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Gaugino Mass without Singlets*

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Abstract

In models with dynamical supersymmetry breaking in the hidden sector, the gaugino masses in the observable sector have been believed to be extremely suppressed (below 1 keV), unless there is a gauge singlet in the hidden sector with specific couplings to the observable sector gauge multiplets. We point out that there is a pure supergravity contribution to gaugino masses at the quantum level arising from the superconformal anomaly. Our results are valid to all orders in perturbation theory and are related to the 'exact' beta functions for soft terms. There is also an anomaly contribution to the A terms proportional to the beta function of the corresponding Yukawa coupling. The gaugino masses are proportional to the corresponding gauge beta functions, and so do not satisfy the usual GUT relations.

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1 Introduction

Supersymmetry (SUSY) is arguably the most attractive mechanism to stabilize the hierarchy between the fundamental scale (*e.g.* the Planck scale $M_* \sim 10^{18}$ GeV) and the electroweak scale ($M_W \sim 100$ GeV). However, superpartners of the standard-model particles have not been observed up to energies of order M_W, so SUSY must be broken at or above the weak scale. The phenomenology of SUSY depends crucially on the mechanism of SUSY breaking and the way that SUSY breaking is communicated to the observable sector.

Communication of SUSY-breaking effects by supergravity (SUGRA) interactions is in some ways the most attractive scenario. In models of this type, SUSY is broken in a hidden sector and gravitational-strength interactions communicate SUSY breaking to the observable sector. The main advantage of this scenario lies in its theoretical appeal: the key ingredients are either present of necessity (*e.g.* SUGRA) or very well-motivated (*e.g.* hidden sectors are generically present in string theories). The main disadvantage of this scenario is that at present there is no convincing explanation for the degeneracy of squark masses required to avoid large flavor-changing neutral current effects. In the context of string theory and SUGRA models with singlets, there are also cosmological problems related to the existence of uncharged fields with almost flat potentials and interactions suppressed by powers of the Planck scale.

In order to explain the origin of the SUSY breaking scale (and hence the weak scale) the most attractive scenario is that SUSY is broken dynamically [1, 2, 3]. In recent years, it has been found that this occurs in many asymptotically-free supersymmetric gauge theories. In these models, dimensional transmutation generates the hierarchy between the SUSY breaking scale $\mu_{\rm SUSY}$ and the Planck scale, and the SUSY-breaking masses are of order $\mu_{\rm SUSY}^2/M_*$. The most important challenge of constructing phenomenologically viable models of dynamical SUSY breaking in the hidden sector is generating sufficiently large gaugino masses [3, 4]. In models without gauge singlets in the hidden sector, the gaugino mass is conventionally believed to be extremely suppressed, at most of order $\mu_{\rm SUSY}^3/M_*^2 \simeq 1$ keV. There have been a variety of solutions discussed in the literature [5, 6, 7], all of which involve gauge singlets with SUSY-breaking VEV's, and require more or less complicated model-building. It is not at all clear whether any of these solutions can work in the context of string theory, where one singlet field, the dilaton, couples to all gauge kinetic terms. Obtaining realistic gaugino masses in string theory therefore appears to require a large F component for the dilaton (in addition to the usual dilaton stabilization problem), which does not occur in conventional mechanisms for (locally) stabilizing the dilaton.[1] More generally, the presence of gauge singlet fields also causes a variety of concerns, such as cosmological problem [8, 9, 5] or destabilization of hierarchy [10].

In this paper, we point out a completely model-independent contribution to the gaugino mass whose origin can be traced to the conformal anomaly. This contribution is always present even if there are no gauge singlet fields that generate the gaugino masses at the tree-level. Therefore, no model-building gymnastics is necessary to generate gaugino masses at order $1/M_*$. This contribution to the gaugino mass is given *exactly* (to all orders in perturbation theory) by

$$m_\lambda = \frac{\beta(g^2)}{2g^2} m_{3/2}, \tag{1}$$

where $\beta(g^2) = dg^2/d\ln\mu$ is the gauge beta function. In models without singlets (or models in which the singlets do not couple to the gauge fields in the required way), Eq. (1) gives the leading effect in the gaugino mass. This has interesting phenomenological consequences. First, the gaugino mass ratios are given by ratios of beta functions, a very different result from the usual 'unified' relation. Other aspects of the phenomenology depend crucially on the scalar masses. The simplest assumption is that the scalar masses are of order $m_{3/2}$, which is much larger than the gaugino masses in Eq. (1). This scenario unfortunately suffers from quite severe fine-tuning required for electroweak symmetry breaking, but has a predictive and interesting phenomenology that we will discuss below. An alternative possibility is that the scalar masses are naturally suppressed compared to $m_{3/2}$. For example, this occurs in models with Heisenberg symmetry [11], *i.e.*, models of 'no-scale' type [12].

In complete analogy to Eq. (1), we also show that the A-terms arise proportionally to the β-function of the corresponding Yukawa coupling.

Contributions to gaugino masses that are proportional to the corresponding β-functions have been previously found in the string-based models of Ref. [13], using the results in Ref. [14]. However, those contributions depend on the moduli and therefore, unlike Eq. (1), their normalization is not purely fixed by the gravitino mass. We emphasize that the contribution considered here exists in any model, and becomes the dominant one in particular classes.

This paper is organized as follows. In Section 2, we review the important features of dynamical SUSY breaking in the hidden sector, and comment on previous work on

[1] One could still use F-component of moduli fields which appear in the gauge kinetic function at the one-loop level. Here again the stabilization is an issue, and the cosmological problem is there as well.

gaugino masses. Section 3 contains our main results. We derive formulae for gaugino masses and other $\mathcal{O}(m_{3/2})$ SUSY-breaking parameters to all orders in perturbation theory in models without gauge singlets in the hidden sector. In Section 4 and 5, we consider phenomenology and the 'μ problem.' Section 6 contains our conclusions.

2 Dynamical SUSY Breaking in the Hidden Sector without Singlets

In this section, we review the main features of SUGRA models with dynamical SUSY breaking in the hidden sector and no singlets. Our primary motivation for dynamical SUSY breaking is that it is the simplest mechanism for generating the SUSY breaking scale, and hence explaining (rather than simply stabilizing) the hierarchy between the weak scale and the Planck scale. (In fact, if we want to have a SUSY breaking scale well below the Planck scale, and we assume that the Kähler potential is 'generic', it can be shown that SUSY must be broken in the flat limit [15, 7].) We consider models without singlets because we will show below that they are not necessary to obtain large gaugino masses.

