been shown for superconducting undulators, with successful tests reported [1]–[3]. The major advantage of the superconducting technology is to offer, either a higher operating field for the same period length, or a smaller period for the same operating field. This leads to access to higher energy radiation and larger tuning ranges. The radiation wavelength is tuned with the main coils current. The Lawrence Berkeley National Laboratory (LBNL) is developing an undulator technology using superconducting Nb_3Sn coils tailored to the needs of soft X-ray FELs. Three proof-of-principle short prototypes were previously tested [4]–[6] and showed promising results.

The correction of the electron trajectory errors through the undulator is a critical step for the proper operation of the FEL. These errors originate from machining and winding imperfections. Several correction concepts have already been proposed. Some are passive, such as magnetic shims [7], inductive shimming coils [8], or indentations into the poles [9]. Other are active, such as superconducting coils [5], [10] and a superconducting switch network [11]. The concept of correction with line current pairs has been proposed in [12]. The switching device has been successfully tested, but a full correction network has not been fabricated yet. The correctors consist in thin HTS tapes with patterned current paths triggered with heaters. These tapes can be glued on the poles. It will be shown in this paper how the errors can be corrected dynamically with corrector coils. First, the device used for the study will be presented in Section II. Then, a tolerance study (Section III) will describe the different types of geometrical errors and their effect on the trajectory. Finally, the Section IV will explain how these errors can be corrected with a set of line current pairs.

Manuscript received July 17, 2013; accepted September 17, 2013. Date of publication October 9, 2013; date of current version November 6, 2013. This work was supported by the Director, Office of Science, High Energy Physics, U.S. Department of Energy under contract DE-AC02-05CH11231.

The authors are with Lawrence Berkeley National Laboratory, Berkeley, CA 94720 USA (e-mail: etienne.rochepault@gmail.com).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TASC.2013.2285095

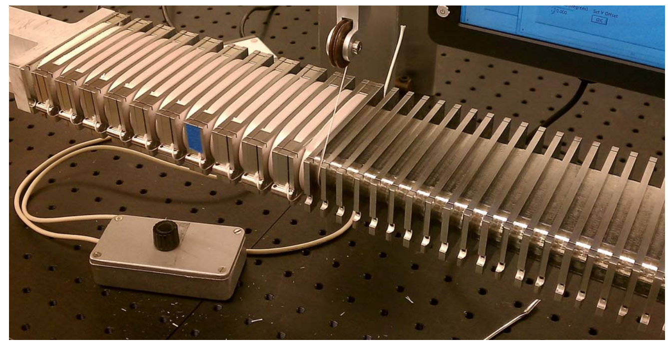

Fig. 1. Photograph of the winding process of the Nb_3Sn undulator prototype developed at LBNL.

II. Description of the Device

A. Main Parameters

The device used in this study is a superconducting undulator prototype currently in fabrication at LBNL and aimed at studying the field quality. It is a 25-period undulator (0.5 m), composed of Nb_3Sn coils and a ferromagnetic yoke, as depicted in Fig. 1. The conductor used is a 0.48 mm diameter Modified Jelly Roll (OST) strand, with a copper to noncopper ratio of 1.13 and a $J_c(12\ \mathrm{T}, 4.2\ \mathrm{K}) = 8060$ A/mm2 [6]. The gap between the two halves is 7.5 mm. The nominal engineering current density is $J_E = 1610$ A/mm^2 (80% of the short sample limit), corresponding to a peak field of 2.22 T and a deflection parameter $K = 4.14$.

