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MEASUREMENTS OF THE TRANSMISSION OF LIGHT FROM AN UNDERWATER SOURCE HAVING VARIABLE BEAM-SPREAD

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1. INTRODUCTION AND SUMMARY

Measurements of the transmission of monochromatic light from a submerged uniform point source have been reported.* The spectral irradiance H_r at any distance r from the source was found to be represented by the semi-empirical equation

$$H_r = J \frac{e^{-\alpha r}}{r^2} + J \frac{u(1 + ve^{-Kr})\, Ke^{-Kr}}{4\pi r} \qquad (1)$$

where J is the spectral radiant intensity, α is the attenuation coefficient for non-scattered light (monopath transmission), K is the attenuation coefficient for scattered light (multipath transmission) and the empirical constants u and v have the values $u = 2.5$ and $v = 7$ for the data from which the equation was evolved.

The present report describes underwater measurements of the transmission of light from a submerged source of variable beam-spread. Angular beam-widths down to 20° were used. It was found that the irradiance due to sources producing uniform circular light beams of

* Duntley, S. Q., "Measurements of the Transmission of Light from an Underwater Point Source" Report No. 5-11, Bureau of Ships, Contract NObs-72039, October 1960.

total plane angular width $\theta \gtrsim 20^{\circ}$ can be represented by equation (1) if the empirical constants u_{θ} and v_{θ} are, respectively,

$$u_{\theta} = u_{2\pi} - \frac{3}{2} \log \frac{2\pi}{\theta} \tag{2}$$

and

$$v_{\theta} = v_{2\pi} \left(\frac{2\pi}{\theta}\right)^{1/2} , \tag{3}$$

where $u_{2\pi} = 2.5$ and $v_{2\pi} = 7$ for the data from which these equations were evolved.

An important implication of the foregoing equations is that, from the standpoint of efficiency, underwater lighting systems should employ narrow-beam sources to the maximum extent practicable, even at ranges so great that the monopath transmission is negligible.

The following sections of this report describe the experiments and present the data from which equations (2) and (3) and the foregoing conclusions have been evolved.

2. DESCRIPTION OF THE EXPERIMENT

2.1 Introduction

The irradiance data discussed in this report were obtained at the Visibility Laboratory's Diamond Island Field Station in Lake Winnipesaukee, New Hampshire where a unique combination of favorable experimental conditions permitted the work to be performed with simple apparatus and at low cost. A photoelectric radiance photometer was mounted at an underwater window of an anchored, floating barge which was specially constructed at the field station in 1948 for use in underwater research. A train of nine black-painted wooden rafts, each ten feet long, was attached to the barge in front of the photometer window, as shown in Figure 1. These rafts served to support the light source at selected distances from the photometer and also to eliminated specular reflection at the water surface. The measurements were made on a moonless night when no ambient light was detectable by the photometer.

2.2 The Underwater Light Source

A variable beam-spread light source was constructed by enclosing a 1000 watt incandescent "diving lamp" manufactured by the General Electric Company by a rectangular box as shown in Figure 2. The lamp, designed for underwater burning, had a spherical envelope three inches in diameter. It was sprayed with W. P. Fuller No. 7786 gloss white lacquer in order to produce a uniform translucent white

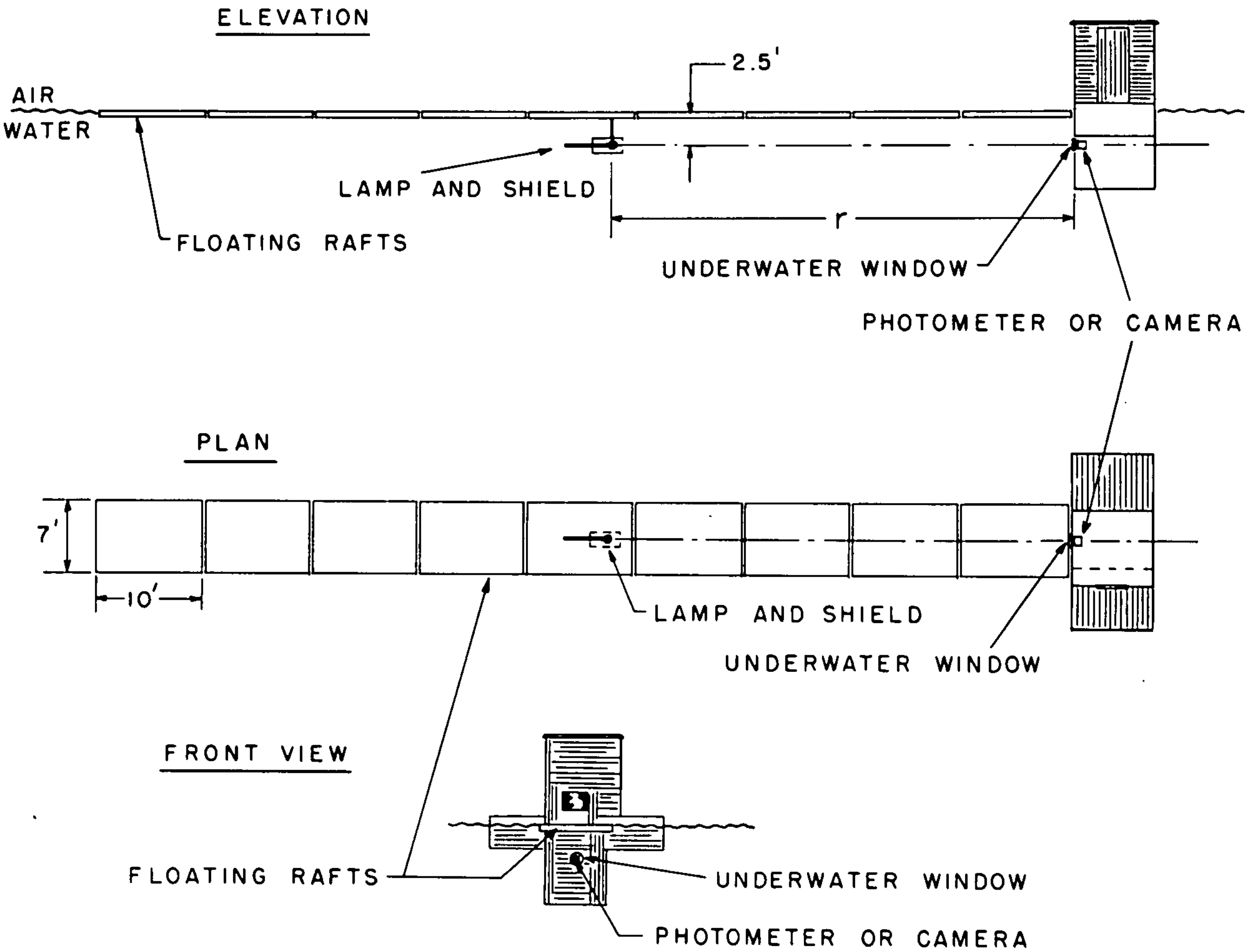
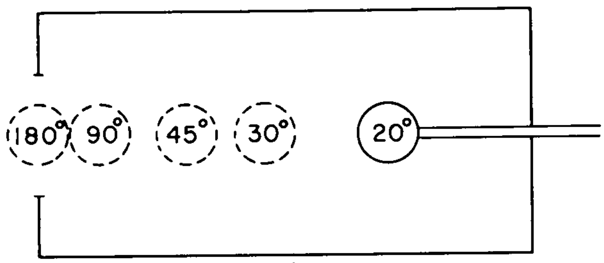