We therefore consider a model that breaks SUSY dynamically at a scale $\mu_{\rm SUSY}$ in the flat limit $M_* \to \infty$, and couple it to SUGRA. Since the model has a stable vacuum in the flat limit, we do not expect any Planck-scale VEV's.[2] This is to be contrasted with the situation in conventional hidden sector models, in which generally there are fields with VEV's of order (or larger than) the Planck scale [16, 17, 18]. In models without Planck-scale VEV's, the SUGRA scalar potential simplifies drastically. By keeping the leading $\mathcal{O}(\mu_{\rm SUSY}^4)$ terms of an expansion in $\mu_{\rm SUSY}/M_*$, one finds

$$V = |W_z|^2 - \frac{3}{M_*^2}|W|^2 + D\text{-terms} + \mathcal{O}(\mu_{\rm SUSY}^5), \tag{2}$$

irrespective of the form of the Kähler potential as long as it has a Taylor expansion with canonical kinetic term as its lowest order term: $K = z^* z + \mathcal{O}(z^3/M_*)$. (A linear term is absent if there are no singlets.) The first term is equivalent to the case of globally supersymmetric theories and has a finite (positive) value as long as SUSY is broken. The second term is used to fine-tune the cosmological constant by adding a constant term in the superpotential, related to the gravitino mass by $\langle W \rangle = m_{3/2} M_*^2$.

The soft terms in the observable sector described by the fields ϕ come from the cross terms in $(K_i W + W_i)^* K_{ij}^{-1} (K_j W + W_j) = |W_i|^2 + m_{3/2}(\phi_i W_i + \text{h.c.}) + \mathcal{O}(m_{3/2}^2)$ and $-3|W|^2 = -3 m_{3/2} W + \text{h.c.} + \mathcal{O}(m_{3/2}^2)$. Therefore, $\mathcal{O}(m_{3/2})$ terms are completely

[2] Even for models with non-renormalizable interactions suppressed by powers of Planck scale, the expectation values are often much smaller than the Planck scale.

model-independent,

$$m_{3/2}(\phi_i W_i - 3W), \tag{3}$$

and hence $A = 0$, $B = -m_{3/2}$ and $C = -2m_{3/2}$ [19].[3] The scalar masses are $\mathcal{O}(m_{3/2}^2)$ and depend on the form of the Kähler potential up to $\mathcal{O}(z^2/M_*^2)$. For instance, a term $z^* z \phi^* \phi / M_*^2$ in the Kähler potential gives additional contributions to the ϕ scalar mass squared if z has an F-component expectation value.

If there were a gauge-singlet field with an F-component VEV $F_z = \mathcal{O}(\mu_{\rm SUSY}^2)$, it could be used to generate gaugino masses in the observable sector of the same order of magnitude as the other soft SUSY breaking parameters by coupling it to the gauge kinetic function:

$$\int d^2\theta \frac{z}{M_*} \,\mathrm{tr}\, W^\alpha W_\alpha + \text{h.c.} \tag{4}$$

This operator cannot appear if the model does not contain singlets, and the standard conclusion is that the leading contribution to the gaugino mass in such models comes from higher-dimensional operators, and is therefore $\mu_{\rm SUSY}^3/M_*^2 \sim 1$ keV or smaller. Even if a model does contains singlets, the operator in Eq. (4) may be forbidden by symmetries, such as a $U(1)_R$ symmetry.

In fact, this has been regarded as one of the most serious problems in models of dynamical SUSY breaking in the hidden sector, since most of these models do not contain gauge singlets. One possibility is to use vector-like models of SUSY breaking with gauge-singlet fields having a non-generic superpotential [6]. Another possibility is to couple singlets to a model with dynamical SUSY breaking in such a way that SUSY is not restored and the singlets aquire F components [7]. Another proposal is to use a mechanism similar to the messenger $U(1)$ [24, 25] to generate expectation values for the F-component of a gauge singlet fields at two-loop order [5]. These proposals show that gaugino masses can be generated at order $\mathcal{O}(M_*^{-1})$ in models with singlets, but it remains true that a generic model of dynamical SUSY breaking appears to give extremely small gaugino masses.

Another natural possibility would be that a gaugino mass is generated at 1-loop order from massive vector-like chiral superfields with a SUSY-breaking mass term $B = -m_{3/2}$ from SUGRA. Indeed, a direct calculation appears to confirm this, giving a gaugino mass $\sim g^2 m_{3/2}/(16\pi^2)$. However, it was pointed out in Ref. [4] that one should be able to integrate out the massive vector-like matter, and write an effective low-energy theory in which the gaugino mass (if any) appears as a local operator. But we have seen that all such operators give gaugino masses suppressed by additional

[3] It is interesting that this particular form of the soft SUSY breaking parameters belongs to the ansatz in Ref. [20] that automatically extends a fine-tuning in the superpotential to the full theory.

powers of M_*^{-1}, and so the effect should be absent. In fact, Ref. [4] showed that a careful one-loop calculation using Pauli–Villars regulator gives a vanishing gaugino mass, because the Pauli–Villars regulator also has a SUSY-breaking mass from SUGRA that precisely cancels the contribution from the vector-like multiplet. The fact that the regulator necessarily breaks SUSY in models of SUSY breaking in the hidden sector is one of the ways of deriving the results we present below.

3 Gaugino Mass from Light Multiplets

In this section, we show that in models with no gauge singlets the gaugino masses in the observable sector are proportional to $\beta m_{3/2}$, where β is the beta function for the corresponding gauge group. In a similar way, the A-terms are proportional to the anomalous dimension of the corresponding Yukawa coupling.[4] The key point in our analysis is that there is no local operator that can give a gaugino mass or A term proportional to $1/M_*$. This implies that the $\mathcal{O}(M_*^{-1}) = \mathcal{O}(m_{3/2})$ contributions to these quantities (if present) are completely finite and calculable in the low-energy effective theory, since there is no counterterm for the effect. We will establish a nonzero quantum contribution to the gaugino masses and A terms using several different methods. First, we show by explicit calculation that the effect arises when we use locally supersymmetric regulators for matter loops in the observable sector. Then we give a general operator analysis that shows that the effect appears in the 1PI effective action as a direct consequence of local supersymmetry. Finally, we show that the effect can be directly understood in terms of the conformal anomaly multiplet.

3.1 Explicit Calculations

We begin by explaining how gaugino masses are generated at the quantum level when we carefully regulate the theory. (We will discuss A terms only in the next section, where we give more general arguments.) Since we are not interested in loops of SUGRA fields, it is sufficient to regulate matter and gauge loops in the presence of a fixed SUGRA background.

We would like to write an effective theory for the observable sector with the hidden sector fields integrated out. Note that we cannot integrate out the full SUGRA multiplet, since the graviton is massless. However, the contribution to the gaugino mass and A-terms we are interested in are $\mathcal{O}(M_*^{-1})$, while the exchange of propa-

[4] For gaugino masses this result holds *both* for the holomorphic and '1PI' definitions of the gaugino mass, while for the A terms there is only a 1PI definition.

gating supergravity fields is $\mathcal{O}(M_*^{-2})$.[5] At order $\mathcal{O}(M_*^{-1})$ we can therefore drop the propagating SUGRA fields and keep only the VEV of the scalar auxiliary field of the SUGRA multiplet proportional to $m_{3/2}$.