B. Magnetic Design

1) Trajectory Errors: Through the undulator, the electrons undergo magnetic deflections and exit with an angle and/or displacement. The force F_y exerted on an electron of mass m, charge q, Lorentz factor γ, and speed v_z is proportional to the magnetic field B_x according to the Lorentz force law

$$F_x = -qv_z B_y = -\gamma m \frac{d^2x}{dt^2} = -\gamma m v_z^2 \frac{d^2x}{dz^2}. \qquad (1)$$

Then the position derivative is given by the first integral of the field

$$\frac{dx}{dz}(z) = -\frac{q}{\gamma m v_z} \int_{-\infty}^{z} B_y dz^2. \qquad (2)$$

And the position is obtained with the second integral

$$x(z) = -\frac{q}{\gamma m v_z} \int\int_{-\infty}^{z} B_y dz^2. \qquad (3)$$

In order to meet the FEL requirements, the electron trajectory must remain within a certain tolerance band along the entire length of the undulator. In addition, so as to maximize the radiation intensity, the electron wave must be in phase with the radiation wave it is producing. Every field perturbation will add a phase advance or delay to the trajectory, resulting in phase errors [7]. For the following of the study, the LCLS project requirements [7] are assumed: $|I_1| < 40 \times 10^3$ T.mm, $|\int\int B_x dz^2| < 50$ T.mm^2, and a RMS (Root Mean Square) of the phase error $Phase_{RMS,err} < 10°$.

2) Coil Pattern: The undulator ends for the prototype described here have been designed so as to produce no kick and no displacement. In order to produce no net first and second integral under ideal conditions (no saturation for instance), the pole potentials follow the binomial expansion pattern [13] (0, $-1/4$, $+3/4$, -1, $+1$, etc.). The corresponding pattern for the number of turns per coil is ($-1/8$, $+4/8$, $-7/8$, $+1$, -1, etc.).

3) End Correctors: First, the trajectory errors, due to the approximation in the end pattern and the nonlinear magnetic behavior of the yoke, have to be corrected. Also, due to the odd number of poles and the yoke magnetization, a dipole is created in the undulator. This dipole has to be canceled, since it will provide an additional and undesired displacement. Two sets of independent corrector coils are first placed in the ends to correct the dipole and end kicks. The residual displacement could also be corrected with a third set of coils. However, in practice, this displacement is low, and a dedicated set of correctors would complicate the structure. If the residual displacement does not fulfill the requirements, it can be corrected further by the line current correctors presented in Section IV.

III. Tolerance Study

The undulator ends are not the only source of trajectory errors. All the imperfections in yoke machining and winding will create local perturbations in the magnetic field and so in the trajectory. All these errors accumulated can lead to a nonnegligible drift of the electrons, undesired for the FEL operation.

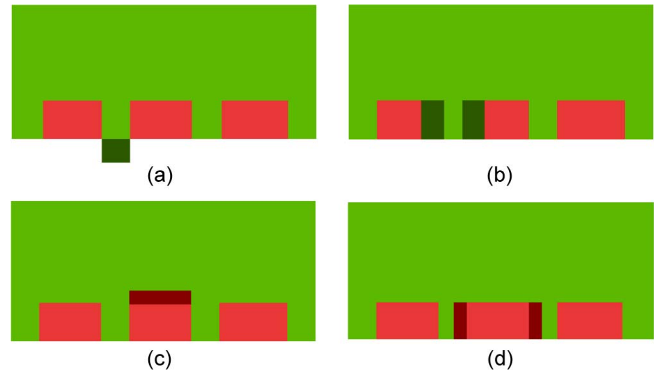

Fig. 2. Four different types of pole errors. (a) The pole is higher. As the flux is high through this part, the field will be strengthened locally. (b) The pole is thicker, which will increase also the field locally. Consequently, the adjacent pockets are thinner and the current density of the corresponding coils is increased to keep a constant current. (c) The pocket is deeper, and the coil current density is lower. (d) The pocket is wider, and the coil current density is lower. It is assumed that the same amount of pole material from each neighboring pole is removed, which leads to a symmetric signature. Note that this error is also equivalent to two consecutive pole errors (b) with the same amplitude.