ELEVATION
2.5'
AIR
WATER
LAMP AND SHIELD
FLOATING RAFTS
r
UNDERWATER WINDOW
PHOTOMETER OR CAMERA
PLAN
7'
10'
LAMP AND SHIELD
UNDERWATER WINDOW
FRONT VIEW
FLOATING RAFTS
UNDERWATER WINDOW
PHOTOMETER OR CAMERA

Figure 1

covering. After being painted, the lamp was found to produce the same radiant intensity in all directions to within $\pm$ 7 percent except toward the base, which was always away from the opening in the box (see Figure 2). The lamp was attached by its base to a length of hollow pipe through which the electric leads passed. The position of the lamp within the box could be adjusted by sliding the pipe through the supporting gland at the rear of the device, and the beam-spread produced by the system could thus be varied at will. The upper portion of Figure 2 indicates the lamp positions corresponding with various commonly used beam spreads. The indicated nominal values of the beam spread θ are as measured from the center of the lamp. This geometry is illustrated by the lower portion of Figure 2, which shows that the source produced extreme rays with a spread of 28° when the nominal beam spread θ was 20°.

Provision was made for mounting the lamp beneath any raft, i.e., at virtually any desired distance from the photometer window. The depth of the lamp was 30 inches; this depth corresponded with that of the center of the window in the barge.

The lamp was operated at rated current and voltage through a Sorenson 3000S regulator. Careful checks made with the irradiance photometer showed that the luminous output of the source was free from detectable warm-up effects, long-term, or short-term fluctuations.


180°
90°
45°
30°
20°

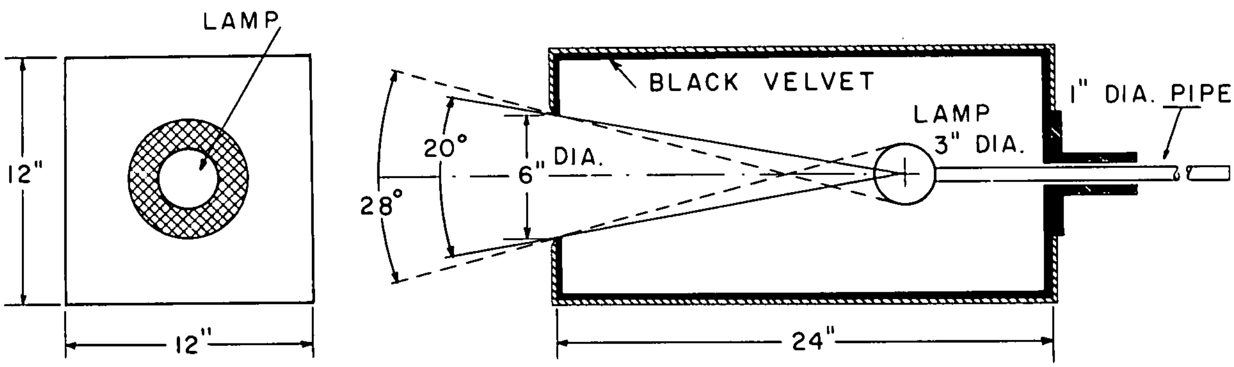

LAMP
12"
12"
BLACK VELVET
1" DIA. PIPE
LAMP
3" DIA.
20°
6" DIA.
28°
24"