The $\mathcal{O}(M_*^{-1})$ terms have a very simple form, which is easiest to understand using the superconformal calculus formulation of SUGRA [29]. In this formulation, one first constructs an action invariant under local superconformal transformations, and then breaks local superconformal symmetry explicitly down to local super-Poincaré symmetry to define the lagrangian. Every field is assigned a Weyl weight (scaling dimension), and conformal invariance is broken explicitly by a 'compensator' field $\mathcal{E}$ with Weyl weight +1. $\mathcal{E}$ is taken to have value $\mathcal{E} = 1 + H\theta^2$, where H is the auxiliary scalar field of SUGRA, with $\langle H \rangle = m_{3/2}$. The important feature for our purposes is that H appears only in $\mathcal{E}$, and so the H dependence is determined entirely by dimensional analysis. This gives the $\mathcal{O}(m_{3/2})$ SUGRA effects in a very simple form:

$$\begin{aligned} \mathcal{L} \; = \; & \int d^2\theta d^2\bar{\theta} \sum_{\Omega} \left[1 + \tfrac{1}{2}\left(2 - \dim(\Omega)\right) m_{3/2}(\theta^2 + \bar{\theta}^2)\right] \Omega \\ & + \left(\int d^2\theta \sum_{\Xi} \left[1 + (3 - \dim(\Xi)) m_{3/2}\theta^2\right] \Xi + \text{h.c.} \right) + \mathcal{O}(m_{3/2}^2), \end{aligned} \tag{5}$$

where 'dim' denotes the total mass dimension of fields and derivatives in the operators Ω and Ξ (*i.e.* the coupling constants do not contribute to the dimension). The close connection between the coefficient of the linear term in $m_{3/2}$ and the dimension is a key ingredient in our results. Note that we reproduce the well-known fact that there is no $\mathcal{O}(m_{3/2})$ contribution to the gaugino mass or trilinear scalar couplings in the local lagrangian of supergravity.

The universal nature of the $m_{3/2}$ dependence given above means that if we regulate the theory in a supersymmetric manner, the regulator will depend on $m_{3/2}$ in a well-defined way. This SUSY breaking in the regulator sector will induce finite SUSY-breaking effects at loop level that give the contribution to the gaugino mass we are discussing.

For example, we can regulate SUSY QCD with $F \leq 2N$ flavors by imbedding it in a finite $\mathcal{N} = 2$ theory.[6] We can add $2N - F$ vector-like quarks and an adjoint chiral multiplet Φ together with the superpotential $W = \sqrt{2}\bar{Q}\Phi Q$ to obtain a finite $\mathcal{N} = 2$ theory. Adding mass terms for Φ and the extra quarks breaks $\mathcal{N} = 2$ SUSY down to $\mathcal{N} = 1$ maintaining finiteness of the theory, while only the desired degrees of freedom

[5] This is true even if we take into account the constant term in the superpotential proportional to M_* that is needed to cancel the cosmological constant.

[6] These theories are known to be finite even non-perturbatively, but this is not important for our analysis.

survive at low energy. We then compute the physical gaugino mass in this theory at 1 loop, including the contribution from the regulator fields.[7] Because the B-term for all massive fields is $-m_{3/2}$, the adjoint contributes $(g^2/16\pi^2)Nm_{3/2}$ at one loop, while the additional vector-like quarks contribute $(g^2/16\pi^2)(2N - F)m_{3/2}$. (These contributions can be viewed as gauge-mediated SUSY breaking [24, 25, 30] from the regulator sector.) The result at one loop is therefore

$$m_\lambda = \frac{g^2}{16\pi^2}(3N - F)m_{3/2}. \tag{6}$$

note that the result is proportional to the 1-loop beta function coefficient $b_0 = 3N - F$ of the low-energy theory. We will show in the next subsection that this result generalizes to arbitrary theories (with arbitrary regulators) and to all orders in perturbation theory.

We can also compute the contributions of vector-like chiral multiplets using Pauli–Villars regularization. When computing the physical gaugino mass at one loop, the massive Pauli–Villars fields give a contribution to the gaugino mass of $-g^2T_r m_{3/2}/(16\pi^2)$, where T_r is the index of the representation and the minus sign comes from the 'wrong' statistics of the Pauli-Villars field. Again this is consistent with Eq. (6).

The contribution of the gauge multiplet can also be obtained by imbedding the theory into an $\mathcal{N} = 4$ theory. We introduce 3 additional chiral multiplets Φ_j in the adjoint representation with superpotential $W = \sqrt{2}\,\mathrm{tr}(\Phi_1[\Phi_2, \Phi_3])$, and add mass terms for the Φ's to break the theory down to $\mathcal{N} = 1$. At one loop, the regulator fields give a contribution to the gaugino mass $-3g^2Nm_{3/2}/(16\pi^2)$, where the factor of 3 comes from the 3 adjoints.

Finally we can consider dimensional reduction [31], in which the d-dimensional superconformal invariance modifies Eq. (5) to

$$\begin{aligned}\mathcal{L} \;=\; &\int\! d^2\theta d^2\bar\theta \sum_\Omega \left[1 + \tfrac{1}{2}\left(d - 2 - \dim(\Omega)\right) m_{3/2}(\theta^2 + \bar\theta^2)\right]\Omega \\ &+ \left(\int\! d^2\theta \sum_\Xi \left[1 + (d - 1 - \dim(\Xi)) m_{3/2}\theta^2\right]\Xi + \text{h.c.}\right) + \mathcal{O}(m_{3/2}^2), \end{aligned} \tag{7}$$

where we define the Weyl weights of fundamental superfields to be equal to their mass dimension in $d = 4 - \epsilon$ dimensions. Note that the vector superfield is dimensionless, and so the gauge biliner $W_\alpha W^\alpha$ has dimension 3 for all d. Therefore by Eq. (7) the bare gauge kinetic term is

$$\mathcal{L}_{\text{gauge}} = \int\! d^2\theta\,(1 - \epsilon m_{3/2}\theta^2)\frac{1}{4g_0^2}W_A^\alpha W_{\alpha A}, \tag{8}$$

[7] See the appendix for an explanation of why the auxiliary equation of motion does not produce an additional contribution to the gaugino mass via the Konishi anomaly.

where g_0^2 is the bare coupling and A is a gauge index. This lagrangian contains a bare gaugino mass equal to $-m_{3/2}\epsilon$ that combines with the $1/\epsilon$ terms in the bare gauge coupling to give a finite gaugino mass. At one loop, we obtain

$$m_\lambda = \left(m_{3/2}\epsilon\right)\left(\frac{b_0 g^2}{16\pi^2}\frac{1}{\epsilon}\right). \tag{9}$$

We could consider other supersymmetric regulators, such as higher-derivative regulators [32] or an infinite tower of Pauli–Villars regulators [33], but we will stop here.

We can gain additional insight into this result if we note that the proportionality between the gaugino mass and the beta function of the low-energy effective field theory is preserved across effective field theory thresholds. This can be seen by direct 1-loop calculation, but it follows more directly from the method of 'analytic continuation into superspace' [34, 35]. At 1-loop order, the gauge coupling and gaugino mass can be grouped into a chiral superfield

$$S(\mu) = \frac{1}{2g^2(\mu)} - \frac{i\Theta}{16\pi^2} - \frac{m_\lambda(\mu)}{g^2(\mu)}\theta^2, \tag{10}$$

and the effects of a threshold at the scale M is calculated using the 1-loop RG equation $\mu dS/d\mu = b/(16\pi^2)$:

$$S_{\rm eff}(\mu) = S(\mu_0) + \frac{b}{16\pi^2}\ln\frac{M}{\mu_0} + \frac{b_{\rm eff}}{16\pi^2}\ln\frac{\mu}{M}. \tag{11}$$

Here $\mu_0 > M$ is the renormalization scale used to define the fundamental theory, and $\mu < M$ is the renormalization scale in the effective theory. In all cases of interest, the scale M can be written as a chiral superfield. For example, if we are integrating out a massive vector-like chiral field, its mass M appears in the superpotential and can be analytically continued to a full chiral superfield. The other possibility is that the mass threshold is due to the VEV of a chiral superfield, which can partially break the gauge symmetry and/or give mass to some vector-like multiplets. In all cases, it is easily checked that Eq. (11) is correct in the limit of unbroken supersymmetry.