A. Different Error Types

The four main types of yoke geometrical errors were investigated; they are illustrated in Fig. 2. Of course, these errors can be considered positively or negatively. Moreover, it is assumed that local winding errors are negligible compare to yoke errors, and will be neglected in this study. The signature of each error is computed by subtracting the case with the considered error to the case without error. The computations are carried out with Opera to take into account the saturation effects. Fig. 3 thus shows that a Gaussian curve can be made to fit the error signature very accurately. For the rest of the study, the following fits are applied to simulate an error signature. The pole errors (a), (b), are characterized by even functions

$$B_{pole\ err} = c \exp\left(\frac{z^2}{2d^2}\right) \qquad (4)$$

whereas the coil errors (c), (d), feature odd functions

$$B_{coil\ err} = c\frac{z}{d^2} \exp\left(\frac{z^2}{2d^2}\right). \qquad (5)$$

c and d have different values according to the error type. The error signatures have been computed with different operating fields. 100 μm errors have been considered, assuming small errors which justifies a linearization. The coil errors produce only a displacement. The total displacement being the sum of displacement errors, the standard deviation of coil errors scales with the $N_{err}^{1/2}$. The number of errors N_{err} equals the number of poles, which is proportional to the undulator length L_{und}. Coil errors then scale with the $L_{und}^{1/2}$. Moreover, for the poles errors, the total displacement is given by the sum of kick errors times the length of the drift. By calculating the variance of a random process with kicks and drift sections, one can derive that pole errors scale with $L_{und}^{3/2}$. This means that all the errors will grow with the undulator length and that pole errors will feature an even bigger effect.

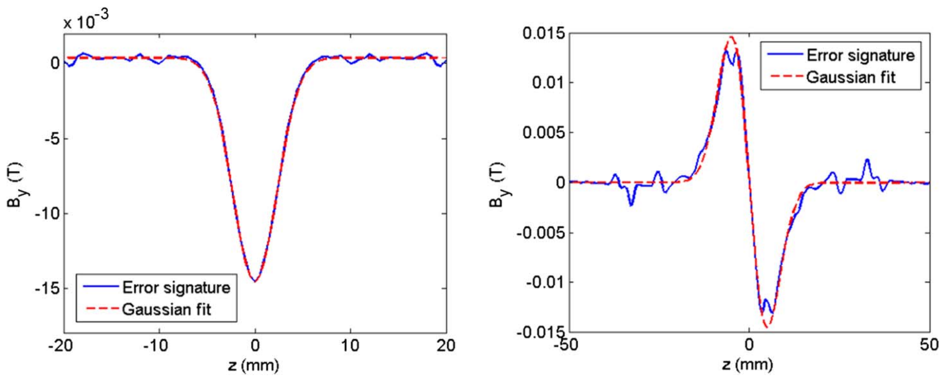

Fig. 3. Gaussian fits for two types of error signatures at 2.22 T and with an error size $s = s_0 = 0.1$ mm. Left: type (a), right: type (c).

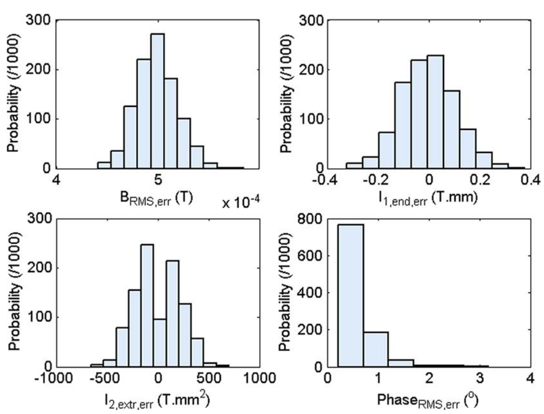

Fig. 4. Histograms of the errors, generated for 1000 cases with an operating field of 2.22 T. $\mathrm{B}_{\mathrm{RMS,err}}$: RMS value of the field errors. $\mathrm{I}_{1,\mathrm{end,err}}$: end value of the first integral errors. $\mathrm{I}_{2,\mathrm{extr,err}}$: extremum value of the second integral errors. $\mathrm{Phase}_{\mathrm{RMS,err}}$: RMS value of the phase errors.