Figure 2

2.2.1 Minimization of Reflection Effects

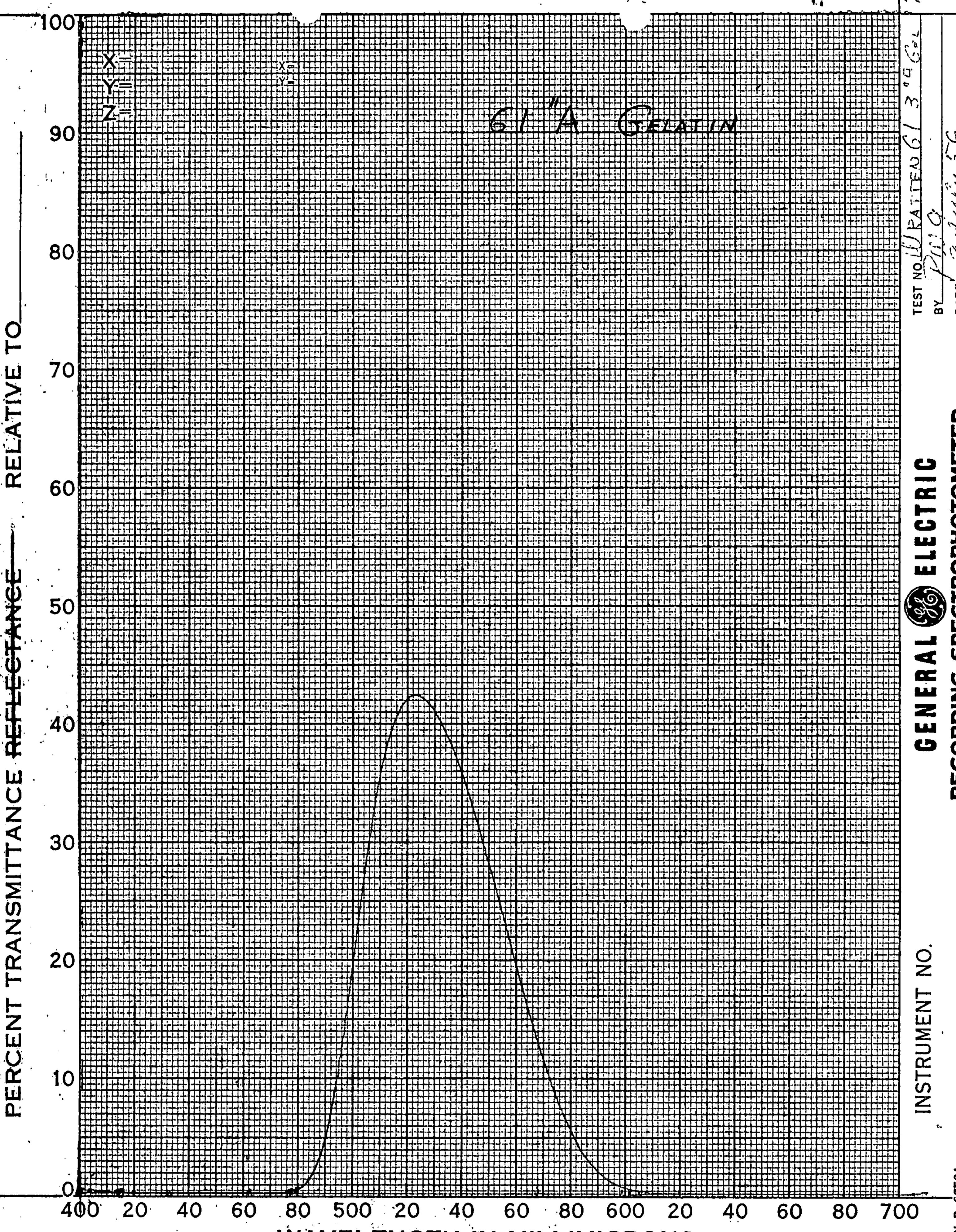
An optically boundless, homogeneous body of water with no reflecting surfaces to affect the light-field produced by the submerged lamp is required for the experiment described by this report. This condition was approximated as closely as practical limitations permitted. The available barge had its observation window 2.5 feet beneath the water surface and only horizontal or near-horizontal viewing was possible. The lamp, therefore, was held at a depth of 2.5 feet. Reflection effects were minimized and, it is believed, adequately eliminated (1) by providing a "roof" of floating rafts to eliminate the air-water boundary and (2) by painting the under surfaces of these rafts and the side of the barge with matte black paint having a submerged reflectance of approximately 1.5 percent, a value chosen to match the measured reflectance function of the water. The visual impression gained by looking out through the barge window was of a highly uniform and symmetric light field with no evidence of specular glints or other unwanted reflected light.

2.3 The Irradiance Photometer

The irradiance photometer consisted (1) of a sheet of translucent white plastic mounted in the water just outside the barge window and (2) a selected 931-A multiplier phototube in a light-tight metal housing placed just inside the barge window. An aperture in the housing admitted light from the rear surface of the white plastic sheet to the phototube. Neutral filters, made from pieces of uniformly fogged photographic film, were inserted as necessary between the photocell housing and the window in order to keep the photoelectric readings on scale. The multiplier phototube was connected to a Sweet-type 4-cycle logarithmic photometer circuit which had been linearized to within ± 0.02 logarithmic units by means of a standard lamp and an inverse-square-law attenuator. A Brown strip-chart recording potentiometer was used to record the data. The strip-charts were read with a specially constructed rule embodying the detailed calibration data for the photometer. The reproducibility of the photometer and the stability of its calibration was such that the over-all photometric precision is believed to have been approximately ± 1 percent except at the lowest light levels where noise was appreciable.

2.4 The Color Filter

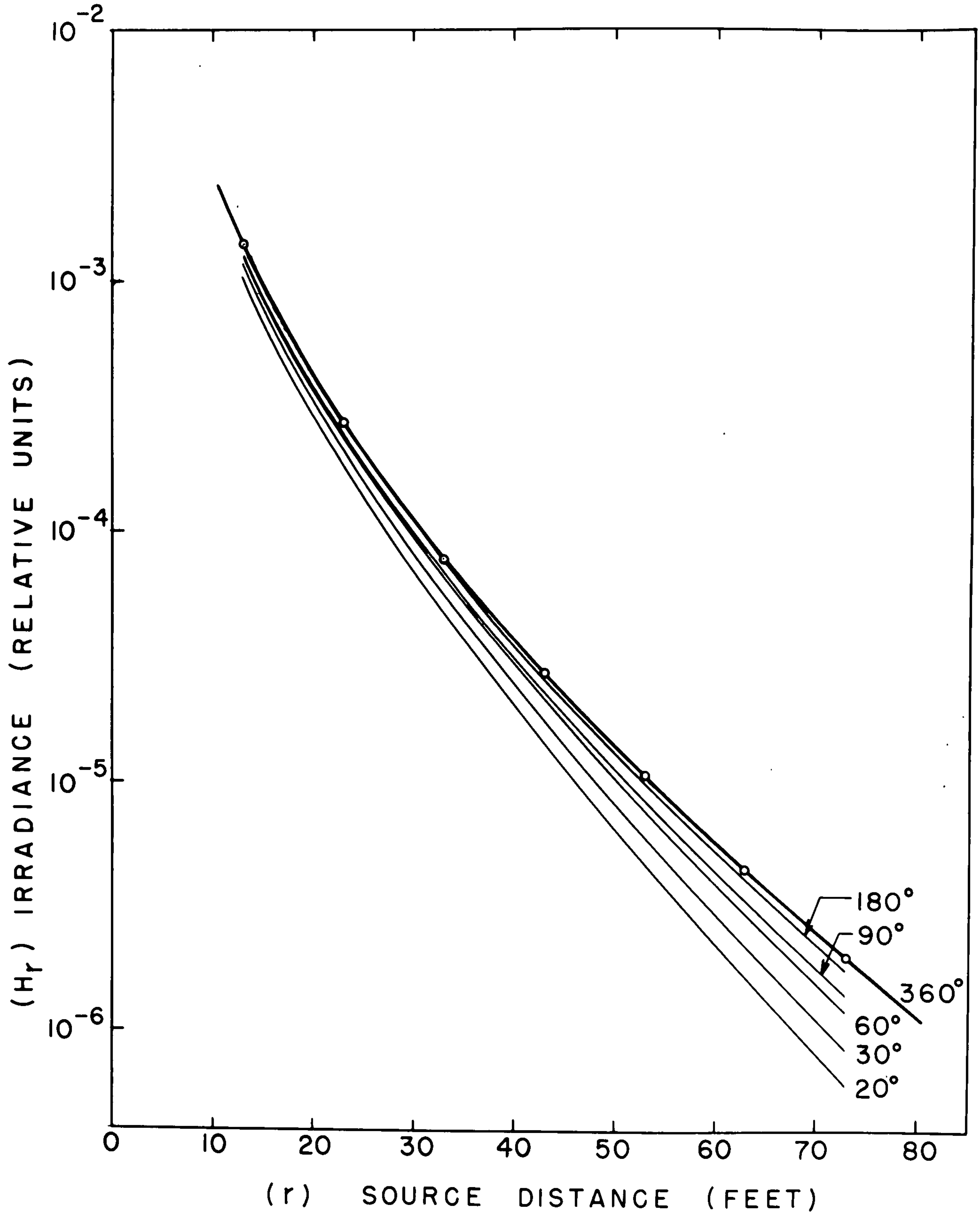
A Wratten No. 61 gelatin filter was fastened to the surface of the 931-A multiplier phototube in order to limit the spectral response of the photometer to a narrow band in the green portion of the spectrum, as shown in Figure 3. This filter was chosen in the belief that the water exhibited minimum absorption at the wavelength interval it transmits.