If $m_{3/2} \ll M$, the theshold at the scale M is approximately supersymmetric. In this case, Ref. [34] showed that Eq. (11) remains correct in the presence of SUSY breaking if the θ-dependent components of M are included. (There are additional subtleties beyond 1 loop; see Ref. [35].) By Eq. (5), this amounts to the substitution $M \to M(1 + m_{3/2}\theta^2)$, which gives

$$\frac{m_{\lambda,\rm eff}(\mu)}{g^2_{\rm eff}(\mu)} = \frac{m_\lambda(\mu_0)}{g^2(\mu_0)} + \frac{b_{\rm eff} - b}{16\pi^2}m_{3/2} = \frac{b_{\rm eff}}{16\pi^2}m_{3/2}. \tag{12}$$

Note that this result includes the correct 1-loop RG evolution down to the scale μ.

To make this more explicit, consider for example $SU(N)$ gauge theory with one flavor broken down to $SU(N-1)$ by the Higgs mechanism. We take a superpotential $W = \lambda X(Q\bar{Q} - v^2)$, where X is a singlet and Q, $\bar{Q}$ are one flavor in the fundamental of $SU(N)$. In the SUSY limit, we find $\langle Q \rangle = \langle \bar{Q} \rangle = v$. In the presence of soft SUSY breaking terms, the potential is

$$V = \lambda^2 |Q\bar{Q} - v^2|^2 + \lambda^2(|Q|^2 + |\bar{Q}|^2)|X|^2 + 2m_{3/2}(\lambda X v^2 + \text{h.c.}). \tag{13}$$

We find $\langle X \rangle = -m_{3/2}/\lambda$, and hence $F_Q = F_{\bar{Q}} = -\lambda\langle X \rangle v = m_{3/2}v$. The low-energy effective superfield coupling is

$$S_{\text{eff}}(\mu) = S(\mu) + \frac{1}{16\pi^2}\ln(\frac{Q\bar{Q}}{\mu^2}) \tag{14}$$

where $S(\mu)$ is the coupling of the high energy theory. The F components in Q and $\bar{Q}$ modify the gaugino mass by

$$\Delta\left(\frac{m_\lambda}{g^2}\right) = -\frac{1}{16\pi^2}\left(\frac{F_Q}{Q} + \frac{F_{\bar{Q}}}{\bar{Q}}\right) = -\frac{2}{16\pi^2}m_{3/2}. \tag{15}$$

This factor of 2 is the difference in beta-function coefficients, so the resulting low-energy gaugino mass is precisely what one obtains with our formula (6) applied to the effective $SU(N-1)$ gauge theory.

3.2 General Argument

We have seen that at one loop the gaugino mass is proportional to the beta function of the low-energy theory. This strongly suggests that there is a close connection between the effect we are discussing and the conformal properties of the theory. We now give a general argument that shows this connection explicitly, and generalizes the results of the previous subsection to arbitrary theories and to all orders in perturbation theory.

The starting point is a definition of the 1PI gaugino mass using an operator analysis in superspace, following Ref. [35]. A useful definition of the 1PI gauge coupling and gaugino mass can be obtained by considering the 1PI gauge 2-point function expanded at short distances (compared to m_λ^{-1}). The leading term in the expansion in $1/\Box$ is

$$\Gamma_{\text{1PI}} = \int d^4x \int d^2\theta d^2\bar{\theta}\, W_A^\alpha R(\Box)\left(-\frac{D^2}{8\Box}\right) W_{\alpha A} + \text{h.c.} + \cdots \tag{16}$$

The function $R(\Box)$ has a logarithmic dependence on $\Box$ that is the source of the 1PI renormalization group. The identity

$$\int d^2\theta d^2\bar{\theta}\, W_A^\alpha \left(-\frac{D^2}{8\Box}\right) W_{\alpha A} = \frac{1}{2}\int d^2\theta\, W_A^\alpha W_{\alpha A} \tag{17}$$

shows that the leading term in Eq. (16) is local in coordinate space even though it is nonlocal in superspace. A general operator analysis [35] can be used to show that all other operators that contribute to the gauge 2-point function are suppressed by powers of $1/\Box$. This shows that the superfield R contains the 1PI gauge coupling and gaugino mass as its lowest components:

$$R(\Box = -\mu^2) = \frac{1}{g^2(\mu)} - \left(\frac{m_\lambda(\mu)}{g^2(\mu)}\theta^2 + \text{h.c.}\right) + \cdots \tag{18}$$

For a more complete discussion (including the meaning of the $\theta^2\bar{\theta}^2$ components of R) see Ref. [35].

We can now write the covariant generalization of Eq. (16) in a SUGRA background using the results quoted in Eq. (5). Since $W_\alpha(D^2/\Box)W^\alpha$ has dimension 2, the $\mathcal{O}(m_{3/2})$ terms are obtained simply by making the replacement

$$R(\Box) \to R(\Box[1 - (m_{3/2}\theta^2 + \text{h.c.})]). \tag{19}$$

Expanding the terms linear in $m_{3/2}$, we obtain the gaugino mass

$$m_\lambda = \frac{g^2 m_{3/2}}{2}\mu\frac{dR}{d\mu} = -\frac{m_{3/2}}{2g^2}\mu\frac{dg^2}{d\mu} = -\frac{\beta(g^2)}{2g^2}m_{3/2}. \tag{20}$$

Note that g and m_λ are 1PI renormalized couplings, defined in a 'superfield' scheme where they are the components of a real superfield. This result generalizes our previous result to all orders in perturbation theory.

This argument shows very directly the connection between the quantum contribution to the gaugino mass and the conformal anomaly. The point is that SUGRA covariance relates the $\mathcal{O}(m_{3/2})$ soft breaking terms to the scaling dimension of the operators in the SUSY limit. At tree level, this relation is given in Eq. (5); our analysis above shows that this relationship is preserved at the loop level as well, so that the $\mathcal{O}(m_{3/2})$ terms depend on the quantum scaling dimension of the operators. This arises because the physical gaugino mass must be read off from the 1PI effective action in the presence of a SUGRA background. SUGRA covariance mandates the replacement $\Box \to \Box[1-(m_{3/2}\theta^2\text{h.c.})]$, which means that the SUGRA covariant version of $\ln\Box$ contains a local SUSY-breaking piece.