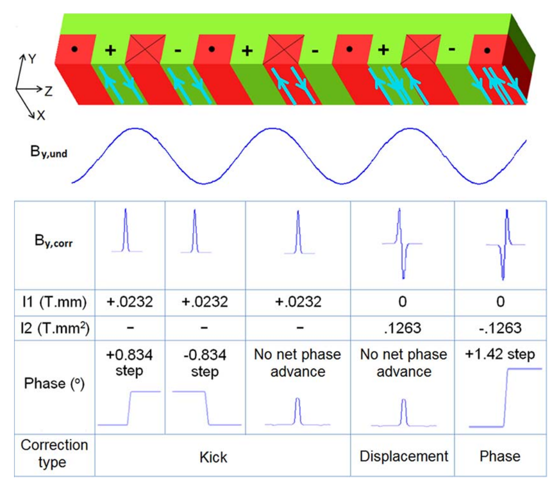

Fig. 5. Schematics of the different corrector combinations with their field, first integral, second integral, and phase signatures.

B. Monte Carlo Simulation

In order to determine if an additional correction is needed to compensate for the tolerance errors, a statistic study has been carried out. First, the geometrical errors have been measured on the yoke prototype with a Coordinate Measurement Machine. The standard deviations are 2.9 μm for the pole heights, 0.84 μm for the pole lengths, 1.9 μm for the coil heights, and 0.82 μm for the coil lengths. Once all the error signatures are recorded, a Monte Carlo simulation is performed using Matlab. A 165-period undulator (3.3 m) is considered in this study, corresponding to the NGLS target. The measured standard deviations are then implemented in this error analysis. For each case, four types of errors per period are generated with random sizes. The trajectory will depend a lot on the error distribution. The errors can either compensate each other or accumulate, resulting in high exit angle and displacement. Fig. 4 illustrates the statistics of the trajectory errors obtained with the Monte Carlo simulation. The average RMS field error is 5.00×10^{-4} T and its standard deviation is 1.98×10^{-5} T. The standard deviations of the exit angle and the maximum trajectory walk-off are, respectively 0.11 T.mm and 220 T.mm^2. These values are above the specifications defined in Section II-B1, and need to be corrected with an additional set of correctors. The standard deviation of the phase errors is only 0.6°, and does not need to be corrected.

IV. LINE CURRENT CORRECTORS

A. Different Corrector Combinations

The concept of line current correctors proposed in [12] is used. However, the correction scheme proposed here considers correctors with an unidirectional current and no heater. All the correctors are powered in series with a maximum current of 50 A. Consequently, the degrees of freedom for the correction are the corrector locations and the current. The choice of these parameters will be clarified in the next sub-section. Different correctors can also be combined to act differently on the first integral, the second integral, or the phase. Fig. 5 illustrates the location of the correctors and their effect on the trajectory.

1) Kick Correctors: The signature of a single corrector can be approximated with a Gaussian. Single correctors will provide a pure kick. A corrector located on a positive pole will give a phase advance, whereas it will give a phase delay if located on a negative pole. If placed on a coil, a corrector will have no impact on the net phase advance.

2) Displacement Correctors: If two correctors are placed head to tail, the provided angle will be zero but they will exhibit a net displacement. There is no phase correction if such a pair is placed on a pole.

3) Phase Correctors: If placed on a coil, a pair of correctors will also add a phase advance.

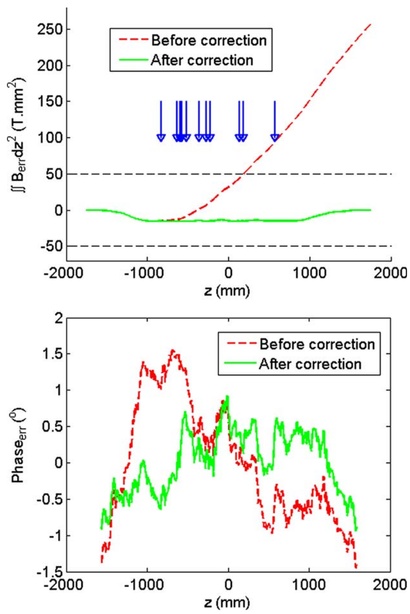

Fig. 6. Trajectory and phase errors before and after correction. The location of the correctors is symbolized with blue arrows.

