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Author
Bergstrom, Ted

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The Welfare Loss from Price Distortions–*Econometrica, 1976*

by

Ted Bergstrom

University of Michigan

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One of the more disconcerting "negative" results in the theory of welfare economics was articulated by Richard Lipsey and Kelvin Lancaster in 1956 in their paper "The General Theory of Second Best". They demonstrated that if there are "distortions" in more than one market, even in a one-consumer economy, it may not be beneficial to remove a distortion in a single market if distortions remain in other markets. To policy economists this observation might seem a devastating criticism of recommendations for "piecemeal reform". In 1970, Edward Foster and Hugo Sonnenschein discovered a remarkable result that removes some of the sting of the Lipsey-Lancaster observation. They proved that under reasonably general circumstances, at least one kind of "piecemeal reform", namely "radial" or proportional reductions in all distortions, would improve welfare in a one-consumer, general equilibrium economy.

This paper by Trout Rader extends the Foster-Sonnenschein result in important and interesting ways. [1] Foster and Sonnenschein required that the production possibility set be the intersection of a half-space with the non-negative orthant. (For this to be the case, not only must there be constant returns to scale, but essentially there also must be no more than one non-produced factor of production.) They also required convexity of preferences and normality of all goods. Rader's theorem dispenses with all of these assumptions. [2]

The Theorem

This theorem is a true "general equilibrium theorem" in which full account is taken of the fact that a distortionary tax not only will insert a "wedge" between producers' prices and consumers' prices, but may also change the equilibrium price received by producers. Let $p \in R^n$ be a price vector and $t \in R^n$ where t is not proportional to p. Let $\eta(p)$ be the supply correspondence and and let $\xi(p, M)$ be the aggregate demand correspondence where M is aggregate income. [3] A competitive equilibrium with price distortion t consists of a price vector p and an output vector x such that $x \in \eta(p) \cap \xi(p + t, (p + t)x)$.

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[1] K. Kawamata (1974) independently developed a similar extension of the Foster-Sonnenschein theorem. Kawamata uses somewhat different assumptions, including convexity of preferences, and has a considerably more complicated proof.

[2] Foster and Sonnenschein use the extra assumptions to establish uniqueness of equilibrium for any vector of distortions, which appears to be necessary if we want a result that says that *every* equilibrium corresponding to a radial increase in distortions is worse than *every* equilibrium before the increase. Rader's theorem states conditions under which for every equilibrium corresponding to the initial distortion there is some equilibrium corresponding to the radially expanded distortion which is worse. If equilibrium is unique, then these two requirements are the same.

[3] Rader, like Foster and Sonnenschein, will assume that aggregate demand independent of the distribution of income.

Rader's version of the theorem can be stated as follows. Suppose preferences are continuous and that aggregate demand satisfies the weak axiom of revealed preference. Suppose also that production possibility sets are bounded and that the preferences are locally nonsatiated. Since aggregate demand satisfies the weak axiom of revealed preference, this aggregate demand is rationalized by some transitive preference relation P. Then if the consumption vector x is an equilibrium with distortion t and if $0 < \theta < 1$, there must exist a consumption vector y which is an equilibrium with the (smaller) distortion θt such that yPx.

The Proof

Rader's proof is clean and elegant, but his presentation is terse. It may save a reader some time to see a slightly more verbose "preview" of how the proof works. Because Rader's proof is fully rigorous and general, I can afford in this explanation to be just a little sloppy.

The Sonnenschein-Foster proof and the Rader proof can each be divided into the following 3 steps.

Step 1. Zero distortion is better than any non-zero distortion. Proof of this step is pretty trivial.

Step 2. If x is an equilibrium under the distortion t and y is an equilibrium under the distortion θt, where $0 < \theta < 1$, then it can not be that x is indifferent to θ. This is the central argument of the proof. Rader uses a "revealed profitability" argument for the firm and a "revealed preference" argument for the consumers. I will explain the idea of the proof below using a diagram to illustrate the two-good case.

Step 3. From Steps 1 and 2, it follows that a radial increase in distortion reduces the consumer's utility.

Here is a sketch of a proof of Step 2 for the special case where there are only two goods.[4] Consider a tax vector $t = (\tau, 0)$, where $\tau > 0$. In Figure 1, let x and y denote two distinct price distorted equilibria corresponding respectively to distortions t and θt and suppose that x is indifferent to y Let p and $\bar{p}$ be the corresponding equilibrium prices faced by producers. The dotted lines through x and y represent producer's isoprofit lines at prices p and $\bar{p}$, respectively. Since x must be at least as profitable as y at prices p and since y must be at least as profitable as x at prices $\bar{p}$, it must be that the dotted line through y is at least as steep as the dotted line through x. The solid lines through x and y represent the consumer's budget lines at prices $p + t$ and $p + \theta t$ respectively. Since x is indifferent to y, it follows from the weak axiom of revealed preference that the solid line through x is steeper than the line through y. The two arrows in the figure denote the tax wedges, t and θt respectively. But notice that these arrows point in opposite directions. This is not possible given our assumptions since $0 < \theta$.

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[4] But this proof could be applied to the case of n goods by applying the same arguments used here to the projections of the appropriate hyperplanes into the plane determined by the origin and the two points x and y that are discussed here.

If equilibrium is unique for each vector of distortions and continuous in the distortion, then Step 3 is very easy. See Figure 2, which shows the utility of the single consumer as a function of the parameter θ. From Step 1 it follows that the curve is higher at $\theta = 0$ than at $\theta = 1$. If utility is not monotonically decreasing in θ, there will be distinct θ and $\bar{\theta}$ which result in equal utilities for the consumer. But this would violate Step 2. Where equilibrium is not unique, a more sophisticated argument is needed. Rader cleans up this possibly very messy situation with a short but clever argument based on the continuity of correspondences.

Remarks on the Proof

If a theorem has a clever proof, it is interesting to look at the proof not only to see that the theorem is true, but also to see if there are tricks that are likely to be useful for proving other things. Rader's proof uses (at least) three rather surprising and clever devices. Credit for the first of these ideas goes to Foster and Sonnenschein. They noticed that they can prove their proposition if they can show that no two distinct radial distortions can lead to indifferent outcomes. This is a powerful strategy because the assumption of indifference has very strong implications under revealed preferences. Second there is Rader's observation that general production structures can be handled by conjoining revealed preference with "revealed profitability". Third there is Rader's elegant treatment of Step 3 in the case where there may be multiple equilibria.

Aside from the intrinsic interest of the results and its practical value as a source of tricks that may be borrowed, economic theorists can enjoy the Foster-Sonnenschein theorem and its extensions by Rader and Kawamata for aesthetic reasons. These are beautiful examples of the use of the principle of revealed preference to yield general propositions in comparative statics and welfare economics.

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