We now briefly consider A terms. For a dimension n term in the superpotential of the form $W = \lambda\Phi_1 \cdots \Phi_n$, Eq. (5) gives a tree-level soft-breaking term $V_{\text{soft}} = (n-3)m_{3/2}\lambda\phi_1 \cdots \phi_n$. We now read off the quantum corrections to this from the kinetic terms in the 1PI effective action. The leading term in the expansion in $1/\Box$ is

$$\int d^2\theta d\bar{\theta}^2\, \Phi_r^\dagger Z_r(\Box[1-(m_{3/2}\theta^2 + \text{h.c.})])\Phi_r + \mathcal{O}(m_{3/2}^2). \tag{21}$$

The 1PI renormalized wavefunction and A terms can be defined by appropriate components of $Z_r(\Box = -\mu^2)$. We then find that the A-type terms renormalized at a scale μ^2 are

$$A_n(\mu) = \left(n - 3 - \frac{1}{2}\sum_{r=1}^{n}\gamma_r(\mu)\right) m_{3/2} \tag{22}$$

where

$$\gamma_r(\mu) = \mu\frac{d\ln Z_r}{d\mu} \tag{23}$$

is the anomalous dimension. Notice that the right-hand-side of Eq. (22) is proportional to the quantum dimension of the chiral operator minus 3. We see that trilinear soft terms are proportional to the beta function of the corresponding Yukawa coupling:

$$A_3 = -m_{3/2}\mu\frac{d\ln\lambda}{d\mu}. \tag{24}$$

The results we have quoted above are valid to all orders in perturbation theory, and we make some comments on scheme dependence. The preceding derivation makes clear that the results hold in any scheme in which the SUSY-breaking couplings are treated as higher components of superfield couplings. In Ref. [35, 38] it was shown that such a definition is always possible to all orders in perturbation theory, and this class of schemes were called ‘superfield coupling schemes’. In the literature there are many examples of ‘exact’ results for soft terms whose derivation is based on the all-orders beta function of Novikov, Shifman, Vainshtein and Zakharov (NSVZ) [36]. If these results truly depended on the precise form of the all-orders beta function, they would be valid only in the NSVZ scheme where the beta function takes the form of Ref. [36]. However, the study in Ref. [35] shows that these results are in fact valid in any superfield coupling scheme. One example of such an ‘exact’ relation is [37]

$$\frac{g^2 m_\lambda}{\beta} + \frac{1}{b_0}\sum_r T_r\left(\ln Z_r(\mu^2)|_{\theta^2} - \frac{\gamma_r g^2 m_\lambda}{\beta}\right) = \text{RG invariant}. \tag{25}$$

In the class of theories we are considering, the second term on the left-hand-side vanishes by the results for A terms derived above. We then obtain

$$\frac{g^2 m_\lambda}{\beta} = -\tfrac{1}{2}m_{3/2} = \text{RG invariant}. \tag{26}$$

It was pointed out in Ref. [37] that this relation is in general valid only in the absence of Yukawa interactions. Our results imply that this relation is true in minimal supergravity even in the presence of other interactions, and hold in any superfield scheme.

3.3 Superconformal Anomaly Multiplet

In this subsection, we present an alternative argument which justifies Eqs. (20) and (22). The argument assumes the existence of a manifestly supersymmetric and holomorphic regularization, as those based on finite $\mathcal{N}=2$ or $\mathcal{N}=4$ theories. However, it does not depend on the details of the regularization procedure.

In an explicitly regulated theory, the ultraviolet cutoff is provided either by the mass of the regulators (*e.g.*, Pauli–Villars fields or extra adjoint and quark fields in $\mathcal{N}=2$ regularization) or by the inverse mass scale of the higher-dimensional terms (*e.g.*, higher-derivative regularization), or both. For our purposes, we refer to the ultraviolet cutoff generically by M. The assumption of a holomorphic regularization implies that the cutoff M can be regarded as a chiral superfield spurion appearing in the superpotential. From Eq. (5) it is easy to see that the effect of supersymmetry breaking is a simple replacement $M \to M(1+m_{3/2}\theta^2)$, independent of details of the regularization procedure. Because of manifest holomorphy, the dependence on the cutoff M fixes the effect of supersymmetry breaking at $\mathcal{O}(m_{3/2})$.

The Wilsonian renormalization group invariance states that one can change the cutoff M without changing low-energy physics as long as one changes the bare parameters in the Lagrangian in a specific manner. To be explicit, the statement is that for any (physical) correlation function G

$$M\frac{d}{dM}G = \left(M\frac{\partial}{\partial M} + \frac{b_0}{16\pi^2}\frac{\partial}{\partial S} + \sum_i M\frac{d\ln Z_i}{dM}\frac{\partial}{\partial \ln Z_i}\right) G = 0 \qquad (27)$$

where S is defined in Eq. (10). Here, the index i runs over all chiral superfields in the theory, and b_0 is the one-loop beta function coefficient of the gauge coupling constant. We also assumed that there is no dimensionful coupling constant in the theory; if any, one can trivially extend the analysis by including the dimensionful terms as an explicit breaking of scale invariance to be added to the right-hand side of Eq. (27). Note that $M d\ln Z_i/dM = \gamma_i$ to the lowest order in θ, $\bar{\theta}$.

To find the effect of the replacement $M \to M(1+m_{3/2}\theta^2)$ in the presence of supersymmetry breaking in the hidden sector, one can use the renormalization group invariance and integrate Eq. (27) from a constant M to $M(1+m_{3/2}\theta^2)$. This is equivalent to the technique of 'analytic continuation to superspace' [34, 35]. The derivative with respect to the cutoff M inserts the trace of the energy-momentum tensor Θ^μ_μ to the correlation function. It has been known for more than two decades [39] that the trace of the energy-momentum tensor belongs to a chiral superfield Φ called 'anomaly-multiplet' whose F-component is $\Theta^\mu_\mu + i\frac{3}{2}\partial_\mu j^\mu_R$, where j^μ_R is the $U(1)_R$ current. From the above M derivative in the Wilsonian effective action, we get

$$\Phi = \frac{b_0}{8\pi^2} W_\alpha W^\alpha + \sum_i \frac{\gamma_i}{16\pi^2} \bar{D}^2(\phi_i^\dagger e^V \phi_i). \tag{28}$$

Note that the first term can be fixed by the $U(1)_R$ anomaly, while the second term gives a total derivative to the imaginary part of Φ and hence cannot be determined from the $U(1)_R$ anomaly. One can easily derive this from Eq. (27) by noting that derivatives with respect to S and $\ln Z_i$ pulls down the $W_\alpha W^\alpha$ and $\bar{D}^2(\phi_i^\dagger e^V \phi_i)$ operators from the action in the path integral. This equation is exact to all orders, once the one-loop gauge beta function b_0 is used.

Now it is easy to see that the integration of Eq. (27) from a constant M to $M(1 + m_{3/2}\theta^2)$ produces the gaugino mass and the A-terms. The lowest component of the anomaly multiplet Φ is the sum of the gaugino-bilinear $\lambda_\alpha\lambda^\alpha$, and the operator $F_i^* A_i$ which gives the A-terms upon solving the auxiliary equations of motion for F_i. Here, A_i (F_i) is the lowest (highest) component in the chiral superfield ϕ_i. This immediately justifies Eqs. (20) and (22).