61 "A" GELATIN
X=
Y=
Z=
PERCENT TRANSMITTANCE ~~REFLECTANCE~~ RELATIVE TO
100
90
80
70
60
50
40
30
20
10
0
400
20
40
60
80
500
20
40
60
80
600
20
40
60
80
700
"A"
TEST NO WRATTEN 61 3"G GEL
BY
GENERAL ELECTRIC
INSTRUMENT NO.

2.5 The Data

Photoelectric irradiance data were obtained with the source at 13', 23', 33', 43', 53', 63', and 73' from the photometer. At each of these distances the source was operated with beam spreads of 20°, 30°, 45°, 60°, 75°, 90°, 180°, and 360°. Unavoidable delays due to mechanical problems of several kinds, wind, rain squalls, and moonlight combined to prevent a complete run of this experiment from being made on any one of the few nights available for the work. Minor changes in the optical nature of the water occurred from night to night due, it is believed, to changes in the standing crop of organisms in the water caused primarilly by variations in water temperature. The optical properties of the water were monitored by means of a hydrophotometer and a nephelometer and first order corrections were applied to the data with the intent of nullifying the small night-to-night changes. The resulting smoothed data are represented by the curves of irradiance H_r vs source distance r shown in Figure 4. Data points have been shown on the 360° curve in order to identify the source distances at which all of the data were taken. The 360° curve is the only one for which all points were obtained on a single night; it is, therefore, regarded as the most accurate of the data.



10^-2
10^-3
10^-4
10^-5
10^-6
(H_r) IRRADIANCE (RELATIVE UNITS)
0
10
20
30
40
50
60
70
80
(r) SOURCE DISTANCE (FEET)
180°
90°
360°
60°
30°
20°

Figure 4

2.5.1 Efficiency

It is evident from Figure 4 that an increase in beam-spread from 20° to 360° produces only a minor increase in irradiance, particularly at small source distances. Even at 73 feet the increase in irradiance is only 5-fold. Since the radiant power (i.e., the total light) entering the water from the light source is linearly proportional to the solid angle of the beam, the 5-fold increase in irradiance at 73 feet results from a 132-fold increase in power output from the source. It is obvious, therefore, that increasing the beam-spread of the light source is a poor way to increase the irradiance on an underwater object. From the standpoint of efficiency, underwater lighting systems should employ narrow-beam sources to the maximum extent practicable, even at ranges so great that multipath (glow) transmission accounts for virtually all of the irradiance. A more quantitative discussion of this conclusion is given in Section 3.3 of this report.

2.5.2 Trends of the Data

The details of the curves shown in Figure 4 are better displayed by the cross-plot shown in Figure 5, which depicts the irradiance H_r at various source distances as a function of the beam-spread θ. The circled points in this figure relate to the subject matter of Section 3.2 of this report and are of no relevance to the present discussion. The curves represent the same data shown in Figure 4.