B. Correction Algorithm

Considering the nominal parameters of the undulator (2.2 T operating field, correctors powered with 50 A), a corrector produces an operating field of 3.8 mT. In the worst case, a pole error can be 2.7 mT in total, which means that there is potentially enough correctors to correct all the errors. Anyways, there is no need to correct locally every error, since a lot of errors cancel each other. Consequently, the number of correctors can be reduced. The Monte Carlo simulation presented above allowed to estimate the average number of correctors to 9.3 (for a total of 331 poles). Then an algorithm is used to compute the corrector locations while fulfilling the initial specification. The main idea is to cancel the exit angle, while minimizing the trajectory walk-off, by adding necessary kicks. The correctors used are line current pairs located at the poles. This algorithm can be described by the following basic steps:

1) The correctors are placed so as to minimize the trajectory walk-off. If the local extremum is negative (positive), correctors providing a positive (negative) kick are placed.
2) Since the number of correctors is an integer, the exit angle will not be exactly zero. However, it can be canceled by adapting the current.
3) Once the correctors are glued at their fixed positions, the current is tuned to cancel the exit kick at every operating field.

Note that other algorithm methods could use less correctors, but would result in higher exit angle and trajectory walk-off. To test the algorithm, a random case is generated. It has been chosen so as the errors are representative of the errors computed with the Monte Carlo simulation. The first and second integrals are, respectively 0.13 T.mm and 257 T.mm^2 at the exit. The RMS phase error is 0.8°. Only 11 correctors are required to cancel the exit kick while keeping the second integral within the specifications. Fig. 6 shows the corrected trajectory. After correction, the exit kick is exactly zero for all cases, the maximum walk-off is -15.8 T.mm^2, and the RMS phase error is only 0.4°. As a last step, the current is tuned to cancel the exit kick at any operating field.

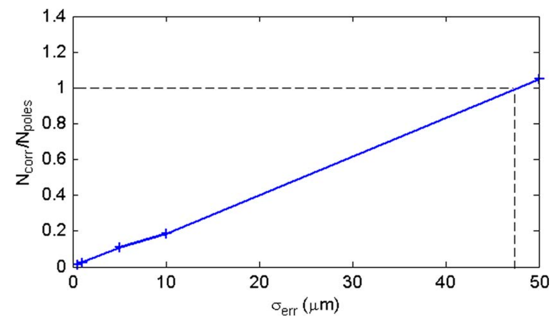

Fig. 7. Normalized number of correctors versus the standard deviation of errors.

This algorithm is mainly dedicated to the correction of the second integral (trajectory), but if the requirements were to be more stringent (decrease the phase errors for instance), other combinations of correctors presented in Fig. 5 could be used.

The example presented above has been generated with good tolerances errors, and required few correctors. However, a good machining tolerance might not be guaranteed in all cases. The average number of correctors required to correct the trajectory has been estimated for different error levels, assuming 2.2 T operating field and a 50 A corrector current. Fig. 7 shows that the number of correctors is proportional to the errors. The measured errors (0.8 to 2.9 μm) require to cover only a few percent of the poles. Moreover, if a corrector is placed on each pole (331), it is possible to correct 50 μm errors.

V. Conclusion

Undulator tuning is an essential component for the good operation of FELs. A field correction scheme, applicable to superconducting undulators, has been presented. First, two independent sets of corrector coils are used in the ends to cancel both the exit slope and displacement created by end effects. In addition, the yoke imperfections disrupt locally the magnetic field. A model of the magnetic signature of these errors have been established and can be used to predict the field errors. Then, line current pairs are implemented to correct the electron trajectory. Different combinations can be used to correct kick, displacement, and phase errors. An algorithm is used to determine the corrector locations. The locations are fixed, but the current can be tuned to adapt to the undulator operating field. Finally, with their respective power supply, these three sets of correctors offer three degrees of freedom to correct the end errors and the random local errors, at any operating field. The method presented can be easily used to correct the field errors in undulators: it is adaptable and especially can save a lot of time during the tricky process of tuning.

Acknowledgment

The authors would like to thank R. Connors, S. Myers, and R. Oort, for their kindness and their technical support.

References

[1] I. Ben-Zvi, Z. Y. Jiang, G. Ingold, L. H. Yu, and W. B. Sampson, "The performance of a superconducting micro-undulator prototype," *Nucl. Instrum. Methods Phys. Res. A, Accel. Spectrom. Detect. Assoc. Equip.*, vol. 297, no. 1/2, pp. 301–305, Nov. 1990.