The above argument leads to Eq. (20) at the one-loop level, which is exact in the 'holormorphic' definition of the gauge coupling constant and the gaugino mass employed here but not in the 'canonical' definition which admits a more direct physical interpretation. The justification of Eq. (20) requires an additional step to go from the 'holomorphic' definition to the 'canonical' definition by changing the normalization of the chiral and gauge multiplets to the canonical normalization. This rescaling of the vector multiplet induces an anomalous Jacobian in the Fujikawa measure which changes both the gaugino mass and the gauge beta function in the same manner [37].[8]

4 Phenomenology

In absence of singlet fields in the hidden sector, we have seen that gaugino masses are generated, but they turn out to be of order $\alpha m_{3/2}$ rather than $m_{3/2}$. Since we expect

[8]This is not the way it was discussed in [37]. The explanation here is based on an extension of the analysis in [40]. In a manifestly supersymmetric calculation, the contribution of the rescaling of the gauge multiplet comes from the Konishi anomaly of the b-ghost chiral superfield [40], by going from the original Lagrangian $\int d^4\theta (S + S^\dagger)\bar{b}b$ to the canonical one.

squark and slepton masses to be of order $m_{3/2}$, the phenomenology is quite different from conventional hidden sector models.

Before entering into this discussion it is worth questioning whether the scalar masses are necessarily of order $m_{3/2}$ without additional suppressions. Unlike the gaugino masses and A terms, the scalar masses and $B\mu$ terms are not calculable in the low-energy effective theory due to the presence of the counterterms

$$\int d^2\theta d^2\bar{\theta}\, \frac{z^\dagger z}{M_*^2} Q^\dagger Q \quad \int d^2\theta d^2\bar{\theta}\, \frac{z^\dagger z}{M_*^2} H_u H_d, \tag{29}$$

where z are hidden-sector fields, Q are observable-sector matter fields, and $H_{u,d}$ are Higgs fields. The coefficients of these terms can be adjusted so that the scalar masses and $B\mu$ terms are of order $\alpha^2 m_{3/2}^2$ rather than $m_{3/2}^2$. This appears to be a fine-tuning of order $\alpha^2 \sim 10^{-4}$, but it is possible that it could be the consequence of a more fundamental theory such as string theory. (For example, the scalar masses are naturally suppressed in 'no-scale' models [12, 11].) More generally, it is worth noting that if the counterterms are chosen to make all soft terms of the same order, there is no fine-tuning evident in the low-energy effective theory below the Planck scale. In such a theory, the main differences with conventional hidden-sector models are that the gravitino is much heavier than the other superpartners, and that gaugino masses satisfy the specific relations discussed below.

We now turn to the phenomenological consequences of the (probably more likely) scenario in which scalar masses are of order $m_{3/2}$. In the case of the minimal supersymmetric extension of the Standard Model, the gaugino masses at the weak scale are

$$M_1 = \frac{11\alpha}{4\pi \cos^2\theta_W} m_{3/2} = 8.9 \times 10^{-3} m_{3/2}, \tag{30}$$

$$M_2 = \frac{\alpha}{4\pi \sin^2\theta_W} m_{3/2} = 2.7 \times 10^{-3} m_{3/2}, \tag{31}$$

$$M_3 = -\frac{3\alpha_s}{4\pi} m_{3/2} = -2.6 \times 10^{-2} m_{3/2}. \tag{32}$$

Electroweak gaugino masses receive also contributions from finite one-loop diagrams with Higgs and Higgsino exchange. If the supersymmetric Higgs mass μ is of the same order of $m_{3/2}$, this contribution is comparable to those of Eqs. (30)–(31). In the limit in which M_W is much smaller than both μ and the pseudoscalar Higgs mass m_A, the total result for the electroweak gaugino masses becomes

$$M_1 = \frac{\alpha}{4\pi \cos^2\theta_W} m_{3/2} \left[11 - f(\mu^2/m_A^2)\right], \tag{33}$$

$$M_2 = \frac{\alpha}{4\pi \sin^2\theta_W} m_{3/2} \left[1 - f(\mu^2/m_A^2)\right], \tag{34}$$

$$f(x) = \frac{2x \ln x}{x-1}. \tag{35}$$

The present LEP bound on the chargino mass requires $M_2 \gtrsim M_W$. This translates into a lower bound on the gravitino mass $m_{3/2}$ of about 30 TeV for $\mu^2/m_A^2 = 1$. This bound decreases for larger values of μ^2/m_A^2 and it is about 8 TeV for $\mu^2/m_A^2 = 8$. The gluino mass is heavier than 200 GeV as long as $m_{3/2} > 8$ TeV. As mentioned above, the scalar masses are expected to be of the same order of magnitude as the gravitino mass. This then implies somewhat dishearteningly large squark and slepton masses, and requires considerable fine-tuning in the electroweak symmetry breaking.

For $\mu^2/m_A^2 \gtrsim 3$, we find $|M_1| < |M_2|$ and an almost pure B-ino is likely to be the lightest supersymmetric particle (LSP). In this case, the LSP relic abundance overcloses the Universe. For instance, assuming that the three families of sleptons are degenerate with mass $m_{\tilde{\ell}}$ and the squarks are heavier, the LSP contribution to the present energy density, in units of the critical density, is

$$\Omega_{\rm LSP} h^2 \simeq 90 \left(\frac{100~{\rm GeV}}{m_{\chi^0}} \right)^2 \left(\frac{m_{\tilde{\ell}}}{\rm TeV} \right)^4, \tag{36}$$

where h is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}. The problem of the large $\Omega_{\rm LSP}$ could be resolved if some sfermion masses or μ are much smaller than the typical scale $m_{3/2}$. The LSP annihilation cross section can then be increased either by the light sfermion exchange in the t-channel or by the s-channel Z exchange induced by gaugino-higgsino mixing. A more radical solution is to invoke early LSP decay caused by some R-parity violation in the theory.

For $\mu^2/m_A^2 \lesssim 3$, we find the more unconventional possibility that the W-ino is lighter than the B-ino. The mass splitting between the neutral and charged W-inos belonging to the same $SU(2)$ triplet is induced by electroweak breaking, but occurs (both at the classical and the quantum level) only at order M_W^4. In the limit $\mu \gg M_{1,2}, M_W$, the charged and neutral W-ino masses are

$$m_{\chi^\pm} = M_2 - \frac{M_W^2}{\mu} \sin 2\beta + \frac{M_W^4}{\mu^3} \sin 2\beta, \tag{37}$$

$$m_{\chi^0} = M_2 - \frac{M_W^2}{\mu} \sin 2\beta - \frac{M_W^4 \tan^2 \theta_W}{(M_1 - M_2)\mu^2} \sin^2 2\beta, \tag{38}$$

where $\tan\beta$ is the ratio of Higgs vacuum expectation values. From Eqs. (37)–(38) we infer that the neutral W-ino state is the LSP. W-ino annihilation in the early Universe is very efficient, since two neutral W-inos can produce W boson pairs and charged and neutral W-inos can produce fermion pairs via W exchange. Neglecting for simplicity the co-annihilation channels, we find

$$\Omega_{\rm LSP}h^2 \simeq 5\times 10^{-4}\left(\frac{m_{\chi^0}}{100\ {\rm GeV}}\right)^2 . \tag{39}$$

The neutralino does not cause any problem with relic overabundance, but cannot be used as a cold dark matter candidate [41].