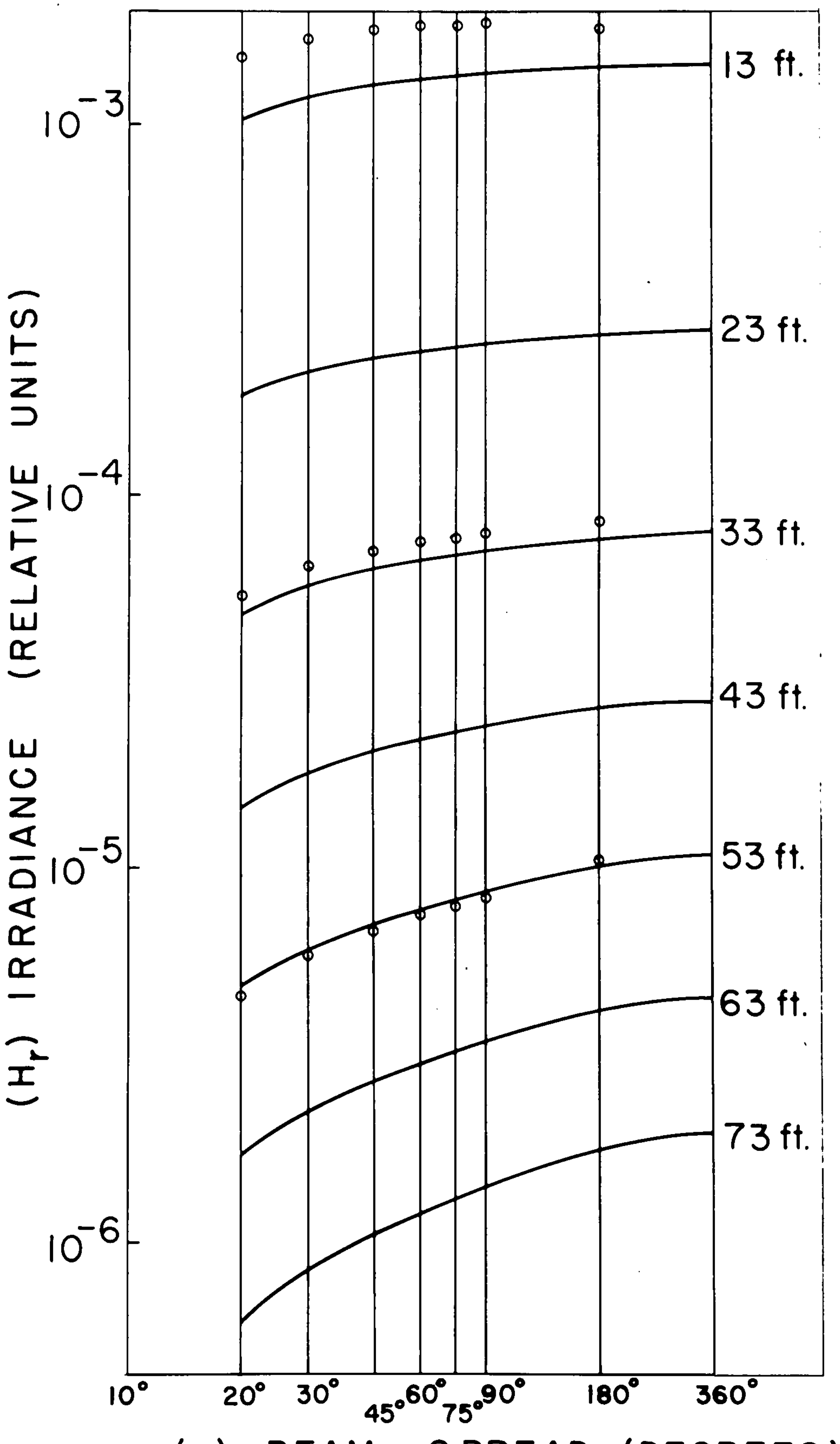

13 ft.
23 ft.
33 ft.
43 ft.
53 ft.
63 ft.
73 ft.
10^{-3}
10^{-4}
10^{-5}
10^{-6}
(H_r) IRRADIANCE (RELATIVE UNITS)
10°
20°
30°
45°
60°
75°
90°
180°
360°
(θ) BEAM SPREAD (DEGREES)

Figure 5

It will be noted that there is a progressive change in curve shape such that the irradiance is more dependent upon beam spread at the longer distances. This progression is illustrated by Figure 6, wherein all of the curves in Figure 5 have been superimposed in such a way that they coincide at beam-spread 360°.

A primary goal of the research described by this report is the evolution of a useable empirical equation capable of specifying the irradiance produced by a distant underwater light source of restricted beam-spread in terms of measurable inherent or apparent optical properties of the water. Progress toward that goal is described in the following section.