[2] T. Hezel, B. Krevet, H. O. Moser, J. A. Rossmanith, and R. Rossmanith, "A superconductive undulator with a period length of 3.8 mm," *J. Synchrotron Rad.*, vol. 5, pt. 3, pp. 448–450, May 1998.

[3] A. Geisler, A. Hobl, D. Krischel, R. Rossmanith, and M. Schillo, "First field measurements and performance tests of a superconductive undulator for light sources with a period length of 14 mm," *IEEE Trans. Appl. Supercond.*, vol. 13, no. 2, pp. 1217–1220, Jun. 2003.

[4] S. O. Prestemon, D. R. Dietderich, S. A. Gourlay, P. Heimann, S. Marks, G. L. Sabbi, R. M. Scanlan, R. Schlueter, B. Wang, and B. Wahrer, "Design and evaluation of a short period Nb3Sn superconducting undulator prototype," in *Proc. Part. Accel. Conf.*, 2003, pp. 1032–1034.

[5] S. O. Prestemon, D. R. Dietderich, S. E. Bartlett, M. Coleman, S. A. Gourlay, A. F. Lietzke, S. Marks, S. Mattafirri, R. M. Scanlan, R. D. Schlueter, B. Wahrer, and B. Wang, "Design, fabrication, test results of undulators using Nb_3Sn superconductor," *IEEE Trans. Appl. Supercond.*, vol. 15, no. 2, pp. 1236–1239, Jun. 2003.

[6] D. R. Dietderich, A. Godeke, S. O. Prestemon, P. T. Pipersky, N. L. Liggins, H. C. Higley, S. Marks, and R. D. Schlueter, "Fabrication of a short-period Nb_3Sn superconducting undulator," *IEEE Trans. Appl. Supercond.*, vol. 17, no. 2, pp. 1243–1246, Jun. 2007.

[7] Z. Wolf, "Algorithms to automate LCLS undulator tuning," SLAC Nat. Accel. Lab., Menlo Park, CA, USA, Tech. Rep. LCLS-TN-06-8, 2006.

[8] D. Wollmann, A. Bernhard, P. Peiffer, and T. Baumbach, "Experimental demonstration of the induction-shimming concept in superconductive undulators," *Phys. Rev. ST. Accel. Beams*, vol. 12, no. 4, pp. 040702-1–040702-6, Apr. 2009.

[9] S. Chunjarean, J. C. Jan, C. H. Hwang, P. H. Lin, and H. Wiedemann, "A new field correction scheme for superconducting undulators by modification the iron pole geometry," *Supercond. Sci. Technol.*, vol. 24, no. 5, pp. 055013-1–055013-7, Apr. 2011.

[10] D. Wollmann, A. Bernhard, S. Casalbuoni, M. Hagelstein, B. Kostka, R. Rossmanith, F. Schöck, M. Weißer, E. Steffens, G. Gerlach, and T. Baumbach, "A concept on electric field error compensation for the ANKA superconductive undulator," in *Proc. EPAC*, Edimburgh, U.K., 2006, pp. 3577–3579.

[11] A. Madur, F. Trillaud, D. Dietderich, S. Marks, S. Prestemon, and R. Schlueter, "Superconducting switch concept applied to superconducting undulator phase-error correction," in *Proc. AIP Conf.*, 2010, vol. 1234, pp. 552–555.

[12] D. Arbelaez, D. Lee, H. Pan, T. Koettig, P. Bish, S. O. Prestemon, D. R. Dietderich, and R. D. Schlueter, "Magnetic field correction concepts for superconducting undulators," *IEEE Trans. Appl. Supercond.*, vol. 23, no. 3, p. 4100104, Jun. 2013.

[13] K. Halbach, "Desirable excitation patterns for tapered wigglers," *Nucl. Instrum. Methods Phys. Res. A, Accel. Spectrom. Detect. Assoc. Equip.*, vol. 250, no. 1/2, pp. 95–99, Sep. 1986.