The chargino search at LEP is more difficult in the case of a pure W-ino LSP than in the case of B-ino LSP [42]. Because of the small mass difference between $\chi^\pm$ and χ^0 (see Eqs. (37)–(38)) chargino production leads to extremely soft final states, and detection could require a photon-tagging technique (see *e.g.* the analysis in Ref. [43]). For very small mass difference, the chargino is so long-lived that it could be observed through anomalous ionization tracks with little associated energy deposition in calorimeters. Indeed, the average distance travelled by a chargino with energy E is

$$L=\left(\frac{\rm GeV}{m_{\chi^\pm}-m_{\chi^0}}\right)^5\left(\frac{E^2}{m^2_{\chi^\pm}}-1\right)^{1/2}\times 10^{-2}\ {\rm cm}. \tag{40}$$

This distance could well be macroscopic and exceed the detector size when μ is of the order of the gravitino mass, since $m_{\chi^\pm}-m_{\chi^0}\sim M_W^3/\mu^2$, see Eqs. (37)–(38). Quasi-stable electromagnetically charged particles can also be searched at hadron colliders. Moreover, at hadron colliders, the search can also proceed through the conventional missing energy signature, which can now be renforced by an effectively invisible chargino decay, whenever the chargino decays promptly. In particular, the most promising missing energy signal comes from gluinos, which are strongly produced and decay into a pair of quarks and a neutral or charged invisible W-ino.

The most unpleasant feature of the scenario presented here is the large hierarchy between scalar and gaugino masses. Heavy scalars, however, help weakening the problem with flavor-changing neutral current processes from supersymmetric loop effects. A certain degree of degeneracy among scalars is still required, but this can well be a consequence of a flavor symmetry. This problem is common to all hidden sector models and not special to this particular framework. Here it has been alleviated at the price of more fine tuning in the electroweak breaking condition.

It should be noted that even when supersymmetry breaking is mainly in the D-terms rather than F-terms in the hidden sector, the squark degeneracy is not guaranteed, contrary to the claim of Ref. [27]. If there is a large D-term expectation value D, the auxiliary equation of motion insures that there is at least one scalar field which generates $D=z^*Qz\sim\mathcal{O}(\mu^2_{SUSY})$, where Q is the charge of the z field under the gauge generator. Then the operator $\int d^4\theta(z^*e^Vz)\phi^*\phi/M_*^2$ gives contributions to the observable field ϕ scalar mass of order $m_{3/2}$, which do not preserve squark and slepton degeneracy even in models without F-term.

An interesting feature of the scenario is that one can naturally justify the absence of new phases in the soft breaking terms and therefore satisfy the experimental constraints on CP violating processes. In the minimal supersymmetric model, there are five possible sources of CP-violating phases: μ, $B\mu$, M_i $(i = 1, 2, 3)$, A. The physically observable phases are only those combinations that are invariant under $U(1)_R$ and Peccei–Quinn phase rotations. In our framework there is just one parameter, $m_{3/2}$, that breaks $U(1)_R$ and just one, μ, that breaks PQ, therefore there is no physical phase. This makes the constraints on neutron and electron electric dipole moments automatically satisfied.

If the origin of $B\mu$ is different than the universal B-term in Eq. (3), this feature may be spoiled. The question then becomes somewhat model dependent, and it is connected with the μ problem that will be addressed in the next section.

Our framework does not address the structure of the scalar masses, which depends on the specific form of the Kähler potential, and therefore nothing can be said about possible imaginary parts of the squark mass matrix. These phases can lead to CP violation in flavor-violating processes, like ϵ_K, and depend on the underlying flavor theory.

Finally we want to point out a major cosmological advantage of the theories discussed here. Since there is no light ($\sim m_{3/2}$) gauge singlet field with Planck-scale expectation value, there is no cause for the cosmological Polonyi problem [8, 9, 5].

5 μ Problem

An important virtue of hidden sector supersymmetry breaking is the ease of generating the μ parameter in the low-energy superpotential at the correct order of magnitude. This mechanism [44] relies on the operator

$$\int d^4\theta \frac{z^*}{M_*} H_u H_d, \tag{41}$$

where z is a hidden sector field with an F-component, $F_z = \mathcal{O}(\mu_{SUSY}^2)$. Then this operator produces $\mu = \mathcal{O}(\mu_{SUSY}^2/M_*)$, which is appropriately of the order of the weak scale. If there is no gauge-singlet field in the hidden sector, however, the above operator is forbidden. Then a natural question is what alternatives are possible.

If the hidden sector model is vector-like [21, 22], the μ term can be generated by the operator

$$\int d^2\theta \frac{QQ}{M_*} H_u H_d. \tag{42}$$

For instance the fields Q can be chosen to belong to the $SP(N)$ gauge group in the hidden sector together with the superpotential $W = \lambda S_{ij} Q^i Q^j$. If $N_f = N_c + 1$, the quantum modified constraint $\mathrm{Pf}(Q^i Q^j) = \Lambda^{2N_f}$ does not allow a supersymmetric vacuum consitent with the requirement $F_S = 0$. In a limit where λ can be regarded as perturbative, the quantum modified constraint forces many of the $Q^i Q^j$ meson operators to acquire expectation values of $\mathcal{O}(\Lambda^2)$. The supersymmetry breaking scale is $\mu^2_{SUSY} \sim \lambda\Lambda^2$. If λ is order unity, the quantum modified constraint may not be satisfied exactly; one cannot reliably calculate the meson operator expectation values. Still, we expect the meson operators to have expectation values of the same order of magnitude. This operator then gives rise to a μ-parameter of weak-scale size.

The operator in Eq. (42) also generates a $B\mu$ term $\mathcal{O}(\mu_{SUSY} m_{3/2})$, if the Q fields acquire non-vanishing vacuum expectation values in their auxiliary components. This may seem a generic feature, but this is not always the case. If the supersymmetry-breaking sector possesses an R-symmetry unbroken at the vacuum, then the fields Q cannot acquire non-vanishing vacuum expectation values in their auxiliary components. In fact, an effective theory analysis suggests that this is indeed the case [45]. A μ term $\mathcal{O}(m_{3/2})$ is generated, but no $B\mu$ terms $\mathcal{O}(\mu_{SUSY} m_{3/2})$. The B term is originated from the universal contribution $B = -m_{3/2}$, and hence its phase is always related to the gaugino mass. The assumed R symmetry is a property of the supersymmetry-breaking sector in the flat limit. The complete supergravity theory breaks explicitly the R symmetry, in particular in the constant term in the superpotential chosen to fine tune the cosmological constant.

Notice also that the operator

$$\int d^4\theta \frac{zz^*}{M_*^2} H_u H_d \tag{43}$$

generates a $B\mu$ term, after supersymmetry breaking. This term is of the correct order of magnitude, the weak scale, and therefore it is not phenomenologically dangerous. However it spoils the simple relation between the gaugino mass and the B term and it can introduce irremovable phases. Nevertheless it is easy to imagine that Peccei-Quinn-like symmetries of the underlying supergravity theory forbid the occurance of this operator.