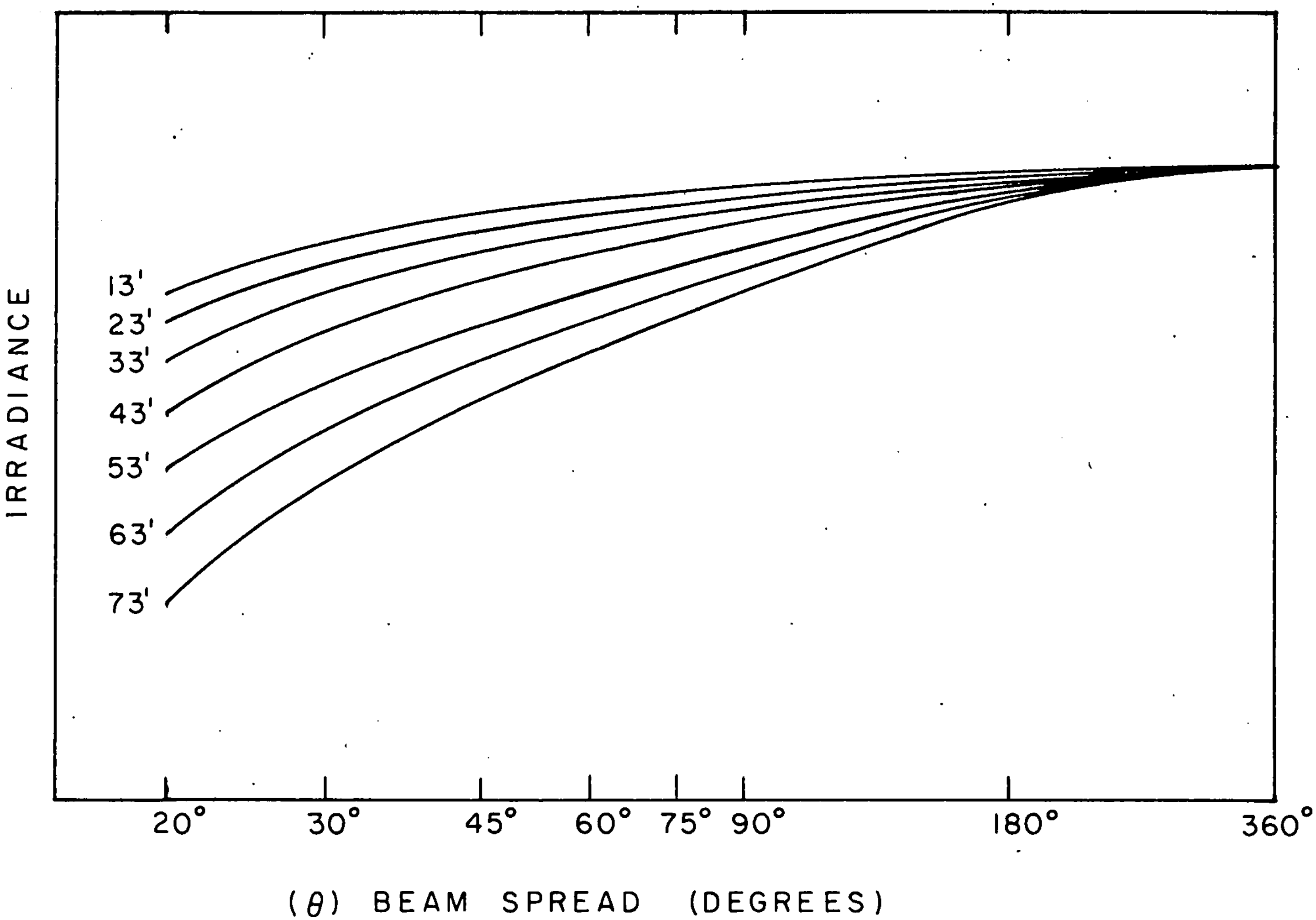

IRRADIANCE
13'
23'
33'
43'
53'
63'
73'
20°
30°
45°
60°
75°
90°
180°
360°
(θ) BEAM SPREAD (DEGREES)

Figure 6

3. Empirical Relations

3.1 Introduction

A rigorous treatment of the submerged point source has been given by Preisendorfer using his discrete-space method.[1] The resulting solution to this imposing problem in radiative transfer theory is iterative in character and application of it to any practical problem is a major task even for the largest electronic computers. A program for the IBM 7090 computer at Wright-Patterson Air Force Base is soon to be prepared by an Air Force contractor, but even after this advanced technique is available to predict the underwater radiance and irradiance distributions produced by any arbitrary underwater light source, a simple approximate empirical irradiance equation in closed form will be of use.

Such a relation for the special case of an unrestricted, uniform point source ($\theta = 360^\circ$) has been reported.[2] The spectral irradiance H_r at any distance r from the source was found to be represented by the semi-empirical equation

$$H_r = J \frac{e^{-\alpha r}}{r^2} + J \frac{u(1 + ve^{-Kr})\,Ke^{-Kr}}{4\pi r} \qquad (1)$$

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[1] Preisendorfer, R. W., "Two Fundamental Methods of Solving Point-Source Problems in Discrete-space Radiative Transfer Theory," SIO Ref. No. 59-71, December 1959.

[2] Duntley, S. Q., "Measurements of the Transmission of Light from an Underwater Point Source," Contract NObs-72039, Report No. 5-11, October 1960.

The data described in Section 2 of this report provide the basis for an empirical extention of equation (1) to include the effect of restricted beam-spread θ.

3.2 Effect of Beam Spread

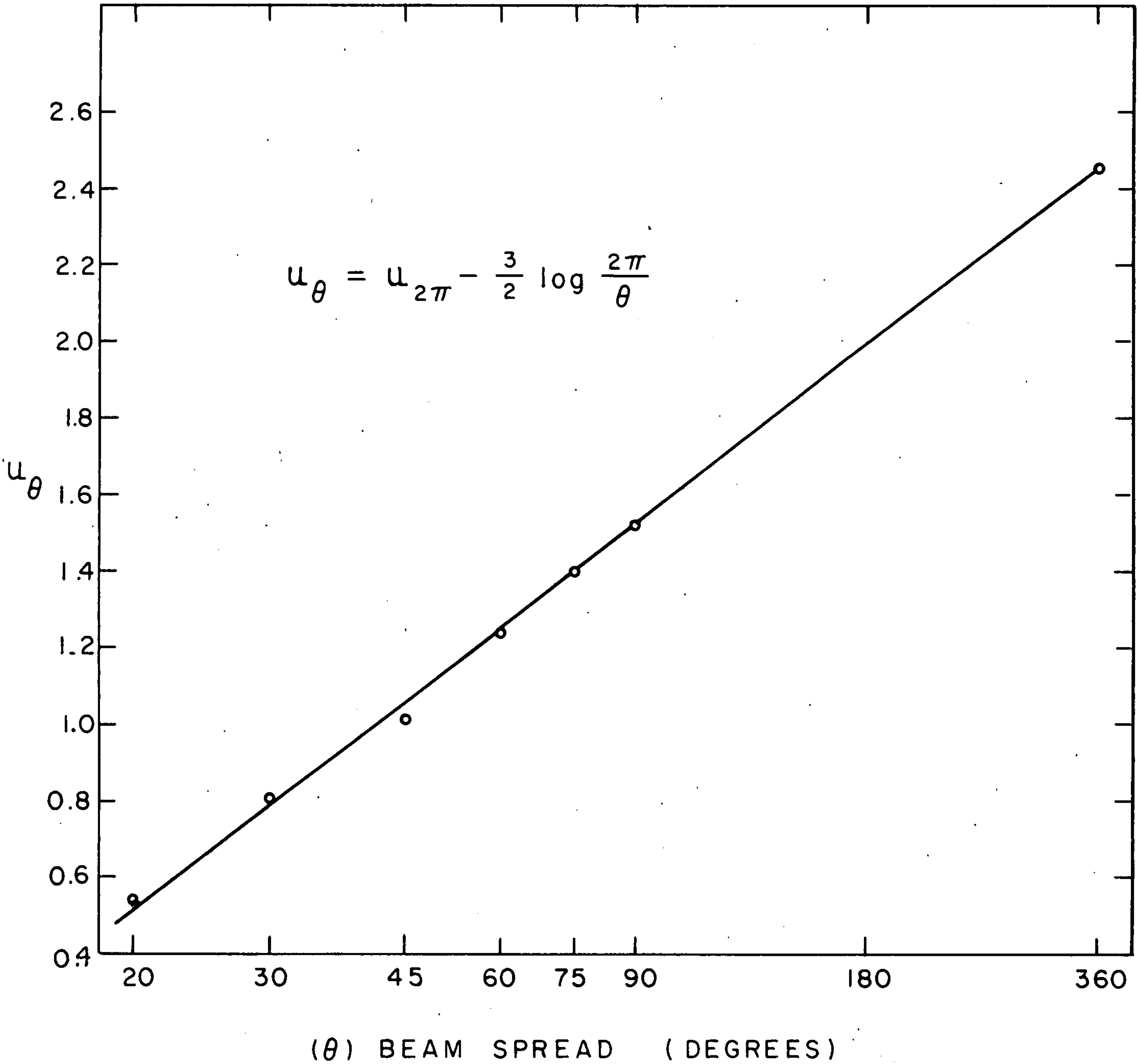
The constants u and v in equation (1) were modified by a trial-and-error process until the equation fitted the data portrayed by Figures 4 and 5 of this report. At the conclusion of this lengthy task it was found that the required values of u and v varied in a systematic way with the beam-spread θ. This is illustrated by figures 7 and 8, the plotted points on which represent values of u_θ and v_θ respectively required to make equation (1) represent the irradiance data. In each case a straight line representing a simple function of θ has been drawn through the points. These functions are:

$$u_\theta = u_{2\pi} - \frac{3}{2} \log_{10} \frac{2\pi}{\theta} \tag{2}$$

and

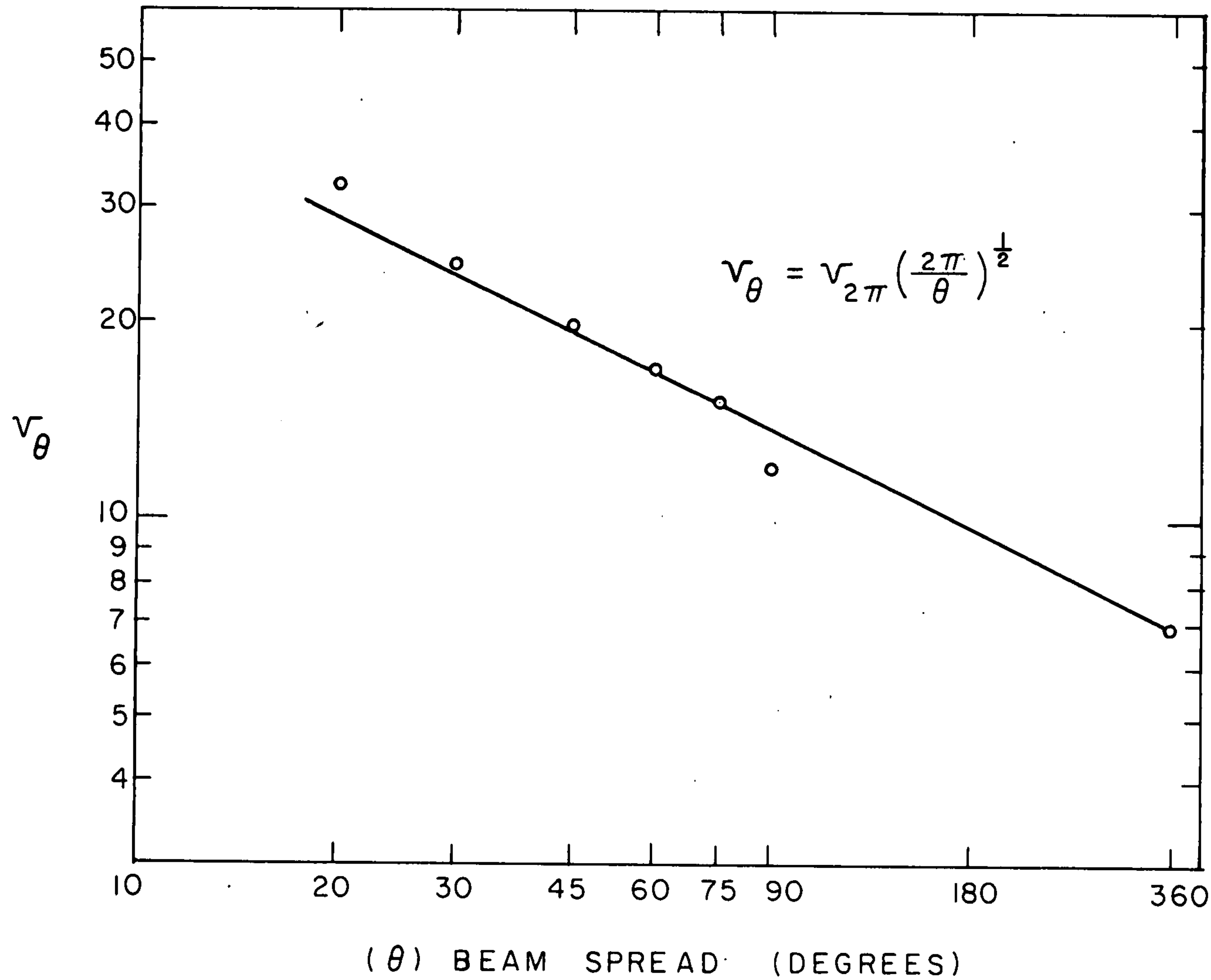
$$v_\theta = v_{2\pi} \left(\frac{2\pi}{\theta}\right)^{1/2} \quad \text{respectively.} \tag{3}$$

In these equations $u_{2\pi} = 2.5$ and $v_{2\pi} = 7$ for figures 7 and 8.



$u_\theta = u_{2\pi} - \frac{3}{2} \log \frac{2\pi}{\theta}$
2.6
2.4
2.2
2.0
1.8
u_θ
1.6
1.4
1.2
1.0
0.8
0.6
0.4
20
30
45
60
75
90
180
360
(θ) BEAM SPREAD (DEGREES)

Figure 7



50
40
30
20
V_θ
10
9
8
7
6
5
4
$V_\theta = V_{2\pi}\left(\frac{2\pi}{\theta}\right)^{\frac{1}{2}}$
10
20
30
45
60
75
90
180
360
(θ) BEAM SPREAD (DEGREES)

Figure 8

It is obvious from the form of equation (2) that negative values of u_Θ are possible when the beam-spread is small. Actually, these occur at $\Theta \leq 8$ degrees (approximately) and produce unrealistic implications. Equations (2) and (3) should not be trusted outside of the range of the data upon which they are based. Experiments with sources having smaller beam-spreads should provide the basis for better empirical functions than equations (2) and (3). Even the present data may suggest better functions or better-chosen constants for the present functions. It can only be said that equations (2) and (3) are the best that have been evolved thus far. It is believed that they enable the irradiance produced by sources having beam spreads of 20° or more to be calculated with sufficient accuracy for most practical purposes. The circles in Figure 5 represent the predictions of equation (1) when combined with equations (2) and (3); they illustrate that agreement is best at large distances from the source.