If the hidden sector is chiral, we cannot find an operator that generates the μ-term at the desired order of magnidue. There are at least three possibilities to consider in this case. One is the generation of the μ-parameter from loop diagrams, the second is from large expectation values and non-renormalizable interactions, and the third is the Next-to-Minimal Supersymmetric Standard Model (NMSSM).

The μ-parameter can be generated by a one-loop diagram of vector-like fields with a B-term [46]. For instance, one can introduce vector-like fields with the same (opposite) quantum numbers of left-handed quarks Q ($\bar{Q}$) and right-handed down-quarks D ($\bar{D}$). With the superpotential

$$W = QDH_d + \bar{Q}\bar{D}H_u + m_Q\bar{Q}Q + m_D\bar{D}D, \tag{44}$$

together with the universal B-terms for m_Q and m_D, one generates both μ and $B\mu$. Due to an accidental cancellation [47] (which was later interpreted in Ref. [34]), $m_{H_u}^2$ or $m_{H_d}^2$ are not generated at the one-loop level.

Another possibility is to employ a field with a flat potential lifted only by non-renormalizable interactions such that it acquires a large expectation value. The global symmetry of the model restricts the possible terms in the superpotential, which then generates the μ-term at the desired magnitude. The first of such example was given in Ref. [48], with a global Peccei–Quinn symmetry imposed on the model, which gives an DSFZ-type axion. All quark, lepton superfields carry the PQ charge $+1/2$, while the H_u and H_d -1. The μ-term is forbidden in the superpotential. The model has two standard-model singlet fields $P(-1)$ and $Q(n)$ and right-handed neutrinos N. The charge n is a model-dependent integer. The allowed superpotential is then

$$W = QdH_d + QuH_u + LeH_d + LNH_u + PNN + \frac{1}{M_*^{n-2}}P^nQ + \frac{1}{M_*^{n-2}}H_uH_dP^{n-2}Q. \tag{45}$$

Here, we suppressed all coupling constants and retained only the dependence on the cutoff-scale M_*. The Yukawa coupling PNN induces a negative mass squared for the P field, which together with the $|P^n|^2/M_*^{2n-4}$ potential from P^nQ/M_*^{n-2} term in the superpotential generates an expectation value $\langle P\rangle = \mathcal{O}(m_{3/2}M_*^{n-2})^{1/(n-1)}$. The supersymmetry breaking effects in Eq. (3) give a term $(n-2)m_{3/2}P^nQ/M_*^{n-2}$ in the potential, which also foces Q to acquire an expectation value $\langle Q\rangle = \mathcal{O}(m_{3/2}M_*^{n-2})^{1/(n-1)}$. Then the μ-parameter is automatically of the desired order of magnitude, $\mu = P^{n-2}Q/M_*^{n-2} = \mathcal{O}(m_{3/2})$. Furthermore with the choice $n = 4$, the right-handed neutrino mass is in the interesting range for the atmospheric neutrino and the Peccei–Quinn symmetry breaking scale (axion decay constant) for the axion CDM.

This type of mechanism is not specific to the Peccei–Quinn symmetry, but it is desirable to have a symmetry that forbids Planck-scale μ-term to begin with. Similar mechanisms were used in Refs. [49, 25].

The NMSSM is presumably also possible to generate the μ-term at the weak-scale. In this scheme, however, it suffers from the possible tadpole problem for the singlet [50], especially in the connection to the triplet-doublet splitting in grand-unified theories. Even if the theory is not grand-unified, it still needs to avoid the gravitational instability problem [10]. If both of the problems are avoided by an appropriate global symmetry, the NSSM can be a viable option.

6 Conclusions

We have shown that there is a completely model-independent contribution to the gaugino masses from SUSY breaking in the hidden sector whose origin can be traced to the conformal anomaly. This contribution to the gaugino mass is given *exactly* by

$$m_\lambda = \frac{\beta(g^2)}{2g^2} m_{3/2}, \tag{46}$$

where $\beta(g^2)$ is the (1PI) beta function. Trilinear soft SUSY breaking terms are generated by the same mechanism.

This mechanism opens up the possibility of hidden-sector models without singlets, which had been regarded as essential to get gaugino masses of order $m_{3/2}$. Models without singlets may be attractive for a variety of reasons, including simplicity, absence of cosmological problems, and the absence of instabilities to maintaining the hierarchy. In models without singlets, then the conformal anomaly contribution gives the leading contribution to the gaugino mass, predicting gaugino mass ratios that are very different from the conventional GUT relations. This is a very general prediction that can be tested if superpartners are observed in future experiments.

One issue that must be addressed in models without singlets is the fact that the gaugino mass is suppressed by a loop factor compared to $m_{3/2}$. Generally, one expects that the scalar masses are of order $m_{3/2}$, which requires fine-tuning to get acceptable electroweak symmetry breaking. This scenario has a phenomenology that is very different from the conventional one, and we have analyzed some of the main features in the paper.

Note added: While completing this paper, we received a paper by L. Randall and R. Sundrum [52] which also considers anomalous contributions to the gaugino mass. These authors also consider an interesting mechanism to suppress scalar masses compared to $m_{3/2}$.

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A Konishi Anomaly Subtlety

In this appendix, we discuss a subtlety concerning the soft supersymmetry breaking parameters $A = 0$, $B = -m_{3/2}$ discussed in section 2. The two contributions were given by $-3\langle W\rangle$ from the $-3|W|^2$ term, and ϕW_ϕ from the cross term in $|\phi^* W + W_\alpha|^2$. The latter contributions are actually a consequence of the kinetic operator $\int d^4\theta (m_{3/2}\theta^2)\phi^*\phi$, and this operator in general contains the gaugino mass operator as well [51] when the equation of motion is used to rewrite the auxiliary component F in terms of $F_\phi = (\partial W/\partial\phi)^*$:

$$\phi F_\phi^* = \frac{\partial W}{\partial \phi} + \phi \frac{g^2}{16\pi^2} T_F \lambda\lambda. \tag{47}$$

However, this contribution to the gaugino mass from the Konishi anomaly is absent in a fully regulated theory. Since it was necessary to use fully regulated theory in order to understand the origin of the gaugino mass, the Konishi anomaly effect is always absent and this concern is a red herring.

The simplest case to see this is when a matter field is accompanied by the Pauli–Villars regulator. In this case the regulator field has the same supersymmetry-breaking effect in the kinetic term that cancels the Konishi anomaly.

To check the same cancellation in the $\mathcal{N} = 2$ or $\mathcal{N} = 4$ regularization is somewhat trickier. For instance with $\mathcal{N} = 4$ regularization, it appears that the adjoint chiral multiplets produce gaugino mass from the Konishi anomaly. However, the gauge multiplet needs a gauge fixing in a manifestly supersymmetric manner, which requires three Faddeev–Popov ghost chiral supermultiplets in the adjoint representation. Their kinetic terms produce the opposite Konishi anomaly. The same cancellation can be checked with the $\mathcal{N} = 2$ regularization as well.

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