3.3 Efficiency

One measure of the geometrical efficiency of an underwater light source is the ratio of the monochromatic irradiance H_r produced at source-distance r to the total monochromatic power P radiated by the source. This is a function of r and Θ. In the special case of the uniform conical beam of spread Θ produced by the source considered in this report, the efficiency ratio H_r/P can be expressed by means of equations (1), (2), and (3) by the relation

$$\frac{H_r}{P} = \frac{\dfrac{e^{-\alpha r}}{r^2} + \dfrac{\left(2.5 - \frac{3}{2}\log\frac{2\pi}{\theta}\right)\left(1 + 7\left(\frac{2\pi}{\theta}\right)^{1/2} e^{-Kr}\right)Ke^{-Kr}}{4\pi r}}{2\pi(1 - \cos\theta/2)} \qquad (4)$$

where α = 0.200 natural log-units per foot and K = 0.0570 natural log-units per foot for the water in which the data depicted by Figures 4 and 5 were obtained.

Figure 9, a plot of equation (4), shows the geometrical efficiency ratio H_r/P as a function of beam spread θ for four values of source-distance r, corresponding to 1, 2, 4, and 8 attenuation lengths or mean-free-photon-paths, $1/\alpha$. This plot demonstrates quantitatively that, from the standpoint of efficiency, underwater lighting systems should employ narrow-beam sources whenever practicable, even at ranges so great that the monopath transmission is negligible and the multipath (glow) transmission contributes virtually all of the irradiance.

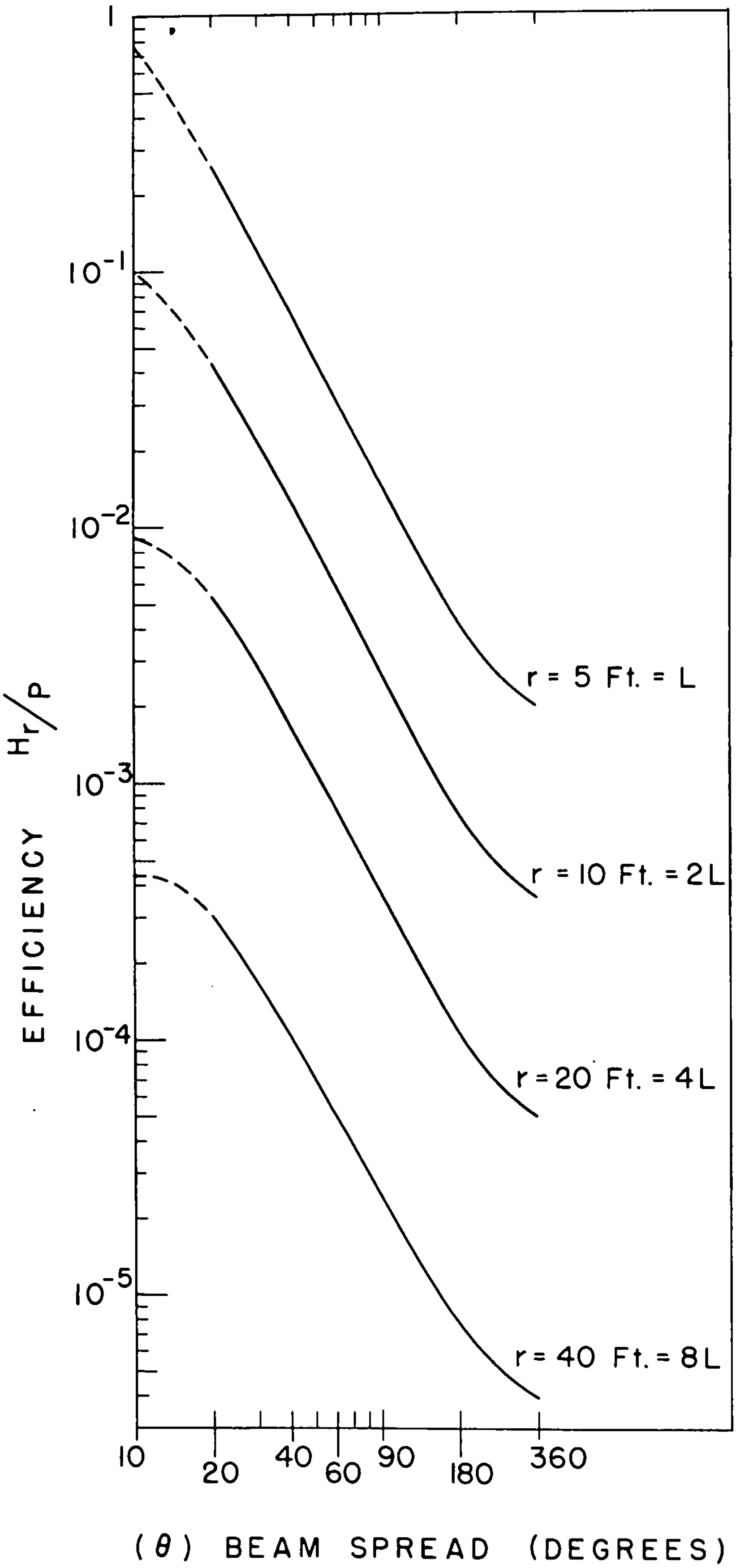

EFFICIENCY H_r/P
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
r = 5 Ft. = L
r = 10 Ft. = 2L
r = 20 Ft. = 4L
r = 40 Ft. = 8L
10
20
40
60
90
180
360
(θ) BEAM SPREAD (DEGREES)

Figure 9

3.4 Applicability to Other Natural Waters

Equations (1) through (4) with the constants given in this report describe data obtained in only one natural water. Their applicability to other waters is unknown. Inasmuch, however, as the first term of equation (1) is exact and the second term is founded upon a well established diffusion theory model, there is a basis for the expectation that the general form of the expressions will serve as a useful approximation relation for most natural waters and the hope that the constants other than α and K will be insentive to water type.

Important next steps are, obviously, to test equations (1) through (4) by means of irradiance data collected in other types of natural waters and to include sources having smaller beam-spreads.